CIRJE-F-144

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January 2002

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Does e-Commerce Always Increase Social Welfare in the Long Run? *

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January 13, 2002

Abstract

We examine the effect of electronic commerce ("e-commerce") on social welfare, in the framework of conventional spatial competition models. We consider the case where both conventional and electronic retailers coexist in equilibrium. We show that e-commerce does not necessarily increase social welfare in the long run. In particular, when electronic retailers have clear cost advantage over conventional retailers, then the advent of e-commerce is shown to *reduce* social welfare.

*We are grateful for Masayoshi Maruyama, Tatsuhiko Nariu, Takuo Imagawa and participants of seminars at Nanzan, Hitotsubashi, Kobe, Yokohama National and Osaka Prefecture University for their comments on earlier versions of this paper.

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1 Introduction

Since the stock value of so-called B-to-C internet firms soared in the NASDAQ market, electronic commerce ("e-commerce") has attracted much attention in terms of their potential to change century-old retailing of neighborhood stores and large retail outlets. Although the subsequent burst of these internet "bubbles" in the NASDAQ market somewhat lessens the public interest in e-commerce, its impacts have been increasingly apparent in everyday life of many countries from North America to Asia and Europe. Consumers are buying on line in increased numbers. The popular consensus seems to emerge: e-commerce is considered to provide ideal transactions in a "frictionless" market and to be desirable for our economy.[1]

To put this popular consensus in economic terms, Smith, Bailey, and Brynjolfsson [2000] offer four dimensions to measure efficiency in electronic markets compared to conventional markets: price levels, price elasticity, menu costs, and price dispersion. Then, they survey recent empirical researches (Bailey [1998], Brynjolfsson and Smith [2000], Goolsbee [2000] etc.)[2] and conclude that electronic markets are more efficient than conventional markets with respect to price levels, price elasticity and menu costs.[3]

However, lower prices, higher price elasticity and smaller menu costs do not necessarily imply social welfare is higher. As welfare economics teaches us, we should examine both sides of the market: retailers as well as consumers to get a complete picture of the impact of e-commerce on social welfare. Moreover, we should look at long-run consequence of e-commerce after entry and exit of retailers under free entry. To do so, we need a model where both electronic retailers and conventional retailers compete for demand of consumers in a retail market. Unfortunately, to our knowledge, there has been no such attempt to evaluate retail market equilibrium where conventional and e-commerce retailers coexist.

The purpose of this paper is to construct such a model and then to

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[1] See quotations in Brynjolfsson and Smith [2000], for example.

[2] See Smith, Bailey, and Brynjolfsson [2000] for a complete list of research papers on e-commerce. They also provide an annotated bibliography of selected papers.

[3] However, they report that price dispersion remains in electronic markets although price information is considered to be disseminated smoothly in those markets. They offer various hypotheses to explain it, such as product and retailer heterogeneity, convenience and shopping experience.

examine the impact of e-commerce on social welfare explicitly. To do this, we choose the simplest and commonly-used model of spatial competition as a starting point. Our model is a variant of Salop [1979]'s circular city model with neighborhood stores.[4] We call these stores as conventional retailers. We incorporate into this framework electronic retailers powered by Internet technology. We characterize e-retailing as having a large fixed cost and a negligible menu cost. On the one hand, e-retailers have to engage in extensive advertisement in order to obtain consumer recognition and to build sophisticated distribution network to cater dispersedly-located consumers. These costs are large and fixed in their nature. On the other hand, e-retailers can adjust their prices more frequently and effectively than conventional retailers, suggesting their menu costs are negligible.

The result is a striking one. In the conventional circular-city framework, we show that if electronic commerce has clear cost advantage (*i.e.*, lower marginal cost) and that there are many conventional retailers before the advent of e-commerce, then long-run equilibrium social welfare is *unambiguously lower* after e-commerce is introduced into the market. Although it counters popular conviction, it has an intuitive reason. Cost advantage of e-commerce implies many entries in the cybermarket, and this drives a large number of conventional retailers out of markets. This means that consumers who once patronized these neighborhood stores lose their favorite stores and are forced to buy from Internet retailers. If there were many neighborhood stores before, then consumers negatively affected by e-commerce outnumber those who benefit from e-commerce (*i.e.*, those who located far away from neighborhood stores). Moreover, under free entry, Internet retailers' profits dissipate in the long run and thus there is no producer's surplus. Thus, only negative (distributional) effect on consumer's surplus remains in the market, making e-commerce welfare-reducing. Although the model of this paper is only one of many possible models of e-commerce, the result of this paper suggests that the present euphoria about e-commerce is somewhat overdue.

The remainder of this paper is organized as follows. In section 2, we first explain a conventional circular-city model as a preliminary step, and then consider features of e-commerce. In section 3, we incorporate

[4] A similar model is developed in Nishimura [1995] who incorporates large discount stores located at the beltway just outside the circular city of conventional small neighborhood stores.

these features of e-commerce into the circular city model of Section 2 and construct a model of coexisting e-commerce and conventional commerce. In section 4, we characterize equilibrium. In section 5, we examine welfare properties and present main results. Concluding remarks are given in Section 6.

2 The Circular City and e-Commerce

2.1 The Circular City Model

To examine the impact of e-commerce on the retail market, we begin with the conventional circular city model of Salop [1979], which we briefly explain here for the sake of completeness.[5] Consider a circular city of a unit circumference, where households are distributed uniformly with unit density. Conventional retailers, which we hereafter call *c-retailers*, are located equidistantly on the circle. Figure I depicts this market. In what follows, we only consider symmetric equilibrium in which all retailers charge the same price.

In the short-run, the number of c-retailers is fixed and equal to n. Thus, the distance between neighboring retailers is $1/n$. Each household buys at most one unit of a commodity (*e.g.* a book or a music CD), whose value is assumed to be v.[6] If she buys the commodity at price p from a c-retailer located at a distance x from her house, she obtains a benefit $v - p - cx$, where c is a trip cost per distance.

A consumer located at a distance x_i from c-retailer i posting price p_i is indifferent between purchasing from this c-retailer and a neighboring c-retailer posting price p if $v - p_i - cx_i = v - p - c[(1/n) - x_i]$. This implies a demand at the ith c-retailer such that

$$d_i = 2x_i = \frac{1}{c}\left(p + \frac{c}{n} - p_i\right), \quad i = 1, 2, .., n. \tag{1}$$

Each c-retailer's marginal cost of supplying one unit of the commodity is assumed to be constant and equal to m. Each c-retailer maximizes its profit (ignoring a fixed cost), $(p_i - m)\,d_i$, by setting their price p_i appropriately.

[5] See Salop [1979] and Tirole [1988, pp. 282–285] for details.

[6] We assume that v is sufficiently large so that each consumer buys one unit in equilibrium.

Thus, competition among c-retailers is Bertrand one with spatial product differentiation.

We consider symmetric equilibrium where all c-retailers charge the same price. Then, the short-run equilibrium price in this circular city model is given as

$$p_S = \frac{c}{n} + m, \tag{2}$$

while the profit each c-retailer obtains in the equilibrium is given as

$$\Pi_S = \frac{c}{n^2}.$$

In the long-run, free entry determines the number of c-retailers, n.[7] There is a fixed cost f of operating in this retail market, so that the long-run profit is $\Pi_S - f$, which must be zero in the long-run equilibrium. For analytic simplicity, we treat the number of firms as a real number in stead of an integer. Then, the long-run equilibrium number of c-retailers is given as

$$n_S = \sqrt{\frac{c}{f}}. \tag{3}$$

In order that this equilibrium number of stores is consistent with the derivation of demand function (1), we should have $n_S \geq 2$, since the derivation assumes at least two stores are in the circular city. This condition is satisfied if

$$c \geq 4f. \tag{4}$$

We assume (4) throughout this paper. Then, substitute (3) into (2), we obtain the long-run price as follows

$$p_{S,\,long-run} = \sqrt{cf} + m. \tag{5}$$

To distinguish the above variables with corresponding ones in our model later, we added subscript S, which indicates "Salop", to the former variables.

Finally, to clarify conditions about parameters in our main proposition, we use the following long-run maximum trip cost in the circular city model:

$$\widehat{cx} \equiv c * \left(\frac{1}{2n_S}\right) = \frac{1}{2}\sqrt{cf}. \tag{6}$$

[7] For analytic simplicity, we assume that c-retailers can relocate themselves equidistantly with no cost.

2.2 e-Commerce

Let us now consider e-commerce. We hereafter call a retailer in the Internet an *e-retailer*. In this paper, we are concerned with large-scale e-retailers who command a substantial market share. We do not consider so-called SOHO-type e-retailers, since their impact on conventional retailing is rather insignificant. We focus on the following four characteristics of large-scale e-commerce.

Firstly, the nature of consumers' purchase cost in e-commerce is different from that in conventional shopping. In the conventional retail market, consumers visit a store in order to buy a product and return home with the product. The trip cost depends on the location of consumers (distance from the store), and thus it is heterogeneous as formulated in the circular city model. In contrast, there is no physical store that consumers visit in the e-retail market, so that there is no trip cost. Instead, consumers incur delivery charges in order to get the product, but these costs usually do not depend on their particular location in the city. Consumers also incur costs to be connected to the Internet, and these costs also do not depend on particular locations of consumers.[8] Thus, the purchase cost in e-retailing is homogeneous among consumers.

Secondly, an e-retailer incurs a substantial fixed cost just to make themselves known to potential consumers. While the presence of a store is apparent in the local market, it is not in the cyberspace. To get consumers' recognition and to induce consumers to visit their web site, e-retailers usually spend a considerable money in advertising in TVs, radios, and other medias. This implies that the fixed cost that an e-retailer incurs to be profitable in the cyberspace is not small, and may be substantially greater than the fixed cost of a c-retailer.

Thirdly, consumers can get information quickly from the Internet. In addition, e-retailers can change their price more flexibly due to small menu costs.[9] Therefore, once an e-retailer succeeds in getting consumers' recog-

[8]Connection costs are negligible in the United States where unlimited services for local calls are common. The costs are not negligible in other countries where metered services are common.

[9]Bailey [1998, Table 5.8] finds that e-retailers change their price more frequently than c-retailers in the U.S. markets for books, CDs and software. Moreover, Brynjolfsson and Smith [2000] find that e-retailers' price changes are up to 100 times smaller than the smallest price changes made by c-retailers in the markets for books and CDs. These findings indicate that menu costs that e-retailers incur are smaller than those c-retailers do.

nition, information about the e-retailer's change in price is transmitted to potential customers much faster than that about the c-retailer's, who wait for consumers to pass by the store to see their window advertisement or to read price advertisement in the next Sunday's newspapers.

Fourthly, since e-retailers cater dispersedly-located consumer demand, they have to establish an effective network of distribution and inventories to respond consumer demand quickly and effectively. Thus, advanced planning of their capacity to satisfy consumer demand becomes important. Many episodes of stock-outs and failed delivery on time in Christmas seasons in the United States in recent years highlight the importance of this capacity consideration.

3 A Model of Local Retail Markets with e-Commerce

In this section, we incorporate the characteristics of e-retailers examined in the previous section into the circular city model, then construct a model where both e-retailers and c-retailers compete in a retail market simultaneously. Hereafter, $*$ denotes e-retailing. The number of c-retailers is n as before and that of e-retailers is now n^*. In the short run, n and n^* are assumed to be fixed. In the long run, they are determined by free entry.[10]

The discussion in the previous section revealed two kinds of difference between conventional retailing and e-retailing. On the one hand, price information diffuses rather slowly in the conventional retailing, so that c-retailers (that is, conventional retailers) have to advertise their prices in advance, while e-retailers can change their prices rather easily to adjust them to current market conditions. On the other hand, e-retailers catering a large market area have to plan well in advance how much and where they stock their merchandise in their local distribution centers. Thus, as a stylized description, e-retailers first determine their quantity (stock of merchandise) and then adjust their prices to sell them, while c-retailers first determine their prices and to satisfy demand that these prices generate. In other words, e-retailers compete with one another and with c-retailers in a Cournot way,[11] while c-retailers compete with one another and e-retailers

[10]Here we retain the assumption that c-retailers can relocate themselves equidistantly with no cost.

[11]The result of Kreps and Scheinkman [1983] suggests that it is appropriate to assume Cournot type quantity competition among e-retailers in our framework and even though

in a Bertrand way. Consequently, we consider a one-shot game where c-retailers use price and e-retailers use quantity (capacity) as their strategic variables in the short run.

3.1 Demand Structure with e-Commerce

Let us first examine demand structure in the presence of e-retailers. We assume that c-retailers locate equidistantly around the circular city as before, while we assume that e-retailers set up their web sites in the Internet which are equally accessible to all consumers. As in Section 2, we consider symmetric equilibrium, in which all c-retailers charge the same price and all e-retailers supply the same quantity.

The Internet brings consumers an additional choice of shopping in the cyberspace. That is, they can buy a product either from a conventional market or from an electronic market. Figure II depicts the choice. On the one hand, if she buys the product at price p from a c-retailer located at a distance x from her, then she obtains a benefit $v - p - cx$, where c is a trip cost per distance. On the other hand, if she buys it at price p^* from an e-retailer[12], then she obtains $v - p^* - c^*$ where c^* is a delivery charge, which is the same for all consumers. We assume a competitive transportation industry with constant marginal delivery cost c^* and no fixed cost. Then, the delivery charge is equal to the marginal cost of delivery, as shown here.

We assume that consumers know all price in the market before deciding a shop from which they buy the product. This assumption is justified because they can know the prices of c-retailers and e-retailers quite easily especially in the case of homogenous products such as books and music CDs. In the case of c-retailers, they obtain price information, for example, from c-retailers' window advertisement when they pass by c-retailers' store. In the case of e-retailers, price information of a particular store can be obtained almost instantly with a negligible additional cost if consumers know that particular e-retailer.

A consumer located at a distance x_i from c-retailer i posting price p_i is indifferent between purchasing from this retailer and purchasing from the

e-retailers have the flexibility to change their price, Bertrand-type price competition where price equals marginal cost does not occur because of the capacity constraint.

[12]Note that we have assumed Cournot competition among e-retailers so that one price, p^*, prevails in e-market.

e-market if $v - p_i - cx_i = v - p^* - c^*$, or equivalently, $x_i = (1/c)(p^* + c^* - p_i)$. Thus, c-retailer i faces demand d_i such that

$$d_i = 2x_i = \frac{2}{c}(p^* + c^* - p_i), \quad i = 1, 2, .., n. \tag{7}$$

for $\widehat{p} < p_i < p^* + c^*$ where $\widehat{p}$ is defined in Appendix A. Appendix A shows that the optimal c-retailer's price is in fact in this range in equilibrium. C-retailer i maximizes its profit

$$\Pi_i = (p_i - m)\, d_i = (p_i - m)\left(\frac{2}{c}\right)(p^* + c^* - p_i), \tag{8}$$

with respect to p_i, taking e-retailers' price p^* as given. From (8), it is apparent that the c-retailer's profit is a strongly concave function of p_i.

The demand in the e-retailer submarket is the residual demand, so that we immediately have the total e-retailer submarket demand D^* from (7) such that

$$D^* = 1 - \sum_{i=1}^{n} d_i = 1 + \frac{2n}{c}[\overline{p} - (p^* + c^*)], \tag{9}$$

where $\overline{p}$ is the average price of c-retailers: $\overline{p} \equiv (1/n)\sum_{i=1}^{n} p_i$. (9) is appropriate for $\overline{p} - c^* < p^* < \overline{p} - c^* + \frac{c}{2n}$. It can be shown that p^* is in fact in this range in equilibrium (see Appendix A).

Since competition among e-retailers is Cournot one, it is convenient to express the total demand of e-retailers (9) in the form of the inverse demand function. Let d_j^* denote the quantity e-retailer j supplies. Then, the total demand of e-retailers (9) in the form of the inverse demand function is given as follows

$$p^* = \overline{p} - c^* + \frac{c}{2n}(1 - D^*), \tag{10}$$

where $D^* \equiv \sum_{j=1}^{n^*} d_j^*$. E-retailer j maximizes its profit

$$\Pi_j^* = (p^* - m^*)\, d_j^* = \left\{\overline{p} + \frac{c}{2n}\left[1 - \left(D_{-j}^* + d_j^*\right)\right] - (m^* + c^*)\right\} d_j^*, \tag{11}$$

where $D_{-j}^* = \sum_{k \neq j} d_k^*$, with respect to its quantity d_j^* taking other e-retailers' quantity $d_k^* : k \neq j$ and the average price of c-retailers $\overline{p}$ as given. It is easy to show that (11) is a strongly concave function of d_j^*.

Let us examine the difference between demand in the conventional market without e-commerce (1) and that with e-commerce (7). The price sensitivity of the demand (that is, the first-order derivative of the demand with respect to its own price) is $1/c$ in the conventional market without e-commerce (1), while it is $2/c$ in the market with e-commerce (7). Thus, the presence of e-retailers increases price sensitivity of demand. Moreover, price sensitivity of demand in the cyberspace (9) is $2n/c$, which is substantially higher than that in the conventional market. They are consistent with recent empirical studies of e-commerce surveyed by Smith, Bailey and Brynjolfsson [2000].

4 Equilibrium with e-Retailers

In this section, we first examine price and quantity in Nash equilibrium in the short-run where both c-retailers and e-retailers coexist and their numbers are fixed. Then, we investigate long-run equilibrium with free entry, especially equilibrium numbers of c-retailers and e-retailers, as well as long-run equilibrium price and quantity. As in the circular city model of Section 2, we treat n and n^* as real numbers. However, again as in the circular city model, we only consider cases in which $n \geq 1$ and $n^* \geq 1$ in equilibrium, since at least one store in the conventional market and one store in electronic market are assumed in the derivation of demand functions in Section 3. We examine conditions on parameters of the model that guarantee them.

In the following discussion, the following definition is shown to be useful.

Definition 1 (Degree of Absolute Cost Advantage of e-Retailers)

$$z \equiv m - (m^* + c^*)\,. \tag{12}$$

Here $m^* + c^*$ is the sum of e-retailers' marginal cost m^* and the delivery cost of e-retailing c^*, which is the social marginal cost of purchasing from e-retailers (common to all consumers). In contrast, m is c-retailers' marginal cost, which the minimum social marginal cost of purchasing from c-retailers since the trip cost cx is not included. To put it differently, m is the social marginal cost of purchasing from c-retailers for consumers located just in front of c-retailers' store. Thus, the term z can be interpreted as a measure of absolute cost advantage of e-retailers. If e-retailers are efficient in their procurement of merchandise, m^* may substantially lower than m. However, the delivery cost c^* may be large to offset the e-retailers' cost advantage. Thus, z can be positive or negative depending on the relative size of the marginal cost that two groups of retailers face, and the delivery cost.

4.1 Short-Run Equilibrium

In this market, c-retailers and e-retailers simultaneously choose their price (in the case of c-retailers) and their quantity (capacity) (in the case of e-retailers). C-retailer i maximizes (8) with respect to p_i given p^*. From the first-order condition, we have

$$p_i = \frac{1}{2}(m + p^* + c^*).$$

Using (10), we can rewrite the above equation as follows

$$p_i = \frac{m}{2} + \frac{1}{2}\left[\overline{p} + \frac{c}{2n}(1 - D^*)\right]. \tag{13}$$

E-retailer j maximizes (11) with respect to its quantity d_j^* taking other e-retailers' quantity $d_k^* : k \neq j$ and the average price of c-retailers $\overline{p}$ as given. From the first-order condition, we have

$$d_j^* = \frac{n}{c}[\overline{p} - (m^* + c^*)] + \frac{1}{2}\left(1 - D_{-j}^*\right). \tag{14}$$

In this paper, as we mentioned before, we focus our attention on a symmetric equilibrium where all c-retailers set the same price and all e-retailers set the same quantity,

$$p_i = p, \ \ i = 1, 2, \ldots, n,$$
$$d_j^* = d^*, \ \ j = 1, 2, .., n^*.$$

Combining the conditons above with (13) and (14), we obtain equilibrium price for c-retailers and equilibrium quantity for e-retailers such that

$$p = \frac{1}{2n^* + 1}\left(\frac{c}{2n} - n^* z\right) + m, \tag{15}$$

$$d^* = \frac{1}{2n^* + 1}\left(\frac{2n}{c}\right)\left(\frac{c}{n} + z\right). \tag{16}$$

In Appendix A, we show that all retailers have no incentive to deviate from (15) or (16). Then, substituting (15) and (16) into (10), we obtain an equilibrium price in the e-market,

$$p^* = \frac{1}{2n^* + 1}\left(\frac{c}{n} + z\right) + m^*. \tag{17}$$

In addition, substituting (15) and (17) into (7), we obtain an equilibrium quantities for c-retailers,

$$d = \frac{2}{(2n^* + 1)\,c}\left(\frac{c}{2n} - n^* z\right). \tag{18}$$

Since we are concerned with equilibrium where both c-retailers and e-retailers are present: the price-cost margin of both c-retailers and retailers should be positive and at the same time the demand at their (physical or virtual) stores should be positive. From (15)-(18), both requirements are satisfied if the following inequalities

$$-\frac{c}{n} < z < \frac{c}{2nn^*} \tag{19}$$

are satisfied. In the next section, we examine conditions on parameters (m, m^*, c, c^*, f, f^*) that guarantee that this is the case in long-run equilibrium.

4.2 Long-Run Equilibrium

Let us now examine long-run equilibrium with free entry. Substituting (15), (16) and (17) into (8) and (11), we obtain the equilibrium gross profits of each c-retailer and each e-retailer which are functions of n and n^* as follows, respectively;

$$\Pi(n, n^*) \equiv \frac{2}{c}\left[\frac{1}{2n^* + 1}\left(\frac{c}{2n} - n^* z\right)\right]^2, \tag{20}$$

$$\Pi^*(n, n^*) \equiv \frac{2n}{c}\left[\frac{1}{2n^* + 1}\left(\frac{c}{n} + z\right)\right]^2. \tag{21}$$

In the long-run equilibrium, the number of both types of retailers is determined by zero-profit conditions, that is,

$$\Pi(n, n^*) = f, \tag{22}$$

$$\Pi^*(n, n^*) = f^*. \tag{23}$$

From (22) and (23), we obtain the equilibrium number of c-retailers and that of e-retailers (see Appendix B)

$$n = \frac{cf^*}{2\left(\sqrt{2cf} + z\right)^2}, \tag{24}$$

$$n^* = \frac{\sqrt{2cf} + z}{f^*} - \frac{\sqrt{2cf}}{2\left(\sqrt{2cf} + z\right)}. \tag{25}$$

From (24) and (25), we find that n is small and n^* is large when f^* is small, while n is large and n^* is small when f is small as expected.

Next, we consider long-run equilibrium prices and quantities. Substitute (24) and (25) into (15)-(18), we obtain

$$p_{long-run} = \sqrt{\frac{cf}{2}} + m, \tag{26}$$

$$p^*_{long-run} = \left(\sqrt{2cf} + z\right) + m^*, \tag{27}$$

$$d_{long-run} = \sqrt{\frac{2f}{c}}, \tag{28}$$

$$d^*_{long-run} = \frac{f^*}{\sqrt{2cf} + z}. \tag{29}$$

Equilibrium prices and quantities (26) through (29) reveal that the fixed cost of the e-retailer f^* has the effect only on the demand of the e-retailer, $d^*_{long-run}$. As expected, when f^* is large, the number of entering e-retailers is small and, as a result, $d^*_{long-run}$ is large. However, the total demand in e-market becomes small when f^* is large since it holds that

$$D^*_{long-run} \equiv (n^* d^*)_{long-run} = 1 - \frac{\sqrt{2cf}\, f^*}{2\left(\sqrt{2cf} + z\right)^2}. \tag{30}$$

from (25) and (29).

As for $p_{long-run}$ and $p^*_{long-run}$, an increase in f^* has two effects; one through a decrease in n^*, the other through an increase in n. The former raises these prices, while the latter decreases them under (19). (See (15) and (17).) However, since these two effects are just canceled out in our model, these prices are not influenced by changes in f^*.[13]

[13]Similarly, since the relationship, $p_{long-run} + c\left[(1/2)\, d_{long-run}\right] = p^*_{long-run} + c^*$, holds in the long run equilibrium, demand for each c-retailer, $d_{long-run}$, are not affected by changes in f^* in the long-run.

We are concerned with the coexistence equilibrium, that is, long-run equilibrium in which both c-retailers and e-retailers coexist. In the coexistence equilibrium, we have $n \geq 1$ and $n^* \geq 1$, price-cost margins should be positive for both retailers, and demand at both (physical or virtual) stores should be positive. If model parameters (m, m^*, c, c^*, f, f^*) satisfy the following two assumptions, then we have the coexistence equilibrium:

Assumption 1

$$-\sqrt{2cf} < z < \frac{c - 3\sqrt{2cf}}{2},$$

Assumption 2

$$\frac{2}{c}\left(\sqrt{2cf} + z\right)^2 \leq f^* \leq \frac{2\left(\sqrt{2cf} + z\right)^2}{3\sqrt{2cf} + 2z}.$$

Proposition 1 *Under Assumptions 1 and 2, c-retailers and e-retailers coexist in long-run equilibrium ($n \geq 1$ and $n^* \geq 1$).*

Proof. From (26)-(29), it is immediate to see that if the left-hand-side inequality of Assumption 1 is satisfied then price-cost margins are positive and demand is positive.[14] It is apparent from (24) and (25) that the left-hand-side inequality of Assumption 2 guarantees that $n \geq 1$, while the right-hand-side inequality implies that $n^* \geq 1$. It is also immediate to see that the right-hand-side inequality of Assumption 1 implies $c > 3\sqrt{2cf} + 2z$, which is needed for Assumption 2 to be meaningful. **Q.E.D.**

Let us now compare equilibrium prices between equilibrium without e-retailers (5) and with e-retailers (26) and (27). We immediately get

$$p_{long-run} - p_{S,\, long-run} = \left(\frac{1}{\sqrt{2}} - 1\right)\sqrt{cf} < 0.$$

Thus, c-retailers' prices are lower in equilibrium with e-retailers than without e-retailers. However, e-retailers' equilibrium prices may be greater or smaller than equilibrium prices without e-retailers, since we have

$$p^*_{long-run} - p_{S,\, long-run} = \left(\sqrt{2} - 1\right)\sqrt{cf} - c^*.$$

[14] In the previous section, we have assumed (4). Then, it is easy to show that $-\sqrt{2cf} < \left(c - 3\sqrt{2cf}\right)/2$. Thus, there exists z satisfying Assumption 1.

Thus, if the delivery cost c^* in e-commerce is sufficiently large compared with the trip cost c of conventional retailing and the conventional retailers' fixed cost f, equilibrium e-retailers' price is lower than the equilibrium prices without e-retailers, and *vice versa.* Thus, there is no *a priori* reason that suggests e-retailers prices are always lower than equilibrium prices without e-commerce.

A recent empirical study by Brynjolfsson and Smith [2000] shows prices in the Internet tend to be lower than conventional retail markets. However, at the same time, many e-retailers are not able to earn positive profits, and their long-run survival comes under question. Thus, the their results are not likely to be a good description of long-run equilibrium, but rather a snap shot of transition to long-run equilibrium. There is no *a priori* reason to assume that prices are lower in the cyberspace, especially if competition among e-retailers is Cournot one based quantity (capacity) competition as assumed in this study.

5 Welfare Properties of Equilibria

In this section, we examine resource allocation of our retail market with e-commerce. First, we compare social welfare between equilibrium with e-retailers (Section 4) and without e-retailers (Section 2). Next, we compare the equilibrium allocation to the optimum allocation where a planner maximizes social welfare, then detect inefficiencies in market equilibrium.

5.1 Market Equilibrium with and without e-Retailers

As a measure of social welfare, we adopt the total surplus, that is, the sum of consumer's and producer's surpluses. Firstly, the total surplus in market equilibrium only with c-retailers (in Section 2) is defined as

$$TS_S\left(n_S\right) \equiv 2n_S \int_0^{1/(2n_S)} (v - m - ct)\, dt - n_S f. \tag{31}$$

where n_S is the long-run number of c-retailers (3). Secondly, the total surplus in market equilibrium with both c- and e-retailers (Section 4) is defined as

$$TS(n,n^*) \equiv \left[2n\int_0^x (v-m-ct)\,dt - nf\right] \tag{32}$$
$$+\left[2n\int_x^{1/(2n)} (v-m^*-c^*)\,dt - n^* f^*\right],$$

where the term in the first brackets is the total surplus of n c-retailers and consumers who buy from c-retailers, while that in the second brackets is the total surplus of n^* e-retailers and consumers who buy from e-retailers. The term x in (32) is the equilibrium border satisfying $p_{long-run} + cx = p^*_{long-run} + c^*$. That is, x is the distance between the marginal consumer (indifferent between buying from a c-retailer and from an e-retailer) and the nearby c-retailer. Since $x = (1/2)\,d_{long-run}$, we have from (28) that

$$x = \sqrt{\frac{f}{2c}}. \tag{33}$$

Substituting (3) into (31), we obtain

$$TS_S(n_S) = v - m - \frac{5}{4}\sqrt{cf}. \tag{34}$$

Similarly, substituting (24), (25) and (33) into (32), we get

$$TS(n,n^*) = v - m - \sqrt{2cf} + \frac{cff^*}{4\left(\sqrt{2cf}+z\right)^2}. \tag{35}$$

Then, from (34) and (35), we have

$$TS(n,n^*) - TS_S(n_S) = \frac{(2cf)}{8\left(\sqrt{2cf}+z\right)^2}\left[f^* - \left(8-5\sqrt{2}\right)\frac{\left(\sqrt{2cf}+z\right)^2}{\sqrt{2cf}}\right]. \tag{36}$$

From the above equation, we immediately get the following proposition.

Proposition 2 *Under Assumptions 1 and 2, if*

$$f^* < \left(8-5\sqrt{2}\right)\frac{\left(\sqrt{2cf}+z\right)^2}{\sqrt{2cf}} \tag{37}$$

then e-commerce reduces social welfare. That is, equilibrium social welfare with e-retailers is SMALLER than the that without e-retailers: $TS(n,n^*) < TS_S(n_S)$. *If otherwise, e-commerce increases social welfare.*

This proposition shows that, if the fixed cost of e-retailers is sufficiently small to satisfy (37), then social welfare is lower in equilibrium with e-retailers than without e-retailers. Thus, it is not always true that the advent of e-commerce increases social welfare.

Although this proposition is simple, it is rather cumbersome to determine whether a particular set of parameters (m, m^*, c, c^*, f, f^*) satisfies Assumptions 1 and 2 and the condition (37). The following corollary of the proposition presents an easy procedure to determine whether the advent of e-commerce increases social welfare or not. The proof of this corollary is delegated to Appendix C.

Corollary 1 *Under Assumptions 1 and 2, two market equilibria have the following welfare characteristics.*

1. *Suppose that the equilibrium number of retailers when there is no e-retailing n_S (see (3)) is sufficiently small that*

$$n_S \leq \frac{2\sqrt{2}}{8-5\sqrt{2}} \approx 3.045$$

is satisfied. Then e-commerce increases social welfare: $TS_S(n_S) < TS(n, n^*)$

2. *Suppose the contrary that n_S is sufficiently large that*

$$3.045 \approx \frac{2\sqrt{2}}{8-5\sqrt{2}} < n_S$$

Then we have two cases.

   (a) *Suppose that absolute cost advantage of e-retailers, z (see (12)), is sufficiently large that*

$$-1.2\widehat{cx} \approx -\left(\frac{3}{2} - \frac{1}{8-5\sqrt{2}}\right)\sqrt{2cf} < z$$

   is satisfied (where $\widehat{cx}$ is defined in (6)). Then e-commerce reduces social welfare: $TS(n, n^*) < TS_S(n_S)$.

(b) *Suppose the contrary that absolute cost advantage of e-retailers is sufficiently small that*

$$z \leq -\left(\frac{3}{2} - \frac{1}{8-5\sqrt{2}}\right)\sqrt{2cf} \approx -1.2\widehat{cx}$$

is satisfied. This case is divided into two sub-cases.

i. *Suppose that e-retailers' fixed cost f^* is sufficiently small that*

$$f^* < \left(8-5\sqrt{2}\right)\frac{\left(\sqrt{2cf}+z\right)^2}{\sqrt{2cf}}$$

Then e-commerce reduces social welfare: $TS(n,n^*) < TS_S(n_S)$

ii. *Suppose that e-retailers' fixed cost $f^*$is sufficiently large that*

$$\left(8-5\sqrt{2}\right)\frac{\left(\sqrt{2cf}+z\right)^2}{\sqrt{2cf}} \leq f^*$$

Then e-commerce increases social welfare: $TS_S(n_S) \leq TS(n,n^*)$.

Of particular interest is the case that e-retailers have absolute cost advantage, that is, $m^* + c^* < m$, or equivalently, $z > 0$. In this case, we have the following Corollary from (2.a) of the above Corollary.

Corollary 2 *Suppose that Assumptions 1 and 2 hold. Then if e-retailers have absolute cost advantage ($z > 0$), then equilibrium social welfare with e-retailers is smaller than that without e-retailers: $TS(n,n^*) < TS_S(n_S)$. That is, e-commerce REDUCES social welfare if e-retailers have absolute cost advantage.*

Corollary 2 is a striking result.[15] It often takes it granted that if entrants have clear cost advantage over incumbents, then entry of them increases social welfare. Corollary 2 shows that it is not always the case, especially in the case of e-commerce. In what follows, we investigate what makes e-commerce reduce social welfare.

[15]For the coexistence equilibrium to hold under $z > 0$, the right-hand-side in Assumption 1 must be positive. This is true if we have $c > 18f$.

First, note that retailers' profits are zero in the long run for both c- and e-retailers under free entry. Consequently, the effect of cost advantage of e-retailers dissipates in the long run through an increase in e-retailers. Only the effect on consumers' surplus remains in the long run. Let $CS(n, n^*)$ and $CS_S(n_S)$ denote consumer's surplus in equilibrium with e-retailers and without e-retailers, respectively. Then, we can rewrite (31) and (32) as follows,

$$
\begin{aligned}
TS_S(n_S) &= CS_S(n_S) \equiv 2n_S \int_0^{1/(2n_S)} \left(v - p_{S,\,long-run} - ct\right) dt, \\
TS(n, n^*) &= CS(n, n^*) \\
&\equiv 2n \int_0^{x} \left(v - p_{long-run} - ct\right) dt + 2n \int_x^{1/(2n)} \left(v - p^*_{long-run} - c^*\right) dt.
\end{aligned}
$$

From (5) and (26), we have

$$
p_{S,\,long-run} - p_{long-run} = \left(1 - \frac{1}{\sqrt{2}}\right)\sqrt{cf} > 0.
$$

Thus, consumers located close to c-retailers benefit from entry of e-retailers, *so long as these c-retailers stay in the market*. Moreover, from (3), (5) and (27), we find that

$$
p_{S,\,long-run} + c\left(\frac{1}{2n_S}\right) - \left(p^*_{long-run} + c^*\right) = \left(\frac{3}{2} - \sqrt{2}\right)\sqrt{cf} > 0.
$$

This means that consumers located far away from c-retailers also benefit from entry of e-retailers. However, from (5) and (27), we get

$$
p_{S,\,long-run} - \left(p^*_{long-run} + c^*\right) = \left(1 - \sqrt{2}\right)\sqrt{cf} < 0.
$$

Thus, consumers lose substantially if they were located close to c-retailers whom e-retailers drive out of the market.

From the above arguments we find that there are three types of consumers who are differentially affected by e-commerce. If there are many consumers of the first and the second types compared to those of the third type, namely the effects through falling price dominate the effect through exit of some c-retailers, then e-commerce enhances social welfare in total. However, if there are many consumers of the third type, those whose neighborhood stores are driven out of the market, then social welfare is likely to be reduced.

In the short-run where the number of conventional retailers is fixed, cost advantage of entering e-retailers may contribute to the total surplus by increasing e-retailers' profits, although it reduces existing conventional retailers' profits. However, in the long run, some conventional retailers have to exit from the market because of competitive pressure while other e-retailers enter the market to push down e-retailers' profits to zero. Consequently, the direct cost-advantage effect on profits dissipates and only an indirect effect remains on the consumer's surplus through the change in the number of conventional retailers. If consumers negatively affected by e-commerce through exit of their neighborhood stores outnumber those benefited from falling c-retailers' price, e-commerce has a negative impact on social welfare.

5.2 Market Equilibrium and Command Optimum

To investigate inefficiencies in market equilibrium, let us consider a command economy where a planner controls the number and the market area of both types of retailers to maximize the total surplus. We focus on a symmetric policy under which the planner locates c-retailers equidistantly on the circular city. Thus, the objective function of the planner is

$$\begin{aligned}
TS &\equiv \left[2n\int_0^y (v-m-ct)\,dt - nf\right] \\
&\quad + \left[2n\int_y^{1/(2n)} (v-m^*-c^*)\,dt - n^* f^*\right] \\
&= v - 2n\left[\left(my + \frac{1}{2}cy^2\right) + (m^*+c^*)\left(\frac{1}{2n} - y\right)\right] - nf - nf^* \qquad (38)
\end{aligned}$$

where consumers located within distance y from c-retailers are ordered to buy from these c-retailers, and other consumers are ordered to buy from e-retailers. (That is, $2y$ corresponds to a c-retailer's market area.) The planner maximizes (38) with respect to y, n and n^* subject to the following constraints:

$$0 \leq y \leq \frac{1}{2n},\ n \geq 0,\ n^* \geq 0.$$

We assume that the planner solves the above maximization problem in the following sequence: firstly, it maximizes (38) with respect to n and n^*, and secondly it does with respect to y given n and n^*. The former is the maximization of social welfare in the long run, while the latter is that in the short run.

Let us first consider y. From (12) and (38), we obtain

$$TS = v - m + (1 - 2ny) z - ncy^2 - nf - n^* f^*. \tag{39}$$

Differentiating (39) with respect y, then we find that $\partial TS/\partial y = -2n(z + cy)$. Therefore, given n and n^*, the optimum y is such that

$$y = \begin{cases} \frac{1}{2n}, \; if \;\; z < -\frac{c}{2n} \\ -\frac{z}{c}, \; if \;\; -\frac{c}{2n} \leq z < 0 \\ 0, \; if \;\; z \geq 0 \end{cases}. \tag{40}$$

Next, we examine n and n^*, taking (40) into account. From (39) and (40), we find that, when $z \geq 0$,

$$TS = v - (m^* + c^*) - nf - n^* f^*.$$

It is obvious that the optimum n and n^* are such that $n = 0$ and $n^* = 1$ since an e-retailer's cost is smaller than a c-retailer's.[16] Thus, all the market is served by one e-retailer in the optimum.

In contrast, when $z < 0$, we have two cases. If $-c/(2n) \leq z < 0$ or equivalently, $0 < n \leq c/2(-z)$, then we have

$$TS = v - (m^* + c^*) + \left[\frac{(-z)^2}{c} - f\right] n - n^* f^*, \tag{41}$$

while if $z < -c/(2n)$, or equivalently, $n > c/2(-z)$, then we get

$$TS = v - m - \frac{c}{4n} - nf - n^* f^*. \tag{42}$$

Differentiating (41) with respect n, then we find that $\partial TS/\partial n = (-z)^2/c - f$. Thus, there are three cases: Case 1 ($(-z)^2/c > f$), Case 2 ($(-z)^2/c = f$) and Case 3 ($(-z)^2/c < f$). In Case 1, the total surplus increases as n^*

[16] Here we assume implicitly that $f^* < v - (m^* + c^*)$ to assure that it is optimal to have an e-retailer.

decreases and n increases. Thus, the optimal $n^* = 0$. The planner's problem is reduced to the command optimum problem of the conventional circular city model using (42) as the objective function. Thus, the optimum n satisfies

$$n = \frac{1}{2}\sqrt{\frac{c}{f}}.$$

In Case 2, n does not affect social welfare. Thus, the optimal n is indeterminate, while the optimum n^* is unity, since e-retailers have the same constant marginal cost and a fixed cost. In Case 3, the total surplus increases as n decreases. Thus, it is optimal to have $n = 0$ and $n^* = 1$.

Thus, we have the following proposition:

Proposition 3 *There are three cases:*

1. *Suppose that $z < -\sqrt{cf}$. Then, it is optimal that all market areas are served by c-retailers, and their optimum number is $(1/2)\sqrt{c/f}$.*
2. *Suppose that $z = -\sqrt{cf}$. Then, it is optimal to have one e-retailer and c-retailers. The optimum number of c-retailers is indeterminate.*
3. *Suppose that $z > -\sqrt{cf}$. Then, it is optimal that all market areas are served by one e-retailer.*

Proposition 3 shows that coexistence of c-retailers with e-retailers in the market is not optimal except for a knife-edge case of $z = -\sqrt{cf}$.[17]

Comparing market equilibrium in the previous section and the optimum allocation in this section, we find two sources of inefficiency. First, there is a classical over-entry inefficiency. Private benefits of entry (the gross profit of a retailer) deviates from its social benefits (increase in the total surplus). Second, non-competitive behavior of e-retailers adds another inefficiency. This can be explained easily by considering the case of e-retailers' absolute cost advantage ($z > 0$). Although it is optimal to have only one e-retailer, both c-retailers and e-retailers coexist in market equilibrium. E-retailers with

[17]In Nishimura (1994), he shows that coexistence of different types of retailers is optimal. This is because his model assumes that the fixed cost of neighborhood stores, which corresponds to c-retailers in our model, is heterogeneous among neighborhood stores, while we assume homogeneous fixed costs among c-retailers. His model also assumes no fixed cost of a discount store which corresponds to an e-retailer in our model.

cost advantage do not want to capture all market areas but rather allow c-retailers operate in the market. This is because to serve only a part of the market and to maintain large price-cost margin is more profitable than to capture the whole market by narrowing price-cost margin. As a result, conventional retailers with cost disadvantage can remain in the market in market equilibrium.

6 Concluding Remarks

In this paper, we have constructed a simple model where both conventional retailers and electronic retailers ("e-retailers") compete in a retail market. We have shown that e-commerce does not necessarily increase social welfare in the long run under free entry. Indeed, it has revealed that in a conventional model of spatial competition, if electronic retailers have absolute cost advantage over conventional retailers, then e-commerce reduces, rather than increases, social welfare in the long run.

The results of this paper have several empirical implications. First, in order to examine whether e-commerce enhances social welfare, we should not only investigate retail price data but also various "hidden costs of purchase" such as trip costs in conventional retailing and Internet-connection and delivery costs in e-commerce, as well as cost structures of e-retailers. Second, the long-run consequence of e-commerce may be very different from its short-run impact. Third, it is utmost important to examine whether total demand increases when e-retailers appear. It should be noted that our simple spatial competition model concerns only with the trade-diverting effect of new entry, and it does not consider possible demand-creation effects. Thus, if e-commerce creates entirely new demand, then the conclusion of this paper may be reversed and e-commerce is more likely to increase social welfare.

Finally, we list some issues which we can not treat in this paper. First, we assume implicitly that all household can purchase goods from e-retailers. That is, we assume that they not only have personal computers or similar tools that are connected to the Internet but also have their abilities to use them when shopping in e-markets. However, this assumption is not satisfied in reality. We should take into account the heterogeneity of household, or in other words, *digital divide*. Second, the issue of digital divide is related to the government's infrastructure policy because the government can affect the number of retailers and welfare of household by accumulating various public capital which reduces trip costs when households go shopping at conventional retail outlets and/or communication costs when they go shopping at electronic outlets. Incorporating these features to examine the optimal government policy is an important research agenda of future research.

Appendices

A Proof that (15) and (16) are equilibrium price and quantity

In this appendix, we prove that (15) and (16) are equilibrium price and quantity under (19). That is, we show that all retailers have no incentive to deviate from (15) or (16).

(1) c-Retailers.

Let us consider c-retailer i examining whether its profit increases by changing its own price given other c-retailers' price, p, and e-retailers' capacity, d^*. Obviously, there is no incentive to charge a higher price than (15). In the following, we examine whether the c-retailer has incentive to charge a lower price than (15).

By decreasing its price, c-retailer i can gain more and more customers of e-retailers. When its price is $\widehat{p}$ such that

$$\widehat{p} + c\left(\frac{1}{n} - \frac{d}{2}\right) = p^* + c^*, \tag{A.1}$$

then there is no customer of e-retailers in its neighborhood. Substituting (17) and (18) into (A.1) yields

$$\widehat{p} = \frac{1}{2n^* + 1}\left[(1 - 4n^*)\left(\frac{c}{2n}\right) - 3n^* z\right] + m. \tag{A.2}$$

When the c-retailer decreases its price further from $\widehat{p}$, then it faces competition with neighboring c-retailers. Thus, the demand is now the same as in Section 2 such that

$$d_i = \frac{1}{c}\left(p + \frac{c}{n} - p_i\right).$$

If c-retailer i reduces further and hits $\underline{p}$ such that

$$\underline{p} + c\left(\frac{1}{n}\right) = p, \tag{A.3}$$

then the demand discontinuously increases and it obtains the entire market area of a nearby c-retailer. However, from (15) and (A.3), we obtain

$$\underline{p} = -\frac{n^*}{2n^*+1}\left[\left(2+\frac{1}{2n^*}\right)\left(\frac{c}{n}\right)+z\right]+m, \tag{A.4}$$

which is less than the marginal cost, m, under (19). That is, to undercut its own price to get the entire market area of a nearby c-retailer pushes its profit to be negative. Thus, the c-retailer has no incentive to set its price below $\underline{p}$.

Therefore, what we have to examine is whether the c-retailer's profit is greater than the equilibrium profit when its price is sufficiently low so that it faces competition with neighboring c-retailers, rather than e-retailers. Then, the marginal profit change is

$$\frac{\partial \Pi_i}{\partial p_i} = \frac{1}{c}\left[\left(p+\frac{c}{n}+m\right)-2p_i\right].$$

From (15), the above equation is rewritten as

$$\frac{\partial \Pi_i}{\partial p_i} = \frac{2}{c}\left\{\frac{1}{2\,(2n^*+1)}\left[(4n^*+3)\left(\frac{c}{2n}\right)-n^*z\right]+m-p_i\right\}. \tag{A.5}$$

Substituting (A.2) into p_i in (A.5) yields that

$$\left.\frac{\partial \Pi_i}{\partial p_i}\right|_{p_i=\widehat{p}} = \frac{5n^*}{(2n^*+1)\,c}\left[\left(\frac{6}{5}+\frac{1}{10n^*}\right)\left(\frac{c}{n}\right)+z\right] > 0,$$

as long as (19) holds. Thus, if the c-retailer decreases its price below $\widehat{p}$, its profit decreases. As a result, we find that each c-retailer has no incentive to deviate from (15) given other retailers' price and quantity.

(2) e-Retailers.

Let us consider e-retailer j examining whether its profit increases by changing its own capacity given other e-retailers' capacity, d^*, and c-retailers' price, p. Obviously, there is no incentive to decrease its capacity smaller than (16). So we examine whether the e-retailer has incentive to increase it.

By increasing its capacity and selling larger amount of goods, e-retailer j can obtain more and more customers from other e-retailers and c-retailers. As a result, from (10), price in e-market, p^*, falls.

Suppose that e-retailer j 's capacity is $\widehat{d}^*$ and a corresponding e-market price $\widehat{p}^*$ such that

$$p = \widehat{p}^* + c^*. \tag{A.6}$$

Then, from (10), (15) and (A.6), it holds that

$$\widehat{d}^* + (n^* - 1)\, d^* = 1.$$

That is, there is no customer of c-retailers in our economy (except on sites located by c-retailers). When the e-retailer increases its capacity beyond $\widehat{d}^*$, then it faces competition only with other e-retailers. However, it is easily shown that increasing capacity beyond $\widehat{d}^*$ simply results in lowering p^*, and reducing the e-retailer's profit. Thus, there is no incentive for each e-retailer to increase its capacity larger than (16), given other retailers' price and quantity.

B Long Run Equilibrium

In this appendix, we derive the equilibrium number of both type retailers from (22) and (23). The sequence of the entry does not affect the results. For exposition, we assume that first c-retailers enter the market and next e-retailers enter the market. Given n , the number of e-retailers which can enter the market, n^*, is determined by (23), which yields

$$n^*(n) = \frac{1}{2}\left[\sqrt{\frac{2n}{cf^*}}\left(\frac{c}{n} + z\right) - 1\right]. \tag{B.1}$$

Substituting (B.1) into (20) and rearranging, we obtain

$$\Pi(n) \equiv \Pi(n, n^*(n)) = \frac{1}{2c}\left(\sqrt{\frac{cf^*}{2n}} - z\right)^2. \tag{B.2}$$

Taking (B.2) into consideration, each c-retailer decides whether it enters the market. Thus, the number of c-retailers which can enter the market, n, is determined by the free entry condition,

$$\Pi(n) = f,$$

which yields[18]

$$n = \frac{cf^*}{2\left(\sqrt{2cf}+z\right)^2}. \tag{B.3}$$

Substituting (B.3) into (B.1), we obtain

$$n^* = \frac{\sqrt{2cf}+z}{f^*} - \frac{\sqrt{2cf}}{2\left(\sqrt{2cf}+z\right)}. \tag{B.4}$$

C Proof of Corollary 1

Hereafter we define $\overline{f^*}$ and $\underline{f^*}$ such that

$$\overline{f^*} \equiv \frac{2\left(\sqrt{2cf}+z\right)^2}{3\sqrt{2cf}+2z}, \tag{C.1}$$

$$\underline{f^*} \equiv \frac{2}{c}\left(\sqrt{2cf}+z\right)^2. \tag{C.2}$$

for notational simplicity. Then Assumption 2 is simply $\underline{f^*} \le f^* \le \overline{f^*}$.

In addition, we define $\widehat{f^*}$ such that

$$\widehat{f^*} \equiv \left(8-5\sqrt{2}\right)\frac{\left(\sqrt{2cf}+z\right)^2}{\sqrt{2cf}}, \tag{C.3}$$

Then Proposition 1 is that under Assumptions 1 and 2, if $f^* < \widehat{f^*}$ holds, then $TS(n,n^*) < TS_S(n_S)$, and *vice versa*.

1. Suppose that

$$n_S \le \frac{2\sqrt{2}}{8-5\sqrt{2}}$$

holds. Then from (3) we have

$$\frac{8-5\sqrt{2}}{2} \le \sqrt{\frac{2f}{c}}.$$

[18] $\Pi(n)$ and f have an intersection as long as Assumption 1 holds.

Note that from (C.2) and (C.3), we have

$$\widehat{f^*} - \underline{f^*} = \frac{2\left(\sqrt{2cf} + z\right)^2}{\sqrt{2cf}} \left(\frac{8 - 5\sqrt{2}}{2} - \sqrt{\frac{2f}{c}} \right) \leq 0.$$

Therefore, if the set of parameters satisfies Assumptions 1 and 2, then such f^* is always equal to or greater than $\widehat{f^*}$. Consequently, e-commerce increases social welfare for such parameter sets.

2. Suppose that

$$n_S > \frac{2\sqrt{2}}{8 - 5\sqrt{2}} \tag{C.4}$$

holds. Then we have $\underline{f^*} < \widehat{f^*}$.

(a) From (C.1) and (C.3), we find that

$$\widehat{f^*} - \overline{f^*} = \frac{2\left(\sqrt{2cf} + z\right)^2 \left(8 - 5\sqrt{2}\right)}{3cf + z\sqrt{2cf}} \left[\left(\frac{3}{2} - \frac{1}{8 - 5\sqrt{2}} \right) \sqrt{2cf} + z \right].$$

Consequently, if we have

$$\left(\frac{3}{2} - \frac{1}{8 - 5\sqrt{2}} \right) \sqrt{2cf} + z > 0,$$

then we get $\overline{f^*} < \widehat{f^*}$. Therefore, if the set of parameters satisfies Assumptions 1 and 2, then such f^* is always smaller than $\widehat{f^*}$. Consequently, e-commerce decreases social welfare for such parameter sets.

(b) Suppose that

$$\left(\frac{3}{2} - \frac{1}{8 - 5\sqrt{2}} \right) \sqrt{2cf} + z \leq 0$$

holds. Then we have $\underline{f^*} < \widehat{f^*} \leq \overline{f^*}$. Then, if

$$f^* < \left(8 - 5\sqrt{2}\right) \frac{\left(\sqrt{2cf} + z\right)^2}{\sqrt{2cf}}$$

holds, we have $f^* < \widehat{f^*}$ so that e-commerce decreases social welfare for such parameter sets. Otherwise, e-commerce increases social welfare.

References

[1] Bailey, Joseph, Intermediation and Electronic Markets: Aggregation and Pricing in Internet Commerce. (1998) Ph.D. Thesis, Technology, Management and Policy, MIT, Cambridge, MA. (available at http://www.rhsmith.umd.edu/tbpp/jbailey/)

[2] Brynjolfsson, Erik and Michael D. Smith "Frictionless Commerce?" *Management Science*, 46 (2000) , 563-585.

[3] Goolsbee, Austan "In A World Without Border: The Impact of Taxes on Internet Commerce" *Quarterly Journal of Economic***s**, 115 (2000), 561-576.

[4] Kreps, D., and J. Scheinkman "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes" *Bell Journal of Economics*, 14 (1983), 326-337.

[5] Nishimura, Kiyohiko. G. (1995) "Product Innovation with Mass-production: Insufficient or Excessive?" *Japan and the World Economy* 7, 419-42.

[6] Salop, S. C. "Monopolistic Competition with Outside Goods" *Bell Journal of Economics*, 10 (1979) , 483-498.

[7] Smith, Michael D., Joseph Bailey, and Erik Brynjolfsson "Understanding Digital Markets: Review and Assessment" in Erik Brynjolfsson and Brian Kahin, eds *Understanding the Digital Economy*, MIT Press (2000).

[8] Tirole, Jean. *The Theory of Industrial Organization*, MIT Press (1988).

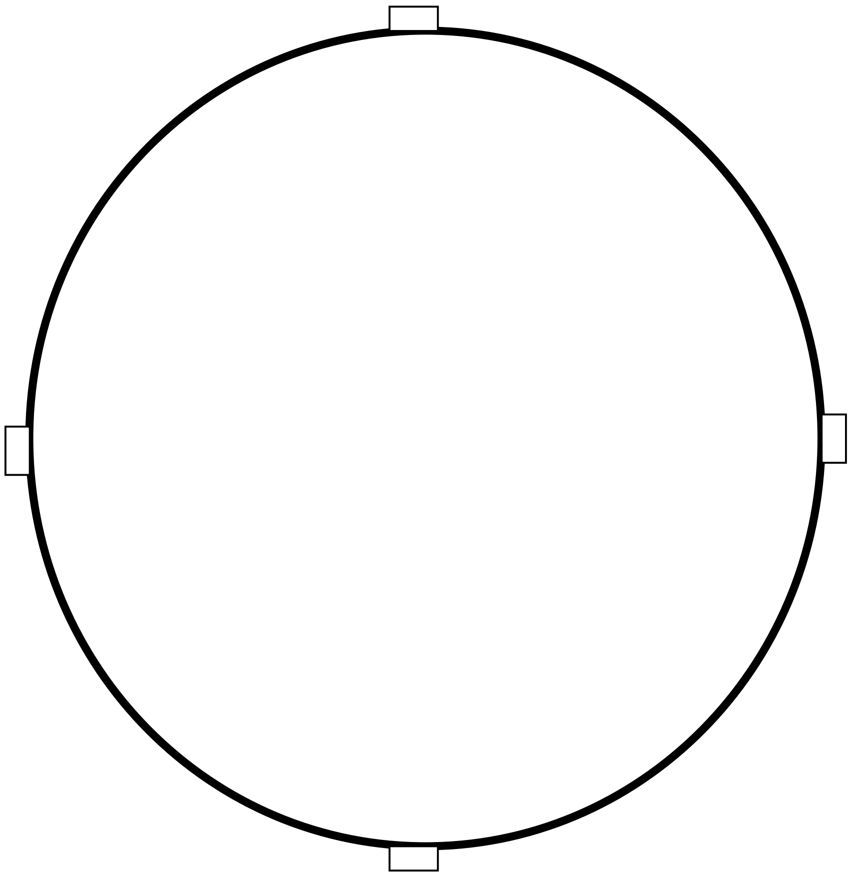

Figure I:
A Circular City Only With C-retailers

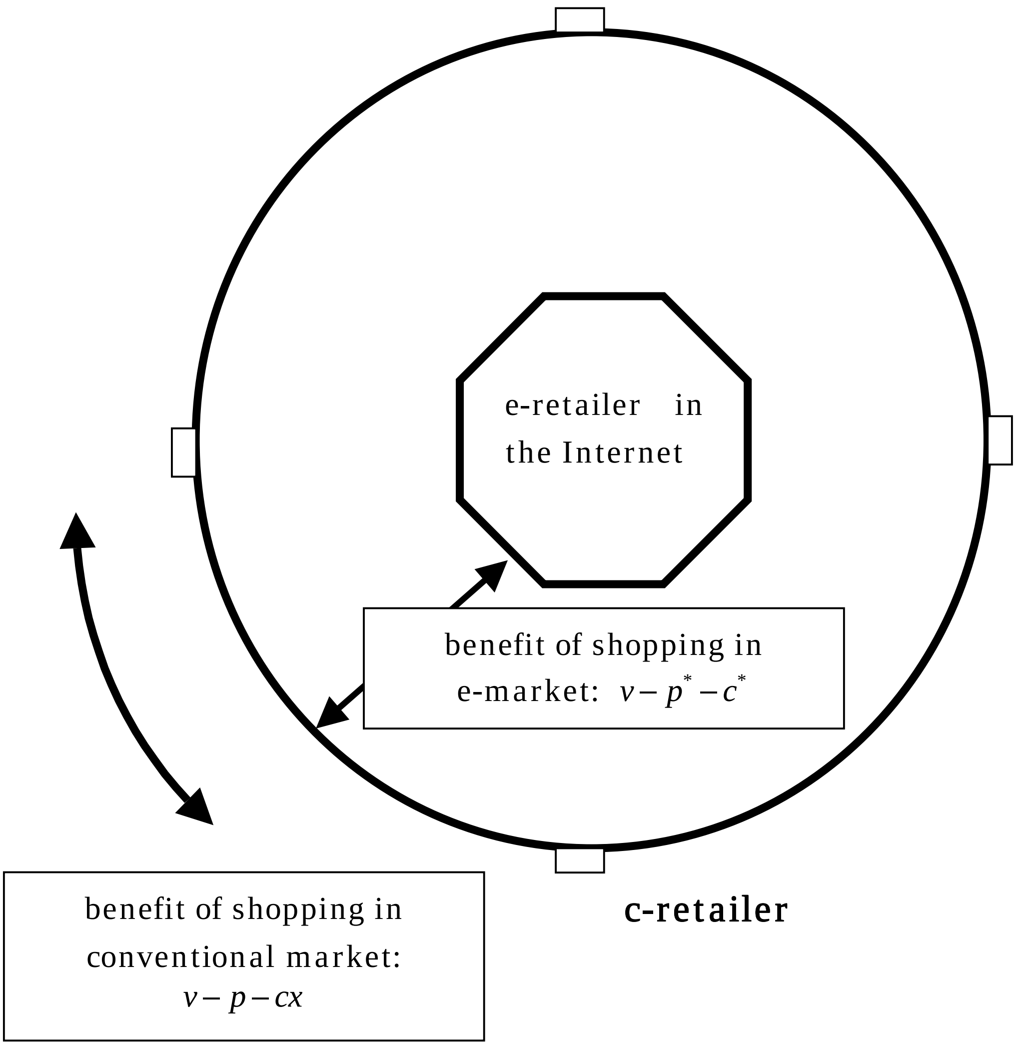

Figure II:
A Circular City With C- and E-Retailers