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On Modelling Negotiations within a Dynamic Multi-objective Programming Framework:
Analysis of Risk Measurement with an Application to Large BOT

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Abstract: The dynamic and multi-objective programming is used here to establish a risk measurement model. We develop an iterative algorithm and the convergence conditions for the model solution. The results obtained from the model developed here show that the sum of the interactive utility value (IUV) could determine whether or not the interactive relationship is characterized by independence among negotiators. In addition, the numerical example shows that this risk measurement model of the negotiation group can reflect risk assessment by the negotiation group for certain events and can analyze interaction characteristics among negotiators. We show the feasibility and applicability of the model and the exact solution algorithm, and their policy relevance for analyzing BOT projects.

Key Words: *BOT;Dynamic Models; Dynamic Programming; Interactive utility; Multi-objective programming; Risk measurement*

1. Introduction

The main purpose of this paper is to develop a risk measurement model that can be applied to analyze large scale infrastructure projects, among others. BOT (Build, Operate and Transfer)[1] is a process where the private sector is granted a concession to plan, design, construct, operate and maintain a project. This technique is useful because, generally speaking, there are high financial risks for major infrastructure projects (Tiong, 1995). Thus, private sector enterprises are invited by the government to participate in a BOT project in order to share the potential risks that occur in the project development

[1] Usually BOT is described as a kind of privatization process; but theoretically the ownership pattern is less important for efficiency analysis than the engineering and economic aspects. However, if the characteristics of the owners to whom the transfer part of BOT applies vary systematically depending on whether they are private or public entities, then the negotiation process will clearly be affected in systematic ways. In this paper, we focus on transfer to private ownership.

process.

In the BOT concept proposed by Walker and Smith (1996), after completion of the tendering stage for the major infrastructure of a BOT project, the enterprise which receives first priority for contract negotiation will form a BOT Concession Company, which is a team enterprise. The Concession Company will then go through concession contract negotiation with the government in order to discuss the risk factors, their explicit description and possible ways of sharing them. The results are then put into relevant documents and the contract. Therefore, the purpose of risk sharing is achieved through contract negotiation between the government and Concession Company. The concession contract negotiation is accomplished through BOT negotiation team and the government negotiation team, whereas the BOT concession contract negotiation process includes public and private participation and repeated discussion (Tiong, 1997).

Before negotiation, the negotiation groups from both the Concession Company and the government department conduct risk measurement internally regarding those uncertain factors existing in the contract. Such internal risk measurements naturally reflect, among other things, their experience and the information collected(the information set, to use the terminology of Game Theory). This is done in order to better account for the different types of risks and determine which are primary and which are secondary risks. Generally speaking, after risks have been evaluated by the decision-makers, the negotiation group will discuss internally to determine the risk events. Such "discussion" within the negotiation group can be formalized via the idea of a utility interaction among negotiators.

Since the initial and by now, classical, proposal from Bernoulli, the utility function has been utilized widely in decision-maker's risk analysis. Among recent contributions, Bell (1995), shows that the maximum value of expected utility function can reveal the characteristics of high return and high risk. The Multi-attribute Utility (MAU) proposed by Keeney and Raiffa (1993) has been adopted for the study of decision-making behavior (Bose, *et al.*, 1997), and for risk analysis of engineering projects (William and Crandall, 1982). Although the MAU model has additive utility and multiplicative utility, it assumes that the decision-maker's preference map[2] is independent and cannot be used to explain interactive behavior during the negotiation process. Feng and Kang (1999, 2000), Feng, Kang, and Tzeng (2000) adopted the MAU theory to study risk measurement for BOT

[2] For a theorem giving the exact conditions under which the preference map can be represented by a real-valued utility function, see Debreu(1959).

concession contracts.[3] Those studies include risk preferences of the negotiators and determine the primary and secondary risks associated with a BOT project. However, those studies do not investigate risk measurement for utility interaction between negotiators. The purpose of this study is to investigate utility interaction among the negotiators, and to develop a utility dependence model that can be applied for risk measurement.

In the past, studies regarding utility interaction have included analysis of utility dependence (i.e, the Monte Carlo method), Team Theory (Marschak and Radner, 1972; McGuire and Radner 1986;Kim and Roush, 1987) and mathematical programming (Haimes, 1998; Orlovski, 1990). As to the simulation approach used in applied utility analysis, Carbone (1997) adopted the Monte Carlo method to develop rank-dependent expected utility theory, and to distinguish the decision-making behavior for various models, such as pair-wise choice utility, expected utility theory, prospective reference theory and weight utility theory. One difficulty here is that when utility is simulated by the Monte Carlo method, all the utility values, simulated parameters and probabilities have to be pre-set. However, when utility values of any one of the decision-makers interact with those of other decision-makers, it is not easy to determine a priori which values of parameter and probability distribution should be pre-set.

In the mathematical analysis of utility dependence, the question of whether or not the utility of event is related to probability requires further investigation (Belichrosdt and Quiggin, 1997; Daniels and Keller, 1990). The rank-dependent utility theory proposed by Belichrosdt and Quiggin (1997) assumes that probability is one of the endogenous variables of utility function, and investigates the relationships between utility and joint or marginal probability. If the probability of utility function is a joint one, then the event utility will not be obtained from the expected utility value. In addition, Quiggin (1991) relaxes the condition of independence for unrelated outcomes because he believes that a specific utility may have certain relationships with those outcomes that are not completely independent from one another. Thus the concept of probability weight and an associated linear transformation can be adopted. Applying a scale constant between 0~1, we can combine various utility outcomes and compute expected values to obtain the total utility and rank preferences. This is the basic idea behind a rank-dependent expected utility theory (RDEU).

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[3] In a different context, Gang and Khan(1989, 1990,1991,1993,1999), Khan and Hoshino(1992) and Khan(1994,1995a,b, 1996-97,1997, forthcoming a, b) use loss minimization for decisionmakers with bounded rationality.

The team theory proposed by Kim and Roush (1987)[4] adopted the concept of coordination to investigate the interaction issues between decision-maker and environmental factors. This was done by integrating the parallel team, chain team, coordination team, and search team to analyze the interactive behavior between the decision-maker and the factors of environment. Kim and Roush (1987) focus their studies on the impacts of environmental factors associated with the team utility, and they did not discuss the issue of interaction issue among decision-makers.

Recently in the area of risk measurement via mathematical programming, Haimes (1998) has proposed the risk measurement concept of multi-objective programming and a dynamic programming approach. On the other hand, Orlovski (1990) proposed the concept of fuzzy bi-level programming approach to analyze the two-person game problem. Reviewing these studies, we can see that mathematical programming is another approach for risk measurement that can also be applied in two-person game problems or coordination problems.

In this paper, we first discuss the background of our research in Section 2. Section 3 presents the assumptions for developing a dependent-utility model for individual negotiators and negotiation groups. In this section, we also develop dependent-utility models for individual negotiators and negotiation groups. This is followed by Section 4, where we develop the iterative algorithm and converging factors for the dependent utility negotiation group. In Section 5, we proposed a numerical example to examine the risk measurement for a BOT project. Finally, we draw some conclusions and present some thoughts for future research.

[4] As noted earlier mathematical economists Marschak and Radner (1972)are the original contributors here. But we follow the more recent exposition.

2. Background: Negotiations and Utility

In this section, for concreteness, we assume that a transportation infrastructure project will be implemented using BOT, and that the Concession Company and government each will carry out the contract negotiation process. Usually and without loss of generality, we can assume that the government negotiation team includes the members representing transportation, environmental agencies, and local officials. Meanwhile, the Concession Company negotiation team includes lawyers, financial consultants, the initiator, and the engineering experts. The principal negotiator from each team is in charge of the negotiation process. Naturally, if the negotiation fails, the concession contract will not be valid. The negotiation process aims to discuss possible uncertainties in the contract, define the individual rights and obligations of each party and, finally, write all agreements in a concessional format. Figure 1 presents a conceptual diagram of this process.

Before signing the concession contract, the government and BOT Concession Company undergo the so-called risk-sharing negotiation steps to determine which risk events will be included in the contract. In this negotiation process, if both parties cannot accept a specific risk event, then it will result in the topic of negotiation in the next meeting; and the negotiation group will conduct internal discussion regarding the subject risk items and re-assess the risk items. In addition, if a negotiator completes preliminary risk measurement for a specific event, the negotiation group should continue future discussion regarding the risk level of the risk events.

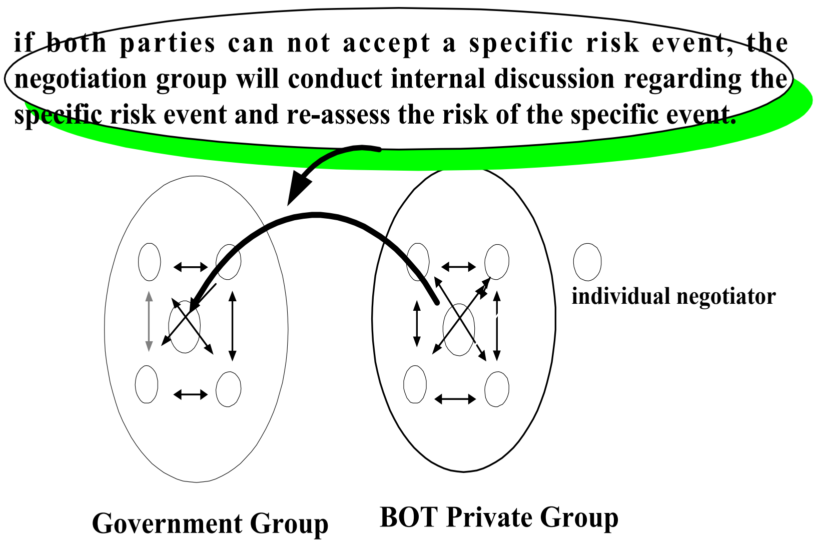

Figure 1: the conceptual map of interactive utility among negotiators

It should be emphasized that there is discussion within the negotiation groups. This implies that other participants' decision variables may affect the utility results of a specific negotiator. In addition, it also implies that the utility of the subject negotiator may affect the utility results of other participants. Thus, we face almost a classic type of externality problem. The factors that involve human interaction can be modeled by certain types of (non-)cooperative game theory. There are two different types of interaction that occur during decision-making: one is the mutually independent variety (concept of Fig.1); the other is characterized by a process of negotiation existing during the decision-making process (concept of Fig.2). The second type is clearly amenable to modeling according to cooperative game theory, and is the topic that is studied in this paper. We offer a rather strikingly simple way to model a complex process in what follows.

Generally speaking, whenever there is discussion among the negotiators, an utility interaction can be said to exist. The status of utility interaction may become stable after several rounds of discussion, and a domestic "consensus" of the negotiation group for a specific event can thus be obtained. The characteristics of the utility interaction among the negotiators include rules of binary interactive, feedback and expansion. If there is no utility interaction among the negotiators, then the utility is independent; otherwise the utility is dependent. The conceptual model for internal discussion within a negotiation group is shown in Figure 2.

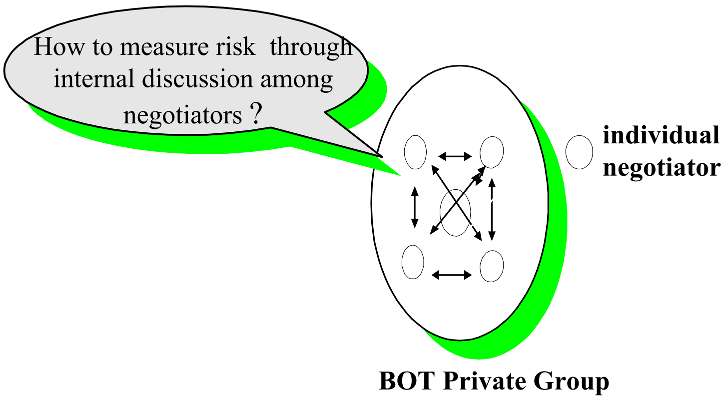

Figure 2 Conceptual map of risk measurement through internal discussion among negotiators

3. A Model

In this section we describe the assumptions and develop a dependent utility model in a setting for both individual negotiators and negotiation groups.

3.1 The assumptions

The assumptions for our model are as follows:

(1) Agency relationships exist between negotiators and the parties they represent.

However, we assume for simplicity that the agent's costs are independent of the negotiators' utility.

(2) The utility function of the negotiator is a continuous real valued function.

(3) The negotiator makes decisions rationally, i.e. s/he optimizes in a risky environment.

(4) The probability distribution of attribute-outcome occurrence is a Bernoulli experiment. Where the probability of occurrence is regarded as the probability of success.

Assumption (1) indicates that the negotiator is authorized by a specific organization. However, if the agency cost is not equal to zero, adverse-selection behavior might occur. Assumption (2) implies that the utility function satisfies the N-M (Von Neumann-Morgenstern) axioms. Assumption (3) satisfies the principle of maximizing utility while minimizing risk. Finally, Assumption (4) ensures that the negotiator assesses the attribute outcome, state and probability of the event based on previous experience or factual information in a consistent way according to the statistical decision theory.

3.2 Definition of risk-state

We assume that there are n uncertain states for a specific event, say s_1 , s_2 ,Λ , s_n . Let s_j indicate the j th state, where $j = 1,2,...,n$; and let p_j indicate the occurrence probability of the state j . Every state corresponds to an outcome of attribute x_j ; and every outcome of attribute x_j corresponds to a utility value $u_j(x_j)$. In addition, $p_j \times u_j(x_j)$ is the utility value for every state and the corresponding outcome of an attribute. $\overline{u}(x)$ is the average utility value for all of the states.

Based on the risk defined by Buhlmann (1996) and the risk preference concept defined by Keeney and Raiffa (1993), we define the risk-state as following: a risk state exists when for the decision-maker, the actual utility of a specific event under a certain state is less than the average utility of all the states, as shown in Eq. (1).

$$R_j \equiv u_j(x_j) < \overline{u}(x) \text{，} \forall j \qquad (1)$$

Where the direction of the inequality indicates that state j is a risk state; $u_j(x_j)$ is the utility value of the negotiator regarding state j for a specific event; and $0 \le u_j(x_j) \le 1$; $0 \le p_j \le 1$.

It is worth reemphasizing that eq. (1) tells us that, for a specific negotiator,and a certain event, if the utility value of the outcome of attribute x_j is less than the average utility value of all the outcomes of attribute, then the subject event is a risk event under state s_j. The relationships among state, attribute and probability of an event are shown in Table 1.

Table 1 Representation among event state, attribute outcome, and utility

	State (S)
	s_1 , s_2 ,..., s_j ,..., s_n
Outcome of attribute x	x_1 , x_2 ,..., x_j ,..., x_n
Probability p	p_1 , p_2 ,..., p_j ,..., p_n
Utility u	u_1 , u_2 ,..., u_j ,..., u_n

According to eq. (1), although $0 \le u_j(x_j) \le 1$, $0 \le p_j \le 1$, because the difference of measured utility value can be great, therefore the averaged utility value may be greater than 1. In this case it can not meet the requirement that the averaged utility value should be in the range of [0,1]. For simplified comparison, we utilize the transformation of utility proposed by Keeney and Raiffa, and normalize eq. (1). As shown below ,

$$u_j^*(x_j) = \frac{p_j \times u(x_j) - \min\limits_j \{p_j \times u(x_j)\}}{\max\limits_j \{p_j \times u(x_j)\} - \min\limits_j \{p_j \times u(x_j)\}}, \quad \max_j \{p_j \times u(x_j)\} \ne \min_j \{p_j \times u(x_j)\}, \forall j \quad (2)$$

where $u_j^*(x_j)$ is the normalized utility value.

Since $0 \le u_j(x_j) \le 1$ and $0 \le p_j \le 1$, the normalized utility value meets the constraint of lying between 0 and 1. When $\max\limits_j \{p_j \times u(x_j)\} = \min\limits_j \{p_j \times u(x_j)\}$, then $u_j^*(x_j) = 0$. $u_j^*(x_j) <$ $\overline{u}^*(x)$ means that the negotiator believes there is risk for a specific event under state s_j and outcome of attribute x_j. In other words, the outcome of attribute x_j for the specific event under state s_j belongs to a risk state. Notice the strict inequality characterizing the necessary and sufficient condition for the risk event.

3.3 Dependent utility model for negotiation group

In this section, we present the conception of utility linear transformation, and construct a dependent utility model for both an individual negotiator and a group negotiator.

3.3.1. Concept of utility linear transformation

Assume there are three negotiators in the negotiation group, and define $\alpha_{2,1}(x_j,t)$ as the interactive utility value (IUV), where $\alpha_{2,1}(x_j,t)$ means that during discussion $\#t$ for the outcome of attribute x_j of event f, the utility value of negotiator #2 affects the utility of negotiator #1. In addition, f is defined as an event or decision-making policy of the BOT private group. Similarly the others $\alpha_{1,3}(x_j,t)$, $\alpha_{3,1}(x_j,t)$, $\alpha_{2,3}(x_j,t)$, $\alpha_{3,2}(x_j,t)$, $\alpha_{1,3}(x_j,t)$, $\alpha_{3,1}(x_j,t)$, $\alpha_{2,1}(x_j,t)$, $\alpha_{1,2}(x_j,t)$, $\alpha_{2,3}(x_j,t)$, and $\alpha_{3,2}(x_j,t)$ are constants on the closed interval [0,1].

Assume that the utility function satisfies the N-M axiom. Based on the utility linear transformation concept of Fishburn (1990), we can state the following:

if the utility function satisfies the continuity, transitivity and weak independent axioms, *then the utility function can be a linear transformation*, i.e. $\exists\alpha \quad ,\alpha\in[0,1]$, and $u_1^f(t),u_2^f(t)\in U^f$, such that $U^f(t)=\alpha u_1^f(t)+(1-\alpha)u_2^f(t)$, the $U^f(t)$ still satisfies " ϕ " (binary preference relation).

It is now necessary to combine the concepts of utility linear transformation and utility normalization. We can thus carry on the utility linear transformation for the utility value of those three negotiators in our example. We have according to the notation introduced earlier,

$\alpha_{1,2}(x_j,t)$, $\alpha_{2,1}(x_j,t)$, $\alpha_{2,3}(x_j,t)$, $\alpha_{3,2}(x_j,t)$, $\alpha_{1,3}(x_j,t)$, and $\alpha_{3,1}(x_j,t)$.

The transformed utility model is shown in eqs. (3) to (5).

$$u_1^f(x_j,t+1)=(1-\alpha_{2,1}(x_j,t)-\alpha_{3,1}(x_j,t))u_1^f(x_j,t)+\alpha_{2,1}(x_j,t)u_2^f(x_j,t)+\alpha_{3,1}(x_j,t)u_3^f(x_j,t) \qquad (3)$$

$$u_2^f(x_j,t+1)=(1-\alpha_{1,2}(x_j,t)-\alpha_{3,2}(x_j,t))u_2^f(x_j,t)+\alpha_{1,2}(x_j,t)u_1^f(x_j,t)+\alpha_{3,2}(x_j,t)u_3^f(x_j,t) \qquad (4)$$

$$u_3^f(x_j,t+1)=(1-\alpha_{1,3}(x_j,t)-\alpha_{2,3}(x_j,t))u_3^f(x_j,t)+\alpha_{1,3}(x_j,t)u_1^f(x_j,t)+\alpha_{2,3}(x_j,t)u_2^f(x_j,t) \qquad (5)$$

where $u_1^f(x_j,t)$ is the utility value of negotiator #1 for attribute-outcome x_j of event f at discussion # t ; $u_2^f(x_j,t)$ is the utility value of negotiator #2 for attribute-outcome x_j of event f at discussion # t ; $u_3^f(x_j,t)$ is the utility value of negotiator #3 for attribute-outcome x_j of event f at discussion # t ; and

$$u_q^f(x_j,t)=\frac{p_j\times u(x_j,t)-\min_j\{p_j\times u(x_j,t)\}}{\max_j\{p_j\times u(x_j,t)\}-\min_j\{p_j\times u(x_j,t)\}},\ \max_j\{p_j\times u(x_j,t)\}\neq\min_j\{p_j\times u(x_j,t)\},\forall\, q=1,2,3;$$

where t is discussion number, $t=0,1,2,\Lambda\ ,T$.

As shown in eqs. (3) to (5), previous discussion of a specific negotiator may affect the utility of other negotiators through IUV. This is analogous to working with a time-series with memory. But the additional complication arises from intertemporal externalities. As for the other negotiators, they will affect that specific negotiator during current discussion through IUV so that there is indeed an interactive game with feedback features.

3.3.2. A Dependent utility model

Assume there are q negotiators in the negotiation group, $q=1,2,\Lambda\ ,Q$. The negotiator q has the utility of outcome of attribute x_j for event f during discussion # t and # $t+1$. $u_q^f(x_j,t+1)$ and $u_q^f(x_j,t)$, where $0\le u_q^f(x_j,t+1)\le 1$ and $0\le u_q^f(x_j,t)\le 1$. After linear transformation of utility functions of negotiator #1 and other negotiators, the resulting utility function is shown in eq. (6). The conceptual diagram for dependent utility among q negotiators negotiation group is shown in Figure 3.

$$u_1^f(x_j,t+1)=(1-\alpha_{2,1}(x_j,t)-\ldots-\alpha_{k,1}(x_j,t)-\alpha_{k+1,1}(x_j,t)-\ldots-\alpha_{Q,1}(x_j,t))u_1^f(x_j,t)$$
$$+\alpha_{2,1}(x_j,t)u_2^f(x_j,t)+\Lambda+\alpha_{k,1}(x_j,t)u_k^f(x_j,t)+\alpha_{k+1,1}(x_j,t)u_{k+1}^f(x_j,t)$$
$$+\ldots+\alpha_{Q,1}(x_j,t)u_Q^f(x_j,t)=(1-\sum_{q=2}^{Q}\alpha_{q,1}(x_j,t))u_1^f(x_j,t)+\sum_{q=2}^{Q}\alpha_{q,1}(x_j,t)u_q^f(x_j,t) \quad (6)$$

where $\alpha_{q,1}(x_j,t)$ is the interactive utility value for the outcome of attribute x_j toward event f, and the utility value of negotiator $\#q$ affects the utility of negotiator #1 at discussion $\#t$;

$\alpha_{1,q}(x_j,t)$ is the interactive utility value for outcome of attribute x_j toward event f, and the utility value of negotiator #1 affects the utility of negotiator $\#q$ at discussion $\#t$; $u_q^f(x_j,t)$ is the utility value of negotiator $\#q$ for outcome of attribute x_j toward event f at discussion $\#t$; $\forall q=2,\ldots,Q$.

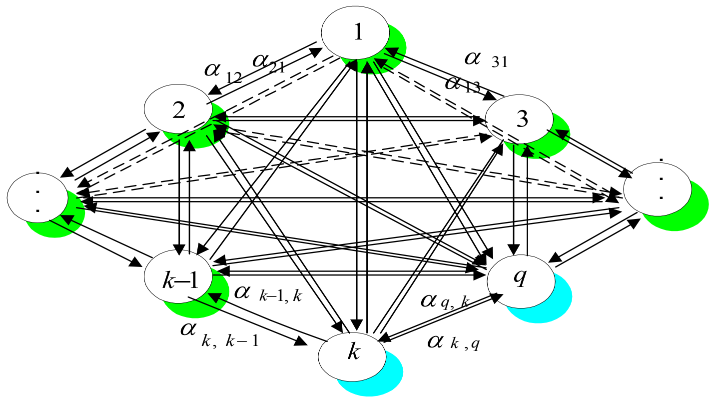

Figure 3. Diagram of the interactive utility among q negotiators

Similarly, the utility functions of negotiators #2, $\#k$, and $\#Q$ can undergo linear transformation, after which their utility functions are as shown in eqs. (7) to (9).

$$u_2^f(x_j,t+1)=(1-\sum_{q=1,q\neq 2}^{Q}\alpha_{q,2}(x_j,t))u_2^f(x_j,t)+\sum_{q=1,q\neq 2}^{Q}\alpha_{q,2}(x_j,t)u_q^f(x_j,t)$$
$$=(1-\sum_{q=1}^{Q}\alpha_{q,2}(x_j,t))u_2^f(x_j,t)+\sum_{q=1}^{Q}\alpha_{q,2}(x_j,t)u_q^f(x_j,t) \quad (7)$$

$$u_k^f(x_j,t+1)=(1-\sum_{q=1}^{Q}\alpha_{q,k}(x_j,t))u_k^f(x_j,t)+\sum_{q=1}^{Q}\alpha_{q,k}(x_j,t)u_q^f(x_j,t) \quad (8)$$

$$u_Q^f(x_j,t+1)=(1-\sum_{q=1}^{Q}\alpha_{q,Q}(x_j,t))u_Q^f(x_j,t)+\sum_{q=1}^{Q}\alpha_{q,Q}(x_j,t)u_q^f(x_j,t) \quad (9)$$

For eq. (6), $u_1^f(x_j,t+1)$ will change as both $\sum_{q=2}^{Q}\alpha_{q,1}(x_j,t)$, $u_1^f(x_j,t)$ and $\sum_{q=2}^{Q}\alpha_{q,1}(x_j,t)u_q^f(x_j,t)$ change. As a result, other negotiators and interactive utility values (IUV) among the negotiators will also affect the utility of negotiator #1 at discussion $\#t$.

Since $u_1^f(x_j,t)$ and $u_q^f(x_j,t)$ satisfy the conditions of $0\le u_1^f(x_j,t)\le 1$ and $0\le u_q^f(x_j,t)\le 1$, therefore, $u_1^f(x_j,t+1)$ will fall in the closed interval [0,1] and the inequalities $0 \le \sum_{q=2}^{Q} \alpha_{q,1}(x_j,t) \le 1$ will also hold. Let $\sum_{q=2}^{Q} \alpha_{q,1}(x_j,t)=0$, then $u_1^f(x_j,t+1)=u_1^f(x_j,t)$. This means that the utility assessed by negotiator #1 is unchanged for attribute x_j of event f at discussion $\# t+1$ and $\# t$. It also indicates that negotiator #1 is unaffected by other negotiators, i.e., he/she is utility-independent. However, when $\sum_{q=2}^{Q} \alpha_{q,1}(x_j,t) \ne 0$, it means that other negotiators are related to negotiator #1 via interdependent utilities. When $\sum_{q=2}^{Q} \alpha_{q,1}(x_j,t)=1$, it means that negotiator #1 was affected completely by other negotiators, and gave up his/her original measured value. When $0 < \sum_{q=2}^{Q} \alpha_{q,1}(x_j,t) < 1$, then eq. (6) is the dependent utility model for negotiator #1 and other negotiators. When $\sum_{q=2}^{Q} \alpha_{q,1}(x_j,t) \to 1$, the degree of utility-dependence is higher; contrarily, when $\sum_{q=2}^{Q} \alpha_{q,1}(x_j,t) \to 0$, the degree of utility-dependence is lower. For eqs. (7) to (9), whether or not the negotiators are dependent will depend on whether or not variables $\sum_{q=1}^{Q} \alpha_{q,1}(x_j,t)$ □ $\sum_{q=1}^{Q} \alpha_{q,2}(x_j,t)$ □ $\sum_{q=1}^{Q} \alpha_{q,k}(x_j,t)$, and $\sum_{q=1}^{Q} \alpha_{q,Q}(x_j,t)$ are zero.

In eqs. (6) to (9), there is linear relationship between utility of negotiator q at discussion $\# t+1$ and utility of other negotiator at discussion $\# t$. And since Q negotiator affect others utility each other through IUV, so IUV is the endogenous variable of the utility function of the individual negotiator and also the negotiation group. Which is $GU^f(x_j,t+1) = U(u_1^f(\sum \alpha_{q,1}(x_j,t)), u_2^f(\sum \alpha_{q,2}(x_j,t)), \Lambda, u_k^f(\sum \alpha_{q,k}(x_j,t)), \Lambda, u_Q^f(\sum \alpha_{q,Q}(x_j,t)))$. Since after linear transformation, $u_q^f(x_j,t)$ and $u_k^f(x_j,t)$ satisfy the N-M Axiom, $GU^f(x_j,t+1)$ still satisfies the binary preference relation (Fishburn, 1990). Therefore the utility of the negotiation group can be represented by expected utility value, and the expected utility value can be obtained by the concept of the preference decomposition theory (Bleichorodt and Quiggin, 1997). Therefore, summing up the utility function of individual negotiator, and obtaining the negotiation group's utility value we get $GU^f(x_j,t+1)$, as shown in eq. (10).

$$
\begin{aligned}
GU^{f}(x_{j},t+1) &= U(u_{1}^{f}(x_{j},t),u_{2}^{f}(x_{j},t),\Lambda\ ,u_{Q}^{f}(x_{j},t)) = \sum_{q=1}^{Q} u_{q}^{f}(x_{j},t) \\
&= [(1-\sum_{q=1}^{Q}\alpha_{q,1}(x_{j},t))u_{1}^{f}(x_{j},t)+\sum_{q=1}^{Q}\alpha_{q,1}(x_{j},t)u_{q}^{f}(x_{j},t)]+[(1-\sum_{q=1}^{Q}\alpha_{q,2}(x_{j},t))u_{2}^{f}(x_{j},t) \\
&+\sum_{q=1}^{Q}\alpha_{q,2}(x_{j},t)u_{q}^{f}(x_{j},t)]+\Lambda\ +[(1-\sum_{q=1}^{Q}\alpha_{q,\lambda}(x_{j},t))u_{\lambda}^{f}(x_{j},t)+\sum_{q=1}^{Q}\alpha_{q,\lambda}(x_{j},t)u_{q}^{f}(x_{j},t)]+\Lambda \\
&+[(1-\sum_{q=1}^{Q}\alpha_{q,Q}(x_{j},t))u_{Q}^{f}(x_{j},t)+\sum_{q=1}^{Q}\alpha_{q,Q}(x_{j},t)u_{q}^{f}(x_{j},t)] \\
&= \sum_{q=1}^{Q}[(1-\sum_{k=1}^{Q}\alpha_{q,k}(x_{j},t))u_{k}^{f}(x_{j},t)]+\sum_{q=1}^{Q}(\sum_{k=1}^{Q}\alpha_{q,k}(x_{j},t))u_{q}^{f}(x_{j},t) \\
&\forall\ k=1,2,\Lambda\ ,\lambda,\Lambda\ ,Q;\ q=1,2,\Lambda\ ,\lambda,\Lambda\ ,Q.
\end{aligned}
\tag{10}
$$

However, there is an iterative and recursive relation between $GU^{f}(x_{j},t+1)$, $u_{k}^{f}(x_{j},t,t+1,\Lambda,t+n-1,\Lambda,T)$ and $u_{q}^{f}(x_{j},t,t+1,\Lambda,t+n-1,\Lambda,T)$. So it can be handled by either forward or backward dynamic programming. The backward procedure of dynamic programming is applied in this paper to handle the iteration and recursion relation as follows. Substitute eqs. (6) to (9) into Eq. (10) to obtain Eq. (11).

$$
\begin{aligned}
GU^{f}(x_{j},t+1) &= \sum_{q=1}^{Q}[((1-\sum_{k=1}^{Q}\alpha_{q,k}(x_{j},t))^{t}u_{k}^{f}(x_{j},1))+((1-\sum_{k=1}^{Q}\alpha_{q,k}(x_{j},t))^{t-1}\sum_{k=1}^{Q}\alpha_{q,k}(x_{j},t)u_{q}^{f}(x_{j},1)) \\
&+((1-\sum_{k=1}^{Q}\alpha_{q,k}(x_{j},t))^{t-1}\sum_{k=1}^{Q}\alpha_{q,k}(x_{j},t)u_{k}^{f}(x_{j},1))+((1-\sum_{k=1}^{Q}\alpha_{q,k}(x_{j},t))^{t-3}\sum_{k=1}^{Q}\alpha_{q,k}(x_{j},t)u_{q}^{f}(x_{j},1)) \\
&+\sum_{k=1}^{Q}\alpha_{q,k}(x_{j},t)((1-\sum_{k=1}^{Q}\alpha_{q,k}(x_{j},t))^{t-3}u_{k}^{f}(x_{j},1))+((1-\sum_{k=1}^{Q}\alpha_{q,k}(x_{j},t))^{t-3}(\sum_{k=1}^{Q}\alpha_{q,k}(x_{j},t))^{t-2}u_{q}^{f}(x_{j},1)) \\
&+((1-\sum_{k=1}^{Q}\alpha_{q,k}(x_{j},t))^{t-3}(\sum_{k=1}^{Q}\alpha_{q,k}(x_{j},t))^{t-2}u_{k}^{f}(x_{j},1))+(\sum_{k=1}^{Q}\alpha_{q,k}(x_{j},t))^{t}u_{q}^{f}(x_{j},1)] \\
&\forall\ k=1,2,\Lambda\ ,\lambda,\Lambda\ ,Q;\ q=1,2,\Lambda\ ,\lambda,\Lambda\ ,Q,k\neq q;
\end{aligned}
\tag{11}
$$

Eq. (11) is the utility value of the negotiation group when attribute-outcome x_{j} of event f at discussion $\#t+1$ is realized. This value can be obtained through weighting of utility of the individual negotiator at discussion #1. If it converges after first discussion among negotiators, that indicates that there is no dispersion or iteration during the discussion process. We can substitute $t=1$ into the first and second items of the right side of eq. (11) and obtain $GU^{f}(x_{j},2)=\sum_{k=1}^{Q}(u_{k}^{f}(x_{j},1)+\sum_{q=1}^{Q}\alpha_{q,k}(x_{j},1)u_{q}^{f}(x_{j},1))$. When $t=1$ and $\sum_{k=1}^{Q}\alpha_{q,k}(x_{j},1)=0$, Eq. (11) becomes $GU^{f}(x_{j},2)=\sum_{q=1}^{Q}u_{q}^{f}(x_{j},1)$, and the utility value of the negotiation group is the sum of individual negotiators' utility. In other words, when a negotiator is utility-independent, the utility value of the negotiation group can be obtained through addition of the original utility of the individual negotiator. This result turns out to be the same as the additive-utility of Bleichorodt and Quiggin (1997), and the additive independent-utility of Luce and Fishburn (1995)[5].

When $t=1$ and $\sum_{q=1}^{Q}\alpha_{q,k}(x_j,1)=1$, eq. (11) becomes $GU^f(x_j,2)=\sum_{q=1}^{Q}\sum_{k=1}^{Q}\alpha_{q,k}(x_j,1)u_q^f(x_j,1)$, which shows that the utility value of the negotiation group from a realization of the outcome of attribute x_j of event f is the sum of the IUV values multiplied by the utility of the individual negotiator. *This shows that although there is no independence phenomenon among the negotiators, but if the negotiators reach consensus during first discussion, there is no iteration relation among the negotiators.* At this point $GU^f(x_j,2)$ reverts back to the weight of individual's utility and the IUV value. *When* $t\geq 3$*, then* $0<\sum_{k=1}^{Q}\alpha_{q,k}(x_j,1)<1$*, which means when discussion among the negotiators is completed three times, the utility value of the negotiation group toward the outcome of attribute* x_j *of event* f *will be affected and changed by the utility of the individual negotiator, the IUV values, and variable* t. Since $0<\sum_{k=1}^{Q}\alpha_{q,k}(x_j,t)<1$, $0<u_k^f(x_j,t)<1$ and $0<u_q^f(x_j,t)<1$, then the utility of negotiation group will decrease with increasing numbers of discussion and the IUV values will become stable. This completes our demonstration. We now turn to some technical issues in the context of the multi-objective programming paradigm.[6]

4. Some considerations within the framework of a multi-objective programming model

Event f with n states has to be integrated into one utility value for the negotiation groups regarding event f. In order to simplify the analysis, we assume that event f is independent from other events. Under the assumption that each individual negotiator pursues maximization of the utility, we assume that the negotiation group also maximizes utility.

Generally speaking, when a negotiation group discusses event f, members will first

[5] We could have written out the above as a theorem with the proof just given. However, the proposition and the somewhat novel demonstration via dynamic programming given here may gain from the emphasis on economic meaning in this form of exposition.

[6] We use the word paradigm here in a Kuhnian sense. The avalanche of work in this area does form a paradigm within which the "normal science" of modeling decisions with many objectives is carried out.Specific models can count as so many artifacts used during the business of carrying out 'puzzle solving' under the overarching paradigm.

discuss the outcome of attributes under each state of the event; and then obtain the group utility of event f. This discussion-behavior of a negotiation group regarding event f can be formulated as a multi-objective programming problem. Eq. (12) is objective function of a negotiation group regarding event f for all states and attributes. Equation (13) is obtained based on the concept described in eq. (11), which is based on the utility value of the negotiation group regarding the outcome of attribute x_j for each state of event f at discussion # $t+1$. In addition, as event f has n states, let $\psi(x_j) = \max\{p(x_1), p(x_2), \Lambda, p(x_j), \Lambda, p(x_n)\}$, $\psi(x_j)$ be defined as the maximum probability value for every negotiator regarding every state of event f. Eq. (12) represents the utility measured by the negotiation group regarding event f. When the $GU(t+1)$ value is in the closed interval [0,1], it is easily seen that eqs. (14) and (23) can affect eqs. (12) and (13).[7]

Eqs. (14) and (15) represent the utility value of negotiators q and k at discussion $\#t+1$ respectively. The utility function related to the utility of other negotiators at discussion $\#t$, and the utility of negotiators q and k at discussion $\#t+1$ will affect the utility of other negotiators in the next discussion. In addition, Constraints (16) and (17) are to ensure that the utility of all the negotiators can satisfy the condition of being in the closed interval [0,1]. Equation (18) is to ensure that the sum of the IUV values among negotiators satisfies the condition being in the same closed interval.[8] Equation (19) is also to ensure that the utility value of the negotiation group is not negative. Since eqs. (18) and (19) meet the constraint of being limited between 0 and 1, so the utility value of the negotiation group in eq. (20) meets the constraint of being limited between 0 and 1. Equation (21) shows that the IUV value among the negotiators have to be between 0 and 1, and $\alpha_{k,q}(x_j,t)$ is the decision variable. Equation (23) shows that if consensus cannot be reached through discussion among negotiators, the utility value will be calculated as if at an independent state; if consensus is reached during discussion, then the utility value after the discussion can be substituted. The complete formulation of the maximization problem subject to the above constraints then is as follows:

$$Max \quad GU(t+1) \tag{12}$$

[7] Further generalizations are possible when the metric chosen is not simply the Euclidean metric; but we do not pursue this here.

[8] Of course, this ensures (the proof is immediate and trivial, but the result is nontrivial) that the utility value of individual negotiator is not negative.

$$
\begin{aligned}
Max\ \ GU^{f}(x_j,t+1) = & \sum_{q=1}^{Q}[((1-\sum_{k=1}^{Q}\alpha_{q,k}(x_j,t))^{t}u_k^{f}(x_j,1))+((1-\sum_{k=1}^{Q}\alpha_{q,k}(x_j,t))^{t-1}\sum_{k=1}^{Q}\alpha_{q,k}(x_j,t)u_q^{f}(x_j,1) \\
& +((1-\sum_{k=1}^{Q}\alpha_{q,k}(x_j,t))^{t-1}\sum_{k=1}^{Q}\alpha_{q,k}(x_j,t)u_k^{f}(x_j,1))+((1-\sum_{k=1}^{Q}\alpha_{q,k}(x_j,t))^{t-3}\sum_{k=1}^{Q}\alpha_{q,k}(x_j,t)u_q^{f}(x_j,1)) \\
& +\sum_{k=1}^{Q}\alpha_{q,k}(x_j,t)((1-\sum_{k=1}^{Q}\alpha_{q,k}(x_j,t))^{t-3}u_k^{f}(x_j,1))+((1-\sum_{k=1}^{Q}\alpha_{q,k}(x_j,t))^{t-3}(\sum_{k=1}^{Q}\alpha_{q,k}(x_j,t))^{t-2}u_q^{f}(x_j,1)) \\
& +((1-\sum_{k=1}^{Q}\alpha_{q,k}(x_j,t))^{t-3}(\sum_{k=1}^{Q}\alpha_{q,k}(x_j,t))^{t-2}u_k^{f}(x_j,1))+(\sum_{k=1}^{Q}\alpha_{q,k}(x_j,t))^{t}u_q^{f}(x_j,1)]
\end{aligned} \tag{13}
$$

$$
s.t.\quad u_q^{f}(x_j,t+1) = (1-\sum_{q=1}^{Q}\alpha_{q,k}(x_j,t))u_k^{f}(x_j,t)+\sum_{q=1}^{Q}\alpha_{q,k}(x_j,t)u_q^{f}(x_j,t) \tag{14}
$$

$$
u_k^{f}(x_j,t+1) = (1-\sum_{q=1}^{Q}\alpha_{q,k}(x_j,t))u_k^{f}(x_j,t)+\sum_{q=1}^{Q}\alpha_{q,k}(x_j,t)u_q^{f}(x_j,t); \tag{15}
$$

$\forall\ q,k \in Q,\ t \in T,\ j = 0,1,\Lambda\ N.$

$$
0 \le u_q^{f}(x_j,t), u_q^{f}(x_j,t+1) \le 1, \forall q \in Q, t \in T; \tag{16}
$$

$$
0 \le u_k^{f}(x_j,t), u_k^{f}(x_j,t+1) \le 1, \forall k \in Q, k \ne q;\ t \in T; \tag{17}
$$

$$
0 \le \sum_{q=1}^{Q}\alpha_{q,k}(x_j,t), \sum_{k=1}^{Q}\alpha_{q,k}(x_j,t) \le 1, \forall q,k \in Q, t \in T; \tag{18}
$$

$$
0 \le \sum_{q=1}^{Q}\sum_{k=1,q\ne k}^{Q}\alpha_{q,k}(x_j,t) \le 1, \forall q,k \in Q,\ t \in T; \tag{19}
$$

$$
0 \le GU^{f}(x_j,t+1),\ GU_{mau}(x_j,t+1), GU^{f}(x_j,t+1) \le 1; \tag{20}
$$

$0 \le \alpha_{q,k}(x_j,t) \le 1,\ \forall\ q,k \in Q; q \ne k, t \in T;$ decision variable; (21)

$0 \le u_q^{f}(x_j,0) \le 1$ the utility value of individual negotiator; (22)

$$
GU(t+1) = \begin{cases} GU(x_j,t+1)) \times \psi(x_j),\ if\ \sum_{q,k\ne q}\alpha^{*}_{q,k}(x_j,t+1) = \sum_{k,k\ne q}\alpha^{*}_{k,q}(x_j,t+1) \\ GU_{mau}(x_j,t+1)) \times \psi(x_j),\ if\ \sum_{q,k\ne q}\alpha^{*}_{q,k}(x_j,t+1) \ne \sum_{k,k\ne q}\alpha^{*}_{k,q}(x_j,t+1) \end{cases}. \tag{23}
$$

where $GU^{f}(t+1)$ is the utility value of negotiation group for event f at discussion # $t+1$;

$GU^{f}(x_j,t+1)$ is the utility value of a negotiation group for the outcome of attribute x_j toward event f at discussion # $t+1$;

$GU_{mau}(x_j,t+1)$ is the utility value of a group for the outcome of attribute x_j toward event f at discussion # $t+1$;

$u_q^{f}(x_j,t+1)$ is the utility value of individual negotiator # q for the outcome of attribute x_j toward event f at discussion # $t+1$;

$u_k^{f}(x_j,t+1)$ is the utility value of individual negotiator # k for the outcome of attribute x_j toward event f at discussion # $t+1$;

$u_q^{f}(x_j,t)$ is the utility value of individual negotiator # q for the outcome of attribute x_j toward event f at discussion $\# t$;

$u_k^f(x_j,t)$ is the utility value of individual negotiator $\# k$ for the outcome of attribute x_j toward event f at discussion $\# t$;

$u_q^f(x_j,1)$ is the utility value of individual negotiator $\# q$ for the outcome of attribute x_j toward event f at discussion #1;

$u_k^f(x_j,1)$ is the utility value of individual negotiator $\# k$ for the outcome of attribute x_j toward event f at discussion #1;

$\sum_{q=1}^{Q} \alpha_{q,k}(x_j,t)$ is the sum value of IUV's where the utility value of negotiator # q affects negotiator #1 at discussion $\# t$, $\forall q,k \in Q$; $k \neq q$, the sum value of IUV's is constant and located between 0 and 1;

$\sum_{k=1}^{Q} \alpha_{q,k}(x_j,t)$ is the sum value of IUV's where the utility value of negotiator # q affects negotiator $\# k$ at discussion t, $\forall q,k \in Q$; and $k \neq q$, the sum value of IUV's is constant and located between 0 and 1;

$\alpha_{k,q}(x_j,t)$ is the utility value of negotiator $\# k$, which affects the utility of negotiator $\# q$ for the outcome of attribute x_j of event f at discussion t, $\forall q,k \in Q$; and $k \neq q$; the value of IUV is located between 0 and 1;

$\alpha_{q,k}(x_j,t)$ is the utility value of negotiator $\# q$, which affects the utility of negotiator $\# k$ for the outcome of attribute x_j of event f at discussion $\# t$, $\forall q,k \in Q$; and $k \neq q$; the value of IUV is located between 0 and 1;

$\psi(x_j)$ is maximum probability value for all states of event f by q negotiators, $j = 1,2,\ldots,n$;

t is the index of the discussion number, $t \in \{0,1,2,\Lambda\ ,T\}$.

This completes the analytical discussion. What remains to be done is to formulate an appropriate algorithm and provide a numerical illustration of the applicability of the approach developed here.

5. The algorithm

From the dynamic multi-objective programming model, the decision variables are $\alpha_{q,k}(x_j,t)$, $\alpha_{k,q}(x_j,t)$, $GU^f(x_j,t+1)$, $GU_{mau}(x_j,t+1)$ and $GU^f(t+1)$. Therefore the decision variables cannot be obtained by solving the simultaneous-equation system. By considering the dispersion and feedback characteristics of the model, we develop an iterative algorithm, for which the algorithm steps are described as below.

Step 0: Set the number of discussions and input the $\psi(x_j)$ value

Generally speaking, when solving the mathematical programming model, the simulation frequency is related to the optimal solution through the level and rate of convergence. However, since the number of discussions can not be infinite in the real world, to simplify the analysis we let the number of discussions be finite, and input the probability value $\psi(x_j)$.

Step 1: Set the initial utility value

Let $u_q^f(x_j,0)$ and $u_k^f(x_j,0)$ represent the utility before discussion for negotiator $\# q$ and $\# k$ respectively; $u_q^f(x_j,1)$ and $u_k^f(x_j,1)$ represent the utility values for discussion #1 for negotiators q and k, $q=1,2,\Lambda,k,\Lambda,Q$.

Step 2:Obtain the initial interactive utility value (IUV)

When discussion is proceeded by the negotiation group, assume that the main negotiator (such as the chairman or key negotiator) speaks first; thus we can obtain the initial IUV value for the key negotiator. Then, calculate the initial IUV's value for other negotiators. The calculation procedure is as fellows.

Apply values for $u_q^f(x_j,0)$ and $u_k^f(x_j,0)$ together with $u_q^f(x_j,1)$ and $u_k^f(x_j,1)$ to eqs. (A-4) and (A-5) in appendix A, to obtain the initial interactive utility value among the negotiators, as shown in Eq. (24). Take the absolute value of eq. (24), to make both $\alpha_{q,k}(x_j,t+1)$ and $\alpha_{k,q}(x_j,t+1)$ satisfy the non-negativity condition.

$$\alpha_{q,k}(x_j,t+1)=\left|\frac{u_k^f(x_j,t+1)-u_k^f(x_j,t)}{(u_q^f(x_j,t+1)-u_k^f(x_j,t))}\right|, \alpha_{k,q}(x_j,t+1)=(1-\alpha_{q,k}(x_j,t+1))\left|\frac{u_q^f(x_j,t)-u_q^f(x_j,t+1)}{u_k^f(x_j,t+1)-u_q^f(x_j,t)}\right| \quad (24)$$

When $u_k^f(x_j,t+1)=u_k^f(x_j,t)$ or $u_q^f(x_j,t)=u_k^f(x_j,t+1)$, then let $u_k^f(x_j,t+1)-u_k^f(x_j,t)=\varepsilon$ or $u_q^f(x_j,t)-u_k^f(x_j,t+1)=\varepsilon$, $\varepsilon=1\times 10E-06$.

Step 3: Normalization of the interactive utility value

This step is to simulate conditions with respect to those negotiators who strongly affect others by their own view (strong minded negotiators) as well as those who are easily affected by others (obedient negotiators). In either case, it will be difficult model to converge. Therefore, we normalize step 2, which is as shown in eq. (25).

$$\alpha_{k,q}^{adj}(x_j,t+1)=\frac{\alpha_{k,q}(x_j,t+1)-\alpha_{\min}(x_j,t+1)}{\alpha_{\max}(x_j,t+1)-\alpha_{\min}(x_j,t+1)}, \alpha_{q,k}^{adj}(x_j,t+1)=\frac{\alpha_{q,k}(x_j,t+1)-\alpha_{\min}(x_j,t+1)}{\alpha_{\max}(x_j,t+1)-\alpha_{\min}(x_j,t+1)} \quad (25)$$

Step 4: Solve the IUV that occurs after discussion

After normalizing the $\alpha_{k,q}^{adj}(x_j,t+1)$, substitute $\alpha_{q,k}^{adj}(x_j,t+1)$ into the dynamic multi-objective programming model and obtain the negotiator's utility after discussion,

$u_q^{f*}(x_j,t+1), u_k^{f*}(x_j,t+1)\ \forall k \neq q, k,q \in Q$. Since $\alpha_{k,q}^{adj}(x_j,t+1)$ and $\alpha_{q,k}^{adj}(x_j,t+1)$ are the pre-discussion IUV values, then obtain $\alpha_{k,q}^{*}(x_j,t+1)$ and $\alpha_{q,k}^{*}(x_j,t+1)$ to represent values after discussion, in accordance with step 2.

Step 5: Determine whether the IUV will converge or not

We have to check if $u_q^{f*}(x_j,t+1)$, $\alpha_{k,q}^{*}(x_j,t+1)$ and $\alpha_{q,k}^{*}(x_j,t+1)$ as obtained from steps 1 to 4, converge or not. The convergence condition can be verified as in appendix B. As shown in the appendix B, when $\alpha_{k,q}^{*}(x_j,t+1)=\alpha_{q,k}^{*}(x_j,t+1)$ or $\sum_{q,k\neq q}\alpha_{q,k}^{*}(x_j,t+1)=\sum_{k,k\neq q}\alpha_{k,q}^{*}(x_j,t+1)$ satisfy the convergence condition; this indicates that views among the negotiators are very close. Therefore, $\alpha_{k,q}^{*}(x_j,t+1) = \alpha_{q,k}^{*}(x_j,t+1)=0$ or $\alpha_{k,q}^{*}(x_j,t+1) = \alpha_{q,k}^{*}(x_j,t+1)=1$ is one of the(necessary) convergence conditions. When satisfying the convergence condition, we skip directly to step 9 below to determine the utility value for the negotiation group. If there is no convergence, then proceed to step 6, modify the IUV and start the next discussion.

Step 6: Modify the IUV

When the model cannot converge, then modify the IUV among the negotiators, as below.

When $|\alpha_{k,q}^{*}(x_j,t+1)-\alpha_{q,k}^{*}(x_j,t+1)|>E$, then let $\alpha_{k,q}^{*}(x_j,t+1)=\alpha_{k,q}^{*}(x_j,t)-E$, where E is the assumed allowable tolerance-error value.

Step 7: Modify the individual utility value

Substitute the modified IUV value (done in step 6) $\alpha_{k,q}^{*}(x_j,t+1)$ and $\alpha_{q1,k}^{*}(x_j,t+1)$ into the dynamic multi-objective programming model.

Step 8: Repeat steps 4 to 7 until the model converges or the end of discussion.

Step 9: Calculate utility values for individual negotiators and negotiation groups.

When the obtained solution satisfies the convergence condition, we can obtain $\alpha_{q,k}^{*}(x_j,t)$, $\alpha_{k,q}^{*}(x_j,t)$, t, $u_k^{f}(x_j,t)$, $GU^{f}(x_j,t)$ and $GU^{f}(t)$.

6. A Numerical example

One particular example is described in this section to demonstrate the usefulness of the particular approach to analyze the discussion behavior among negotiators developed here, using data from Feng, Kang and Tzeng (2000).

6.1 Description of the loan credit ratio event

The Concession Company must pay loan interest to the bankers within the concession period. If the credit ratio increases, the interest cost will also increase, meaning increased risk. Let the credit ratio be 6.5%, 7%, 7.5%, 8%, 8.5%, 9%, and 10%. A total of seven states (rc) exists where (rc) represents the level of the loan credit ratio. The attribute outcome for this event is interest cost (ic). Meanwhile, $u(ic)$ is the utility value for the negotiator regarding attribute outcome, and the occurrence probability for each state (rc) is $p(rc)$. In addition, $u(I) = u(ic) \times p(rc)$, where $u(I)$ denotes the utility value for a negotiator regarding attribute and state. Meanwhile, (ic), $u(ic)$ and $p(rc)$ all correspond to each state (rc), so each has eight values. The outcomes of attribute, utility value of each negotiator and the probability of a specific state negotiator for each event are given.

Assume that there are six negotiators in the negotiation group of the BOT Concession Company, and they discuss the bank loan credit ratio. Before discussion, each individual negotiator measures the utility of each state of the event, and the measurement results are shown in Table 2, where utility is calculated by eqs. (1) and (2).

Table 2 Utility value by negotiator regarding the bank loan credit ratio

Negotiator	states						
	6.5%	7.0%	7.5%	8%	8.5%	9%	10%
Negotiator #1	0.1990	0.4701	0.8737	0.4020	0.0613	0.0121	0.0089
Negotiator #2	0.9589	0.8693	0.4330	0.2432	0.0992	0.0449	0.0020
Negotiator #3	0.4680	0.3306	0.2906	0.0546	0.0660	0.0171	0.0090
Negotiator #4	0.1157	0.5478	0.6089	0.4990	0.0024	0.0002	0.0090
Negotiator #5	0.8404	0.6843	0.4235	0.2908	0.0261	0.0018	0.0011
Negotiator #6	0.2016	0.7225	0.7280	0.3570	0.0990	0.0121	0.0001

Source: Feng, *et al*. (2000)

We used Turbo Pascal 7.0 to write the simulation program and to calculate the post-discussion values to obtain IUV, individual negotiators' utility, the number of discussion after convergence and the utility value of the negotiation group. The detailed steps are as follows:

Step 0: Set the number of discussions and the input $\psi(x_j)$ value

Let the number of discussion be finite and set it to be 50, which is $T \leq 50$; and input the probability values $\psi(x_j)$, which are 0.9589, 0.9150, 0.9540, 0.6112, 0.45, 0.2211 and 0.101, respectively; those values are obtained from Feng, Kang and Tzeng (2000).

Step 1:Under the state 9% in Table 2, the utility value of each negotiator is 0.0121, 0.0449, 0.0171, 0.0002, 0.0018 and 0.0121, respectively. Add e value, $e = 0.00001$, to the utility of each negotiator and obtain negotiators' utilities as 0.01211, 0.04491, 0.01711, 0.00021, 0.00181 and 0.01211, respectively, which is the initial utility value during discussion.

Step 2: Substitute the initial utility value of Step 1 into equation (24), and obtain the initial IUV, $\alpha_{q,k}(9\%,1)$, as in Table 3. For example, $\alpha_{2,1}(9\%,1) = 0.00030$ represents that at discussion #1, the utility value of negotiator #2 affecting negotiator #1 is 0.0003, and the rest of the IUV's can be deduced by analogy. As shown in Table 3, the interactive utility value among some negotiators is symmetric, while others are not equal.

Table 3 Initial IUV's of bank credit loan at 9%

Interactive utility value	Negotiator #1	Negotiator #2	Negotiator #3	Negotiator #4	Negotiator #5	Negotiator #6
Negotiator #1	NA	0.00030	0.00200	0.00084	0.00097	0.00000
Negotiator #2	0.00030	NA	0.00032	0.09998	0.00566	0.00087
Negotiator #3	0.00200	0.10000	NA	0.00032	0.00059	0.00180
Negotiator #4	0.00084	0.00022	0.00059	NA	0.00000	0.00000
Negotiator #5	0.00097	0.10000	0.10000	0.00625	NA	0.00087
Negotiator #6	1.00000	0.10000	0.10000	0.00084	0.10000	NA

Step 3: As shown in Table 3, the IUV difference among part of the negotiators is small or they are equal; but the differences between the negotiators is great. This reflects that the point of view among the negotiators is great, therefore we apply Eq. (25) to normalize the IUV's in Table 3.

Step 4: Substitute the normalized IUV and the initial utility of Step 3 into the basic model to obtain the negotiator's utility values after discussion #1, which are $u_1^*(9\%,1)$ =0.0376, $u_2^*(9\%,1)$ =0.0196, $u_3^*(9\%,1)$ =0.05068, $u_4^*(9\%,1)$ =0.11378, $u_5^*(9\%,1)$ =0.22152 and $u_6^*(9\%,1)$ =0.01045, respectively. These are the utility values after discussion #1, and are different from the minor adjusted utility of Step 1. Compared with Step 1, there are obvious changes for the negotiator's utility after discussion #1. Then substitute the utility value of each of the six negotiator into eq. (24), and obtain the IUV, $\alpha^{*}_{q,k}(9\%,1)$, after discussion #1, as shown in Table 4.

Table 4 IUV's among negotiators after discussion #1

Interactive utility value	Negotiator #1	Negotiator #2	Negotiator #3	Negotiator #4	Negotiator #5	Negotiator #6	total
Negotiator #1	NA	0.00000	0.00000	0.00000	0.00000	0.00425	0.00425
Negotiator #2	0.29185	NA	0.34434	0.21415	0.2221	0.29185	1.36429
Negotiator #3	0.26944	0.04846	NA	0.07972	0.08805	0.26944	0.75511
Negotiator #4	1.00000	0.32844	0.86871	NA	1.00000	1.00000	4.19715
Negotiator #5	1.00000	0.29859	0.84112	1	NA	1.00000	4.13971
Negotiator #6	0.00000	0.07850	0.51493	0.21636	0.24997	NA	1.05976
total	2.56129	0.75399	2.56910	1.51023	1.56012	2.56554	NA

Step 5: Apply $u^{*}_{1}(9\%,1), u^{*}_{2}(9\%,1), \Lambda, u^{*}_{6}(9\%,1)$ and $\alpha^{*}_{q,k}(9\%,1)$ of Step 4, to obtain $\sum_{q,k\neq q}\alpha^{*}_{q,k}(9\%,1)$ and $\sum_{k,k\neq q}\alpha^{*}_{k,q}(9\%,1)$. The summation of columns is 0.00425, 1.34629, 0.75511, 4.19715, 4.13971 and the summation of rows are 1.05976; 2.56129, 0.75399, 2.56910, 1.56012 and 2.56554, respectively. This indicates that a specific negotiator's utility that is affected by other negotiators is not equal to the utility of other negotiators that are affected by the specific negotiator; and it does not satisfy the convergence condition of the model, requiring further revision as shown in Step 6.

Step 6: Let the tolerance error, E, be 0.0001; as $\alpha^{*}_{2,1}(9\%,1) - \alpha^{*}_{1,2}(9\%,1)$ >0.0001, let $\alpha_{2,1}(9\%,2) = \alpha^{*}_{1,2}(9\%,1) - 0.0001$. This is the input value for the revised IUV of discussion #2. The revision of other IUV's are similar.

Step 7: Substitute all the $\alpha_{q,k}(9\%,2)$ into the basic model and perform the second simulation.

Step 8: Repeat the calculation from Step 4 to Step 7

Through the repeated simulation of Step 8, the model reaches the convergence condition after discussion #5, where $\alpha^{*}_{q,k}(9\%,5)$ is as shown in Table 5. The IUV shows the symmetry in this case, and the utility impact of negotiator #4 affecting other negotiators is 0, 0, 0, 0, 0 and 0.00425, which is the same as the utility impact of the other negotiators affecting negotiator #4. The sum of IUV's for each negotiator is 0.00001, 0.00001, 0.00425, 0.00425, 0.00000 and 0.00850. This satisfies the condition for model and the discussion can be ended for this state. In another words, these six negotiators reach "consensus" at discussion #5. Then, we can obtain $u_1^*(9\%,5)$ =0.03760, $u_2^*(9\%,5)$ =0.04490, $u_3^*(9\%,5)$ =0.00002, $u_4^*(9\%,5)$ = 0.00002, $u_5^*(9\%,5)$ =0.00002 and $u_6^*(9\%,5)$ =0.00001, for the negotiation group regarding bank credit ratio at 9%. In addition, $GU(9\%,5)$ =0.05068, where $GU(9\%,5)$ is the risk measurement for post-discussion utility, of those six negotiators toward bank credit ratio at 9% of event f .

Table 5 Results of IUV among negotiators under the convergent condition

	Bank Credit Ratio: 9%						
Interactive utility value	Negotiator #1	Negotiator #2	Negotiator #3	Negotiator #4	Negotiator #5	Negotiator #6	total
Negotiator #1	NA	0.00001	0.00000	0.00000	0.00000	0.00000	0.00001
Negotiator #2	0.00001	NA	0.00000	0.00000	0.00000	0.00000	0.00001
Negotiator #3	0.00000	0.00000	NA	0.00000	0.00000	0.00425	0.00425
Negotiator #4	0.00000	0.00000	0.00000	NA	0.00000	0.00425	0.00425
Negotiator #5	0.00000	0.00000	0.00000	0.00000	NA	0.00000	0.00000
Negotiator #6	0.00000	0.00000	0.00425	0.00425	0.00000	NA	0.00850
Total	0.00001	0.00001	0.00425	0.00425	0.00000	0.00850	0.01702

From the above analysis, the model and solution algorithm developed in this paper can describe the discussion behavior among the negotiators, so we can continue to apply this model to the utility measurement and risk measurement of events other loan credit ratios. Following the assumptions made in Feng and Kang (2000), we assume that the state of the event is independent. Simulate the discussion among the negotiators by applying data in Table 2, and obtain the discussion number, utility change and convergence of each state of the event; results are summarized in Table 6.

Simulating all the states of the event for 50 times, the number of discussions for each state are 28, 50, 50, 50, 4, 5 and 6. The utility value of the negotiation group for each state is 0.59044, 3.21145, 2.95137, 1.59945, 0.05949, 0.05068 and 0.53806, respectively. As for credit ratio at 7.0%, 7.5 and 8.0%, it can not reach convergence even after 50 times simulation, so no convergent solution is obtained for the IUV. The utility value for other states are all less than one, meeting the convergence condition.

For utility changing of individual negotiators regarding credit ratio at 6.5%, the post-discussion utility value of negotiators #2, #3, #5 and # 6 is greatly decreased compared with the value before discussion. The variation for credit ratio at 10.0% is also great, except for negotiator #1, the utility value after discussion of all other negotiators is obviously increased. This shows that all six negotiators change their original assessed utility after discussion, as shown in Table 6. In addition, for utility changing of the negotiation group, the pre-discussion $GU_{mau}(x_j)$ is obtained by using the additive MAU model from Feng and Kang, (2000). As for the post-discussion, $GU(x_j,t)$ is obtained by applying the dynamic multi-objective programming model. Compared with the pre- and post-discussion utility value, $GU_{mau}(x_j,t)$ and $GU(x_j,t)$, except for credit ratio at 8.5%, there are obvious changes for all other states. Compared with the pre-discussion $GU_{mau}(x_j,t)$, there is a substantial increase for the post-discussion $GU(x_j,t)$. This shows that during discussion, some negotiator was significantly affected by other negotiators and changed the original risk measurement for the utility, while the others show no significant change.

Table 6 Pre- and post-discussion measured utility value regarding the bank loan credit ratio event

State		The	Loan	Credit	Ratio			
Utility	Discussion	6.5%	7.0%	7.5%	8.0%	8.5%	9.0%	10.0%
Nego.#1	Pre	0.19900	0.47010	0.87370	0.40200	0.06130	0.01210	0.00890
	Post	0.17350	0.44460	0.84820	0.37650	0.03580	0.03760	0.03440
	Variation	-0.02550	-0.02550	-0.02550	-0.02550	-0.02550	0.02550	0.02550
Nego.# 2	Pre	0.95890	0.86930	0.43300	0.24320	0.09920	0.04490	0.00200
	Post	0.11534	0.79091	0.43905	0.25192	0.02369	0.01301	0.05261
	Variation	-0.84356	-0.07839	0.00605	0.00872	-0.07551	-0.03189	0.05061
Nego. #3	Pre	0.46800	0.33060	0.29060	0.05460	0.06600	0.01710	0.00900
	Post	0.06166	0.44195	0.41883	0.19242	0.00000	0.00002	0.17901
	Variation	-0.40634	0.11135	0.12823	0.13782	-0.06600	-0.01708	0.17001
Nego. #4	Pre	0.11570	0.54780	0.60890	0.49900	0.00240	0.00020	0.00900
	Post	0.11981	0.50176	0.41507	0.26065	0.00000	0.00002	0.17901
	Variation	0.00411	-0.04604	-0.19383	-0.23835	-0.00240	-0.00018	0.17001
Nego. #5	Pre	0.84040	0.68430	0.42350	0.29080	0.02610	0.00180	0.00110
	Post	0.07877	0.51609	0.41062	0.26004	0.00000	0.00002	0.46513
	Variation	-0.76164	-0.16821	-0.01288	-0.03076	-0.02610	-0.00178	0.46403
Nego. #6	Pre	0.20160	0.72250	0.72800	0.35700	0.09900	0.01210	0.00010
	Post	0.04136	0.51614	0.41960	0.25792	0.00000	0.00001	0.46513
	Variation	-0.16024	-0.20636	-0.30840	-0.09908	-0.09900	-0.01209	0.46503
Converge /diverge		✓	×	×	×	✓	✓	✓
Discussion No.		28	50	50	50	4	5	5
Pre-Dis.	$GU_{mau}(x_j)$	0.42820	0.56250	0.53640	0.28870	0.05700	0.01360	0.00560
Post-Dis.	$GU(x_j)$	0.59044	3.21145	2.95137	1.59945	0.05949	0.05068	0.53806
	$\psi(x_j)$	0.9589	0.915	0.954	0.6112	0.45	0.2211	0.101

Note :"×" is diverge□"✓" is converge

For risk measurement of the event, after 50 cycles of simulation, the $GU(x_j,t)$ value will be substituted by $GU_{mau}(x_j,t)$ at credit ratio of 7.0%, 7.5% and8.0%. Calculated according to eq. (22), the utility value of the event $GU(t+1)$ is 0.56617, which is greater the average utility value, 0.25689. Therefore, the event of bank loan credit ratio, after discussion among the negotiators, is a non-risk event.

Compared with the study done by Feng and Kang (2000), if the negotiator's utility is independent, the utility measured by the negotiation group toward the loan credit ratio event is 0.4001, and expected utility value is 0.2877 for pre-discussion, which is greater than 0.2877. This implies that the pre-discussion utility measured by the negotiation group toward the bank loan credit ratio appears to be a non-risk event. Compared with the pre- and post-discussion, the utility measured by the negotiation group toward the bank loan credit ratio appears to be a non-risk event. The results are shown in Table 7.

Table 7 Compared utility measurement by pre/ post discussion

The event of Bank loan credit ratio	The utility value of Group negotiator	Expected utility Value	Risk/non-risk
Pre-discussion	0.4001	0.2877	Non-risk
Post-discussion	0.56617	0.25689	Non-risk

From the numerical example, we can summarize the following key points: (1) if the utility difference among the negotiators becomes smaller, the model converges more easily; otherwise, it is difficult for the model to converge and the number of discussions will increase. This makes intuitive sense. (2) With increasing number of discussions, the IUV's among the negotiators will decrease. This is not obvious, but results from the deeper structure of the model itself. Clearly, both the results are nontrivial.

7. Summary and Conclusions:

Dynamic programming and multi-objective programming are adopted in this paper to formulate and test a risk measurement model for interactive negotiators.The numerical example shown in this paper demonstrates the feasibility of this particular model. As shown in the model, algorithm and numerical example presented earlier, the sum of the IUVs will determine whether the utility among negotiators is independent or not. When the sum of the IUVs approaches zero, the level of dependence among the negotiators is lower, which naturally, means greater independence. When the sum of the IUV's approaches one, the level of dependence among the negotiators becomes very strong. In addition, as the utility difference among the negotiators decreases monotonically, the model has a convergent solution. And as the utility difference among the negotiators increases, the number of discussions will also increase leading possibly to divergence.

As demonstrated by the results from the numerical example, risk measured by the negotiation group of a BOT Concession Company with respect to the event of bank loan credit ratio shows that it is a non-risk event both before and after discussion.This is quite a significant finding, and depends crucially on the formal approach adopted. It can be seen also that the dynamic multi-objective programming model developed in this paper can be used as the basis for negotiation discussion, to explain the interactive utility among the negotiators and risk measurement of the negotiation group with respect to some specific event.

We have relaxed the independent utility condition for negotiators, and demonstrated the interactive behavior among negotiators and risk measurement for a specific event. The model developed in this paper treats IUVs as endogenous variables of the negotiator's utility function. Besides the utility interaction among the negotiators, factors that may affect the utility function include learning capability of the negotiator as well as incomplete information. These factors can be incorporated into the model for further study. In the future, the assumption of event independence made in this paper can be relaxed and the model can be revised by developing a risk measurement utility model for multiple events discussed among a negotiation group. Future research can also attempt to improve upon the model algorithm in this paper in order to make the algorithm cmputationally more efficient. However, for many practical purposes(e.g., a BOT project), the model as developed here can be readily applied with reasonable success.

Appendix A: Solving the equations to Derive (Dependent) Utility Values of Negotiators

Assume that the linear dependent utility for negotiators k and q are as Eq. (A-1) and (A-2), and solve for $\alpha_{k,q}(x_j,t)$ and $\alpha_{q,k}(x_j,t)$ by the simultaneous equation system below:

$$\begin{cases} u_k^f(x_j,t+1) = (1-\alpha_{q,k}(x_j,t))u_k^f(x_j,t) + \alpha_{q,k}(x_j,t)u_q^f(x_j,t) & \text{(A}-1) \\ u_q^f(x_j,t+1) = (1-\alpha_{k,q}(x_j,t))u_q^f(x_j,t) + \alpha_{k,q}(x_j,t)u_k^f(x_j,t) & \text{(A}-2) \end{cases}$$

$\forall k,q \in Q,\ t = 0,1,2,\Lambda\ T.\ t$ is discussion number. Substitute Eq. (A-1) into Eq. (A-2) to get Eq. (A-3).

$$u_k^f(x_j,t) = \frac{1}{1-\alpha_{q,k}(x_j,t)}[u_k^f(x_j,t+1) - \alpha_{q,k}(x_j,t)u_q^f(x_j,t)] \qquad \text{(A-3)}$$

Substitute Eq. (A-3) into the variable $u_k^f(x_j,t)$ of Eq. (A-2), yielding $\alpha_{q,k}(x_j,t)$ value, shown as Eq. (A-4); therefore, substitute the $\alpha_{q,k}(x_j,t)$ value into Eq. (A-3) and get the $\alpha_{k,q}(x_j,t)$ value, the result shows as Eq. (A-5).

$$\alpha_{q,k}(x_j,t) = (\frac{u_k^f(x_j,t+1) - u_k^f(x_j,t)}{u_q^f(x_j,t) - u_k^f(x_j,t)})$$

(A-4)

$$\alpha_{k,q}(x_j,t) = (1-\alpha_{q,k}(x_j,t))\left(\frac{u_q^f(x_j,t) - u_q^f(x_j,t+1)}{u_k^f(x_j,t+1) - u_q^f(x_j,t)}\right) \qquad , \qquad \forall\, k,q \in Q;\, k \neq q$$

(A-5)

Appendix B: Derivation of Convergence Condition

To show the convergence condition of iterative algorithm for dynamic multi-objective programming. We modify Eq. (A-1) to (B-1) by applying Eq. (A-1) and (A-2). The left-hand side of Eq. (B-1) represents the incremental utility difference between negotiators k and q. The right-hand-side utility of Eq. (B-1) represents the utility of negotiator k as affected by negotiator q. This can reach equal in IUV's and therefore Eq. (B-1) can be represented by Eq. (B-2).

$$u_k^f(x_j,t+1) - u_k^f(x_j,t) = \alpha_{q,k}(x_j,t)(u_q^f(x_j,t) - u_k^f(x_j,t))$$

(B-1)

$$\Delta u_k^f(x_j,t+1)/\Delta u_q^f(x_j,t) = \lambda n u_{k,q}^f = \alpha_{q,k}(x_j,t)$$

(B-2)

The same as above, modify Eq. (A-2) into (B-3), which represents the difference of incremental utility value of negotiator q and k, which can also be shown as the utility incremental method, such as Eq. (B-4).

$$u_q^f(x_j,t+1) - u_q^f(x_j,t) = \alpha_{k,q}(x_j,t)(u_k^f(x_j,t) - u_q^f(x_j,t))$$

(B-3)

$$\Delta u_q^f(x_j,t+1)/\Delta u_k^f(x_j,t) = \lambda n u_{q,k}^f = \alpha_{k,q}(x_j,t)$$

(B-4)

Equation (B-4) indicates the incremental ratio of the negotiator's utility. If the change of utility becomes stable, then the utility incremental ratios of Eqs. (B-2) and (B-4) tend to be equal, which is $\Delta u_q^f(x_j,t+1)/\Delta u_k^f(x_j,t) = \Delta u_k^f(x_j,t+1)/\Delta u_q^f(x_j,t)$. This implies that the IUV of negotiator k and q tend to be equal, which is $\alpha_{q,k}(x_j,t) = \alpha_{k,q}(x_j,t)$, and development procedure is as below.

As the incremental utility becomes stable, indicating that the smaller the difference between utility incremental ratio of negotiator k and q, the better the case is, which is $\min[(\Delta u_q^f(x_j,t+1)/\Delta u_k^f(x_j,t)) - (\Delta u_k^f(x_j,t+1)/\Delta u_q^f(x_j,t))]$.

$$\min[(\Delta u_q^f(x_j,t+1)/\Delta u_k^f(x_j,t))-(\Delta u_k^f(x_j,t+1)/\Delta u_q^f(x_j,t))]=\min(\lambda nu_{k,q}^f-\lambda nu_{q,k}^f) \qquad \text{(B-5)}$$
$$=\min(\alpha_{q,k}(x_j,t)-\alpha_{k,q}(x_j,t))$$

To keep $0\le\alpha_{q,k}(x_j,t)-\alpha_{k,q}(x_j,t)\le 1$, modify Eq. (B-5) to Eq. (B-6).

$$\min\gamma_{q,k}=\min|\alpha_{q,k}(x_j,t)-\alpha_{k,q}(x_j,t)| \qquad \text{(B-6)}$$

By Eq. (B-6), we can modify Eq. (B-6) to Eq. (B-7).

$$\min\gamma_{q,k}=\min(\alpha_{q,k}(x_j,t)-\alpha_{k,q}(x_j,t))^2=\min(\lambda nu_{q,k}^f-\lambda nu_{k,q}^f)^2 \qquad \text{(B-7)}$$

Equation (B-7) indicates that minimum difference in IUV, so we differentiate $\alpha_{q,k}(x_j,t)$ to get Eq. (8).

$$\min\gamma_{q,k}=\min(\lambda nu_{q,k}^f-\lambda nu_{k,q}^f)^2$$
$$\frac{\partial\gamma_{q,k}}{\partial\lambda nu_{q,k}}=\frac{\partial(\lambda nu_{q,k}^f-\lambda nu_{k,q}^f)^2}{\partial\lambda nu_{q,k}^f}=2\lambda nu_{q,k}^f-2\lambda nu_{k,q}^f \qquad \text{(B-8)}$$
$$Let\ \frac{\partial\gamma_{q1,k}}{\partial\lambda nu_{q,k}}=0,\quad \therefore 2\lambda nu_{q,k}^f-2\lambda nu_{k,q}^f=0\quad\Rightarrow\lambda nu_{q,k}^f=\lambda nu_{k,q}^f$$

i. e. $\Delta u_q^f(x_j,t+1)/\Delta u_k^f(x_j,t)=\Delta u_k^f(x_j,t+1)/\Delta u_q^f(x_j,t)\Rightarrow\alpha_{q,k}(x_j,t)=\alpha_{k,q}(x_j,t)$.

As shown in Eq. (B-8), when the utilities of negotiator k and q become stable, their IUVs tend to be equal. In another words, when $\alpha_{q,k}(x_j,t)=\alpha_{k,q}(x_j,t)$, there is a convergent solution for the model. When $\alpha_{q,k}(x_j,t)$ = $\alpha_{k,q}(x_j,t)=0$, $\alpha_{q,k}(x_j,t)=\alpha_{k,q}(x_j,t)=1$ or $\sum_k\alpha_{q,k}(x_j,t)=\sum_k\alpha_{k,q}(x_j,t)$, this satisfies the convergence condition of the model. The meaning of the convergent solution is that there is no utility change among the negotiators due to discussion, and the negotiators have reached "consensus".

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