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Publicly Listed Parent/Subsidiary Pairs:
Benchmarking to TOPIX and Market Distortion

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Abstract

This paper explores the impact of publicly listed parent/subsidiary pairs on the pricing and volatility of companies' shares.

We construct a noisy rational expectations equilibrium model in which a parent and its subsidiary company are both publicly listed. Two classes of traders participate in the market: institutional investors who have private information on the fundamentals of listed companies, and individual investors who have no private information. A key feature of the model is that institutional investors attempt to optimize the risk-return tradeoff relative to TOPIX, the capitalization-weighted index of the stock market. Individual investors are assumed to act without reference to any performance benchmark.

Within this framework we first establish the rather obvious result that the market portfolio of all outstanding shares is not an efficient portfolio. This result implies that benchmarking to TOPIX, which is the surrogate of the market portfolio without any adjustment for double-counting of parent/subsidiary pairs, generates excessive demand for shares of the subsidiary company. We analyze the equilibrium of our market model and show that (1)the price of the subsidiary company's share is pushed up to a level higher than that implied by its fundamentals, (2) the share price of other companies who are highly correlated with the subsidiary company receive similar effect, (3)the subsidiary company's shares become more volatile and (4)tend to respond more to good news than to bad news.

The results of this paper suggest that using TOPIX as the performance benchmark, which is the prevailing practice in evaluating pension fund managers and other institutional investors, may be causing distortion in share prices and volatilities of subsidiary companies. A new index which corrects for the double counting is worth a serious consideration.

Publicly Listed Parent/Subsidiary Pairs: Benchmarking to TOPIX and Market Distortion

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May 9, 2000

In the Japanese stock market, most institutional investors are evaluated by how they overperform the market index, TOPIX. This is justified by the famous result of the capital asset pricing model, "Market portfolio is efficient." Without any private information, investing in the market portfolio makes the best result. Thus, we expect that investors with additional information will overperform the market portfolio. From this aspect, we can use the performance relative to the market portfolio as the evaluating criterion.

On the other hand, there are several subsidiary companies which have their stocks listed on the Japanese stock market, such as NTT Docomo, NTT Data and seven eleven. Some of these companies have quite large market values.

Majority of these subsidiary companies' outstanding stocks are owned by their parent companies. Moreover, the parent companies do not liquidate their stock holdings of the subsidiary company regardless of price. Thus the real supply of the subsidiary stocks is much less than the total number of outstanding shares. In other words, majority of the subsidiary company's shares are owned by nonspeculative shareholders.

The market portfolio is determined, in principle, by the total number of outstanding shares. With these parent-subsidiary relations, however, it is argued that the market portfolio should be modified. In fact, the market portfolio of other countries are somewhat modified to settle this problem. However, in the Japanese stock market, TOPIX, the most major market portfolio is not modified. Thus it is no longer clear if the claim of the capital asset pricing model holds.

In this paper, we will analyze how the stock market is affected by this parent-subsidiary relation and by the use of the market portfolio as a performance benchmark. We have two questions:

(1) Is the market portfolio efficient?

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(2) How are the stock prices affected by the stock-sharing relation and using the market portfolio as a performance benchmark?

For analyzing these questions, we will use a noisy rational expectations model. In this model, investors have their own information. Their private information is aggregated through the market price. Thus the market price plays the role of information aggregator. With noisy supply, however, the information is not fully revealed in the price.

We will introduce two types of investors into this model: the institutional investors who have private information and use the market portfolio as a performance benchmark, and private investors who have no private information and do not use the market portfolio. We will solve the rational expectations equilibrium under these assumptions, and analyze the effect of these assumptions.

We showed that the market portfolio is not the efficient portfolio. Moreover, we analyzed what the efficient portfolio should be in this situation. We also find that using such inefficient benchmark makes the excess demand for the subsidiary stock and the excess demand affects the price of the subsidiary stock. The price is pushed up, its volatility increase, and its dynamics has some interesting characteristics. Other stocks' prices are also changed.

Our model is similar to the Gennote and Leland(1990)'s model. They also analyzed the effect of non-investment-purposed supply on the stock price. We use their results and extend their model. We show that the price function may become nonlinear with constant excess demand.

This paper is organized as follows. In the following section, we will construct the model. In the section 2, we will answer the first question, and in the section 3 and 4 we will answer the second question. In the section 3, we will analyze the question without considering the effect of short selling constraint and in the section 4 we analyzed the effect of short selling constraint.

1 model

We use a two period noisy rational expectations model. Each asset is traded at date 0 and pays its liquidation value at date 1. Investors have their own information about the liquidation values and trade the stocks using both the private information and the market price.

1.1 Market

There are N companies having their stocks listed on the stock market. Company 1 is the subsidiary company of company 2. Company 2 has θ of the company 1's outstanding shares. No other two companies have such stock-sharing relation. The number of company i's outstanding stocks per capita is ω'_i. The equilibrium price of the stock i is p_i. The "pure" liquidation value, the liquidation value with no stock-sharing relation, of company i per stock at date 1 is δ'_i. Let $\omega' = (\omega'_1, \dots, \omega'_N), \delta' = (\delta'_1, \dots, \delta'_N)$ and $p = (p_1, \dots, p_N)$. δ' is normally distributed with mean vector $\bar{\delta}'$ and covariance matrix Σ'

We assume that the parent company does not buy the holding shares of its subsidiary company regardless of price. Thus the supply of company 1's stock is equal to the number of its outstanding shares minus the number of its shares owned by company 2. Let the supply of company i's stock per capita be ω_i. Then we have $\omega = (\omega_1, \dots, \omega_N) = ((1-\theta)\omega'_1, \omega'_2, \dots, \omega'_N)$.

Similarly, the liquidation value of parent company equals to its "pure" liquidation value plus the liquidation value of the holding shares of subsidiary company. Thus company 2's liquidation value per stock is $\delta_2 = \delta'_2 + \theta \frac{\omega'_1}{\omega'_2} \delta'_1$. The liquidation values of the companies per stock, denoted δ, are given by $(\delta_1, \dots, \delta_N) = (\delta'_1, \delta'_2 + \theta \frac{\omega'_1}{\omega'_2} \delta'_1, \delta'_3, \dots, \delta_N)$. Since the transformation is linear, resulting δ is also normally distributed. We define the mean vector and the covariance matrix of δ as $\bar{\delta}$ and Σ, respectively.

There is also a riskless bond in the market. For simplicity we assume that the bond is of perfectly elastic supply and the interest rate is zero.

1.2 Investors

We introduce two types of investors, the institutional investors (denoted I) and the private investors (denoted U). The institutional investors are different from the private investors in two aspects. First, they have personal information about the liquidation value of the stock. Second, they do not maximize the normal utility function. They are the agent of other investors and their revenues at date 1 is consumed by the principal investors. The institutional investors themselves are evaluated and rewarded on the basis of their relative performance to the market portfolio. Thus they maximize the "utility" for the excess return over the market portfolio. On the other hand, the private investors have no personal information and maximize their utility function. The mathematical setting is provided as follows.

Institutional investors Institutional investor j observes private information $s^j = \delta + \varepsilon^j$ and the market prices of stocks p. We assume that ε^j is normaly distributed with mean 0 and covariance matrix Σ_ε. It is identically and independently distributed across investors, and independent of other random variables.

The expected utility function of the institutional investors j is provided as

$$E[-\exp\{-(d_I^j - d_M)\} |\ s^j, p], \tag{1}$$

where d_M is the payoff of the market portfolio and d_I^j is the payoff of investor j's portfolio. The price of investor j's portfolio should be equal to that of the market portfolio.

Let the stock portfolio and the bond holdings of institutional investor j be w_I^j and b_I^j, respectively. Then we can write his or her maximization problem as

$$\begin{aligned} \max_{b_I^j, w_I^j} E[-\exp\{-(w_I^j - \omega')\cdot\delta - b_I^j\}|\ s^j, p] \\ s.t.\ w_I^j \cdot p + b_I^j = \omega' \cdot p. \end{aligned} \tag{2}$$

Private investors The private investors have no private information. They only observe the market prices of the stocks. Their expected utility function is privided as

$$E[-\exp\{-d_U\}|\ p], \tag{3}$$

where d_U is the payoff of their portfolio. Their maximization problem can be written as

$$\begin{aligned} \max_{b_U, w_U} E[-\exp(-w_U\cdot\delta - b_U)|\ p] \\ s.t.\ w_U \cdot p + b_U = M_U \end{aligned} \tag{4}$$

where M_U is the private investors' initial endowment.

The wait of the institutional investors, the number of the institutional investors divided by the total number of investors, is α. The wait of the private investors is $1-\alpha$.

1.3 Demand and supply

Demand The demand of stock per capita is given by

$$\alpha W_I + (1-\alpha) W_U, \tag{5}$$

where W_I and W_U are

$$\begin{aligned} W_I &= \sum_j \frac{w_I^j}{j} \\ W_U &= \sum_k \frac{w_U}{k} = w_U. \end{aligned} \tag{6}$$

Supply The supply of stocks ,ω , is modified by the noisy supply x. Thus the net supply of the stocks is given by

$$\omega + x \tag{7}$$

The noisy supply x is normally distributed with mean 0 and covariance matrix Σ_x and independent of other random variables. Neither the institutional investors nor the private investors can observe x.

2 The efficient portfolio

In this section we will answer the first question: Is the market portfolio efficient? To address this question, we assume that there are no institutional investors and no noisy supply,

$$\begin{aligned} &\alpha = 0 \\ &\Sigma_x = O. \end{aligned} \tag{8}$$

There is no private information in the market and the net supply of the stocks is observable. All the investors in the market maximize their exponential utility. Under these assumptions, we have the same model as the original capital asset pricing model, except that there is the stock-sharing relation. Using the famous result of the capital asset pricing model, we have the following theorem.

Theorem 1 *The efficient stock portfolio is not the market portfolio ω' but the supply of the stocks ω. The equilibrium price function can be written as*

$$p = \bar{\delta} - \sigma_M \tag{9}$$

where

$$\begin{aligned} \sigma_M &= (Cov[\delta_1, \delta_M], \ldots, Cov[\delta_N, \delta_M]) \\ \delta_M &= \delta \cdot \omega \end{aligned}$$

Proof The proof of the capital asset pricing model is given in many literatures. We simply have to add the assumption that the supply of the stocks is not equal to the market portfolio.

q.e.d.

σ_M is the vector of the covariance between the payoff of the efficient portfolio and that of each stock. The price function takes the familiar form in the capital asset pricing model. The only difference between this model and the original capital asset pricing model is that the market portfolio is not the efficient portfolio.

The institutional investors use the inefficient portfolio as the performance benchmark. This implies that their behavior is also irrational in certain aspects. In the subsequent section, we will analyze in what aspects their behavior is irrational and how the irrationality affects the stock prices.

3 The market equilibrium allowing short selling

In the following analysis, we assume

$$\begin{array}{l} \alpha \in (0,1) \\ \Sigma_x \neq O, \end{array} \tag{10}$$

that is, the institutional investors and the noisy supply exists. There is private information in the market and the net supply of the stocks is not observable. There are also two classes of investors.

Additionally, in this section we assume that investors can short their position arbitrarily. This assumption is not considered explicitly in most asset pricing models. However, as discussed in the next section, this assumption plays important roles in our model

3.1 The optimal behavior of investors

We will assume that investors believe the equilibrium prices of the stocks p is expressed as a function of state variable π, such as

$$p = f(\pi), \tag{11}$$

and the inverse function f^{-1} is well-defined. The state variable π is a linear combination of the payoff of the stocks δ and the noisy supply x, such as $\pi = \delta + Ax$, where A is a constant $N \times N$ matrix. This belief is confirmed in equilibrium.

The private investors The private investors observe the state variable π through the market price p and form their conditional expectations of the payoff of the stocks at date 1. The conditional distribution is the normal distribution with mean $\hat{\delta}_U$ and covariance matrix Σ_U, where

$$\begin{aligned} \hat{\delta_U} &= \bar{\delta} + \Sigma_U \Sigma_p^{-1}(\pi - \bar{\delta}) \\ \Sigma_U &= \left\{ \Sigma^{-1} + \Sigma_p^{-1} \right\}^{-1}. \\ \Sigma_p &= A \Sigma_x A^t \end{aligned} \tag{12}$$

The detail of the derivation of the conditional distribution is summarized in the Appendix.

Using this conditional expectation we can write the optimal portfolio of the private investors w_U as

$$\begin{aligned} w_U &= \Sigma_U^{-1}(\hat{\delta}_U - p) \\ &= \Sigma^{-1}(\bar{\delta} - p) + \Sigma_p^{-1}(\pi - p) \end{aligned} \tag{13}$$

3.2 The institutional investors

The institutional investor j uses the signal from the market price π and the private information s^j to form his or her conditional expectation. The conditional distribution is the normal distribution with mean $\hat{\delta}_I^j$ and covariance matrix Σ_I, where

$$\begin{aligned} \hat{\delta}_I^j &= \bar{\delta} + \Sigma_I \Sigma_p^{-1}(\pi - \bar{\delta}) \\ \Sigma_I &= \left\{\Sigma^{-1} + \Sigma_p^{-1} + \Sigma_\varepsilon^{-1}\right\}^{-1} \end{aligned} \tag{14}$$

Using this conditional expectation, we can write the optimal portfolio of the institutional investor j as

$$w_I^j = \Sigma_I^{-1}(\hat{\delta}_I^j - p) + \omega' \tag{15}$$

The first term of the right-hand side of above equation is the optimal portfolio of institutional investor j without using the market portfolio as the benchmark. The second term is the market portfolio. Thus the optimal behavior of the institutional investors is, "to buy the market portfolio and add on the unconditional optimal portfolio".

The portfolio w_I^j can be written as

$$\begin{aligned} w_I^j &= \Sigma^{-1}(\bar{\delta} - p) + \Sigma_p^{-1}(\pi - p) + \Sigma_\varepsilon^{-1}(s^j - p) + \omega' \\ &= w_U + \Sigma_\varepsilon^{-1}(s^j - p) + \omega'. \end{aligned} \tag{16}$$

The optimal portfolio of the institutional investor j is equal to the optimal portfolio of the private investors plus the modification based on his or her private information plus the market portfolio.

As the number of the institutional investors increases, the average of their optimal portfolios W_I converges to

$$W_I = w_U + \Sigma_\varepsilon^{-1}(\delta - p) + \omega'. \tag{17}$$

3.3 The market equilibrium

Using above result, we have the following theorem. The proof is provided in the Appendix.

Theorem 2 *There exists a rational expectations equilibrium of the form*

$$\begin{aligned} p = &\left\{\alpha\Sigma_I^{-1} + (1-\alpha)\Sigma_U^{-1}\right\}^{-1}\left\{\Sigma_p^{-1} + \alpha\Sigma_\varepsilon^{-1}\right\}\pi \\ &+ \left\{\alpha\Sigma_I^{-1} + (1-\alpha)\Sigma_U^{-1}\right\}^{-1}\Sigma^{-1}\bar{\delta} \\ &- \left\{\alpha\Sigma_I^{-1} + (1-\alpha)\Sigma_U^{-1}\right\}^{-1}(\omega - \alpha\omega') \end{aligned} \tag{18}$$

where $\pi = \delta + \frac{1}{\alpha}\Sigma_\varepsilon x$, $\Sigma_p = Var[\frac{1}{\alpha}\Sigma_\varepsilon x]$

The equilibrium price function is a linear combination of the state variable π, the unconditional expectation of payoff $\bar{\delta}$, the efficient portfolio ω and the market portfolio ω'. The nature of this function is not clear, however.

To make the result simple and clear, we assume that that all the signals have the same covariance structure as that of the unconditional covariance matrix of δ

$$\begin{aligned} \Sigma_p &= \frac{1}{h_p}\Sigma \\ \Sigma_\varepsilon &= \frac{1}{h_\varepsilon}\Sigma \end{aligned} \tag{19}$$

where h_p and h_ε are constant.

Under these assumptions, the covariance structures of the conditional covariance matrices are also the same as that of the unconditional one.

$$\begin{aligned} \Sigma_I &= \frac{1}{h_I}\Sigma \\ \Sigma_U &= \frac{1}{h_U}\Sigma \end{aligned} \tag{20}$$

where $h_I = 1 + h_p + h_\varepsilon, h_U = 1 + h_p$.

h_I and h_U can be thought as the precision of the conditional expectations relative to the unconditional one. The new information only alters the precision of their expectation.

In this case, we have the following proposition.

Proposition 3 *The price function can be written as*

$$p = \left(1 - \frac{1}{h_A}\right)\pi + \frac{1}{h_A}\bar{\delta} - \frac{1-\alpha}{h_A}\sigma_M + \frac{\alpha\theta\omega_1'}{h_A}\sigma_1 \tag{21}$$

where

$$\begin{aligned} &h_A = \alpha h_I + (1-\alpha)h_U \\ &\sigma_1 = (Var[\delta_1], Cov[\delta_2, \delta_1], \ldots, Cov[\delta_N, \delta_1]) \end{aligned}$$

σ_1 is the vector of the covariance between the payoff of the subsidiary company's stock and that of each stock. The price function is written as a linear conbination of the state variable π, the unconditional expectation $\bar{\delta}$, σ_M and σ_1.

The former three terms of the price function can be thought as an extensive form of the price function of the capital asset pricing model. They resemble to the price function in equation (9). The appearance of the first term and the changes in coefficients of these terms merely reflect the effects of introducing the private information and the noisy signal. Moreover, the three terms are not affected by the change of the parameter θ, which represents the degree of stock-sharing relation.

It is the last term that reflects the effect of introducing the stock-sharing relation. We can now characterize the effect. Company i's stock price is modified in proportion to the covariance between the company's stock's payoff and the subsidiary company's. The subsidiary company's stock price always rises and the other stock's price changes according to its similarity to the subsidiary stock.

As showed in equation (16), the institutional investors tend to have the similar portfolio to the market portfolio. When the market portfolio equals to the supply of stocks, this yields no problem. If the stock-sharing relation exists, however, there arises an excess demand on the subsidiary stock, since the market portfolio contains excessive amounts of the subsidiary stock. This excess demand pushes up the subsidiary stock's price. When the price rises, investors reduce their position of the subsidiary stock and buy substitutive stocks. Thus the rise in the subsidiary stock's price in turn generates secondary demands in the subsidiary-stock-like stocks and pushes up these stocks' prices. The σ_1 term expresses these effects.

On the other hand, the parent-subsidiary relation does not affect the dynamics of stock prices. As shown in equation (15), the excess demand for the subsidiary stock is constant. Thus, however the state variable changes, the excess demand does not change and its effect on the prices does not change, either. This result is consistent with the result of Gennote and Leland(1990).

Gennote and Leland(1990) analyzed the effect of the hedging supply on the stock price dynamics. The hedging supply in their model is determined only by the stock price and not affected by the state variable directly. They showed that the hedging supply may make the equilibrium price function nonlinear and discontinuous. However, they also showed that, if the hedging supply is constant or linear function of the stock price, the price function remains linear. Our model is similar to their model. The excess demand in our model is not affected by the state variable, either. Moreover, it is constant and not affected by the prices. Thus using their result, we expect that the price function be linear and the slope of the function unchanged.

However, there is one critical difference between their model and ours. The hedging supply in their model is not so large in amount, while the excess demand in our model is quite large. We will analyze this effect in the next section.

4 The market equilibrium without short selling

Above discussion is based on that investors can short their position arbitrarily. In usual model, investors rarely short their position in equilibrium. Thus the assumption about short selling is not explicitly argued in most case.

However, the excess demand in our model is so large that not a few investors short their position of the subsidiary stock. Substituting equation

(17) in the market-clear-condition equation, we have

$$w_U = (1-\alpha)\omega - \alpha\theta\omega_1' \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} - \alpha\Sigma_\varepsilon(\pi - p) \tag{22}$$

Since $\theta > 0.5$ and the α is sufficiently large in the actual market, the demand for the subsidiary stock may become negative. The assumption about short selling is, thus, critical in our model. In this section, we will assume that the private investors can not short their position of the subsidiary stock.

We additionally assume that institutional investors can short every stock arbitrarily and the private investors can short other stocks arbitrarily. This assumption serves only for simplicity. When the state variable π takes an extreme value, other stocks may be shorted. Moreover, institutional investors with highly noisy private information may short other stocks. Amounts of these short selling are, however, quite small as compared with the amount of the private investors' short selling of the subsidiary stock.

4.1 The optimal behavior of investors

Like the preceding section, we will assume that all the investors believe that the inverse function of the price function $\pi = f^{-1}(p)$ is well-defined. The optimal portfolios of the institutional investors remain the same as derived in the preceding section. The private investors face slightly modified maximization problem as

$$\begin{aligned} &\max_{b_U, w_U} E[-\exp(-w_U \cdot \delta - b_U)|\ p] \\ &\qquad s.t.\ w_U \cdot p + b_U = M_U \quad , w_{U1} > 0 \end{aligned} \tag{23}$$

The solution of this problem is provided as

$$w_U = \begin{cases} \Sigma_U^{-1}(\hat{\delta}_U - p) & if \quad w_{U1} > 0 \\ \begin{pmatrix} 0 & o^t \\ o & \grave{\Sigma}_U^{-1} \end{pmatrix} (\hat{\delta}_U - p) & if \quad w_{U1} = 0 \end{cases} \tag{24}$$

where $\grave{\Sigma}_U$ is the submatrix of Σ_U

$$\grave{\Sigma}_U = \begin{pmatrix} \Sigma_{U(2,2)} & \Sigma_{U(2,3)} & \cdots \\ \Sigma_{U(3,2)} & \ddots & \\ \vdots & & \Sigma_{U(N,N)} \end{pmatrix}$$

and o is the $n-1$ dimensional zero vector.

With short selling constraint, if the private investors' optimal position of the subsidiary stock is positive, the optimal portfolio is the same portfolio as derived in the preceding section. If it goes nonpositive, their optimal portfolio becomes the optimal portfolio with N-1 stocks, the optimal portfolio in the situation that the subsidiary stock does not exist in the market.

4.2 The market equilibrium

Again, we will assume (19) for simplicity. Additionally, we will assume that the $(1,1)$ element of Σ^{-1} is positive. [1] Then we have following theorem. The proof is provided in the Appendix.

Theorem 4 *There exists a rational expectations equilibrium. The price function can be written as the piece wise linear function of the form*

$$p = \begin{cases} g(\pi) & if \quad \phi \cdot \pi < k \\ g(\pi) + \frac{1-\alpha}{\alpha h_I h_A} \begin{pmatrix} 1 & -\grave{\sigma}_1^t \grave{\Sigma}^{-1} \\ o & O \end{pmatrix} \left[\alpha h_\varepsilon (\pi - \bar{\delta}) + h_U \left\{ (1-\alpha)\sigma_M - \alpha\theta\omega_1' \sigma_1 \right\} \right] \\ \qquad\qquad\qquad\qquad\qquad\qquad if \quad \phi \cdot \pi \geq k \end{cases} \tag{25}$$

where

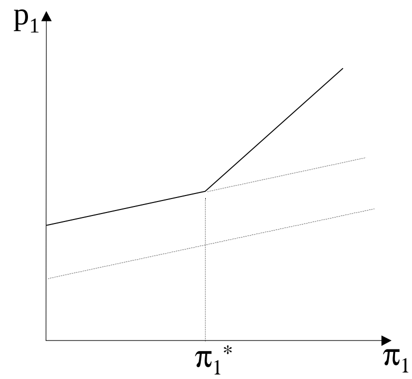
$$\begin{aligned} g(\pi) &= \left(1 - \frac{1}{h_A}\right)\pi + \frac{1}{h_A}\bar{\delta} - \frac{1-\alpha}{h_A}\sigma_M + \frac{\alpha\theta\omega_1'}{h_A}\sigma_1 \\ \grave{\Sigma} &= \begin{pmatrix} \Sigma_{(2,2)} & \Sigma_{(2,3)} & \cdots \\ \Sigma_{(3,2)} & \ddots & \\ \vdots & & \Sigma_{(N,N)} \end{pmatrix} \\ \grave{\sigma}_1 &= \begin{pmatrix} \Sigma_{(2,1)} \\ \vdots \\ \Sigma_{(N,1)} \end{pmatrix} = \begin{pmatrix} Cov[\delta_2, \delta_1] \\ \vdots \\ Cov[\delta_N, \delta_1] \end{pmatrix} \end{aligned}$$

ϕ is a constant N dimensional vector and k is a constant scalar.

$g(\pi)$ is the equilibrium price function without short selling constraint. With short selling constraint, the price function changes. However the change occurs only in the subsidiary stock's part, and all the other stocks' part remains unchanged.

The price function for the subsidiary stock is written as a piecewise linear function of N variables: $\pi_1, \cdots, \pi_N$. Let all the state variables except the subsidiary stock's be given and fixed. Then the price function $p_1 = f_1(\pi_1)$ is of following shape

[1] Introducing this assumption is equivalent to assuming that, when the state variable of the subsidiary stock π_1 rises, the wait of the subsidiary stock in the optimal portfolio should rise, under the condition that all the other parameters remain unchanged. Thus, this assumption is usually satisfied in the actual market.



The kinked bold line is the price function $f_1(\pi_1)$. The below broken line is the price function without subsidiary company's listing. The middle broken line is the price function without short selling constraint. listing subsidiary stock shifts the price curve upwards and short selling constraint kinks the price function, also, upwards.

The upward bending of the price function brings forth two characteristics in the dynamics of the subsidiary stock. First, the volatility of the subsidiary stock increases, and, second, the subsidiary stock's price becomes more sensitive to good news. Since the sensitivity to bad news does not change, the subsidiary stock's price comes to have a tendency to respond to good news more severely than to bad news.

Now we can answer the second question: How are the stock prices affected by the parent-subsidiary relation? There are three effects.

(1) The subsidiary stock's price increase and other stock's price changes according to its similarity to the subsidiary stock.

(2) The volatility of the subsidiary stock increases.

(3) The subsidiary stock's price becomes likely to be affected more sensitively by good news than by bad news.

Why the price function kinks upward? We will give an intuitive explanation for this question in the remaining part of this section. The characteristic of the private investors is critical for the answer.

When δ takes higher value, the institutional investors partially observes it through the private information and increase their position. This pushes up p. When p takes higher value, the private investors know that δ may be high. However, since the net supply is fixed, the equilibrium price is determined at too high level for the private investors to increase their position. In fact, the price is so high that the private investors rather decrease their position. The private investors decrease the position when δ increases, and increase the position when δ decreases. Similarly, the private investors increase their position if x increases, and decrease the position if x decreases.

Thus the the private investors decrease their position when π increase, or when the price increase, and decrease the position when π decrease. This means that the private investor's deal pushes the price to the opposite direction. In other words, the private investors have the role of reducing the volatility of stock prices.

This is the source of the upward bending. When π_1 increases, the private investors decrease their position of the subsidiary stock. When π_1 takes sufficiently high value, the private investors' position becomes zero and they can no longer decrease their position. In this situation, the private investors' volatility reducing function no longer works and the price of the subsidiary stock p_1 becomes more sensitive.

In the Gennote and Leland(1990)'s model, the hedging supply should be the decreasing function of the price so that the price function may be nonlinear and discontinuous. However, we showed that even constant supply, or demand, can make the price function nonlinear, provided that the supply is sufficiently large. This difference is caused by the difference in the logic used in each model. In their model, the non investment-purposed and price-related supply produces additional demand of investors since their additional demand produces additional supply. This makes the price function nonlinear. In our model the excess demand itself does not affect investors' demand. It simply decrease net supply. However, since this decrease is quite large in amount, this can make the price function nonlinear.

5 Conclusion

We analyzed how the stock market is affected by listing a subsidiary company's stock and using the market portfolio as a performance benchmark. We showed that in such a situation, (i)the market portfolio is not the efficient portfolio, (ii)the subsidiary stock's price takes the higher value and other stock' price is modified according to its similarity to the subsidiary stock, and (iii)the volatility of the subsidiary stock' price increases and the subsidiary stock's price has a tendency to respond to good news more sensitively than to bad news.

We introduced several assumptions for this analysis. Among them, following four assumptions are critical to the result. (a)There is a subsidiary company, (b)there are two types of investors, (c)there is private information, and (d)investors can not short their position. The result (i) depends only on the assumption (a), the result (ii) depends on the assumption (a) and (c) and the result (iii) depends on all the assumptions.

We used a similar model to the Gennotte and Lealand(1990)'s model. However, we developed a thoroughly different logic from their model. We showed that even a constant demand can make the price function nonlinear.

The logic developed in our model may be used in analyzing other problems. For example, the price dynamics of low-liquidity stocks may be analyzed using similar logic. Moreover, in the Japanese stock market, stock sharing is broadly practiced and not a monopoly of parent and subsidiary companies. Our model can be applied to analyze the effect of such general stock-sharing relations.

Appendix

A Conditional distribution of investors

Institutional investor j receives following signals

$$\begin{pmatrix} \pi \\ s^j \end{pmatrix} = \begin{pmatrix} I_N \\ I_N \end{pmatrix} \delta + \begin{pmatrix} A & O \\ O & I_N \end{pmatrix} \begin{pmatrix} x \\ \varepsilon^j \end{pmatrix} \tag{26}$$

where I_N is N dimensional identity matrix. In this situation, the conditional distribution on these signals is given as [2]

$$\delta \sim N_N(\hat{\delta}_I^j, \Sigma_I) \tag{27}$$

where

$$\begin{aligned}
\hat{\delta}_I^j &= \delta + \Sigma(I_N\ I_N)F^{-1} \begin{pmatrix} \pi - \bar{\delta} \\ s^j - \bar{\delta} \end{pmatrix} \\
\Sigma_I &= \Sigma - \Sigma(I_N\ I_N)F^{-1} \begin{pmatrix} I_N \\ I_N \end{pmatrix} \Sigma \\
F &= \begin{pmatrix} I_N \\ I_N \end{pmatrix} \Sigma(I_N\ I_N) + \begin{pmatrix} A & O \\ O & I_N \end{pmatrix} \begin{pmatrix} \Sigma_x & O \\ O & \Sigma_\varepsilon \end{pmatrix} \begin{pmatrix} A & O \\ O & I_N \end{pmatrix}^t
\end{aligned}$$

After lengthy but straightforward manipulation of these equations, we have the equation (14). The private investors' conditional distribution can be derived in a similar way.

[2] See, for example, Harvey(1985) Chapter 4.

B Proof of Theorem (2)

Using the equation (5) and (7), we hae the market clear condition

$$\alpha W_I + (1-\alpha)W_U = \omega + x \tag{28}$$

Substituting the equation (13) and (17) in above equation, we have

$$\Sigma^{-1}(\bar{\delta} - p) + \Sigma_p^{-1}(\pi - p) + \alpha\Sigma_\varepsilon^{-1}(\delta - p) + \alpha\omega' = w + x \tag{29}$$

Assuming that $\pi = \delta - \frac{1}{\alpha}\Sigma_\varepsilon x$, we have

$$\Sigma^{-1}(\bar{\delta} - p) + \Sigma_p^{-1}(\pi - p) + \alpha\Sigma_\varepsilon^{-1}(\pi - p) + \alpha\omega' = w \tag{30}$$

This equation is equivalent to the equation (18). Since the equation (18) is the linear function of π, the inverse function is well-defined.

C Proof of Theorem (4)

The equation (25) can be derived in a similar way used in the above proof. Substituting the equation (25) into the (24), we have

$$w_U = \Sigma^{-1}\left\{-\left(1 - \frac{h_U}{h_A}\right)\pi + \left(1 - \frac{h_U}{h_A}\right)\bar{\delta} + \frac{h_U}{h_A}(1-\alpha)\sigma_M - \frac{h_U}{h_A}\alpha\theta\omega_1'\sigma_1\right\} \tag{31}$$

provided that $w_{U1} \geq 0$. Using this equation, we can show that, when $w_{U1} = 0$, following equation holds.

$$\begin{aligned}\frac{1}{h_A(Var(\delta_1) - \grave{\sigma}_1^t\grave{\Sigma}^{-1}\grave{\sigma}_1)}&(1, -\grave{\sigma}_1^t\grave{\Sigma}^{-1})\left[-\alpha h_\varepsilon(\pi - \bar{\delta})\right.\\ &\left.+h_U\left\{(1-\alpha)\sigma_M - \alpha\theta\omega_1'\sigma_1\right\}\right] = 0\end{aligned} \tag{32}$$

On the other hand, two equations in the equation (25) meet when following equation holds.

$$\frac{\beta}{\alpha h_I h_A}(1, -\grave{\sigma}_1^t\grave{\Sigma}^{-1})\left[-\alpha h_\varepsilon(\pi - \bar{\delta}) + h_U\left\{(1-\alpha)\sigma_M - \alpha\theta\omega_1'\sigma_1\right\}\right] = 0 \tag{33}$$

This equation is equivalent to the equation (32). Thus the price function is continuous.

Moreover, transforming the equation (32) we can write ϕ as

$$\phi = l \times (1, -\grave{\sigma}_1^t\grave{\Sigma}^{-1})^t \tag{34}$$

where l is a positive constant.

Finally, using the equation (25) and (34), we can show that any $p_1(\pi)$ which satisfies $\phi{\cdot}\pi \geq k$ is larger than any $p_1(\pi)$ which satisfies $\phi{\cdot}\pi < k$. Since the other part of the price function is linear, this means that for arbitrary p, there exists unique π. Thus the inverse function is well-defined.

References

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[3] Duffie,D., Dynamic Asset Pricing Theory (2nd ed.), Princeton University Press, 1996

[4] Harvey, A.C., *Time Series Models*, Philip Allan, 1981