WPS 1282

POLICY RESEARCH WORKING PAPER 1282

Copper and the Negative Price of Storage

Donald Frederick Larson

Just as the price of a call option contains a premium based on price variability, so the shadow price of inventories contains a dispersion premium associated with the unplanned component of inventories. When inventory levels are low, the value of the premium increases to the point where inventories will be held even in the face of a fully anticipated fall in price.

The World Bank
International Economics Department
International Trade Division
April 1994

Summary findings

Commodities are often stored during periods in which storage returns a negative price. Further, during periods of "backwardation," the expected revenue from holding inventories will be negative.

Since the 1930s, the negative price of storage has been attributed to an offsetting "convenience yield." Kaldor, Working, and later Brennan argued that inventories are a necessary adjunct to business and that increasing inventories from some minimal level reduces overall costs. This theory has always been criticized by proponents of cost-of-carry models, who argue that a negative price for storage creates arbitrage opportunities. Proponents of the cost-of-carry model have asserted that storage will occur only with positive returns. They offer a set of price-arbitrage conditions that associate negative returns with stockouts. Still, stockouts are rare in commodity markets, and storage appears to take place during periods of "backwardation" in apparent violation of the price-arbitrage conditions.

For copper, inventories have always been available to the market regardless of the price of storage. This is true whether the market is broadly defined at the U.S. or world level, or more narrowly defined as the New York Commodities Exchange or the London Metal Exchange.

Larson argues that although inventories may provide a Kaldor cost-reducing convenience yield, inventories also have value because of uncertainty. Just as the price of a call option contains a premium based on price variability so the shadow price of inventories contains a dispersion premium associated with the unplanned component of inventories.

Larson derives a generalized price-arbitrage condition in which either a Kaldor-convenience and/or a dispersion premium may justify inventory holding even during an expected price fall. He uses monthly observations of U.S producer inventories to estimate the parameters of the price-arbitrage condition. The estimates and simulations he presents are ambiguous with regard to the existence of a Kaldor-convenience but strongly support the notion of a dispersion premium for copper. And although the average value of such a premium is low, the value of the premium increases rapidly during periods when inventories are scarce.

This paper — a product of the International Trade Division, International Economics Department — is part of a larger effort in the department to understand international commodity markets. Copies of the paper are available free from the World Bank, 1818 H Street NW, Washington, DC 20433. Please contact Anna Kim, room S7-038, extension 33715 (78 pages) April 1994.

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by

Donald Frederick Larson

I would like to thank Ramon Lopez and Ron Duncan for their guidance and support during this project. I would also like to thank Howard Leathers, Nancy Bockstael, Bruce Gardner, and Karla Hoff for useful comments.

Table of Contents

List of Figures

List of Tables

1.0 Introduction

The purpose of this paper is to explain producer-held inventories of refined copper during an anticipated fall in prices. Commodities are often stored during periods in which storage returns a negative price -- that is, when the expected price change (as indicated by futures prices) does not cover the time-value of money plus storage expenses. In fact, during periods of backwardation (when the settlement price for a near-by future contract is greater than the price for contracts with more distant settlement dates), the anticipated revenue from holding inventories will be negative. In the case of copper, inventories have always been available to the market regardless of the price of storage. This is true whether the market is broadly defined at the US or world level, or more narrowly defined as the New York Commodities Exchange, or the London Metal Exchange. Since the 1930s, the negative price of storage has been attributed to an off-setting "convenience yield". Kaldor (1939), Working (1948) and, later, Brennan (1958) argued that inventories are a necessary adjunct to carrying out business and, at some minimal level, increasing inventories reduces overall costs. This theory has always been criticized by proponents of cost-of-carry models, who argue that a negative price for storage creates arbitrage opportunities. More recently, Williams and Wright (1991) have correctly argued that "convenience yields" have not been derived from a formal optimization model. Proponents of the cost-of-carry model have asserted that storage will occur only with positive returns and offer a set of price-arbitrage conditions. Williams and Wright have augmented the cost-of-carry argument by stressing the effects of stock-outs on price. Nonetheless, Williams and Wright (1991, p.140) note there have been no stock-outs in the Chicago wheat markets during the last 120 years, and concede that storage appears to take place during periods of backwardation, in apparent violation of the price-arbitrage conditions.

This paper argues that while inventories may provide a Kaldor cost-reducing convenience yield, inventories also have value because of uncertainty. Just as the price of a call option contains a premium based on price variability, the shadow price of inventories contains a dispersion premium associated with the unplanned component of inventories. A generalized price-arbitrage condition is derived in which either a Kaldor-convenience and/or a dispersion premium may justify inventory-holding even during an expected price fall. Monthly observations of US producer inventories are used to estimate the parameters of the price-arbitrage condition. The estimation and simulations presented provide little evidence for supporting or dismissing the existence of a Kaldor-convenience, but strongly support the notion of a dispersion premium for copper. Further, while the average value of such a premium is low, the value of the premium increases rapidly during periods in which inventories are scarce.

Following this introduction, the remainder of the paper is organized as follows: Section 2 reviews the behavior of prices for the spot and futures markets in refined copper and discusses backwardation and a negative price for storage in the context of the economic literature. In this section, the arguments of Working, Brennan, Keynes, and Williams and Wright are more fully explained. Section 3 develops the formal model of inventory-holding in terms of a stochastic-control problem for the ending-value of inventories. The generalized price-arbitrage conditions are derived from the optimization problem's first-order conditions. Section 4 contains a description of the data and an estimation of the parameters of the generalized price-arbitrage condition. Section 5 concludes.

2.0 Refined Copper Prices and the Negative Price of Storage

Refining copper is a risky business and refined copper prices are characterized by volatile price movements. Price swings can be large and sudden while low price levels can linger persistently. The movement in refined copper is especially significant given the small profits obtained through refining -- typically 6 to 10 per cent of the final cost of refined electrolytic copper (Brook Hunt & Associates, 1986). Since the refiner must buy scrap or blister copper, the spread between the two prices contains the implicit fee for processing copper (Figure 2.1). Yet, because of the time it takes to process the copper, rapid price movements can dramatically inflate profits or generate losses even when the spread between inputs and output

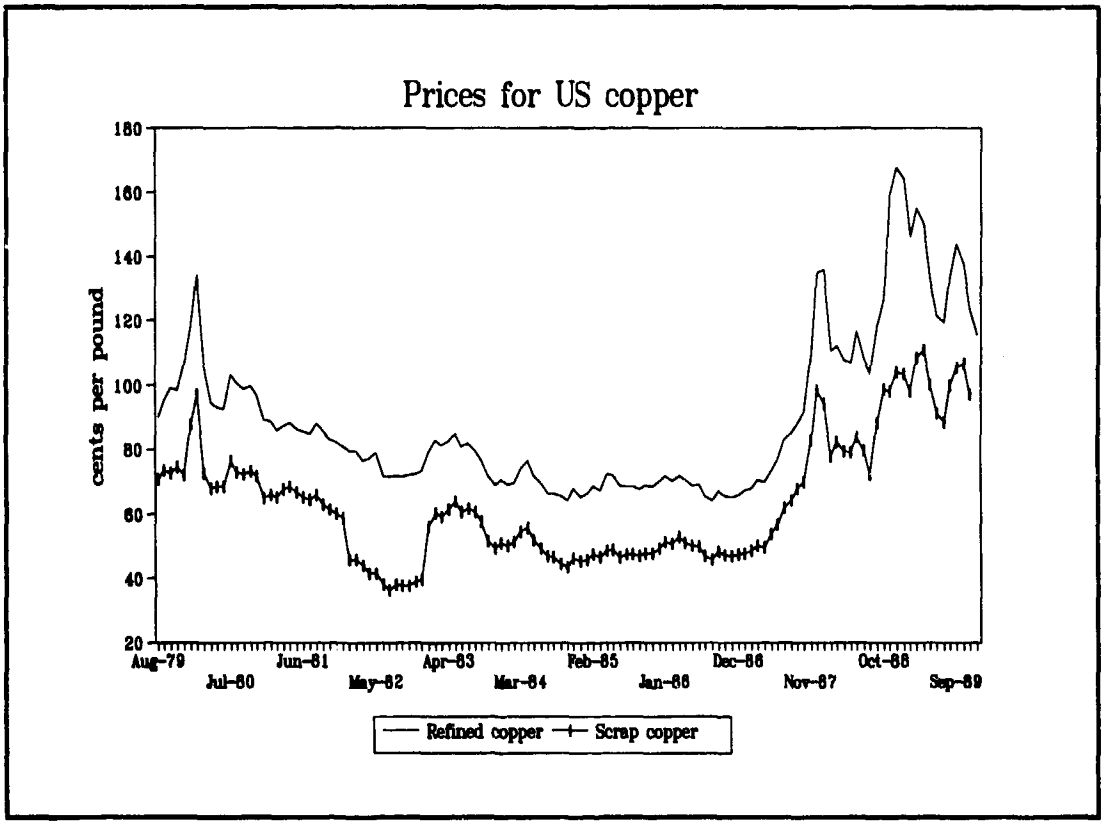

Figure 2.1: Copper prices, August 1978 to December 1989.

price remains constant. There are several options available to the refiner. He can contract the processing services directly, charging a fee but returning the refined copper to the original owner of the blister or scrap copper (tolling); he can implicitly make the same arrangement by buying spot in the blister or scrap market and simultaneously selling (shorting) a futures contract in New York or London; he can contract with a semifabricator for future delivery while buying in the spot market, etc.

The value of finished inventories held by refiners is subject to the same volatility. Gains or losses generated by wide swings in the value of held inventories may affect the firm's profitability more greatly than returns from flows (production and sales). Options open to the refiner include reducing inventories to near-zero levels or contracting to sell inventories either explicitly, or implicitly through the futures market. In addition, even when forward contracts have been written, further revenue can be generated by implicitly "lending" inventories. The mechanics of such an arrangement are discussed later; however the practice has obvious and certain positive returns during periods of market backwardation.

Markets for refined copper and other metals are somewhat distinctive in their proclivity for backwardation in futures pricing. Backwardation occurs when the price of futures contracts with a more distant delivery date are discounted with respect to near-by contracts. A more common and more general condition is when inventories are held at less-than-full carrying charges, which Working (1949) termed a **negative price for storage**. A negative price for storage occurs when the difference between the expected rate of change in the price for immediate delivery and full carrying costs (interest, insurance, warehousing fees, and spoilage) is negative. As a practical matter, a situation in which the difference between the price for near-by and more distant futures prices falls below full carrying charges is taken as

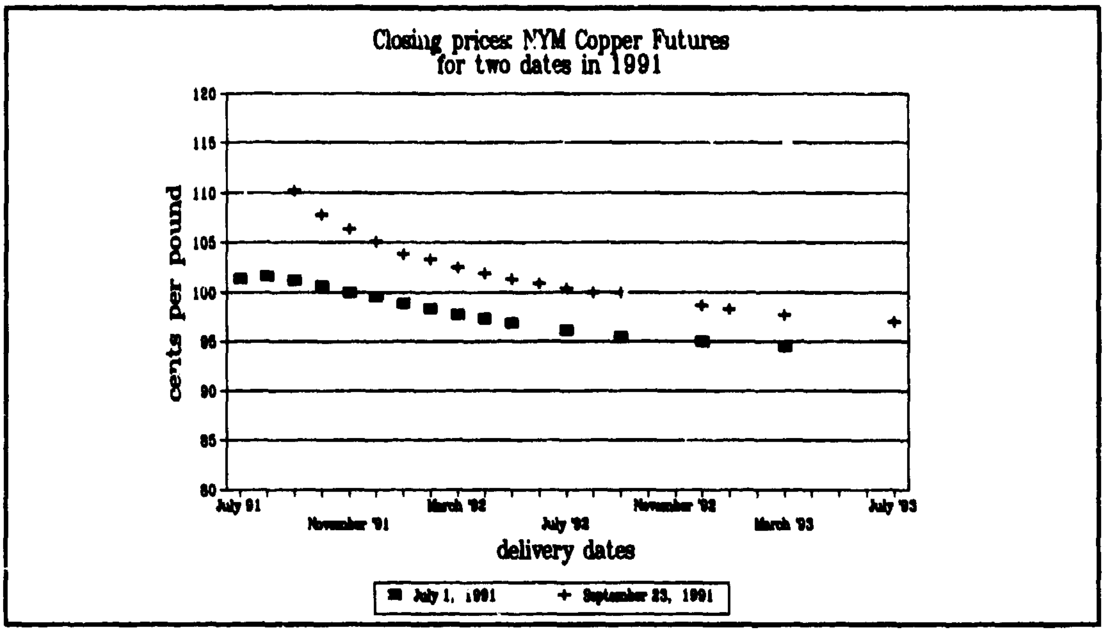

Figure 2.2: Future price profile for high-grade copper.

a negative price for storage. Backward markets necessarily carry a negative price for storage since revenues from storage are negative.

Figure 2.2 gives the profile of closing future prices on high-grade copper for two separate dates in 1991. From July 1 to September 23 the entire price curve rose, while the futures market remained in backwardation. While the situation depicted in Figure 2.2 cannot be characterized as "normal", it is not unique for copper. From 1980 to 1989 the last-day-of-the-month spreads on COMEX copper futures were backward 29 times -- or roughly 24% of the time. A negative price for storage has been even more common. Jeffrey Williams (1986) writes:

> *In nine of the ten years from 1974 through 1983 at least some of the spreads*[1] *in copper failed to display fully carrying charges, sometimes, as in 1974 and 1980, by a considerable margin. That any spreads in copper were less than full*

[1]A spread, in this context, is the price of a futures contract with a more distant delivery date minus the price of a futures contract with an earlier delivery date.

carrying charges is surprising in itself. Because copper is a natural resource, having little seasonality in production or demand, and large reserves of scrap, the presumption would be strong that its price should increase to cover interest and storage costs, as presumed in most models of natural resources.

Fama and French (1988), looking at daily interest-compensated spreads between spot and 3-month, 6-month, and 12-month forward copper contracts on the LME between 1972 and 1983 reported negative **average** spreads for copper.

The general failure of the spreads in futures prices to cover full carrying charges is well known to the literature. Holbrook Working noted the phenomenon in 1934, and Nicholas Kaldor wrote about a "convenience yield" to stocks in 1939. In 1948, Working provided a theory on "inverse carrying charges" which included a supply of storage at a negative price:

> *Another important condition is that for most of the potential suppliers of storage, the costs are joint; the owners of large storage facilities are mostly engaged either in merchandising or in processing, and maintain storage facilities largely as a necessary adjunct to their merchandising or processing business. And not only are the facilities an adjunct; the exercise of the storing function itself is a necessary adjunct to the merchandising or processing business. Consequently, the direct costs of storing over some specified period as well as the indirect costs may be charged against the associated business which remains profitable, and so also may what appear as direct losses on the storage operation itself. For any such potential supplier of storage, stocks of a commodity below some fairly well recognized level carry what Kaldor has aptly called a convenience yield. This convenience yield may offset what appears as a fairly large loss from exercise of the storage function itself.*

The concept of a convenience yield has become a popular and enduring part of the literature on inventory behavior. (See Howell, 1956; Brennan, 1958; Telser, 1958; Weymar, 1974; Gray and Peck 1981; Thompson, 1986; Williams, 1986; Tilley and Campbell, 1988; Thurman, 1988; Fama and French, 1988; and Gibson and Schwarz, 1990.) However, as with many economic terms, there is a conflict in the literature over its definition. Kaldor and Working, for example, apply convenience yield to mean an implicit return to holders of inventories whose market value equals the difference between futures price spreads and full carrying costs. The value to the holder derives from jointness between inventory holding and related merchandising or processing. More recently, "the" convenience yield, has taken a more operational definition, coming to mean the evaluation itself or, more precisely, the array of spreads between futures prices adjusted for carrying costs. Often warehouse fees and transaction costs are ignored so that carrying charges are exclusively composed of interest charges. (See, for example, Fama and French, 1988). In such a case, the convenience yield is an empirical entity, constantly changing much like a bond yield curve. In this paper, I will retain the definition given by Kaldor and Working. A similar idea, in keeping with Working's definition of convenience, is that inventories are essential to production and therefore inventory-holding of inputs is joint with production as well as with marketing. Ramey (1989) used such an argument to justify inclusion of inventory levels in modeling production technology.

Jeffrey Williams and Brian Wright (Wright and Williams, 1989; Williams and Wright, 1991) have been especially critical of literature concerning a convenience yield. Williams and Wright justifiably argue that convenience yields are not derived from "first principles" (i.e. optimization conditions).[2] They go on to argue that the "observation of storage under backwardation is an aggregation phenomenon" and that "a spread below full carrying charges can emerge only when there is no storage of that commodity.... Profit-maximizing storage takes place only at full carrying charges, properly calculated."[3] (Wright

[2]Ramey is an exception not noted by Williams and Wright.

[3]Williams (1986) credits Higinbotham (1976) with suggesting that the paradoxes of spreads result from misleading averaging.

and Williams, pp. 3-4). By including the costs of transporting inventories from one location to another, or the costs of transforming one close substitute to the relevant commodity (say dirty wheat to clean wheat), Wright and Williams argue, the convenience yield disappears. Backwardation in futures markets arises when the probability of a stock-out for the near period is greater than for a later period.[4] In Williams and Wright (1991, p. 140) they soften their stance, stating that "Empirically it does seem that storage takes place when the spot spread is in backwardation, if for no other evidence than the disquieting fact that stock-outs are never observed." At the same time, in their simulation model, they maintain a set of price-arbitrage conditions that precludes storage at less-than-full carrying charges.

Since inventories are often reported aggregated over location, if not over type and grade of commodity as well, it is difficult to test the Wright and Williams assertion in many instances. However, the inventory-holding behavior of certified warehouse inventories of refined copper provides a clear counter-example of the type which Williams and Wright recognized in their 1991 statement. At numerous times throughout their history, both the LME and the COMEX have reported abundant supplies of certified warehouse stocks while simultaneously reporting backwardation in futures pricing. The added value of certification is that certified inventories can be used to fulfill commitments resulting from futures contracts. Should a holder of an open short futures contract in copper decide to deliver copper rather than close his position, he need only transfer a receipt for inventories held in any certified warehouse. No transportation or transformation is involved. Table 2.1 reproduces data reported by Williams (1986) on copper spreads on a single day in January

[4]Bresnahan and Suslow (1986) make a similar argument specifically in the case of copper and backwardation in the London Metal Exchange.

over a ten-year period and supplements it with data on inventory levels. For the observations reported, it was more common than not for refined copper inventory to be held at a negative price, that is, below full carrying costs. At no time did a "stock-out" occur, although inventories were low in January 1974. At the same time, however, it is difficult to argue that effective stock-outs occurred in 1979 and 1980 but did not in 1975 and 1976. And copper is not the only commodity to retain certified stocks in backward markets. Williams (1986) notes that, despite the fact that there is no reason why elevators cannot be completely emptied, certified wheat and soybean stocks, precisely those eligible for delivery on futures contracts, have never fallen to zero. For wheat in Chicago, this includes a period of over 120 years.[5]

While Williams and Wright recognize the tension between the cost-of-carry model and observed periods of negative storage with positive inventory levels, a negative price for storage is routinely excluded from the possible range of solutions in the literature on commodity policy issues. The general inter-temporal arbitrage conditions are often given as:

[5]With regard to the 1973 soybean crop, Williams writes (pp. 36-37):

In 1973, the cost of keeping the more than 3 million bushels in store in Chicago was more than $2.1 million. This expense was even larger nationwide. The total stock of old-crop soybeans as of 1 September 1973 was 60 million bushels, the smallest carryover of the decade (although it was still some 4% of the crop being then harvested.) As of 1 August 1973, the spread between the spot price and the August futures contract, on which deliveries were eligible until the end of the month, was -$1.30 per bushel. This suggests that the holders of those 60 million bushels paid on the order of $78 million, quite apart from physical storage costs, for the privilege of keeping them in store that one extra month.

Table 2.1: Spreads, stocks, and production of refined copper in the United States on the first business day of January, 1974-1983.

Period of spread	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983
January-March	-4.60	.45	.40	.40	.40	.75	1.45	1.30	.80	.60
March-May	-1.40	.55	.50	.45	.50	.75	.15	1.30	.85	.55
May-July	-1.00	.60	.45	.45	.50	.70	.10	.95	.85	.55
July-September	-.60	.60	.45	.45	.45	.55	.15	.85	.85	.60
September-December	-.60	.65	.35	.45	.45	.45	.15	.80	.80	.70
Price of December contract	80.60	58.50	59.60	67.40	54.50	75.30	112.70	99.40	83.40	75.75
Spreads after removal of carrying charges										
January-March	-5.20	.00	.00	.00	-.05	.00	.00	.00	-.10	.00
March-May	-2.10	.00	.00	.00	-.05	.00	-1.30	.00	-.10	-.05
May-July	-1.65	.00	.00	.00	-.05	-.05	-1.30	-.20	-.10	-.05
July-September	-1.20	.00	.00	-.05	-.05	-.25	-1.25	-.30	-.10	-.05
September-December	-1.15	.00	-.05	-.05	-.05	-.35	-1.20	-.40	-.15	.00
Stocks and inventory of refined copper expressed in short tons										
Comex Warehouse stocks	5,873	43,214	100,102	200,953	184,390	179,572	98,856	179,770	186,920	272,999
Total US stocks	49,098	194,851	360,700	473,800	471,100	367,900	186,300	253,000	338,600	484,500
US production in January	157,700	146,000	130,300	140,900	129,000	135,600	161,000	133,500	117,500	97,300

Note: Spreads in copper are based on the closing prices as of the first business day in January and are expressed in cents per pound per month, taken from Williams (1986). Inventory values corresponding to January are December 31 levels. Inventory and production data were compiled from various issues of Metal Statistics.

$$\begin{aligned} p_t &\le p^e_{t+1}/(1+r) - k \rightarrow z_t \ge 0 \\ p_t &> p^e_{t+1}/(1+r) - k \rightarrow z_t = 0. \end{aligned} \tag{2.1}$$

where z is inventory level, p is price, r is a discount rate, k is a constant positive per unit storage cost, and a superscript e denotes expectations. Recent examples include Miranda and Helmberger (1988) and Glauber, Helmberger, and Miranda (1989).

Another argument, related to a negative price for storage, is Keynes's (1931) notion of normal backwardation. Keynes argued that commodity markets could remain backward in equilibrium (at constant price expectations), implying negative revenue for storage, because of the way in which risk is transferred from commodity producers, who are naturally long in the commodity, to speculators. Hedging producers take a short position in the market (to offset their naturally long positions) while speculators take the opposite position. Keynes argued that speculators would decline taking an opposing long position unless the expected rate-of-return on long positions exceeded the riskless rate-of-return. In order for this to be the case, the expected rate-of-return on the futures position would have to be less than the expected spot price and

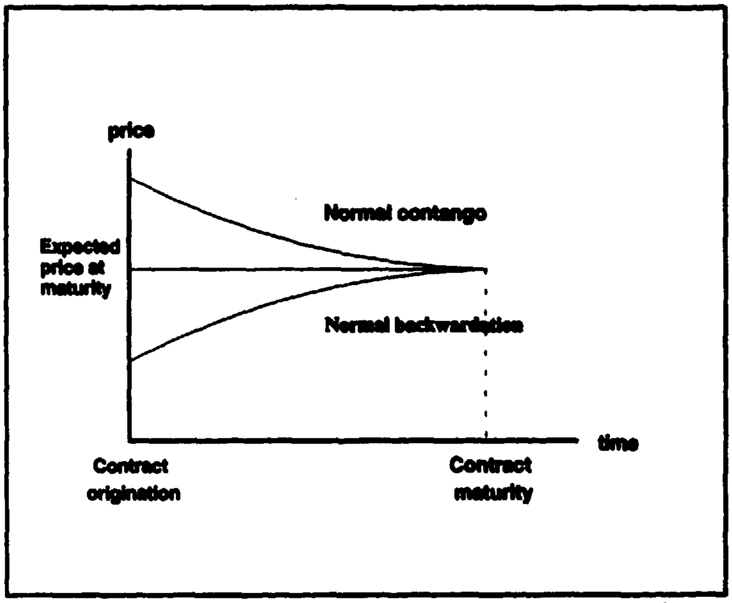

Figure 2.3: Futures prices over time with constant future price expectations.

rise as the contract matured; that is, futures prices would have to be discounted to expected prices. Keynes called this price-time relationship *normal backwardation.* (See Figure 2.3.) Conversely, if hedgers are, on net, long in futures contracts, then speculators would require a higher-than-riskless rate of return on short positions. In this instance, the futures price would have to contain a premium over the expected price, which declines as the contract matures. Keynes referred to this relationship as *normal contango.* Despite its implications of a negative price for storage, Keynes's theory of normal backwardation has little to say about inventories and arbitrage possibilities. This omission has formed the basis of criticism by cost-of-carry proponents (see, for example, Figlewski, 1986).

Regardless of the validity of Keynes's arguments, the theory set off a number of empirical studies aimed at verifying the presence of a downward bias in futures (that is, a tendency for the price of a futures contract to "rise up" to expected price levels as the contract approaches maturity). Houthakker (1957), using trade statistics on corn and wheat, concluded that naive small traders could benefit by blindly following a long-side trade position. Tessler (1958, 1960) found no evidence of normal backwardation. Cootner (1960, 1967) then presented several cases in support of normal backwardation, while Dusak (1973), using a portfolio approach, noted that even gains from arbitrating backward markets were small when compared to investments in the stock market which carried similar levels of risk. Two more recent studies, however, have emerged which provide support for Houthakker's original findings. In the first, Carter, Rausser, and Schmitz (1983) modify Dusak's portfolio approach to allow for systematic risk and found non-zero estimates of systematic risk for most of the speculative return series examined. More recently, using non-parametric techniques, Eric Chang (1990) found statistical support for normal backwardation in a study based on wheat, corn, and soybean futures. In studies relating directly to the market for metals, Hsieh and Kulatilaka (1982), using monthly data on LME forward contracts from January 1970 to September 1980, reported average risk premia of 2.8% for copper, 17% for tin, 12.7% for zinc, and 16% for lead. In a later study, MacDonald and Taylor (1989) reported evidence for a "time-varying premium" in the forward prices for tin and zinc.

While criticisms of Keynes's theory of normal backwardation and Kaldor's convenience yield remain valid, the empirical studies offered in their support are often at odds with the simple cost-of-carry price-arbitrage conditions given in 2.1.

When inventories are being held at less than full carrying charges or when the price on futures contracts is below expected prices, incentives are created to inter-temporally arbitrage the market under the cost-of-carry model. The most obvious course of action would be for holders of inventories to reduce their costs by selling immediately into the spot market, bringing near-prices down relative to more distant delivery dates. Such inter-temporal arbitrating should continue until the expected returns to arbitrage equal the expected returns to inventory holding, or a "stock-out" occurs, when inventories are reduced to some near-zero minimum level.

Similar arbitrage opportunities are available even when inventories have been contracted for future delivery. For example, consider a holder of inventories (say a producer) who has sold copper forward for delivery in 60 days. Regardless of the terms of the forward contract, the seller can generate revenue in a backward market, or reduce holding costs when storage returns a negative price, by "lending" inventories to the market.

Although direct loan markets are currently relatively rare for commodities, they do exist for uranium, and a brokerage firm, the Nuclear Exchange Corporation (NUEXCO), exists to facilitate such loans. In addition, there are active explicit loan markets for equity shares on many stock exchanges.[6] Williams (1986) documents an active loan market in grain warehouse receipts in the United States in the 1860s.

While no explicit loan market exists for refined copper, loan transactions can be accomplished in an equivalent manner using futures and spot markets, obviating the need for an explicit market. Once the decision has been made to hold inventories for a span covering the delivery date of at least one futures contract, lending inventories can be accomplished in a straightforward manner. Holders of inventories can sell into the spot market while simultaneously contracting to repurchase the copper (by purchasing a futures contract) at a future date for a fixed price. In a backward market, this sum will be positive and constitutes a positive interest payment on an implicit copper loan. The effect on the copper market is to increase supplies available for immediate delivery and increase the demand for future deliveries, arbitrating the backwardation. It should be noted that not only does the supplier of inventories receive a payment, but he also eliminates the need to store. As a result, even when the futures market does not exhibit backwardation but does return a negative price for storage, a holder of inventories can reduce his costs by lending supplies to the market. Therefore, in cost-of-carry models, a negative price for storage generates arbitrage opportunities as well, encouraging supplies to be freed from inventories.[7] As with the direct

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[6]The London Exchange provides the most straight-forward example. In London the exchange settles every fortnight rather than every day as in New York. Buyers of stocks have contracted to receive the stocks on a particular day, but, if compensated, may agree to delay taking delivery of the stock. If there is sufficient pressure for immediate delivery, the person agreeing to postpone a contracted delivery may receive either a concessionary rate on margin loans or a fee from the seller. The fee is called a "backwardation" and is equivalent to interest on a loan of deliverable stock.

[7]The flip-side of lending is, of course, borrowing. Because of the two-step nature of the implicit loan procedure given above, first selling near while buying long, each "lender" need not correspond with a single borrower. For example a fabricator may be purchasing the copper from the spot market

inter-temporal arbitrage, holders of inventories can be expected to lend into markets exhibiting backwardation or negative storage prices until the expected return on lending equals the expected return on holding inventories.

The implicit returns to loans from inventories of refined copper were calculated from futures price spreads for September 24, 1991 and are given in Table 2.2. The returns are probably underestimated, as warehouse fees, which would increase the returns to lending, and transaction costs, which would decrease the returns slightly, have been omitted. Recalling that total profits from refining may equal less than 10% of the price of refined copper, the incentives to lend are significant. Put another way, the price of borrowing inventories is high.

In the following section, it is argued that a convex shadow price for inventories, combined with uncertain demand, gives rise to a dispersion premium for inventories. When inventories are low, the effect of a higher-than-expected sales level will have a greater effect on price than when inventories are more plentiful. In addition, when inventories are low, the range of possible price outcomes become more skewed toward higher prices. By carrying inventories into the period, the producer has the option of taking advantage of higher prices, should they materialize, without increasing production and incurring increasingly expensive marginal costs. Because of the asymmetry, the drop in the value of the carried inventories, when sales are correspondingly lower-than-expected, is not as severe. This relationship gives rise to a generalization of the price-arbitrage conditions in which it is rational to hold

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while a toller may be locking in a future sales price for refined copper. However, if desired, the mechanism is readily available for a refiner to borrow inventories which are replenished from future production. This mechanism allows individual firms to hold a negative inventory of copper, even though stocks must be non-negative in the aggregate.

inventories in the face of expected price declines.

In its emphasis on nonlinearities, the model is similar to Gardner's (1979) model of optimal grain storage. Gardner used recursive programming techniques to demonstrate that non-linearities in the shadow price for inventories can result, in part, from nonlinearities in

Table 2.2: Implicit interest from lending refined copper stocks using September 24, 1991 closing COMEX prices for refined copper.

Delivery Date	Settlement Price	1-month	2-months	3-months
		cents per pound (% annualized return)		
September '91	110.25	3.03 (33.0)	4.94 (26.9)	6.69 (23.9)
October	107.75	1.92 (21.4)	3.68 (20.5)	5.42 (20.1)
November	106.35	1.77 (19.9)	3.52 (19.9)	4.56 (17.2)
December	105.10	1.76 (20.1)	2.81 (16.0)	4.10 (15.6)
January '92	103.85	1.06 (12.3)	2.35 (13.6)	3.44 (13.2)
February	103.30	1.30 (15.1)	2.40 (13.9)	3.48 (13.5)
March	102.50	1.10 (12.9)	2.19 (12.8)	3.13 (12.2)
April	101.90	1.10 (13.0)	2.04 (12.0)	2.97 (11.7)
May	101.30	0.94 (11.1)	1.88 (11.1)	2.81 (11.1)
June	100.85	0.94 (11.2)	1.88 (11.2)	2.36 (9.4)
July	100.40	0.94 (11.2)	1.43 (8.5)	
August	99.95	0.49 (5.9)		
September	99.95			

Note: Implicit interest calculated on an annualized 5.98% discount rate where:

$$\text{interest} = p^f(t) - p^f(t+n)e^{-rn},\ n = 1,2,3$$

and where p^f represent settlement prices for futures contracts deliverable at t and t+n.

the underlying objective function. Since Gardner's topic was the optimal storage of grains, the primary source of asymmetry was the fact that supplies of grain can be withheld for future consumption while a symmetric transfer from the future to the present is impossible. Nonetheless, Gardner also showed that nonlinearities in demand, for example, would affect the valuation of the shadow prices as well.

As will be seen later, the convex relationship between price and inventories is suggested by simply plotting the two series and is a standard feature throughout a long history of economic literature. It is often stated as a stylized fact that the marginal value of inventory increases with scarcity, declining in a nonlinear manner to zero as inventory levels increase. The marginally high value of inventories at low levels of physical stocks is usually ascribed to convenience and is often the departure point in textbooks and applied studies (for example, Stein, 1987; Fama and French, 1988; Gibson and Schwartz, 1990). Conversely, when holders of inventories appear willing to retain inventories despite arbitrage opportunities, as in backward markets or other markets with a negative price for storage, the holders are presumed to receive a higher "convenience yield" as compensation. Working (1948) and Brennan (1958) provide a number of reasons why the value of inventories above carrying charges might rise to very high values at scarce levels and fall to zero at sufficiently high stock levels. Working argues primarily that stocks

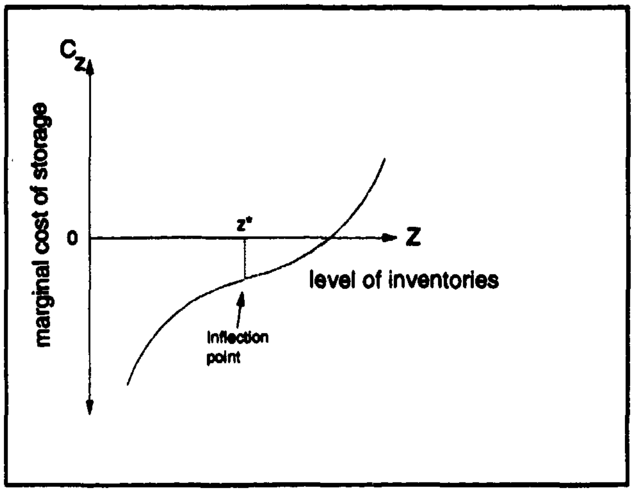

Figure 2.4: The marginal cost-of-storage function.

are often an adjunct of business and provide convenience and cost savings through a reduction in restocking costs and an ability to quickly meet orders. As stock levels increase, the marginal contribution of additional stocks goes to zero. As inventories build, storage facilities reach capacity levels and marginal storage becomes increasingly expensive.

Generally, the marginal cost-of-storage function is depicted as shown in Figure 2.4, where z^* marks the inflection point. However, in addition to the notion of convenience, Working also offered some arguments based on expectations and probability, noting that "Merchants who deal in goods that are subject to whims of fashion, or to sudden obsolescence for other reasons, must lay in stocks and carry them in expectation that some part of the stocks will have to be sold at a heavy loss." Additionally, he points out that as stocks become low the probability of a "squeeze" in the futures market increases. Brennan also argues that the "convenience yield" derives primarily from fewer delays and lower costs (because of less frequent ordering and stocking) in delivering goods to consumers. However, Brennan also discusses what he calls a "risk-aversion factor", noting that the larger the level of inventories, the greater the effect of a price change and the revaluation of held stocks.

In the next section a formal model is developed to show that convex inventory-shadow prices for copper can arise regardless of whether inventories are cost-reducing. The model does allow for a cost-reducing Kaldor-convenience yield as well. A generalized set of price-arbitrage conditions is then derived from the first-order conditions of the model. The model is then used to test empirically for convexity in the shadow price of inventories which gives rise to an estimatable dispersion premium. The existence of a cost-reducing effect for inventories is also examined.

3.0 The Optimization Model and the Price-Arbitrage Condition

In this section, the formal model is summarized from the detailed derivation contained in Annex 1. A generalized price-arbitrage condition is derived from the first-order conditions of the optimization problem which is consistent with inventory-holding during an anticipated price fall. The copper-refining problem is characterized as a continuous two-cycle problem with uncertain future demand. In the current period, the producer knows the current sales price. By deciding how much to produce and sell, he determines how much inventory he will bring into the next period. The expected marginal value, or the shadow price, of the inventory in the next period is not known, but contains a stochastic element since demand is uncertain. The effects of random demand shocks on the shadow price of inventories may be asymmetric -- that is, a positive random shock may increase prices by more than an equally sized negative random shock. In such a case, the shadow price of inventory will carry a dispersion premium so that the shadow price of inventories increases with the variance of the stochastic component of sales. Such a premium is analogous to the volatility premium in an options price and can result in positive inventory levels even when price declines are expected.

The solution to the copper refiners profit maximization problem can be found by solving the Hamiltonian:

$$Max_{s,y} H = [ps - C(y,z)]e^{-rt} + \lambda(y-s) \tag{3.1}$$

where p is the sales price; s is the sales level; C is a joint cost function for storage and production where z is level of inventories and y is the production level; r is the discount-rate. λ is the change in profit due to a marginal change in the inventory level, or the shadow price of inventories. The producer maximizes profits by setting the marginal cost of production and the marginal revenue of sales equal to the value of a marginal change in the level of inventories at the end of the period.

Prices are known in the current period, and inventory levels are determined once sales and production levels are decided, but the ending value of inventories is not known exactly. Rather, the value of stocks, and the shadow price of inventories, λ, are based on expectations about future prices, production, and sales.

When sales contain a random element, inventory levels will also be stochastic and changes in inventories will contain a planned and unplanned component. The difference between planned and actual inventories will be the difference between expected and actual production minus sales. For the moment, assume that the change in inventories can be expressed as the following process:

$$z_t - z_{t-1} = E_{t-1}[y_t - s_{t-1}] + \epsilon_t \tag{3.2}$$

where ϵ has an expected value of zero and a variance σ^2. Rewriting the constraint on inventories in continuous-time notation, the value of ending inventories at time t_1 is the solution to the following infinite-horizon problem:

$$e^{-rt_1} V(z_{t_1}) = Max_{s,y}\, E\int_{t_1}^{\infty} \{[ps - C(y,z)]e^{-r(t-t_1)}\}dt, \quad s.t.\ dz = E(y-s)dt + \sigma dv. \tag{3.3}$$

The term $dv = u(t)dt^{1/2}$ is a Wiener process, where u(t) ~ N(0,1).

The solution to the problem makes use of stochastic calculus, the mechanics of which are somewhat tedius and have been relegated to Annex 1. However the relevant first-order conditions can be summarized graphically. The producer solves for planned production, sales and inventories by setting expected marginal costs equal to expected price equal to the shadow price of inventories, V_z -- which is itself an expected value. Because the producer has the option of either producing more or storing less, he must solve on two margins. This is shown graphically in Figure 3.1 where marginal costs are drawn convex in y and the shadow price of inventories is drawn convex in z. When there is an increase in the expected price, inventories are reduced until the shadow price is equal to the new expected price

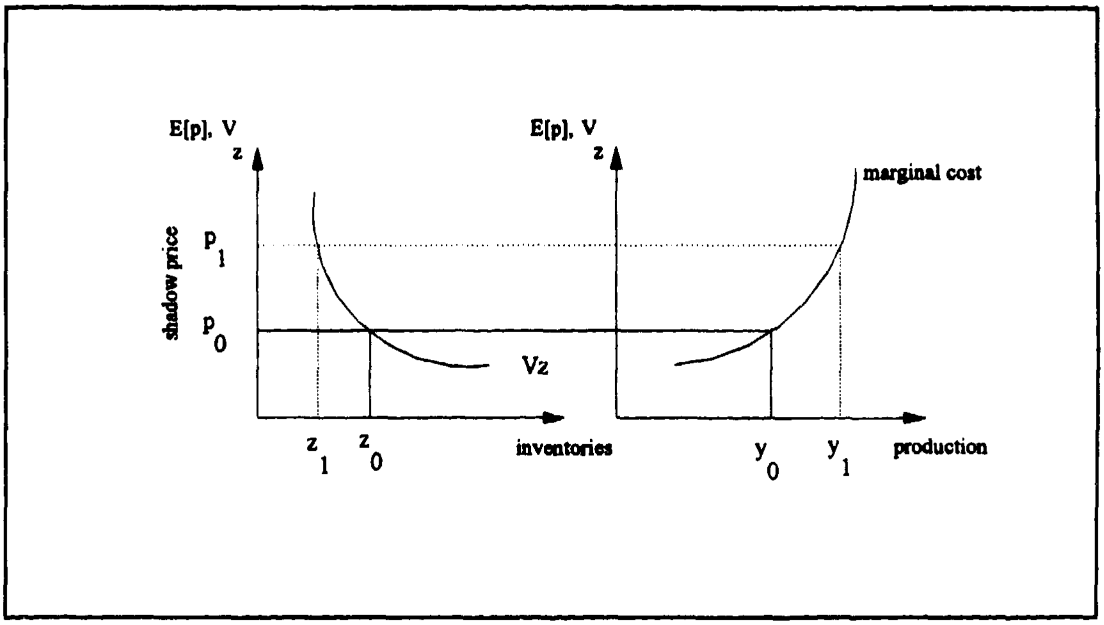

Figure 3.1: The effect of an increase in price on inventories and production.

freeing inventories for current consumption. At the same time, planned production increases and expected marginal costs increase until marginal cost equals expected price.

The first-order conditions can be manipulated to express the optimization conditions in terms of expected price:

$$E[dp/dt] = rE[p] + E\left[C_z\right] - \tfrac{1}{2}V_{zz}\sigma^2, \ \ for\ z,s,y > 0 \tag{3.4}$$

Equation 3.11 is a generalization of price-arbitrage conditions given in cost-of-carry models such as Williams and Wright (1991, p. 27). The arbitrage condition states that the expected change in price will be equal to $rE[p]$ -- interest on investing the money elsewhere -- plus C_z -- the costs of physical storage and any amenity from storage -- minus $\frac{1}{2}V_{zz}\sigma^2$. If V_{zz} is positive, then this last term constitutes a dispersion premium that increases with the variability of the stochastic component of inventories σ^2. The last two components of 3.4 have important implications for holding inventories in the face of less-than-full carrying charges. According to the condition, it still may be optimal to hold inventories when the market is in backwardation -- $E[dp/dt] < 0$ -- if inventories provide a cost-reducing Kaldor-convenience (that is, if C_z is sufficiently negative) and/or the dispersion premium, $\frac{1}{2}V_{zz}\sigma^2$, is sufficiently positive. The two components are not mutually dependent. Kaldor-convenience alone can potentially explain inventory-holding in backward markets, as can a dispersion premium. When $V_{zz} = 0$ and C_z is positive, 3.4 reduces to the cost-of-carry price-arbitrage condition.

For the cost-of-carry model, no inventories are held when the sum of the current price plus a constant physical storage cost is greater than the expected discounted future price. Putting this constraint into a continuous-time counter-part:

$$E[dp/dt] = rp + k, \ \ for\ z > 0. \tag{3.5}$$

The two arbitrage conditions differ in two respects. In 3.12, storage costs are treated as a constant positive marginal cost, and separate from other activities such as sales or production. Cost-of-carry models are usually based on the activities of professional speculators who presumably receive no convenience from holding inventories and do not participate in production. Generally, however, there is nothing fundamental to the derivation of cost-of-carry models which requires fixed marginal storage costs and the marginal storage-cost function could be written in a more flexible manner. The second difference between the two arbitrage conditions is the presence of a dispersion premium in the generalized price-arbitrage conditions. This comes from treating the value of the inventories as stochastic. Cost-of-carry models use expected prices (or futures prices representing expected prices) but do not treat the price changes themselves as a stochastic process. This differs from option-pricing models where the variance of the underlying commodity price enters explicitly into the evaluation of the option.

The dispersion premium, $\frac{1}{2}V_{zzz}\sigma^2$, can be interpreted as the expected difference between the stochastic and deterministic value of inventories. To see this, start with the marginal-value function of inventories defined in terms of a deterministic component (planned inventories) plus a random element, and calculate a Taylor-series approximation:

$$V_z(z^d+\epsilon) \approx V_z(z^d) + V_{zz}\epsilon + \frac{1}{2}V_{zzz}\epsilon^2 + \frac{1}{6}V_{zzzz}\epsilon^3 \qquad (3.6)$$

When ϵ has an expected value of zero, and is symmetrically distributed, where $E[\epsilon] = E[\epsilon^3] = 0$ and $E[\epsilon^2] = \sigma_\epsilon^2$, the expected difference between the stochastic and deterministic component of the shadow price for inventories is approximately the dispersion premium:

$$E\left[V_z(z^d+\epsilon) - V_z(z^d)\right] \approx \frac{1}{2}V_{zzz}\sigma_\epsilon^2 \qquad (3.7)$$

Even when demand and inventories are treated as stochastic, there is nothing in the first or second-order maximization conditions that would require V_{zz} to be positive. In fact, V could certainly be quadratic in z so that V_{zz} need not exist. However, in much of the literature on inventories the shadow price of inventories is often described as convex in z -- at least implicitly.[1] Usually it is stated that there is some type of "pipeline" minimum stock level (Figure 3.2). As stock-levels are drawn down and approach pipeline levels, larger and larger price increases are required to deplete diminished inventories. Usually the convexity is attributed to a convenience yield. According to this argument, pipeline levels are required to carry out business in an orderly fashion, while additional stocks, up to a point, can still facilitate transactions or minimize costs such as re-orders, deliveries, or restocking.

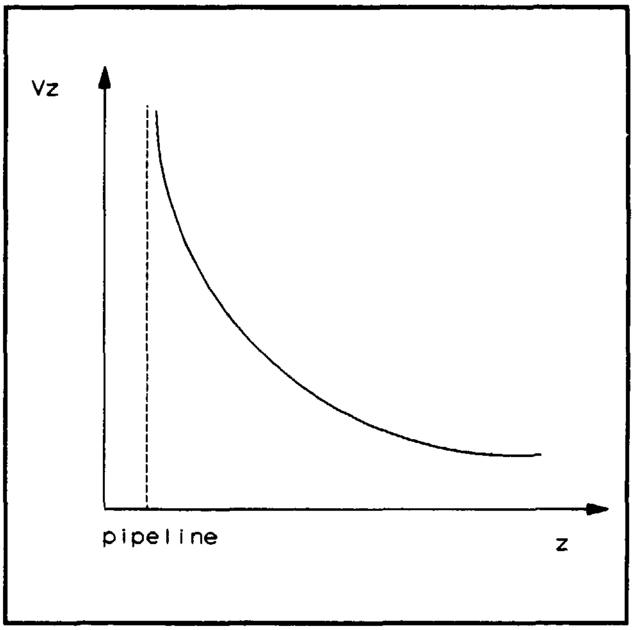

Figure 3.2: Convex shadow price for inventories.

As it turns out, V_{zz} will exists if either of the marginal cost functions, C_z, the cost of physical storage, or C_y, the cost of production, are nonlinear. This is true whether or not the cost function is joint or whether stocks are cost-reducing at any level. A fundamental question is why the marginal cost function should be convex in either storage or production levels. Copper storage is simplicity itself, and requires only a secure and dry area. However, if storage and production are joint and storage, at some

[1]For example, Working (1948), p. 19 and Brennan (1958), p. 54.

level, is cost-reducing, then there is probably some range over which the convenience diminishes rapidly as inventories grow.

In addition, marginal production costs are also likely to be increasing at an increasing rate over some production range, leading to nonlinearities in the cost function. The refining industry is constantly changing and there is a wide diversity to the scale on which refiners operate. For example, one of the newest refining plants in the US opened in 1982 in South Carolina (AT&T Nasaue Recycle Corp) with a relatively small capacity of 70,000 refined tons per year. Since then capacity has grown to 87,500 tons. For these small plants, which attempt to control costs by running near peak capacity, there remains little room for production increases. Alternatively, the country's largest plant, ASARCO's Texas plant, expanded its huge 420,000 ton/year capacity to 456,000 in 1987. At the same time, parts of the facility are quite old. Certainly under normal circumstances ASARCO could probably increase production by 20,000 tons more readily than AT&T Nasaue. Still, for the industry as a whole, increasing production means bringing on line older and increasingly less efficient facilities, operating with double shifts or otherwise using increasingly more expensive inputs, both of which are likely to lead to convex marginal costs.

Earlier it was stated that stochastic demand will give rise to stochastic inventories; however, little was said about the relationship between the means and variances of the two distributions. As it turns out, the problem can be cast in terms of either stochastic demand or stochastic price. This result comes from the finance literature (for example, Cootner, 1964, and especially Merton, 1992). There is long history of decomposing inventories into expected and random components (planned and unplanned inventories), particularly in

Keynesian macroeconomics. Stochastic prices are often employed in finance and capital literature (for example, Black and Scholes, 1973, or Abel, 1983).

For the copper-refining problem at hand, either interpretation is appropriate. Since there are no real restrictions on trade or the recovery of scrap copper, US producers are subject to events and decisions made around the world. On the demand side, copper is used in products such as plumbing components, wiring, and brass. Not only does the derived demand for these goods fluctuate with fluctuations in those associated industries world-wide (for example, the construction of new homes), but demand is also subject to cross-price effects from competing materials (for example, plastic plumbing components, or fiber-optics). In addition, US refiners are not the only agents who hold inventories. Speculators hold inventories at both major exchanges (COMEX and LME), fabricators (for example, brass-mills) often hold inventories at their plants, and non-US refiners hold inventories overseas. Domestic market conditions are also influenced by the flow of new supplies, either in the form of imports or in the supply of recycled copper.

Refiners would prefer to know exactly about events in other sectors which influence price and sales in their own market. Ultimately, this is impossible and some components of final demand and price will not be known with certainty. In an abstract sense, the refiner can be thought of as knowing the deterministic component of his demand schedule and the general properties, but not the value of the stochastic component. The production function presents less of a problem for the producer since, unlike agriculture where weather, disease, and pests create uncertainty, the yield of refined copper from a given set of inputs is known. Still, the final level of production -- after adjustments to unanticipated changes in demand and price -- is not known ahead of time, but is conditional on final demand and price. The initial stochastic term therefore originates in the demand schedule but will ultimately influence sales as well. Normalizing the demand schedule with respect to quantity, sales are stochastic; normalizing with respect to price, price is stochastic.

In the next section, estimation results are presented based on monthly data for US copper refiners. After calculating the money return to storage, $m(t) = p(t)-p(t-1)-r(t-1)p(t-1)$, the following two equations are estimated jointly:

$$
\begin{aligned}
m(t) &= C_z(t) - \tfrac{1}{2}\hat{V}_{zz}\sigma_p^2(t) + u_m(t) \\
p(t) &= C_y(t) + u_p(t)
\end{aligned}
\tag{3.8}
$$

The first equation in 3.8 is a discrete version of the price-arbitrage condition given in 3.11 plus an error term u_m. The second equation is taken from the first-order conditions given in Annex 1, and simply states that the producer will produce to the point where marginal cost equals price.. Merton's result is especially handy in this application, since the second of the two equations provides a formal model for u_p, the stochastic component of price. By using instruments, consistent (in the econometric sense) estimated values ofu_p can then be used to form a logically self-consistent estimate of σ_p^2. As noted earlier, expressing the constraint in terms of the unanticipated component of price rather than sales or inventories is essentially a simple re-scaling of the constraint. The symbol $\hat{V}_{zz}$ is therefore used to reflect the re-scaled value of V_{zz}. The term $\hat{V}_{zz}$ is treated as a constant and estimated directly as a parameter. This treatment is equivalent to assuming that the underlying marginal cost function can be reasonably approximated by a quadratic.[2]

These results are employed in Section 4 of this paper to test for convexity of the

[2]Recall equation A1.15.

shadow price for copper and provide an estimate of the dispersion premium associated with copper inventories. In addition, the model can also provide a test of jointness between production and inventory-holding which forms the economic incentive for a Kaldor-convenience benefit.

4.0 Empirical Results

In this section, results from estimating the price-arbitrage and marginal cost functions, using monthly data on sales, production, and inventory levels for US copper refiners, are presented. Before the estimating procedure and results are discussed however, the underlying data is presented in terms of the price-arbitrage equation developed earlier in Section 3.

Figure 4.1 plots the US producer price for refined copper against inventories held by producers. The data readily suggests a convex relationship. This relationship is not

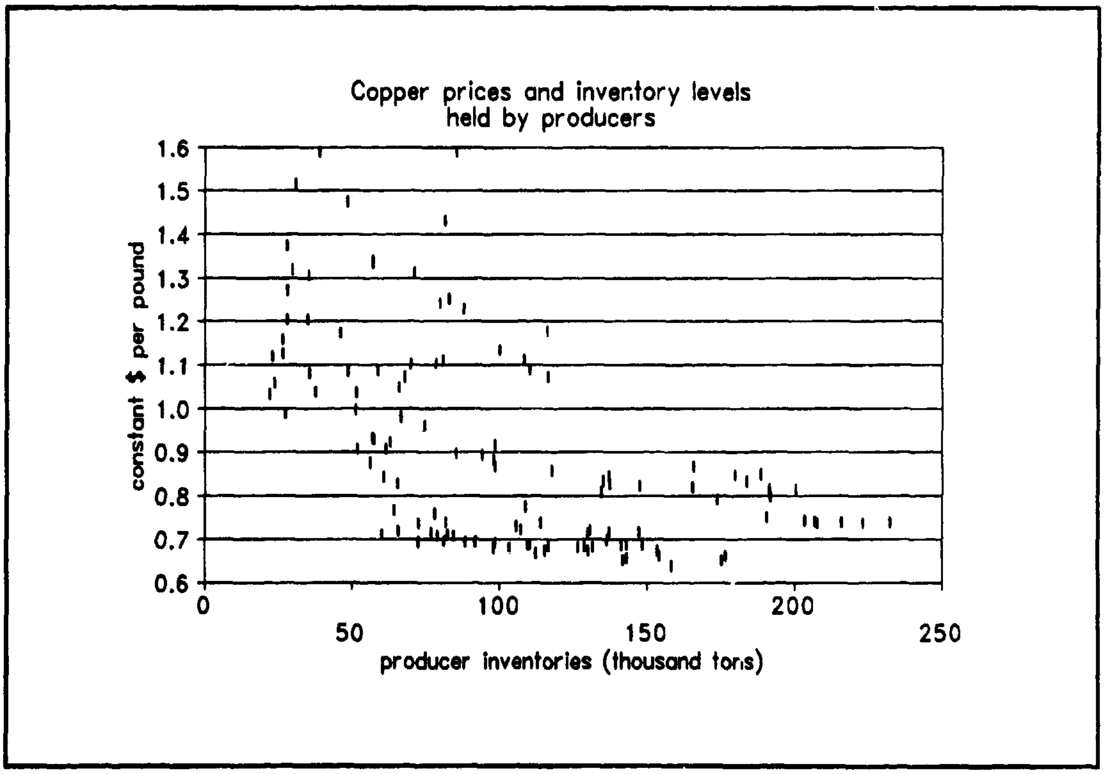

Figure 4.1: Monthly constant US producer prices and producer-held inventories for October 1979 through December, 1989.

unique for copper, and the convex relationship between price and inventories is discussed in much of the literature on inventories. For authors such as Working and Brennan, or more recently Stein and Famma, this relationship is based on a convenience yield. In most cost-of-carry models the relationship is ignored, although an exception is Williams and Wright, who argue that high prices result from the increasing potential for a stock-out.

Figure 4.2 maps the spread[1] between the closing prices of the two nearest COMEX copper futures contracts on the last day of the month against monthly closing producer-held inventories. Figure 4.3 maps the spread between the two futures prices with second and third closest settlement dates. The spreads become negative when inventory levels are low, rising to near-zero positive levels with larger inventory levels. Again, this relationship is not unique to copper. Working (1948) described the same relationship using wheat data for 1926. Brennan's 1958 article provides a number of similar graphs for eggs, cheese, butter, wheat, and oats. Working, Brennan and others attributed such negative spreads to convenience yields and subtracted estimates of the physical storage costs from the spreads to quantify the

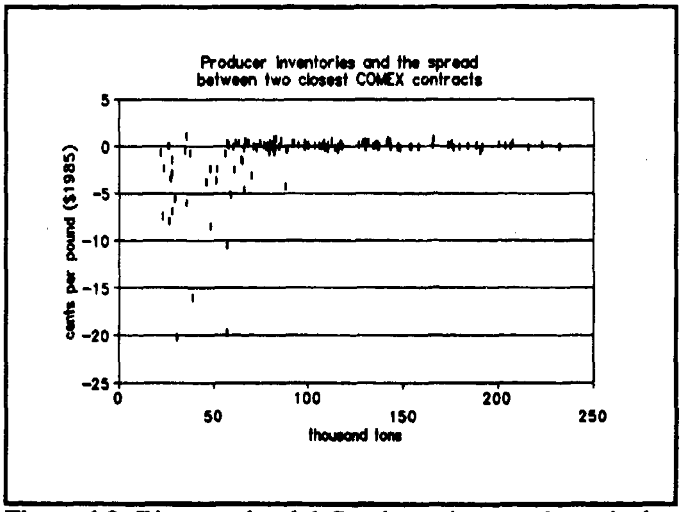

Figure 4.2: Discounted and deflated near-by spreads on the last day of the month mapped against closing producer inventory levels for October 1979 to December 1989.

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[1]The spread is expressed in constant 1985 cents per pound and was calculated as spread = $[F(t+1)/(1+r) - F(t)]/PPI_{85}$, where the future with the closest settlement date is subtracted from the more distant future's price. The difference is then deflated by the monthly US producer price index.

convenience yield. Williams and Wright argue that backwardation in futures prices comes from increased probability of a stock-out. Storage in such circumstances is not fully explained; however, Wright and Williams argue that observed storage is, in some cases, an aggregation phenomena. They argue that while the market in New York may promise negative returns to storage, prices are forward in the geographically diverse markets where the inventories are actually stored.

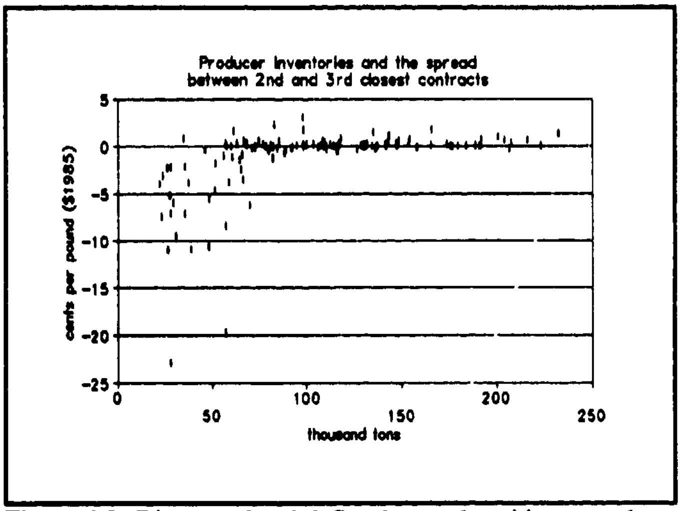

Figure 4.3: Discounted and deflated second-position spreads on the last day of the month mapped against closing producer inventory levels for October 1979 to December 1989.

The generalized price-arbitrage condition developed in Section 3 suggests two possible reasons for storage in backward markets. First, the condition formalizes the arguments put forward by Kaldor and Working. As inventory levels are drawn down, the remaining inventories could potentially confer greater and greater convenience yields -- that is, the marginal cost-of-storage function may become increasingly negative. The second reason is that the convex shape of the shadow price function, which arises from non-linearities in the cost function, conveys a dispersion premium when demand is uncertain. For example, when inventories are low and demand is unexpectedly high, marginal increases in production become increasingly expensive and prices rise increasingly quickly. At the same time, stocks will be drawn down. Following the reduction in stocks, prices will rise even more dramatically should a consecutive period of high demand materialize.

Alternatively, while a drop in demand may be just as likely, the effects on price are not symmetric. From any point on the convex marginal cost curve, the absolute value of the price drop required to lower production by a unit is less than the price increase required to increase production by a unit. Likewise, a greater price change is required to draw down inventories by a single unit than to increase inventories by a single unit. This asymmetry skews the possible outcome of prices (and therefore the shadow price of inventories) toward higher prices and generates a dispersion premium. As a result, not only should prices be higher when inventory levels are low, but the distribution of prices in general should be skewed toward higher values.

The distributional characteristics of the real monthly producer prices as well as the real futures prices (as observed on the last day of the month) for January 1980 to December 1989 are given in Table 4.1.[2] The spot prices are US producer prices (which are used in the empirical estimation reported later in this section), while the futures prices are from the COMEX in New York. Because of the difference in market locations, the means are slightly different.

Table 4.1: Distributional characteristics of monthly producer prices and COMEX end-of-month copper futures prices.

	Mean	Standard Deviation	Skewness
Producer price	0.91	0.24	0.97
Nearby future	0.83	0.25	1.06
One-ahead future	0.83	0.23	1.01

Note: Futures prices are for COMEX high-grade copper while spot prices are US producer prices. All prices are deflated by the producer price index, 1985=100.

[2]The coefficient of skewness given in Table 4.1 is $E\left[(p-\bar{p})^3\right]/\sigma_p^3$, so the expected value for a symmetric distribution is 0.

However, the spot and the futures series exhibit similar standard deviations, and all three series are skewed with the long tail of the distribution extending toward higher prices.

In order to obtain the parameters of the price-arbitrage equation derived in Section 3, as well as the parameters of the marginal cost functions and the dispersion premium, variations of the following two generalized equations were estimated:

$$\begin{aligned} m(t) &= C_z(t) - \frac{1}{2}\hat{V}_{zz}\sigma_p^2(t) + u_m(t) \\ p(t) &= C_y(t) + u_p(t) \end{aligned} \tag{4.1}$$

where u_m and u_p are random errors, where $m(t) = p(t) - p(t-1) - r(t-1)p(t-1)$ is the money-return to storage, p is the producer price, C_z and C_y are marginal cost functions for inventories, z, and production, y. Several functional forms were considered for C_z and C_y and these will be discussed in greater detail later.

As discussed earlier, the variance in the stochastic differential equation associated with the change in copper inventories can be expressed in terms of the variance in the unanticipated component of the price for copper. In turn, the unanticipated component of price is given by u_p in 4.1, which can be used to construct an estimate of $\sigma_p^2(t)$. The estimation process involves several stages. First, to avoid simultaneity biases, instruments were used to provide fitted values for endogenous sales and production. Next, substituting fitted values for endogenous right-hand-side variables, the second equation of 4.1 was estimated using least-squares. One result is an unbiased estimate of the residual vector, $\hat{u}_p$.[3] An l-month moving-variance estimate, $\hat{\sigma}_p^2(t)$, of the variance of the unanticipated

[3]Since $\sigma_p^2(t)$ may vary with time, u_p will be heteroskedastic. Nonetheless, the 2SLS estimates for $\hat{u}_p$, although inefficient, will be unbiased. (See Kennedy, 1992, p. 114.)

component of producer price $\sigma_p^2(t)$ was then constructed from $\hat{u}_p$ where $\hat{\sigma}_p^2(t) = \sum_{t-l-1}^{t-1} [\hat{u}_p(n) - \bar{u}_p(n)]^2/(l-1)$ and $\bar{u}_p(t) = \left(\frac{1}{l}\right) \sum_{t-l-1}^{t-1} \hat{u}_p(n)$ is the moving-mean. Following the construction of $\hat{\sigma}_p^2(t)$, both equations of 4.1 were estimated together simultaneously as the final stage of a three-stage least-squares procedure. As discussed earlier, $\hat{V}_{zz}$ is treated as a constant and estimated directly as a parameter. The entity $\frac{1}{2}\hat{V}_{zz}\hat{\sigma}_p^2$ constitutes an estimate of the dispersion-premium.

The choice of a functional-form for the underlying cost-function, $C(z,y)$, had to meet several criteria. First, the form had to be flexible enough to allow third-order derivatives and jointness between inventories and production, that is C_{yz}, C_{zy}, C_{yyy}, C_{zzz}, etc. had to be potentially non-zero. The following log-linear equations were used initially to approximate the marginal cost functions C_z and C_y from 4.1, since they meet the criterion with a paucity of estimated parameters:

$$\begin{aligned} C_z &= b_0 + b_1 \ln z + b_2 \ln y + b_3 \ln x + b_4 \ln w + b_5 \ln f \\ C_y &= c_0 + c_1 \ln z + c_2 \ln y + c_3 \ln x + c_4 \ln w + c_5 \ln f \end{aligned} \tag{4.2}$$

where the b_i and c_i for $i = 0,1,...5$ are fixed parameters. The choice of functional form has implications -- especially for the underlying marginal cost-of-storage function. These implications, along with more general and more restricted functional forms, are discussed later in this section.

The process of refining copper, explained in Section 2, is relatively simple, which helps limit the number of parameters in 4.2. In addition to inventory levels and production levels, the price of electricity, x, the price of raw input copper, w, and the capacity of the refining plants, f, are given as arguments of the cost function. All prices are deflated by the US producer price index which serves as a proxy for omitted prices (primarily wages). The cost-function $C(y,z)$ is initially modeled as joint. The second-derivative functions, C_{zy} and C_{yz}, are not constrained to be symmetric, although symmetry is later tested.

Monthly data covering a period from September 1978 through December 1989 was used in the estimation. Production, sales, and producer-held inventories were taken from various issues of **World Metal Statistics**. The price of refined copper was taken as the US producer price for refined copper from various issues of **Metal Statistics.** The price of No. 2 Scrap (New York) served as the price of raw input copper and also came from **Metal Statistics.** The price for industrial electricity was taken from various issues of the United States Department of Energy publication **Monthly Energy Review**. The interest rate used was the 30-day US Treasury Bill rate. All prices and the interest rate were converted to constant 1985-dollar values, using the US Producer Price Index. The Treasury Bill and Producer Price data was taken from the International Monetary Fund's Financial Statistics data base. The data on copper refinery capacity is collected by the American Bureau of Metal Statistics, Inc and reported in various issues of **Non-Ferrous Metal Data**. The capacity is reported at the plant level, based on surveys of plant managers. For most periods the combined capacity of all plants greatly exceeded production levels. Plant managers may have an incentive to exaggerate capacity levels (to forestall new entrants) and have no real incentive to be accurate. In addition, the survey is only reported once a year. As a result, the data was treated as suspicious. However, using instruments and a smoothing technique designed to detect errors-in-variables problems did not affect the estimation results materially. Results of this procedure are reported later in this section. During the period a major strike by labor unions dramatically reduced refinery output from July 1980 through October 1980. An intercept-dummy variable, k, was used to designate the observations associated with the strike. These atypical observations were, however, illustrative of the relationship between inventories and uncertainty, as will be seen later during the discussion of the simulation results.

The initial model was estimated from:

$$\begin{aligned} m(t) &= b_0 + b_1 \ln\hat{\ell}(t) + b_2 \ln\hat{y}(t) + b_3 \ln x(t) + b_4 \ln w(t) + b_5 \ln f(t) + b_6 k(t) - \tfrac{1}{2} V_{mm} \hat{\sigma}_p^2(t) + u_m(t) \\ p(t) &= c_0 + c_1 \ln\hat{\ell}(t) + c_2 \ln\hat{y}(t) + c_3 \ln x(t) + c_4 \ln w(t) + c_5 \ln f(t) + c_6 k(t) + u_p(t) \end{aligned} \tag{4.3}$$

In the first-stage of the estimation process, fitted values for $\ln\hat{\ell}(t)$ and $\ln\hat{y}(t)$ were obtained by regressing the log of inventory and production levels on the instruments given in Table 4.2 using OLS. The regressions resulted in R^2s of .93 and .69, respectively. In the second-stage of the estimation process, the fitted values for the log of production and the log of inventories were substituted into the second equation of 4.3, and estimates of the random-component of price, $\hat{u}_p(t)$ were estimated by least-squares. A six-month moving variance, $\hat{\sigma}_p^2$, was constructed from the resulting residuals and included on the RHS of the first equation in 4.3.[4] Simultaneously estimating both equations as the

Table 4.2: Instruments used for fitted values of the log of inventories and the log of sales.

<u>Instruments</u>
interest rate
price of electricity
scrap-copper price
fixed capacity level
log of the price of electricity
log of the scrap-copper price
log of fixed capacity level
month of the observation
year of the observation
lagged production level
lagged inventory level
lag of production level squared
lag of inventory level squared

[4]Results from the six-month moving variance are emphasized here because the model provided the maximum likelihood estimate among the lag-length choices. At a practical level, however, models using four, five, seven, or eight-month moving-variances were not statistically different from the six-month model when subjected to a likelihood ratio test.

third-stage in a three-stage least-squares procedure yielded the results given in Table 4.3, which gives the estimated coefficients, and Table 4.4, which expresses the coefficients as elasticities. The results, for the most part, were insensitive to alternative choices for the length of the lag used to construct $\hat{\sigma}_p^2$ as well as the estimation method. Alternative estimates are discussed later.

Table 4.3: Estimated parameters and t-scores.

Parameter	Estimate	t-score
<u>Dispersion coefficient</u>		
V_{zz}	8.94	4.97
<u>Marginal cost of storage (C_z) coefficients</u>		
b_0, intercept	1.81	4.09
b_1, inventories	0.04	1.89
b_2, production	0.06	1.25
b_3, electricity price	0.43	3.12
b_4, scrap copper price	0.22	4.91
b_5, capacity	-0.11	-2.22
b_6, strike	0.06	1.23
<u>Marginal cost of production (C_y) coefficients</u>		
c_0, intercept	0.39	0.68
c_1, inventories	0.09	4.11
c_2, production	0.16	2.21
c_3, electricity price	-0.46	-2.70
c_4, scrap copper price	0.66	10.82
c_5, capacity	-0.23	-3.66
c_6, strike	0.14	1.86

The estimate of the dispersion premium is positive and significant at a greater-than 99% level of confidence. All of the arguments to the marginal cost functions C_y are significant at 97% + confidence level, with the exceptions of the intercept and the coefficient on the strike dummy. At the mean, marginal production costs are rising with additional production and are reduced by additions to capacity. Evaluated at the mean, additional inventory slightly increases the marginal cost of production. A 1% increase in the price of scrap copper increases marginal production costs by 73%. The elasticity of marginal production costs with respect to the price of electricity is negative, implying the unlikely

Table 4.4: Estimated parameters of marginal cost functions expressed as elasticities, and estimated averaged dispersion premium.

Estimated functions evaluated at series' means:

The elasticity of with respect to:	C_s	C_y
inventories	1.93	0.09
production	3.32	0.18
electricity price	22.75	-0.51
scrap copper price	11.79	0.73
refinery capacity	-5.82	-0.25
strike dummy	3.21	0.16
Average dispersion premium in cents per pound:	1.41	

Note: Elasticities and dispersion premium calculated at means. σ^2_p is constructed by a six-month moving variance of second-stage price-forecast residuals for refined copper.

conclusion that marginal production costs fall as utilities raise their rates. While a negative elasticity is unlikely, it may be that, in the short-run for a given fixed set of plant and equipment, energy costs are relatively fixed so that the elasticity is properly zero. As it turns out, constraining the elasticity on the price of electricity to zero has little effect on subsequent hypothesis testing or simulation results. This topic is discussed more fully later in this section.

The estimates associated with the marginal cost of storage are more mixed. The elasticity with respect to inventory levels has the expected sign, and is significant with a 93%+ level of confidence. The elasticity with respect to production levels and the coefficient on the strike dummy are not significant[5]. The electricity, scrap copper, and refinery capacity elasticities are significant. The sign on the refinery capacity is negative,

[5]Although both strike dummies are not significant individually, when subjected to a Wald-test they are jointly significant with a 99% confidence level.

which is reasonable since larger refinery capacity is probably associated with a larger facility with more storage space. The electricity elasticity is positive, which is also reasonable if the storage facilities are either lighted or heated. Mysteriously, the elasticity associated with the price of scrap copper is also positive and significant. Later, results are presented from constrained versions of the marginal cost-of-storage function including constant marginal storage costs, as asserted in cost-of-carry models, and non-joint costs. As it turns out, the significance and the size of the dispersion premium varies little under alternative versions of the model.

The estimated model was simulated in order to calculate average marginal production costs, average marginal storage costs, and the average dispersion premium. The average levels, along with the minimum and maximum values simulated for these functions, are reported in Table 4.5. The marginal storage costs are on-average low but reasonable at 1.9 cents per pound. The range, from around -5 cents to 11 cents is reasonable and suggests some scope for a convenience-yield. The average levels and ranges of the marginal cost of production and the dispersion premium appeared reasonable as well.

Table 4.5: Simulated values for the marginal cost of production (C_y), the marginal cost of storage (C_z), and the dispersion premium.

	Average	Minimum	Maximum
	------------ cents per pound ------------		
C_z	1.9	-4.8	11.0
C_y	87.7	56.5	126.0
Premium	1.4	0.0	9.4

Figure 4.4 maps the simulated dispersion premium against actual producer inventory levels. With the exception of about 15 observations, the simulated dispersion premiums

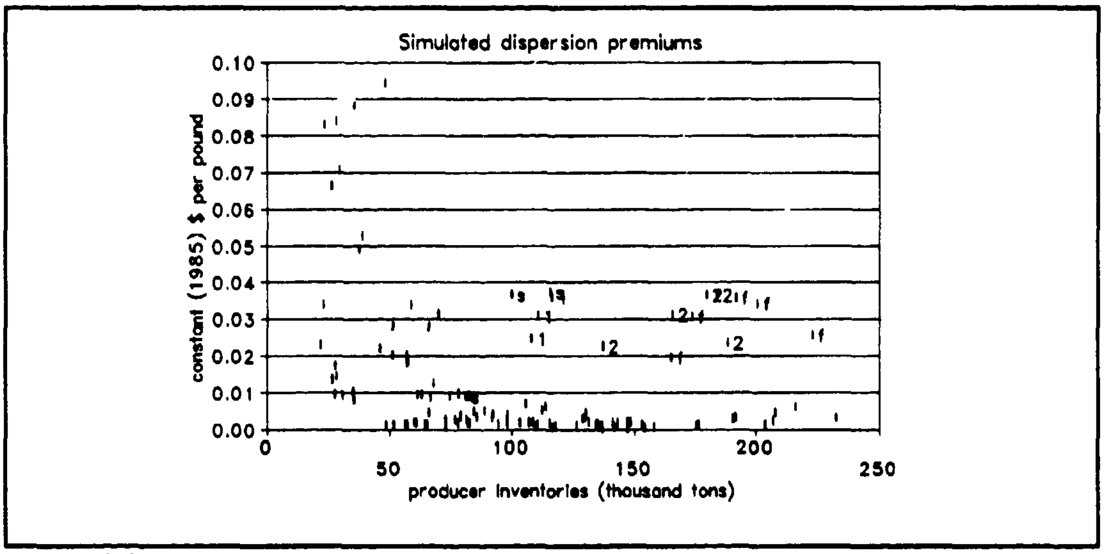

Figure 4.4: Simulated dispersion premiums for March 1980 to December 1989.

behave as expected, climbing when inventories are low and dropping off quickly when inventories grow. The outliers were generated by either a strike, the threat of a strike, or war, all of which resulted in a higher dispersion premium. In the figure, the outliers marked with a 1 represent the three months leading up to the anticipated 1980 copper strike, and observations marked with an s are months during the strike. The observations marked with a 2 represent a period in 1983 when another labor strike seemed likely, but which ended as negotiations were successfully concluded, starting with Kennecott's provisional agreement with its unions in April (Crowser and Thompson, 1984). Finally, the observations marked with an f represent the Falkland crisis (April to June, 1982). For those months, price variability and the dispersion premium grew despite moderate inventories of refined copper.

Although estimated from data on US producer production, price, and inventories, the model behaves as expected vis-a-vis the futures market in New York. The simulation results indicate that, in the case of copper, information about price spreads in New York can be used to predict dispersion premiums for US producers scattered across the country. As shown in Section 3, inventories are rationally held when a price fall is anticipated provided the dispersion premium is sufficiently large. Further, in the absence of a dispersion premium, backwardation in the futures market generates powerful incentives to arbitrage the market intertemporally. As a result, the dispersion premium for inventory-holders, including US producers, should be higher during periods of extended backwardation on the COMEX. Otherwise, inventories would flow to New York to take advantage of arbitrage opportunities. To test this aspect of the model's performance, the sample period was divided into two sub-samples. Observations were categorized as "backward market" observations if the discounted spread between the second and third nearest COMEX contracts (the spreads shown graphically in Figure 4.3) was negative; the observation was categorized as a "forward market" observation if the spread was positive. The spread between the second and third closest contracts was chosen to categorize the observations since it would allow time for the copper to be physically shipped. Of the observations, about 48% were "backward-market" observations. Average spreads and dispersion premiums were calculated from the two samples and are reported on the first line of Table 4.6. In addition, observations associated with wars, strikes, or anticipated strikes were removed from both samples, and means were calculated for the "purged" samples as well. In the "purged" sample, roughly 53% of the observations were

Table 4.6: Average spreads and simulated dispersion premiums during periods of backward and forward COMEX markets.

Sample	backward markets average spread	backward markets average premium	forward markets average spread	forward markets average premium
	---------- cents per pound ----------			
Full	-3.22	1.85	0.44	0.98
Purged	-3.38	1.80	0.40	0.48

Note: In the purged sample, observations associated with anticipated strikes, strikes, or wars were removed. Spreads are discounted and reported in constant cents per pound.

categorized as "backward market" observations. The means from this sample are reported in the second line of Table 4.6. For the "backward market" sample, the spread averaged -3.22 cents and the simulated value of the dispersion premium averaged 1.85 cents. For the "forward market" sample, the spread averaged 0.44 cents and the average dispersion premium dropped in half to 0.98 cents. Purging the samples of war and labor unrest, the difference was more dramatic, with the average dispersion dropping from 1.80 cents in the "backward market" sample to 0.48 cents in the "forward market" sample.

Figure 4.5 maps the simulated marginal costs against production levels. The outliers again are the strike months, when US production was constrained. Interestingly, some of the highest marginal costs occurred in the months prior to the strike as inventories were accumulated to substitute for anticipated drops in production.

Figure 4.6 maps the simulated cost of storage against inventory levels. The estimated elasticity of marginal storage costs with respect to inventory levels is positive but insignificant, and it is difficult to discern any clear pattern from the plotted data. If refined

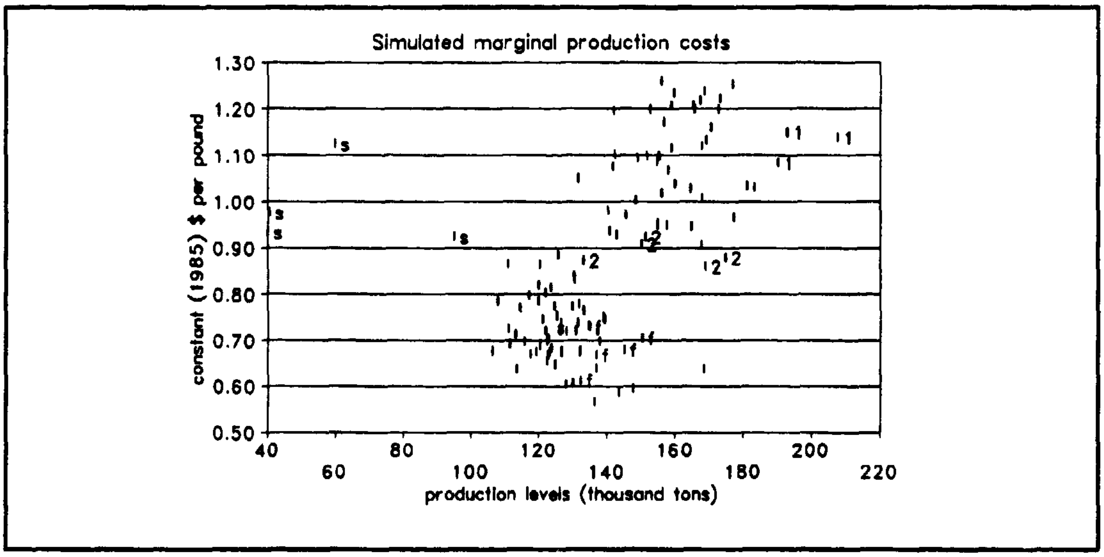

Figure 4.5: Simulated marginal production costs.

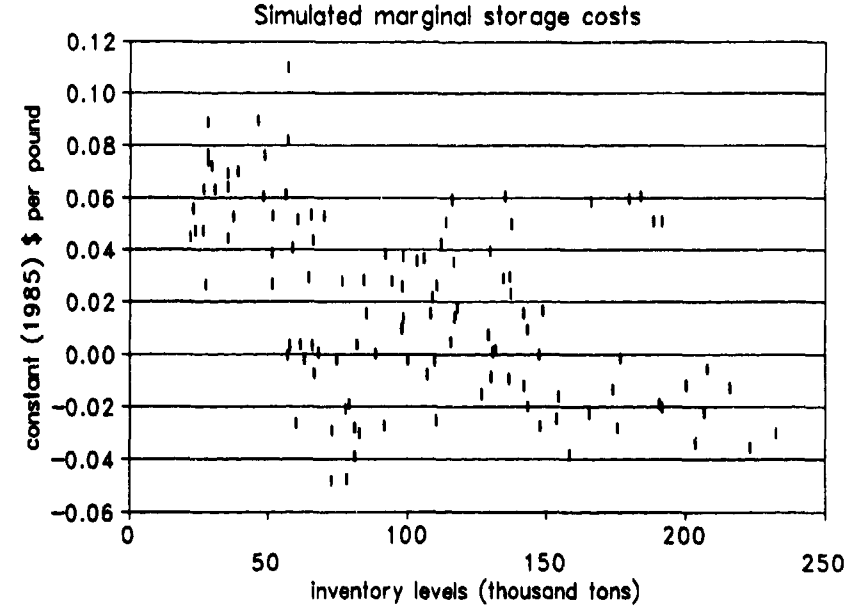

Figure 4.6: Simulated marginal storage costs.

copper inventories do indeed generate a Kaldor-convenience, then negative marginal storage costs should be associated with low levels of inventories. Further, after inventory levels reach some minimal levels, the marginal costs should become positive as the marginal "convenience" disappears. In simulation, however, many of the most negative values for the marginal cost of storage are associated with relatively high inventory levels. Later in this section, the issues of jointness and Kaldor-convenience are examined under alternative specifications.

The estimation and simulation results presented to this point are not sensitive to the lag-length chosen to generate the moving-variance for the random component of producer prices. Nor are the results sensitive to the estimation technique chosen. In addition to the three-stage least-squares estimates presented earlier, the model was also estimated using two

Table 4.7: Summary statistics on the estimated dispersion premium under alternative estimation methods and alternative lag-lengths for the price-variance measure for the log-linear model.

maximum lag on variance	----- 2SLS ----- t-score on V_{zzz}	premium mean	premium std	----- LIML ----- t-score on V_{zzz}	premium mean	premium std	----- 3SLS ----- t-score on V_{zzz}	premium mean	premium std
4	7.01	1.21	2.13	4.96	1.21	2.13	5.94	1.11	1.96
5	5.09	1.29	1.99	5.93	1.29	1.99	4.65	1.22	1.88
6	5.21	1.46	2.09	8.03	1.46	2.09	4.97	1.41	2.00
7	4.81	1.44	1.98	8.03	1.44	1.98	4.67	1.39	1.90
8	3.78	1.33	1.77	7.60	1.33	1.77	3.65	1.28	1.70
9	3.53	1.38	1.77	7.63	1.38	1.77	3.37	1.30	1.68

Note: the hypothesis that the shadow-price of copper inventories is not strictly convex could be rejected with a 99%+ level of confidence under all estimation methods and all moving-variance choices. The mean and standard deviations reported in the table are from simulations of the estimated models.

alternative instrumental variable techniques -- two-stage least-squares and limited-information-maximum-likelihood. Because endogenous variables appear as regressors in 4.1, instruments or full-information likelihood techniques are required to avoid biased estimates. However, all instrumental variable techniques should be consistent. If the model is correctly specified, three-stage least-squares should produce the most efficient unbiased estimates, since the procedure includes information on the contemporaneous variance-covariance matrix of the structural equations' disturbances. In practice, with limited sample sizes, the superiority of one method over another becomes less clear. Kennedy (1992, pp. 166-167) reports the results from several Monte Carlo studies that examine the sensitivity of estimators to changes in sample size, specification errors, multicollinearity, etc. Kennedy concludes that Monte Carlo studies consistently rank two- and three-stage least-squares estimators quite high in terms of robustness. At a practical level, the estimates should be similar. In fact, Hausman (1978) has argued that large differences between estimators that are consistent is, in itself, a sign of misspecification.

Table 4.7 reports the t-score[6] associated with estimates of $\hat{V}_{zz}$ from the three estimators under alternative specifications for $\hat{\sigma}_p^2(t)$. The table also reports the average simulated premium calculated from the estimate, as well as the standard deviation of the simulated premium. The estimated coefficient on the moving-variance, $\hat{V}_{zz}$, was always positive and significant -- that is, the hypothesis that the shadow price of inventories is not strictly positive could be rejected regardless of the lag-length chosen in the variance calculation and regardless of the estimation technique. Further, the distribution of the simulated premiums was quite stable across variance-choices and estimation techniques. This was less true for the extreme values of the various simulations. Minimum values for all simulations were slightly greater than zero, while maximum values ranged between slightly above 7 cents to slightly under 14 cents. Extreme values consistently declined as the lag-length increased, regardless of estimation technique.

As discussed earlier, the unconstrained estimation of the model returned the wrong sign for the price elasticity for electricity. The model was re-estimated under the assumption that energy costs are determined primarily by plant and equipment and that the correct short-run coefficient value should be zero. Since the t-score on the original estimate was significant, the constraint is a binding one; however, imposing the constraint had little practical consequence. Regardless of the estimation procedure or lag-length chosen for the moving variance, $\hat{V}_{zz}$ remained positive and significant. The three-stage least-squares estimates for a range of lag-length choices are reported in Table 4.8. Further, Wald tests

[6]The t-scores reported in Table 4.6 (and later in Table 4.7) for the two- and three-stage least-squares results are computed by TSP Version 6 to be robust in the presence of heteroskedasticity using White's method. This procedure tended to lower the t-score compared to alternative calculations.

Table 4.8: Tests on the effects of excluding the price of electricity from the marginal cost-of-production function.

maximum lag in moving variance	t-score on V_{m}	Wald test score
4	6.51	1.04
5	4.56	0.48
6	4.91	0.55
7	4.62	0.64
8	3.62	0.60
9	3.33	0.58

Note: Under the maintained hypothesis that electricity use is a fixed cost, the hypothesis that shadow prices are not convex can be rejected at a 99%+ level of confidence for all moving-variance choices. The hypothesis that excluding the price of electricity from the marginal cost-of-production function does not change the remaining coefficient estimates cannot be rejected at any reasonable level of confidence. The Wald-test is distributed as χ^2 with 1 degree-of-freedom.

were constructed to see if the values of the remaining fourteen parameters changed when the price of electricity was excluded from the marginal cost-of-production function. The test was constructed by pairing each of the remaining fourteen parameter estimates from the three-stage least-squares estimates of the two models -- one model including the price of electricity, and one model excluding the price of electricity. When the exclusion does not affect the estimates for the fourteen parameters, the sum of the differences between the parameters will be jointly zero. As can be seen in the second column of Table 4.8, the hypothesis that the estimated parameters are not different could not be rejected with any reasonable level of confidence.

As discussed earlier, the existence of the dispersion premium depends in part on non-linearities in the marginal cost-of-storage and production function. The log-linear form chosen for the marginal cost functions (4.2) has the advantage of being nonlinear while requiring few estimated parameters. However, there is a tradeoff between the practical advantage of having fewer parameters to estimate and flexibility in the functional form. This is particularly true given that the marginal cost-of-storage function is usually depicted as sshaped (recall Figure 3.4). For this particular functional form, a positive coefficient on inventories in the cost-of-storage equation implies that the marginal cost-of-storage function is increasing as inventories accumulate, but at a decreasing rate -- that is, $C_{zz} = b_2/z$ and $C_{zzz} = -b_2/z^2$. Often inventory-storage costs are expected to rise first at a declining rate (C_{zz} is at first negative), but then to rise increasingly rapidly (so that C_{zz} is positive) as storage facilities reach full capacity. While storage capacity for some grains or frozen commodities is strictly limited, refined copper is easily stored and not subject to the same type of storage-capacity limits. Nonetheless, in order to test whether the results reported above are dependent on the functional form chosen, a more general form, quadratic with third-order terms for z and y, was estimated as well. More specifically, the following marginal cost functions were substituted for 4.2:

$$
\begin{aligned}
C_z &= b_0 + \sum_1^5 b_i \chi_i + \sum_1^5 \sum_1^5 b_{ij}\chi_i\chi_j + b_7\hat{z}^3 \\
C_y &= c_0 + \sum_1^5 c_i \chi_i + \sum_1^5 \sum_1^5 c_{ij}\chi_i\chi_j + c_7\hat{y}^3
\end{aligned}
\tag{4.2b}
$$

where χ_i, *for* $i = 1,2,...,5$ represents z, y, x, w, and f from 4.2. The estimation procedure was repeated on:

$$
\begin{aligned}
m(t) &= b_0 + \sum_1^5 b_i \chi_i + \sum_1^5 \sum_1^5 b_{ij}\chi_i\chi_j + b_6 k(t) + b_7\hat{z}^3 - \tfrac{1}{2}\hat{V}_{zz}\hat{\sigma}_p^2 + \hat{u}_m(t) \\
p(t) &= c_0 + \sum_1^5 c_i \chi_i + \sum_1^5 \sum_1^5 c_{ij}\chi_i\chi_j + c_6 k(t) + c_7\hat{y}^3 + \hat{u}_p(t)
\end{aligned}
\tag{4.3b}
$$

using the three estimation techniques and the six specification of $\hat{\sigma}_p^2(t)$.

The more general quadratic-plus form generated an additional thirty-two parameters to estimate. For the base model (three-stage least-squares using a six-month lag to generate

$\hat{\sigma}_p^2(t)$, only seventeen of the forty-seven estimated parameters were individually significant at a 90% confidence level. These parameters and their t-scores are given in Annex II. Nonetheless, $\hat{V}_{zz}$ proved relentlessly significant regardless of the estimation technique or lag-length choice. In simulation, the quadratic version produced lower average premiums, primarily due to lower maximum values. The maximum simulated values across the estimated models did not decline as the lag-length on the variance was increased. Rather, the range was more limited, from 4.92 cents to 7.16 cents, and showed no clear pattern. Table 4.9 provides the summary results for the quadratic-plus estimations.

Evaluated at the mean of the series, the linear function C_{zz} turned out to be negative at a 97%+ level of confidence, although, in simulation, C_{zz} did take on small positive values when inventories were extremely high. At the same time, C_{z}, which should be positive, turned out to be negative on average, although not significantly so. In summary, despite the fact that many of its parameters proved insignificant, the estimated quadratic-plus

Table 4.9: Summary statistics on the estimated dispersion premium under alternative estimation methods and alternative lag-lengths for the price-variance measure for the quadratic model.

maximum lag on variance	----- 2SLS ----- t-score on V_{zz}	premium mean	premium std	----- LIML ----- t-score on V_{zz}	premium mean	premium std	----- 3SLS ----- t-score on V_{zz}	premium mean	premium std
4	2.72	0.42	1.12	2.47	0.42	1.12	2.39	0.39	1.02
5	1.65*	0.42	1.01	2.62	0.42	1.01	1.63*	0.42	1.01
6	2.43	0.54	1.26	3.28	0.54	1.24	2.43	0.54	1.24
7	2.69	0.52	1.19	2.61	0.52	1.19	2.68	0.52	1.19
8	2.71	0.51	1.14	2.42	0.51	1.14	2.71	0.51	1.14
9	2.85	0.55	1.18	2.42	0.55	1.18	2.83	0.55	1.18

Note: the hypothesis that the shadow-price of copper inventories is not strictly convex could be rejected with a 99%+ level of confidence under most estimation methods and moving-variance choices. The two scores marked with an * were significant at a 94%+ level of confidence. The mean and standard deviations reported in the table are from simulations of the estimated models.

model supports the notion of a dispersion premium, while suggesting that the extreme-case premiums are quantitatively smaller than those simulated by the log-linear model. At the same time, while the marginal cost-of-storage function was given greater flexibility, it did not trace out the classic path of rising slowly to an inflection point, and then rising more rapidly.

Despite the consistency of the estimates and simulations, one potential problem is shared by all of the specifications. Since industry-wide refinery capacity is only published once a year, the annual figure was repeated for all twelve months of the year when estimating the model. On the one hand, shadow prices for inventories are based on expectations, and it could well be that the market expectations must be based on the published refinery data since no better source of information exists. Alternatively, since the number of refiners is small, industry participants may form better subjective estimates of capacity. Regardless, there is certainly the potential for measurement error in the variable. The practical consequences of an errors-in-variables problem cannot be readily known. If the measurement error is distributed independently of the disturbance terms, then the estimates remain consistent. However, if not, the estimates may be biased even asymptotically. Since the exact nature of an errors-in-variables problem can never be known, the correct course of action cannot be known. In "correcting" the problem, the cure may be worse than the disease -- if indeed a disease exists. The general prescription for an errors-in-variables problem is to use instruments for the problem regressor or to somehow average out the effects of the measurement errors. (Kennedy, 1991 pp. 137-40.) The rationale is that by averaging out the data, the measurement errors are averaged as well, reducing their impact. Table 4.10 reports the parameter estimates after addressing the

Table 4.10: Estimated parameters and t-scores (in parentheses) for base model and model treated for assumed errors-in-variables problem.

	base model	Durbin rank method	in-year smoothing
In equation m(t) coefficient on:			
intercept:	1.81	1.28	1.84
	(4.09)	(2.76)	(4.04)
inventories:	0.04	0.02	0.03
	(1.89)	(1.22)	(1.25)
production:	0.06	0.04	0.06
	(1.25)	(0.80)	(1.06)
electricity:	0.43	0.48	0.42
	(3.12)	(3.58)	(2.62)
scrap copper:	0.22	0.22	0.21
	(4.91)	(4.68)	(4.52)
capacity:	-0.11	0.01	-0.11
	(-2.22)	(1.47)	(-1.49)
price variance:	8.94	8.10	8.91
	(4.97)	(4.96)	(4.50)
strike dummy:	0.06	0.04	0.07
	(1.23)	(0.72)	(1.12)
In equation p(t) coefficient cn:			
intercept:	0.39	-0.87	1.07
	(0.68)	(-1.44)	(1.86)
inventories:	0.09	0.05	0.13
	(4.11)	(3.07)	(5.00)
production:	0.16	0.14	0.14
	(2.21)	(2.03)	2.06
electricity:	-0.46	-0.36	-0.65
	(-2.70)	(-2.24)	(-3.51)
scrap copper:	0.66	0.66	0.64
	(10.82)	(10.62)	(10.61)
capacity:	-0.23	0.04	-0.39
	(-3.66)	(4.1)	(-4.86)
strike dummy:	0.14	0.12	0.14
	(1.86)	(1.82)	(2.04)

errors-in-variables problem two ways. The first column represents the coefficients from the base model, repeated from Table 4.3. The second column lists parameter estimates following a treatment for the errors-in-variables problem suggested by Durbin (see Kennedy,

p. 140, or Johnston, pp. 430-2). Following this procedure, the observations on refining capacity were ranked from highest to lowest. Observations on the rank were then substituted as an instrument for the observations on capacity. Since the capacity was ranked from highest to lowest, the sign on the capacity coefficients switched and, of course, the scale of the capacity coefficients changed. Otherwise, the effects of treating the estimation for measurement errors in the capacity variable were negligible. Column three of Table 4.10 reports estimation results in which the annual observations were smoothed. The smoothing procedure was to measure the difference between refining capacity for any two years and, from the January observation for the first year, add one-twelfth of the difference to each monthly observation. Again, the effects of the smoothing procedure on the estimation results are quite marginal.

As mentioned earlier, Hausman has argued that estimates obtained from alternative but consistent estimators should themselves be consistent. Based on this principal, Hausman proposed the following test for misspecification. Two sets of consistent estimates are differenced and then standardized by the difference in the covariance matrices of the two sets of estimates. The resulting quadratic form is asymptotically chi-squared, with the degrees of freedom equal to the number of linearly independent rows in the differenced covariance matrix. The model fails the test, signaling misspecification of the model, when the hypothesis that the two estimate sets are the same can be rejected. The estimation package TSP Version 6.0 provides a matrix procedure for calculating the Hausman test. Table 4.11 reports the results of the Hausman test based on the difference between the three-stage (consistent and efficient) parameter estimates and both two-stage and limited-information (consistent) parameter estimates. Since different algorithms are used to numerically compute

Table 4.11: Hausman test for misspecification based on LIML, 2SLS and 3SLS estimates for varying moving-variance calculations.

	– three-stage least-squares compared to alternative estimators –			
	two-stage least-squares		limited-information maximum-likelihood	
maximum lag in moving variance	degrees of freedom	test-score	degrees of freedom	test-score
4	2	0.34	6	0.02
5	2	0.21	5	0.01
6	1	0.06	7	0.01
7	1	0.05	8	0.02
8	1	0.01	8	0.02
9	1	0.00	8	0.00

Note: Test-scores are distributed as X^2. The hypothesis that the two sets of estimates are equal could not be rejected with any reasonable level of confidence.

the three estimates, they will in practice yield different estimates. The Hausman test measures whether the estimates are critically different. The test was repeated using various lag-lengths in the moving-variance calculation. Under all versions of the model the hypothesis that efficient and consistent estimates were equal could not be rejected with any reasonable level of confidence.

Earlier simulation and estimation results cast some doubt on the existence of a Kaldor-convenience for refined copper inventories. Kaldor and, later, Working and Brennan argued that a convenience yield exists when storage is a necessary adjunct to business, which, in the case of copper refining, implies a jointness in the cost of production and storage. Further, if this convenience yield effectively explains periods of backwardation and a negative price for storage, then the marginal cost of storage must be negative over some range of low inventory levels. Contrasted against this is the assertion in many cost-of-carry models that marginal storage costs are fixed and positive.

Table 4.12 reports the test results on three hypotheses about inventory costs. All three tests are constructed as Wald tests so the test scores have a chi-square distribution. The symmetry test tests the hypothesis that $C_{zy} = C_{yz}$, evaluated at the mean for the series. If the true underlying cost function is joint, then Young's theorem states that the partial second derivatives should be equal. The hypothesis of symmetry is rejected when the test-score exceeds a critical value. The results of the symmetry test are mixed and are sensitive to the moving variance chosen in the estimation model. For lag-lengths 4 and 9, the hypothesis of symmetry could be rejected at a 90% confidence level.

The second column of Table 4.12 tests the hypothesis that the cost function is actually non-joint. This test is constructed to be quite weak in the sense that only the coefficients on production in the marginal cost of storage and on inventories in the marginal cost of production (b_2 and c_1 in equation 4.2) were constrained to zero. This left, for

Table 4.12: Tests related to the marginal cost of storage.

maximum lag in moving variance	tests on hypotheses of: symmetry	non-joint costs	constant marginal costs
	Wald test scores		
4	5.21°	2.19	15.24**
5	2.04	4.42°	17.93**
6	1.37	7.87**	24.79**
7	1.32	10.86**	27.73**
8	2.30	10.50**	26.02**
9	2.79*	8.77**	26.41**

Note: All tests were constructed as Wald tests distributed as X^2 with one degree of freedom. Test scores denoted with an * are significant at a 90%+ confidence level. Test scores marked with an ° are significant at a 95%+ level of confidence and test scores marked with ** are significant at a greater-than 99% confidence level.

example, the price of scrap copper as an explanatory variable in the marginal cost of storage. Despite the weak definition, the hypothesis of a non-joint cost-function was rejected at a 95% + level of confidence for all but the four-month moving-variance models. Using a stronger definition of non-jointness, which would exclude the price of scrap copper from the marginal cost-of-storage function $c_1 = b_2 = b_4 = 0$, non-jointness was rejected for all models with a 99% + level of confidence.

The third column reports the results on the test that marginal storage costs are constant, as proponents of cost-of-carry models often assert. Constant storage costs imply that $b_i = 0$, for $i = 1,2,3,4,5$ (from equation 4.2) in the estimated model. This hypothesis was rejected at greater-than 99% confidence levels for all moving-variance choices.

Kaldor argued that the adjunct nature of selling and storing provides the economic rationale for convenience. For copper producers this means a jointness between the activities of producing, selling, and storing copper. Others have argued that inventory activity is less complex, and, in the case of most cost-of-carry models, storage costs can simply be best described by a constant marginal cost. The test results provide strong evidence that marginal storage costs are not constant, as asserted in cost-of-carry models. Further, the combined results on the tests of symmetry and jointness provide somewhat less strong support to the notion that costs are joint. Still, a Kaldor-convenience yield relies on more than jointness. Inventories are expected to yield their highest marginal convenience as inventories are initially accumulated -- that is, the marginal cost of inventories is expected to be negative at low levels, but grow positive as additional inventories build. None of the estimated models presented here displayed that characteristic. Rather, the evidence here suggests that, although inventories are a necessary adjunct to business, they generate no savings.

As stated earlier, the existence of a dispersion premium is sufficient to justify storage during an anticipated price decline and strong empirical evidence was presented to support the existence of such a premium. The estimation results reported thus far have come from joint-cost models. Table 4.13 reports tests on the hypothesis that the shadow price for copper inventories is convex under two maintained hypotheses. The first column reports test scores on the hypothesis that the estimated parameter $\hat{v}_{zz}$ is less than or equal to zero under the maintained hypothesis that the cost-function is **not** joint. The results here include the assertion that marginal storage costs have nothing to do with the cost of scrap copper, although similar results were obtained from weaker definitions. The hypothesis that the shadow price of copper inventories is not strictly convex was strongly rejected at a greater-than 99% level of confidence. The second column tests the same hypothesis under the assertion that marginal costs are constant. Again the alternative hypothesis was strongly rejected.

Table 4.13: Tests on convexity of the shadow price for refined copper inventories under non-joint costs and constant marginal storage costs.

	---- maintained hypotheses ----	
maximum lag in moving variance	non-joint cost function	constant marginal cost
	--------- t-scores ---------	
4	4.61	4.69
	3.48	3.88
6	3.91	3.93
7	3.82	3.53
8	2.73	2.41
9	2.32	2.04

Note: The hypothesis that shadow prices are not convex can be rejected at a 99%+ level of confidence under both maintained hypotheses for all moving-variance choices.

Generally, the estimation results provide strong support for the notion that the value of inventories contains a dispersion premium. This finding is empirically robust under a variety of maintained hypotheses and under a variety of estimation techniques. Further, the model behaves in simulation as would be expected even when simulated through periods of rare events, including a war and labor disputes. The existence of a dispersion premium is not guaranteed by the theory developed in Section 3, so empirical tests are crucial in explaining inventories when a fall in price is anticipated. The model does not strongly support or contradict the possibility of Kaldor-convenience, which could also explain the so-called negative price for storage. On the one hand, the hypothesis of jointness between production and inventory-holding for US copper refiners, a pre-condition for Kaldor-convenience, is generally supported by the model results. However, the explanations of the convenience -- savings in delivery costs, re-ordering costs, etc. -- imply that inventories are likely to be marginally less cost-saving as they accumulate. In simulation, the model did generate some negative marginal values for the cost of inventory-holding; however, those values were not consistently associated with lower levels of inventories. Table 4.14 summarizes the empirical results.

Table 4.14: Summary of the empirical results.

Results relating to Kaldor-convenience yields:

-Jointness between production and storage, which provides the economic rationale for a Kaldor-convenience yield is generally supported. The result is not completely robust under alternative models.

-Symmetry between marginal production and inventory-holding costs, an implication of jointness, is generally supported, although the result is not completely robust under alternative models.

-The hypothesis of constant marginal storage costs, a feature of cost-of-carry models which excludes the possibility of jointness, is robustly rejected.

-In simulation, negative marginal storage costs are generated. While negative marginal storage costs imply a Kaldor-convenience, the observations do not appear exclusively when inventories are low, as Kaldor's theory would predict.

Results relating to the dispersion premium

-A dispersion premium will exist if the shadow-price function for inventories is strictly convex in inventories. The alternative hypothesis of concavity is strongly rejected with a high degree of confidence.

-In simulation, the dispersion premium for copper from 1980 to 1989 averaged around 1.4 cents per pound. However, when inventories were low, when strikes appeared imminent, and, during the Falkland crisis, the premium rose. In simulation, the premium ranged from near zero to about 9 cents per pound.

-The estimated value of the premium as well as tests on the convexity of the shadow price for copper inventories were extremely robust under alternative estimation techniques.

-Although the model was estimated exclusively from data on US producers, in simulation, larger-than-average dispersion premiums were associated with periods of backwardation in COMEX copper futures.

-Hausman argues that since two-stage least-squares, limited-information-maximum-likelihood, and three-stage least-squares estimators are all consistent, estimation results from the various estimators should be similar. The model passes Hausman's specification test even under a variety of specifications for the price-error-variance term.

-The test on the convexity of the shadow price and the existence of a dispersion premium was robust under a variety of specifications for the underlying marginal cost functions. Generalizing the functional form greatly increased the number of parameters to be estimated and generated a large number of individually insignificant parameter estimates. Nonetheless, the dispersion premium proved relentlessly significant.

-The simulated dispersion premiums, as well as the tests regarding its existence, were robust even when non-jointness or fixed marginal storage costs were imposed.

5.0 Conclusion

As with many primary commodities, the price of storage for refined copper is sometimes negative and copper futures markets have remained in backwardation for extended periods of time. These market characteristics are in direct violation of frequently used price-arbitrage conditions which maintain that storage only occurs when the price of storage is positive. In Section 3, a generalized price-arbitrage condition is developed which is consistent with observed inventory-holding in the face of an anticipated fall in price. Potentially, there are two reasons why inventories might be held under such circumstances. First, the marginal cost of storage may indeed be negative at certain low levels of inventories -- that is, inventories may produce a Kaldor-convenience yield. In the case of copper, while there was some evidence that inventory-holding are joint activities resulting in a joint cost function, there is little evidence in the simulations of a Kaldor-convenience yield.

The second justification for inventory-holding in backward markets stems from uncertainty and a convex relationship between the shadow-price of inventories and inventory levels. When stock prices are low, an unanticipated positive shift in the demand schedule will result in rapid price gains. While a negative shift may be just as likely, the effect on price is not symmetric. This skews the distribution of potential price outcomes toward higher prices and generates a dispersion premium.

Estimates and simulations in Section 4 provide strong evidence that the shadow price function of refined copper inventories is indeed convex in inventories. Under a variety of assumptions, the key parameter in testing for convexity proved statistically robust. Further, the estimated value of the dispersion premium proved robust as well. The premium remained low on average (around 1.4 cents per pound). Simulations revealed that the low average premium masked the potential importance of the premium. Producer-held inventories for the simulation period ranged from around 25,000 to 230,000 tons. For inventory levels above 80,000 tons, the dispersion premium fell to positive but near-zero levels in the absence of labor strife or war. However, as inventories fell further the premium shot up rapidly to a maximum value of about nine cents per pound. In simulation, the dispersion premium also jumped during the months leading up to the 1980 refiner's labor strike and immediately fell at the conclusion of the strike. Similar movements occurred during a 1983 labor dispute and the 1982 Falkland War. Further, the model simulated larger dispersions during periods of backwardation in the copper futures market in New York (COMEX) even though U.S. producer prices, production, and inventories were used in estimating the model.

Just as the price of a call option contains a premium based on the underlying variability of prices, the shadow price of copper inventories contain a premium based on the variability of the unplanned component of inventories. When inventory levels are low, the value of the premium increases to the point where certain levels of inventories will be held even in the face of a fully anticipated fall in price.

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Annex 1: The Derivation of the Price-Arbitrage Condition

In this annex, the formal model is presented. A generalized price-arbitrage condition is derived from the first-order conditions of the optimization problem which is consistent with inventory-holding during an anticipated price fall. The copper-refining problem is characterized as a continuous two-cycle problem with uncertain future demand. In the current period, the producer knows the current sales price. By deciding how much to produce and sell, he determines how much inventory he will bring into the next period. The expected marginal value, or the shadow price, of the inventory in the next period is not known, but contains a stochastic element since demand is uncertain. The effects of random demand shocks on the shadow price of inventories may be asymmetric -- that is, a positive random shock may increase prices by more than an equally sized negative random shock. In such a case, the shadow price of inventory will carry a dispersion premium so that the shadow price of inventories increases with the variance of the stochastic component of sales. Such a premium is analogous to the volatility premium in an options price and can result in positive inventory levels even when price declines are expected.

The first-period producer problem can be written as:

$$Max_{s,y} \int_{t_0}^{t_1} [ps - C(y,z)]e^{-rt}dt + J(z_1,t_1) \quad s.t. \ \dot{z} = (y-s) \qquad \textbf{(A1.1)}$$

where p is the sales price; s is the sales level; C is a joint cost function for storage and production where z is level of inventories and y is the production level; r is the discount-rate and $J(z_1,t_1)$ is the value of ending stocks. The cost function is treated jointly to allow for a potential cost-reducing Kaldor-convenience for inventories.

The solution can be found by solving the Hamiltonian:

$$Max_{s,y} H = [ps - C(y,z)]e^{-rt} + \lambda(y-s) \tag{A1.2}$$

where the first-order conditions are given by:

$$\begin{aligned}
&i)\ \partial H/\partial s = 0 \rightarrow\ p = \lambda e^{rt},\ \ for\ t_0 \le t \le t_1,\\
&ii)\ \partial H/\partial y = 0 \rightarrow C_y = \lambda e^{rt},\ \ for\ t_0 \le t \le t_1,\\
&iii)\ \partial H/\partial z = -\dot{\lambda} \rightarrow C_z = \dot{\lambda} e^{rt},\ \ for\ t_0 \le t \le t_1,\\
&iv)\ \ \lambda(t_1) = \partial J/\partial z(t_1),\\
&v)\ \ z(t_0) = z_0.
\end{aligned} \tag{A1.3}$$

The producer maximizes profits by setting the marginal cost of production and the marginal revenue of sales equal to the value of a marginal change in the level of inventories at the end of the first period -- that is, the shadow price of inventories at time t_1 (from conditions i, ii and iv in A1.3, evaluated at t_1). Evaluating the solution at $t_1 = 0$, $p = C_y = \lambda = \partial J/\partial z$.

Prices are known in the current period, and inventory levels are determined once sales and production levels are decided, but the ending value of inventories is not known exactly. Rather, the value of stocks J, and the shadow price of inventories, λ, are based on expectations about future prices, production, and sales.

When sales contain a random element, inventory levels will also be stochastic and changes in inventories will contain a planned and unplanned component. The difference between planned and actual inventories will be the difference between expected and actual production minus sales. For the moment, assume that the change in inventories can be expressed as the following process:

$$z_t - z_{t-1} - E_{t-1}[z_t - z_{t-1}] = \epsilon_t = z_t - z_{t-1} - E_{t-1}[y_t - s_t] \qquad \text{(A1.4)}$$

where ϵ has an expected value of zero and a variance σ^2. Rewriting the constraint on inventories in continuous-time notation, the value of ending inventories at time t_1 is the solution to the following infinite-horizon problem:

$$e^{-rt_1} V(z_{t_1}) = Max_{s,y}\ E\int_{t_1}^{\infty}\{[ps - C(y,z)]e^{-r(t-t_1)}\}dt, \quad s.t.\ dz = E(y-s)dt + \sigma dv. \qquad \text{(A1.5)}$$

The term $dv = u(t)dt^{1/2}$ is a Wiener process, where u(t) $\sim$N(0,1). Because inventory changes include a random component, dz/dt does not exist in the usual sense and the rules of stochastic calculus apply.

Evaluated at t_1, the solution to A1.5 gives the value of ending stocks, that is, $J(z_1,t_1) = V(z_1)e^{-rt_1}$. In the language of optimal control theory, the firm's problem is a stochastic infinite-horizon problem, stretching from t_1 onwards. As a result, the shadow price for the end-of-period inventories in the first stage of the producer problem is based on expectations of an on-going process of production amid uncertain demand. Generally, the solution for $J(z_1,t_1)$ can be found by solving Bellman's equation, a partial differential equation: $-\partial J(z_1,t_1)/\partial t = rV(z_1)e^{-rt_1}$. Arbitrarily setting $t_1 = 0$ simplifies the equation somewhat so that the solution to the inventory problem from t_1 onward can be represented by the Hamilton-Jacobi equation of dynamic programming:

$$rV(z_1) = Max_{s,y} E\left[ps - C(y,z) + V_z(y-s) + \tfrac{1}{2}V_{zz}\sigma^2\right] \qquad \text{(A1.6)}$$

The first-order conditions for the maximization of A1.6 are:

$$\begin{aligned} &i)\ E(p - V_z) = 0 \\ &ii)\ E(-C_y + V_z) = 0 \\ &iii)\ E(dz) = E(y-s)dt \\ &iv)\ z(t_1 = 0) = z_1 \end{aligned} \qquad \text{(A1.7)}$$

The producer solves for planned production, sales and inventories by setting expected marginal costs equal to expected price equal to the shadow price of inventories, V_z -- which is itself an expected value.

It is worth noting that the distribution of the error term, especially σ^2, is independent of the decision variables. The variance of the error is regarded as a state of nature and is not subject to choice on the part of the producer. This assumption is implicit throughout the paper and works well empirically. Further, the derivation of stochastic differential equations are premised on the fact that the distribution is dependent solely on current-period information (see, for example, Malliaris and Brock, 1982, p. 67, or Merton, 1992, p. 67). The distinction is a subtle one, since it is reasonable to think of the variance as some function of demand or supply. However, in terms of the optimization problem, it is the expected value of the function, not the function itself, that is relevant.

Several additional assumptions must be made to guarantee that the first-order conditions do indeed provide a maximum. V must be concave in z; the solution values of z, y, and s must be positive[1] (otherwise border solutions must be considered); and the

[1] For the empirical problem at hand, refiner inventories of refined copper are all positive as are quantities sold and produced. Inventories at both the COMEX and LME have remained positive throughout the history of those institutions as well. However, to be complete, stock-outs need to be considered in the theoretical model and non-negativity constraints introduced to the maximization problem. These are given in Annex 2.

transversality-at-infinity condition must hold.[2]

An expression for the marginal value of inventories is found by applying the envelope theory (Lopez, 1989) to the Hamilton-Jacobi equation given in A1.6:

$$rV_z = E\left[-C_z + V_{zz}(y-s) + \frac{1}{2}V_{zzz}\sigma^2\right]. \qquad \text{(A1.8)}$$

At the solution, $E[p] = V_z$, (as part of the first-order conditions in A1.7) so that:

$$E[dp/dt] = V_{zz}E[dz/dt] \qquad \text{(A1.9)}$$

Rewriting part iii of A1.7 provides $E[dz/dt] = E[y-s]$. These results can be combined to form a price-arbitrage condition. First, from rearranging A1.8 and using iii from A1.7:

$$V_{zz}E[y-s] = V_{zz}E[dz/dt] = rV_z + E[C_z] - \frac{1}{2}V_{zzz}\sigma^2. \qquad \text{(A1.10)}$$

Combining A1.10 with A1.9 provides:

$$E[dp/dt] = rE[p] + E\left[C_z\right] - \frac{1}{2}V_{zzz}\sigma^2, \ \textit{for}\ z,s,y > 0 \qquad \text{(A1.11)}$$

Equation A1.11 is a generalization of price-arbitrage conditions given in cost-of-carry models such as Williams and Wright (1991, p. 27). The arbitrage condition states that the

[2]For the infinite-horizon autonomous problem given above, the transversality condition is:

$$\lim_{t\to\infty} V_z(t)z(t)e^{-r(t-t_0)} = \lim_{t\to\infty} J_z(t)z(t)e^{-rt} = 0.$$

Benveniste and Scheinkman (1982) showed that the condition is necessary and sufficient for the solution of A1.6 to be optimal. The logic is that any positive stock level must have no value as the problem approaches infinity. Otherwise, the firm could further increase profits by either producing less or selling more in the last period. Inventories have value because, ultimately, they can be sold. If some price exists at which no copper can be sold, then some upper bound must exist for the shadow price of inventories. If so and if stock levels, z, are limited by physical storage or natural endowments, then discounting will assure that the transversality condition holds. See Brock (1987) for further details on the general condition.

expected change in price will be equal to $rE[p]$ -- interest on investing the money elsewhere -- plus C_z -- the costs of physical storage and any amenity from storage -- minus $\frac{1}{2}V_{zz}\sigma^2$. If V_{zz} is positive, then this last term constitutes a dispersion premium that increases with the variability of the stochastic component of inventories σ^2. The last two components of A1.11 have important implications for holding inventories in the face of less-than-full carrying charges. According to the condition, it still may be optimal to hold inventories when the market is in backwardation -- $E[dp/dt] < 0$ -- if inventories provide a cost-reducing Kaldor-convenience (that is, if C_z is sufficiently negative) and/or the dispersion premium, $\frac{1}{2}V_{zz}\sigma^2$, is sufficiently positive. The two components are not mutually dependent. Kaldor-convenience alone can potentially explain inventory-holding in backward markets, as can a dispersion premium. When $V_{zz} = 0$ and C_z is positive, A1.11 reduces to the cost-of-carry price-arbitrage condition.

For the cost-of-carry model, no inventories are held when the sum of the current price plus a constant physical storage cost is greater than the expected discounted future price. Putting this constraint into a continuous-time counter-part:

$$E[dp/dt] = rp + k, \quad for\ z > 0 \tag{A1.12}$$

The two arbitrage conditions differ in two respects. In A1.12, storage costs are treated as a constant positive marginal cost, and separate from other activities such as sales or production. Cost-of-carry models are usually based on the activities of professional speculators who presumably receive no convenience from holding inventories and do not participate in production. Generally, however, there is nothing fundamental to the derivation of cost-of-carry models which requires fixed marginal storage costs and the marginal storage-cost function could be written in a more flexible manner. The second difference between the two arbitrage conditions is the presence of a dispersion premium in the generalized price-arbitrage conditions. This comes from treating the value of the inventories as stochastic. Cost-of-carry models use expected prices (or futures prices representing expected prices) but do not treat the price changes themselves as a stochastic process. This differs from option-pricing models where the variance of the underlying commodity price enters explicitly into the evaluation of the option.

The dispersion premium, $\frac{1}{2} V_{zzz} \sigma^2$, can be interpreted as the expected difference between the stochastic and deterministic value of inventories. To see this, start with the marginal-value function of inventories defined in terms of a deterministic component (planned inventories) plus a random element, and calculate a Taylor-series approximation:

$$V_z(z^d + \epsilon) \approx V_z(z^d) + V_{zz}\epsilon + \frac{1}{2} V_{zzz}\epsilon^2 + \frac{1}{6} V_{zzzz}\epsilon^3 \qquad \text{(A1.13)}$$

When ϵ has an expected value of zero, and is symmetrically distributed, where $E[\epsilon] = E[\epsilon^3] = 0$ and $E[\epsilon^2] = \sigma_\epsilon^2$, the expected difference between the stochastic and deterministic component of the shadow price for inventories is approximately the dispersion premium:

$$E\left[V_z(z^d + \epsilon) - V_z(z^d)\right] \approx \frac{1}{2} V_{zzz}\sigma_\epsilon^2 \qquad \text{(A1.14)}$$

Even when demand and inventories are treated as stochastic, there is nothing in the first or second-order maximization conditions that would require V_{zzz} to be positive. In fact, V could certainly be quadratic in z so that V_{zzz} need not exist. However, in much of the literature on inventories the shadow price of inventories is often described as convex in z -- at least implicitly.[3] Usually it is stated that there is some type of "pipeline" minimum stock level (Figure A1.2). As stock-levels are drawn down and approach pipeline levels, larger and larger price increases are required to deplete diminished inventories. Usually the convexity is attributed to a convenience yield. According to this argument, pipeline levels are required to carry out business in an orderly fashion, while additional stocks, up to a point, can still facilitate transactions or minimize costs such as re-orders, deliveries, or restocking.

As it turns out, V_{zz} will exists if either of the marginal cost functions, C_z, the cost of physical storage, or C_y, the cost of production, are nonlinear. This is true whether or not the cost function is joint or whether stocks are cost-reducing at any level. To see this, recall that $E[p] = E[C_y] = V_z$ from the first-order conditions (A1.7) and that:

$$d^2p = V_{zz}dz^2 = d^2C_y \qquad \text{(A1.15)}$$

From equation A1.15 it is easy to see that if the joint cost function is linear or quadratic in z and y so that d^2C_y is zero, then $V_{zz} = E[d^2p/dz^2]$ will always equal zero.

Earlier it was stated that stochastic demand will give rise to stochastic inventories; however, little was said about the relationship between the means and variances of the two distributions. As it turns out, the problem can be cast in terms of either stochastic demand or stochastic price. This result comes from the finance literature (for example, Cootner, 1964, and especially Merton, 1992). There is long history of decomposing inventories into expected and random components (planned and unplanned inventories), particularly in Keynesian macroeconomics. Stochastic prices are often employed in finance and capital

[3]For example, Working (1948), p. 19 and Brennan (1958), p. 54.

literature (for example, Black and Scholes, 1973, or Abel, 1983).

For the formal model, the only distinction between the two concepts involves a normalization of the stochastic difference equation, which constrains the optimization problem. This result is due to Merton (1992, pp. 57-75). Consider first the process:

$$v(t) \equiv X(k) - X(k-1) - E_{k-1}\{X(k) - X(k-1)\}, \quad k=1,...n \tag{A1.16}$$

where k denotes observed values, so that the partial sums $S_n \equiv \Sigma_1^n v(k)$ form a martingale. Now let $F(t) = f(X,t)$, where f is some non-linear function. Merton shows that $F(k) - F(k-1)$ will give rise to essentially the same stochastic difference equation as $X(k) - X(k-1)$. This result is used frequently in finance literature -- for example, when valuing instruments such as warrants in terms of the variance of the underlying stock price.

For the problem at hand, the result is useful since it allows the differential equation associated with the optimization problem to be expressed either in terms of stochastic sales or stochastic price. Empirically, it turns out to be convenient to treat price as stochastic since there is an explicit first-order condition from which the price-expectation error can be estimated.

Returning to the copper-inventory problem, when sales are treated as stochastic then $z_t = z_{t-1} + y[p(s)] - s$. Utilizing Merton's result, the stochastic differential equation can be written as:

$$dz = E(y-s)dt + \left(y_p p_s - 1\right)\sigma_s(t)u_s(t)dt^{1/2} \tag{A1.17}$$

When the demand equation is inverted and price is treated as stochastic, $z_t = z_{t-1} + y(p) + s(p)$ and the associated stochastic differential equation can be written as: Either equation A1.17 or A1.18 can be normalized to yield a version similar to the constaint

$$dz = E(y-s)dt + (y_p - s_p)\sigma_p(t)u_p(t)dt^{1/2} \tag{A1.18}$$

in A1.5:

$$dz = E(y-s)dt + \sigma_z(t)u_z(t)dt^{1/2}. \tag{A1.19}$$

In the next section, estimation results are presented based on monthly data for US copper refiners. After calculating the money return to storage, $m(t) \equiv p(t)-p(t-1)-r(t-1)p(t-1)$, the following two equations are estimated jointly:

$$\begin{aligned} m(t) &= C_z(t) - \frac{1}{2}\hat{V}_{zz}\sigma_p^2(t) + u_m(t) \\ p(t) &= C_y(t) + u_p(t) \end{aligned} \tag{A1.20}$$

The first equation in A1.20 is a discrete version of the price-arbitrage condition given in A1.11 plus an error term u_m. The second equation is a combination of i and ii from the first-order conditions given in A1.7 plus an error term u_p. Merton's result is especially handy in this application, since the second of the two equations provides a formal model for u_p, the stochastic component of price. By using instruments, consistent (in the econometric sense) estimated values of u_p can then be used to form a logically self-consistent estimate of σ_p^2. As noted earlier, expressing the constraint in terms of the unanticipated component of price rather than sales or inventories is essentially a simple re-scaling of the constraint. The symbol $\hat{V}_{zz}$ is therefore used to reflect the re-scaled value of V_{zz}. The term $\hat{V}_{zz}$ is treated as a constant and estimated directly as a parameter. This treatment is equivalent to assuming that the underlying marginal cost function can be reasonably approximated by a quadratic.[4]

[4]Recall equation 3.15.

These results are employed in Section 4 of this paper to test for convexity in the shadow price for copper and provide an estimate of the dispersion premium associated with copper inventories. In addition, the model can also provide a test of jointness between production and inventory holding, which forms the ecomonic rationale for a Kaldor-convenience benefit.

Annex 2: Necessary Border Conditions

Let y^*, s^*, be the optimal values for the problem:

$$J(t,z) = Max_{s,y}\, E\int_0^\infty \{[R(s) - C(y,z)]e^{-rt}\}dt \quad s.t.\ dz = (y-s)dt + \sigma dv, \tag{A2.1}$$

as given in Annex 1, and where z, y, and s are constrained to be non-negative. Defining the function:

$$H = E\left[(R - C)e^{-rt} + \tfrac{1}{2}J_{zz}\sigma^2 y^2 + J_z(y-s)\right], \tag{A2.2}$$

and

$$L = H + \lambda^z z + \lambda^y y + \lambda^s s, \tag{A2.3}$$

then, at $z(t)$, $y^*(t)$, $s^*(t)$:

$$L_s = L_y = 0 \tag{A2.4}$$

and the Lagrangian multipliers (λ^i) are such that:

$$\begin{aligned} &i)\ \lambda^y \geq 0;\ \lambda^y y = 0; \\ &ii)\ \lambda^s \geq 0;\ \lambda^s s = 0; \\ &iii)\ \lambda^z \geq 0;\ \lambda^z z = 0,\ when\ s,y \geq 0. \end{aligned} \tag{A2.5}$$

Annex 3: The Quadratic-Plus Model

In order to test whether the results reported above are dependent on the functional form chosen, a more general form, quadratic with third-order terms for z and y, was estimated. Specifically, the following marginal cost functions were substituted for 4.2:

$$\begin{aligned} C_z &= b_0 + \sum_{1}^{5} b_i \chi_i + \sum_{1}^{5} \sum_{1}^{5} b_{ij} \chi_i \chi_j + b_7 \hat{z}^3 \\ C_y &= c_0 + \sum_{1}^{5} c_i \chi_i + \sum_{1}^{5} \sum_{1}^{5} c_{ij} \chi_i \chi_j + c_7 \hat{y}^3 \end{aligned} \tag{A3.1}$$

where χ_i, *for* $i = 1,2,...,5$ represents inventories (z), production (y), the price of electricity (x), the price of scrap copper (w), and refining capacity (f). The estimation procedure was repeated on:

$$\begin{aligned} m(t) &= b_0 + \sum_{1}^{5} b_i \chi_i + \sum_{1}^{5} \sum_{1}^{5} b_{ij} \chi_i \chi_j + b_6 k(t) + b_7 \hat{z}^3 - \tfrac{1}{2} \hat{V}_{zz} \hat{\sigma}_p^2 + \hat{u}_m(t) \\ p(t) &= c_0 + \sum_{1}^{5} c_i \chi_i + \sum_{1}^{5} \sum_{1}^{5} c_{ij} \chi_i \chi_j + c_6 k(t) + c_7 \hat{y}^3 + \hat{u}_p(t) \end{aligned} \tag{A3.2}$$

using the three estimation techniques and the six specifications of $\hat{\sigma}_p^2(t)$. Parameters from one of the eighteen estimations, where a six-month moving average was used to calculate $\hat{\sigma}_p^2(t)$, are given in Table A3.1. The parameters were estimated using three-stage least-squares.

Table A3.1: Parameter estimations for the quadratic-plus model.

Parameter	Estimate	t-score	Parameter	Estimate	t-score
CZ_0	0.72	0.16	CY_0	-0.82	-0.17
CZ_Z	-1.00E-02	-1.45	CY_Z	0.93E-02	1.36
CZ_Y	-0.03	-2.06*	CY_Y	-0.02	-1.05
CZ_X	51.69	0.33	CY_X	13.09	0.08
CZ_W	-7.05	-2.44*	CY_W	7.89	2.97*
CZ_F	0.24E-02	3.10*	CY_F	-0.34E-03	-0.40
CZ_ZZ	-0.39E-04	-1.92*	CY_ZZ	0.40E-05	0.82
CZ_ZY	-0.58E-05	-0.39	CY_ZY	-1.00E-05	-0.67
CZ_ZX	0.27	2.50*	CY_ZX	-0.22	-2.24*
CZ_ZW	1.00E-03	0.47	CY_ZW	-0.11E-02	-0.43
CZ_ZF	0.10E-05	1.44	CY_ZF	0.12E-05	1.47
CZ_YY	0.57E-05	0.48	CY_YY	0.48E-04	0.34
CZ_YX	0.48	2.03*	CY_YX	0.17	0.54
CZ_YW	0.02	2.83*	CY_YW	-0.54E-02	-0.85
CZ_YF	-0.48E-06	-0.20	CY_YF	0.48E-05	1.72*
CZ_XX	-820.33	-0.69	CY_XX	66.20	0.06
CZ_XW	93.13	1.96*	CY_XW	-98.85	-2.32*
CZ_XF	-0.05	-3.36*	CY_XF	0.01	0.86
CZ_WW	1.12	1.62	CY_WW	-0.21	-0.33
CZ_WF	-0.50E-03	-1.68*	CY_WF	-0.43E-03	-1.37
CZ_FF	0.52E-07	0.51	CY_FF	-0.16E-06	-2.12*
CZ_ZZZ	0.84E-07	1.77*	CY_YYY	-0.16E-06	-0.41
CZ_K	0 10	3.00*	CY_K	0.01	0.31
V_ZZZ	6.57	2.43*			

Note: T-scores marked with an * are significant at a 90%+ level of confidence. The parameters are named using the following convention: CZ_W is the parameter in the marginal cost of storage function (C_z) on w, CY_WF is the parameter in the marginal cost of production function (C_y) on the product of w and f, CZ_ZZZ is the coefficient in C_z on z^3, where z is inventories, y is production, x is the price of electricity, w is the price of scrap copper, f is refining capacity, and k is the strike dummy. V_ZZZ is the coefficient on the six-month moving variance, $-\sigma^2(t)$.

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