CIRJE-F-202

A Theory of Optimal Tariffs under a Revenue Constraint

Tatsuo Hatta
The University of Tokyo

Yoshitomo Ogawa
Kinki University

March 2003

A Theory of Optimal Tariffs under a Revenue Constraint

Tatsuo Hatta [a,*], Yoshitomo Ogawa [b]

[a]*Center for Spatial Information Science and Center for International Research on the Japanese Economy, Faculty of Economics, University of Tokyo.*

[b]*School of Business and Economics, Kinki University*

Abstract

This paper examines the optimal tariff structure under a revenue constraint. When a fixed level of tax revenue has to be collected from the tariff alone, no adjustment in tariff rates can achieve an efficient resource allocation, even in a small open economy. Hence, the optimal tariff problem arises under a revenue constraint. We show that the revenue-constrained optimal tariff structure is characterized by the following two rules: (i) the optimal tariff rate is lower for the import good that is a closer substitute for the export good; and (ii) the stronger the cross-substitutability between imports, the closer the optimal tariff is to uniformity. This theoretically explains why empirical studies have shown that the efficiency loss from a uniform tariff structure is negligible.

Keywords: Revenue-Constrained Optimal Tariff; Corlett-Hague Rule; Cross-Substitutability Rule

JEL classification: F11; F13; H21.

---

[*] Corresponding authors. H. Hatta. Address: 705 Faculty of Economics Building, University of Tokyo, 7-3-1, Hongo, Bunkyo-ku, Tokyo, 113-0033, Japan. Tel.&Fax: +81-3-5841-5631. E-mail address: t@hatta.net. Y. Ogawa. Address: School of Business and Economics, Kinki University, 3-4-1, Kowakae, Higashiosaka, Osaka 577-8502, Japan. E-mail address: ogawa@shok.kindai.ac.jp.

1. Introduction

In many developing countries, tariffs are the main source of government revenue.[1] However, if a fixed level of tax revenue has to be collected from tariffs alone, no adjustment in tariff rates can achieve efficient resource allocation, as first pointed out by Dasgupta and Stiglitz (1974). Thus, we need to analyze the tariff structure that attains the second-best resource allocation. We call this *the optimal tariff problem under a revenue constraint*. This problem does not arise in an economy with a lump-sum tax, but it does arise in an economy without a lump-sum tax even when the economy is small.

This problem is entirely different from the more familiar optimal tariff problem in a large economy, which was studied by Kaldor (1940) and Johnson (1954–55) among others. In a large economy, the optimal tariff problem arises even if a lump-sum tax is available.

The optimal tariff problem under a revenue constraint is an extension of the optimal taxation problem pioneered by Ramsey (1927) and Diamond and Mirrlees (1971).[2] This is caused strictly by an institutional framework within which a fixed level of tax revenue has to be collected by tariffs alone. The exportable good in the optimal tariff model plays the role of the leisure good in the optimal commodity tax model, as the untaxed good.[3] Dasgupta and Stiglitz (1974) originated the study of optimal tariffs in a small open economy. Dahl, Devarajan and van Wijnbergen (1986), Heady and Mitra (1987) and Mitra (1992) numerically analyzed

---

1 See Rajaram (1994) for a survey on actual trade policies under a revenue constraint in several developing countries.

2 In the R-D-M model, labor supply is endogenous, and the distortion is generated between goods and leisure. Commodity taxes and wage subsidies at a uniform rate remove this distortion. However, tax revenue is zero. At this point, the optimal commodity tax problem is generated. In the optimal tariff model, we make the labor supply constant, and hence we can disregard this distortion.

3 See Hatta (1994) for a comparison of the two theories in a simple context. In the optimal commodity tax model, labor supply is endogenous and leisure is untaxable. Hence, a distortion is generated between goods and leisure. In the revenue-constrained optimal tariff model, labor supply is assumed to be constant, but the consumption of the exportable goods is untaxed. Hence, a distortion is generated between the exportable and the importable goods. As Hatta (1994) explains, the exportable good in the optimal tariff model plays the role of the leisure good in the

optimal tariff rates for a few developing countries. More recently, this problem has been studied by Hatta (1994), Dahl, Devarajan and van Wijnbergen (1994), Panagariya (1994), Chambers (1994), Mitra (1994) and Keen and Ligthart (2002).[4]

This literature has shown non-uniformity of the optimal tariff structure in a number of ways. Dasgupta and Stiglitz (1974), Dahl, Devarajan and van Wijnbergen (1986) and Panagariya (1994) have derived the inverse elasticity rule in the trade context under the assumption of zero cross-substitutability between importable goods. In addition, simulations have been conducted by assuming Leontif-type production functions in part.

However, the literature has not demonstrated an explicit, general optimal tariff formula for a three-good trade model that allows full technological substitutability. This is in contrast with the literature on optimal tax, where the Harberger expression (1964) gives a fully general optimal tax formula for a three-good economy. A Harberger-like formula has been derived for an optimal tariff in Hatta (1994), but only under an extremely simple assumption about technology.

In the present paper, we derive an explicit formula for a revenue-constrained optimal import tariff in a trade model with full technological substitutability. Our optimal tariff formula is expressed in terms of the elasticity of the *excess* demand function, i.e., the compensated demand function minus the supply function. Thus, the optimal tariff structure depends on the supply elasticities as well as the demand elasticities. This contrasts with the Harberger expression for optimal tax, which is expressed in terms of demand elasticities alone, even if the production possibility frontier is strictly convex, as shown by Auerbach (1985). This difference is caused by a variation in the consumer's budget constraints in the two models: the producer's behavior does not affect the consumer's income in the optimal tax model, while it does in the optimal tariff models.

As special cases of this general formula, we obtain various optimal tariff rules, such as the inverse

---

optimal commodity tax model, as the untaxed goods.

4 Among the literature cited here, Keen and Ligthart (2002) analyzed the welfare effect of a simultaneous change in tariffs and commodity taxes under a revenue constraint. A similar analysis was undertaken by Panagariya (1992),

elasticity rule and the Corlett and Hague-type tariff rule. In particular, we observe that the stronger the cross-substitutability between imports, the closer the optimal tariff is to uniformity. These rules are expressed in terms of the elasticities of excess demand functions. This means that the optimal tariff rules depend upon both producer's and consumer's responses to price changes. This is in contrast with the optimal tax rules, which are expressed in terms of demand elasticities alone.

We investigate two features of the optimal tariff rules through this formula. First, the formula enables us to untangle the puzzle shown by Dahl et al. (1994, p. 222) and Mitra (1992, p. 246), which is that the welfare loss caused by a uniform tariff rather than the optimal tariff is negligible. Indeed, the welfare loss reported by Dahl et al. (1994) is 0.005% of the welfare level under the optimal tariff, and that reported by Mitra is 0.05%. The reason for this negligible welfare loss from a uniform tariff has not been explained. Our formula shows that the virtual optimum is attained by any tariff structure close to uniformity when imports are closely substitutable for each other in consumption and production.

Second, our formula is generalized to complex trade settings. For example, it is extended to the case where a non-tradable good exists. The non-tradable good has an endogenous price, which is adjusted so as to keep the amount of demand equal to that of supply. Even in this case, the implication of our basic formula, that the optimal tariff structure is close to uniformity when imports are closely substitutable, is robust. In addition, our formula is interpretable in the case where one of the imports is an imported input (intermediate good). An imported input is used in the production of final goods, so that imposing a tax on it distorts resource allocation in the production sector.

The imported input and the non-tradable good are peculiar to the optimal tariff framework, and do not appear in the optimal tax framework analyzed in a closed economy model. Our optimal tariff formulae expressed in terms of elasticities of net demand are essential in analyzing the imported input case.

In Section 2, we present the model. Section 3 proves that the first-best tariff (subsidy) policies would yield zero tariff revenue. In Section 4, a formula that characterizes the optimal tariff structure is derived and we examine the optimal tariff structure. Section 5 analyzes the optimal tariff problems in the presence of the

---

who considered an economy where only tariffs can be adjusted and there is an imported input subject to a tariff.

non-tradable good. Section 6 extends the optimal tariff rules to the cases with the export tax (or subsidy) and with an imported input.

2. The Model

We consider an economy that satisfies the following assumptions:

Assumption 1: The economy is small and open. It has perfectly competitive markets for goods and factors. The economy produces three goods: one export good and two import goods. The only inputs of the economy are endowed factors. We denote the export good by 0 and the import goods by 1 and 2.

Assumption 2: There is a representative consumer, who initially possesses all of the factors (whose endowments are fixed), has an income m, consisting of factor incomes,[5] and consumes all of the three final goods. The consumer has a well-behaved utility function $u(\boldsymbol{x})$, where $\boldsymbol{x}' = (x_0,\ x_1,\ x_2)$ is the demand vector of the final goods, and he or she chooses the commodity bundle that maximizes the utility level under given prices and income.[6]

The budget constraint of the consumer is given by:

$$\boldsymbol{q}'\boldsymbol{x} = m\,, \tag{1}$$

where $\boldsymbol{q}' = (q_0,\ q_1,\ q_2)$ is the domestic-price vector.

The consumer's compensated demand function for the i-th good is given by:

---

[5] The profit is zero, because free entry and exit are assumed.

[6] Since the level of the public good provision is fixed in the analysis, it does not enter the utility function as an explicit argument. The function is well behaved if it is: (i) increasing in each argument; (ii) strictly quasi-concave;

$$x_i = x_i(\boldsymbol{q},\ u), \qquad \text{for } i = 0,1,2, \tag{2}$$

where u is the utility level.

Assumption 3: A producer maximizes profit, taking prices as given.

The aggregate of the net revenues of all firms, and hence of all industries, is equal to the income of the consumer. Thus, the aggregate budget equation of the producers is given by:

$$\boldsymbol{q}'\boldsymbol{y} = m, \tag{3}$$

where $\boldsymbol{y}' = (y_0,\ y_1,\ y_2)$ is the output vector.

The supply function of the economy provides the commodity bundle that maximizes the total revenue $\boldsymbol{q}'\boldsymbol{y}$ of the production sector, under a given level of technology and prices. Its i-th element is given by:[7]

$$y_i = y_i(\boldsymbol{q}), \qquad \text{for } i = 0,1,2. \tag{4}$$

Assumption 4: Tariffs are imposed on the two imports.

The relationship between world prices, domestic prices, and import tariffs is given by:

---

and (iii) twice continuously differentiable.

[7] Since the domestic factors are fully employed by the production sector and their supply levels are fixed, they do not enter the supply function as explicit arguments.

$$\boldsymbol{q} = \boldsymbol{p} + \boldsymbol{t}, \tag{5}$$

where $\boldsymbol{p}' = (p_0,\ p_1,\ p_2)$ is the world price vector, and $\boldsymbol{t}' = (t_0,\ t_1,\ t_2)$ is the specific tariff vector.

Assumption 5: The only revenue source of the government is import tariffs. In particular, the government cannot levy commodity taxes or income taxes. The government spends all of the tariff revenue on the purchase of the public good, which is imported from a foreign country.[8]

Thus, the budget equation of the government can be written as:

$$\boldsymbol{t}'(\boldsymbol{x} - \boldsymbol{y}) = r, \tag{6}$$

where r is government spending on the public good and $\boldsymbol{x} - \boldsymbol{y}$ represents the net import vector of the private goods.

Assumption 6: The international balance of payments is in equilibrium, and is written as:

$$\boldsymbol{p}'(\boldsymbol{x} - \boldsymbol{y}) + r = 0. \tag{7}$$

The left-hand side of the equation represents the sum of the international value of the net imports of private goods and the public good.

---

[8] Since the world price and the government revenue are constant, and since the government expenditure is equal to revenue, the quantities imported by the government should also be constants. The public good may be the good traded by the private sector of the home country. In that case, the excess demand for the i-th tradable good is

Assumption 7: The world price of the export good is chosen as the numeraire: $p_0 = 1$.

Equation (7) is the market equilibrium condition. Equations (1), (3) and (6) are the budget equations of the economic agents. However, Equations (1) and (3) can be combined into:

$$\boldsymbol{q}'\boldsymbol{x} = \boldsymbol{q}'\boldsymbol{y}. \tag{8}$$

This equation is the budget constraint of the private sector, and implies that the consumer's expenditure equals the producer's revenue. Since Equations (7) and (8) immediately yield (6), we represent this economy by Equations (2), (4), (5), (7) and (8) in the following.

Definition: An economy satisfying Assumptions 1 through 7 is called a *General Tariff Economy (hereafter, GTE)*. When Equations (2), (4), (5), (7) and (8) are satisfied, it is said that *the GTE is in equilibrium*.

A special case of the GTE is given a name, as stated below.

Definition. An economy satisfying Assumptions 1 through 7 and $t_0 = 0$ is called an *Import Tariff Economy (hereafter, ITE)*. When Equations (2), (4), (5), (7) and (8) are satisfied and $t_0 = 0$, we say that *the ITE is in full equilibrium*.

In the ITE, $q_0 = 1$ holds from Assumption 7.

We define the excess demand functions as:

$$z_i(\boldsymbol{q},\ u) \equiv x_i(\boldsymbol{q},\ u) - y_i(\boldsymbol{q}), \qquad \text{for } i = 0,1,2.$$

---

redefined as $z_i = x_i - y_i - \bar{g}_i$ where $\bar{g}_i$ is the amount the government imports of the i-th good.

and

$$z(q,\ u) \equiv x(q,\ u) - y(q),$$

where $z'(q,\ u) = (z_0(q,\ u),\ z_1(q,\ u),\ z_2(q,\ u))$. By substituting these functions for $x - y$ in (7) and (8), we have:

$$q'z(q,\ u) = 0, \tag{9}$$

$$p'z(q,\ u) + r = 0. \tag{10}$$

In terms of this notation, the GTE is in full equilibrium if and only if it satisfies Equations (5), (9) and (10). The set of Equations (9) and (10) contains four variables, q_0, q_1, q_2 and u, since r is fixed by assumption. When two of the four variables are exogenously given, the two equations determine the remaining variables. For example, if q_0 and u are given, the model determines the remaining variables $(q_1,\ q_2)$. Therefore, from Equation (5), the tariff vector t, which maximizes the utility level u, can be found. The following definition is now required:

Definition: The tariff combination $(t_1,\ t_2)$ that maximizes the utility level u in the model of (5), (9) and (10) for a fixed level of r and for the zero level of t_0 is called *the optimum tariff of the ITE.*

3. Tariff Revenue and Efficient Resource Allocation

A Pareto optimal resource allocation is attained only if the international and domestic prices of the traded goods are proportional. At first, it may seem that the optimal tariff of the ITE must be a proportional tariff structure, which makes the international and domestic prices of the traded goods proportional. It turns out that a proportional tariff yields zero revenue, as is shown below. Hence, any tariff structure that yields positive revenue has to be non-proportional and is inefficient in resource allocation. In other words, the optimal tariff of the ITE is the one that minimizes the inevitable distortion.

We now show that a proportional tax yields zero revenue. To this end, we define the *ad valorem* equivalent rate of t_i by:

$$\tau_i = \frac{t_i}{q_i}, \qquad \text{for } i = 0,1,2. \tag{11}$$

This and (5) yield:[9]

$$q_i = p_i + \tau_i q_i, \qquad \text{for } i = 0,1,2, \tag{12a}$$

and

$$q_i = p_i + \frac{\tau_i}{1-\tau_i} p_i, \qquad \text{for } i = 0,1,2. \tag{12b}$$

From (12a), we also have:[10]

$$q_i = \frac{1}{1-\tau_i} p_i, \qquad \text{for } i = 0,1,2. \tag{12c}$$

A tariff structure is called *proportional* if all tradable (including exportable) goods share an identical

---

9 Let $t_i = \sigma_i p_i$. Then $\sigma_i p_i = \tau_i q_i$, and hence $\sigma_i = \tau_i q_i / p_i$. This and $p_i = (1-\tau_i) q_i$ yield $\sigma_i = \tau_i / (1-\tau_i)$, implying (12b).

10 The definition of the *ad valorem* tariff rate, as shown by (11), ensures that $\tau_i < 1$. Hence, $1-\tau_i > 0$.

ad valorem tariff rate, i.e., if:

$$\tau_i = \varepsilon, \qquad \text{for } i = 0,1,2, \tag{13}$$

holds for some scalar ε. Equation (12c) implies that, under a proportional tariff structure, the domestic prices of both exports and imports are proportionally higher than their world prices. Hence, efficient resource allocation is attained with a proportional tariff. The fact that the domestic price of the exports is higher than the world price implies that a subsidy is given to the export goods at the same rate as the import tariffs. Thus, the "proportional tariff", defined as above, implies a combination of positive tariffs on the imports and a subsidy of equal rate to the export. This means that a proportional tariff structure yields zero revenue, and hence we have the following:

Theorem: *In the GTE, proportional tariff structures achieving efficient resource allocation yield zero revenue.*

Proof. Under proportional tariffs, it follows from (12c) and (13) that:

$$q_i = \frac{1}{1-\varepsilon} p_i . \tag{14}$$

Substituting (14) for q_i in (9) yields:

$$\boldsymbol{p}'\boldsymbol{z} = 0 .$$

From this and (10), we obtain $r = 0$. Q.E.D.

This implies that under a proportional tariff structure, all of the revenue collected by import tariffs is spent on the export subsidy. Therefore, only a non-proportional tariff can raise a positive tariff revenue. In other words, a positive tariff revenue necessarily generates a distortion.[11]

A tariff structure is called *uniform* if all import goods share an identical *ad valorem* tariff rate, i.e., if $\tau_0 = 0, \tau_1 = \varepsilon, \tau_2 = \varepsilon$ for some scalar ε. A uniform tariff structure can raise a positive tariff revenue. However, under a uniform tariff structure, the vector of the domestic prices of the three goods is not proportional to that of the world prices, even though the vector of the domestic prices of the two imports is proportional to that of the world prices. Hence, the resource allocation under a uniform tariff is inefficient. In the next section, we examine how the optimal tariff is different from uniformity under a positive revenue constraint.

4. Optimal Tariff Structure in the ITE

4-1. The Basic Lemma

In this section, we examine the optimal import tariff structure of the ITE, i.e., the economy that has a zero export tariff and subsidy.

Let $z_{ij} = \partial z_i / \partial q_j$. We define the import elasticity of the i-th good with respect to the price q_j by $\eta_{ij} = q_j z_{ij} / z_i$. Then we have the following:

Lemma. *In the ITE, the optimal tariff structure* (τ_1, τ_2) *satisfies the following:*

(i) $$\frac{\tau_1}{\tau_2} = \frac{\eta_{12} - \eta_{22}}{\eta_{21} - \eta_{11}},$$

(ii) $$\frac{\tau_1}{\tau_2} = \frac{\eta_{20} + \eta_{12} + \eta_{21}}{\eta_{10} + \eta_{12} + \eta_{21}},$$

---

[11] Sandmo (1974) discussed this problem in the optimal tax context.

and

$$\text{(iii)} \qquad \begin{aligned} \tau_1 &= (\eta_{20} + \eta_{12} + \eta_{21})\theta , \\ \tau_2 &= (\eta_{10} + \eta_{12} + \eta_{21})\theta , \end{aligned}$$

where $\theta > 0$.

Proof. We must first choose a level of $\boldsymbol{q}$ that maximizes the utility level in the model of (9) and (10) for the fixed level of r:[12]

$$\begin{aligned} &\max_{t_1,\ t_2,\ u} && u, \\ &\text{s.t} && \boldsymbol{q}'\boldsymbol{z}(\boldsymbol{q},\ u) = 0, \\ & && \boldsymbol{p}'\boldsymbol{z}(\boldsymbol{q},\ u) + r = 0. \end{aligned}$$

The Lagrangian of this maximization problem is:

$$L = u - \lambda(\boldsymbol{q}'\boldsymbol{z}(\boldsymbol{q},\ u)) - \delta(\boldsymbol{p}'\boldsymbol{z}(\boldsymbol{q},\ u) + r) ,$$

where λ and δ are Lagrange multipliers. The first-order conditions with respect to q_i are $\lambda z_i + \delta \boldsymbol{p}'\boldsymbol{z}_i = 0$, for $i = 1, 2$, where $\boldsymbol{z}_i' = (\partial \boldsymbol{z} / \partial q_i) = (z_{0i},\ z_{1i},\ z_{2i})$. By using the homogeneity condition, $\boldsymbol{q}'\boldsymbol{z}_i = 0$, of the compensated demand function, this equation can be rewritten as:

$$-\begin{bmatrix} z_{11} & z_{21} \\ z_{12} & z_{22} \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \nu \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, \tag{15}$$

where $\nu = -\lambda/\delta$.[13] Solving (15) for t_1 and noting $z_{12} = z_{21}$ and $\tau_i = t_i/q_i$, we obtain:

$$\tau_1 = (\eta_{12} - \eta_{22}) \cdot \theta, \tag{16a}$$

where

$$\theta = \nu z_1 z_2 / q_1 q_2 \, (z_{11} z_{22} - (z_{12})^2). \tag{17}$$

Similarly, we have:

$$\tau_2 = (\eta_{21} - \eta_{11}) \cdot \theta. \tag{16b}$$

From (16), we obtain Formula (i).

Since:

$$\begin{aligned} \eta_{10} + \eta_{11} + \eta_{12} &= 0, \\ \eta_{20} + \eta_{21} + \eta_{22} &= 0, \end{aligned} \tag{18}$$

Formula (i) immediately yields (ii).

---

12 See Mirrlees (1976) and Hatta (1993) for this formulation of the maximization problem.

13 The proof following this expression adapts the proof of Diamond and Mirrlees (1971, pp. 262-263) for optimal

From (16) and (18), we obtain two equations in Formula (iii). Multiplying (15) by $(t_1,\ t_2)$ gives:

$$-[t_1,\ t_2]\begin{bmatrix} z_{11} & z_{21} \\ z_{12} & z_{22} \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \nu(t_1 z_1 + t_2 z_2)$$

$$= \nu r\,.$$

Since the left-hand matrix is negative semi-definite, ν has the same sign as government revenue r. From this and (17), we obtain $\theta > 0$ since the denominator of (17) is positive.

Q.E.D.

A formula similar to (ii) was first obtained by Harberger (1964) for an optimal commodity tax.[14] However, his formula is expressed in terms of demand elasticities. Even in a model with a fully substitutable production possibility surface, the supply elasticities have no place in the optimal tax formula, as demonstrated by Auerbach (1985).

On the other hand, our formula is expressed in terms of the elasticities of the *excess* demand function, i.e., the compensated demand function minus the supply function. Hence, unlike the optimal tax structure, the optimal tariff structure is affected by supply elasticities.[15]

---

taxation.

[14] This paper was later published as Chapter 2 in Harberger (1974). The Harberger formula is found on p. 49 in Harberger (1974).

[15] We define the compensated demand elasticity as $\phi_{ij} = q_j x_{ij} / x_i$ and the supply elasticity as $\varepsilon_{ij} = q_j y_{ij} / y_i$. From the definition of the excess demand elasticity, we have:

$$\eta_{ij} = \frac{q_j z_{ij}}{z_i}$$

$$= \frac{q_j x_{ij} - q_j y_{ij}}{z_i}$$

The consumer's budget equations are different in the two models. In the optimal tariff model, the budget equation is given by $q'x = q'y$ as in (8); in the optimal tax model, it is given by $q'x = q'\overline{x}$ where $\overline{x}$ represents the initial endowments that the consumer possesses. Hence, the producer's response to a price change affects the consumer's budget in the optimal tariff model, but not in the optimal tax model.

Note that in the optimal tax model, the producer's profit is assumed to be zero either because of free entry or due to a 100% tax on profit, while in the model of the optimal tariff, no such assumptions are made.

4-2. The Inverse Elasticity Rule

We say that import goods are *independent* of each other if $(\partial z_1/\partial q_2) = (\partial z_2/\partial q_1) = 0$, i.e., $\eta_{12} = \eta_{21} = 0$.

Inverse Elasticity Rule: *The optimal tariff rate is inversely proportional to the own elasticity of excess demand if the imports are independent of each other.*

Proof. Independence among the imports implies $\eta_{12} = \eta_{21} = 0$, and hence Formula (i) degenerates into:

$$\frac{\tau_1}{\tau_2} = \frac{\eta_{22}}{\eta_{11}}, \tag{19}$$

which proves this rule. Q.E.D.

$$= \frac{x_i}{z_i} \cdot \phi_{ij} - \frac{y_i}{z_i} \cdot \varepsilon_{ij}.$$

By substituting these for η_{ij} s in the formulae in the Lemma, we find that the optimal tariff rates depend on the compensated demand elasticit<u>ies</u> and the supply elasticit<u>ies</u>.

This rule has been widely used in the literature on optimal tariffs under a revenue constraint. The seminal paper by Dasgupta and Stiglitz (1974, p. 21) showed that the revenue-constrained optimal tariff is not uniform, by directly establishing the inverse elasticity rule. The inverse elasticity rule was also derived by Dahl at al. (1994, p. 217) and Panagariya (1994, p. 234).

Formula (i) of the Lemma immediately implies that if cross-elasticities between the two imports, η_{12} and η_{21}, are not equal to zero, the ratio τ_1/τ_2 would diverge from the inverse elasticity rule. In fact, when $\eta_{12} \neq \eta_{21}$, the optimal tariff rate of the import with the lower own elasticity may become lower than the rate of the other import.

Note, however, that Formula (i) does not imply that the tariff system necessarily approaches uniformity as cross-elasticities become large when $\eta_{12} \neq \eta_{21}$.

4-3. The Corlett–Hague Rule

Definition: We say that the i-th good is *a closer substitute* for the k-th good than the j-th good, if $\eta_{ik} > \eta_{jk}$.

We are in a position to state and prove the following proposition.

Proposition 1: *The optimal tariff rate is lower for the import good that is the closer substitute for the export good.*

Proof. Taking the difference of the two equations in Formula (iii), we have $\tau_1 - \tau_2 = (\eta_{20} - \eta_{10})\theta$. Since $\theta > 0$, this expression shows that $\tau_1 > (<)\tau_2$ if and only if $\eta_{20} > (<)\eta_{10}$. Q.E.D.

This implies that the optimal tariff rate is higher for the good that is more complementary with the export good.

The export is the untaxed good, and hence it is over-consumed. Taxation on the good that is more complementary with the export partially offsets this over-consumption. This shows that the ranking of tariff rates depends upon the relative degree of complementarity between the taxed goods (import goods) and the untaxed good (the export good). Since this was first shown by Corlett and Hague (1953), in the context of commodity taxation, we call this the *Corlett–Hague rule*.

4-4. The Cross-Substitutability Rule

The Corlett–Hague rule shows that optimal tariff structure is non-uniform, while the following proposition shows that the degree of non-uniformity is limited by the degree of cross-substitutability between imports.

Proposition 2: *Assume that* $\tau_1 > 0$ *and* $\tau_2 > 0$. *The stronger the cross-substitutability between imports, the closer is the optimal tariff to uniformity if all the cross-elasticities involving the export good are kept constant.*

Proof. In case $\tau_1 > 0$ and $\tau_2 > 0$, we obtain $\eta_{20} + \eta_{12} + \eta_{21} > 0$ and $\eta_{10} + \eta_{12} + \eta_{21} > 0$, respectively, from Formula (iii). The cross-elasticity of imports $\eta_{12} + \eta_{21}$ is common in both the numerator and the denominator of Formula (ii). When η_{10} and η_{20} are kept constant, the larger is the cross-substitutability, the closer are the values of the numerator and denominator, and hence the closer is the ratio of the tariff to uniformity. Q.E.D.

Proposition 2 may be called the *Cross-Substitutability Rule*. The non-uniformity of the tariff rates will create distortions in the choice among the imports. In particular, when cross-elasticities among imports are high, a non-uniform tariff structure creates strong distortionary effects in the choice among the imports,[16] and hence the optimum tariff structure tends to be close to uniform.

[16] For the case that generalizes the number of taxed goods, see Hatta (1986).

Although the inverse elasticity rule is the most well-known optimal tax rule, its applicability is extremely limited. A good usually has high demand elasticity because it has close substitutes. For example, beef has high demand elasticity because chicken and pork are available as substitutes. However, the inverse elasticity rule assumes that a good has no substitutes among imported goods. In the context of our trade model, this assumption implies that an imported good has high demand elasticity if and only if it is a close substitute for the exported good, and a good has low demand elasticity if and only if it is independent of the exported good.

There is an intriguing puzzle in this literature. Empirical simulations by Dahl at al. (1994, p. 222) and Mitra (1992, p. 246) show that the welfare loss caused by a uniform tariff, rather than the optimal tariff, is negligible.

As was pointed out earlier, the inverse elasticity rule appears contradictory to the finding of Dahl et al. (1986, 1994) and Mitra (1992) that the welfare loss caused by a uniform tariff structure is negligible. This is because their findings suggest that virtual optimality is attained by a uniform tariff structure. However, the puzzle is solved if we realize the unrealistic nature of the assumption in the inverse elasticity rule that the imports are independent of each other in both consumption and production. It is likely that the cross-substitutability term among the imports is dominating, undermining the basic assumption of the inverse elasticity rule.

We assume that import tariffs are positive in Proposition 2. Note that the optimal tariff rate can be negative for an import if the import is a substitute of the export and if it is a complement of the other import.

5. Optimal Tariff Rules under the Presence of a Non-Tradable Good.

In this section, we introduce a non-tradable good to the ITE and examine the optimal tariff structure in such an economy.

We consider the model obtained by adding the non-tradable good to the ITE as the fourth good. We continue to adopt the notations of the ITE, and denote the non-tradable good by n. The market-clearing condition of the non-tradable good is added to the equilibrium conditions in the ITE, and it is expressed as:

$$x_n = y_n, \tag{20}$$

where x_n and y_n denote the demand and supply level of the non-tradable good, respectively. The excess demand function in this economy can be represented by:

$$z_i = z_i(\boldsymbol{q},\ q_n,\ u), \qquad \text{for } i = 0,1,2,n, \tag{21}$$

where q_n is the price of the non-tradable good. From (20) and (21), we obtain:

$$z_n(\boldsymbol{q},\ q_n,\ u) = 0 .$$

This equation determines the price q_n as a function $\boldsymbol{q}$ and u. Suppose that this equation can be solved for q_n. The resulting function may be written as $q_n = q_n(\boldsymbol{q},\ u)$. Substituting this for q_n in (21) yields:

$$z_i^*(\boldsymbol{q},\ u) = z_i(\boldsymbol{q},\ q_n(\boldsymbol{q},\ u),\ u), \qquad \text{for } i = 0,1,2.$$

We call $z_i^*(\boldsymbol{q},\ u)$ the *reduced form of the excess demand function* (hereafter, the *reduced form*). The reduced form has the same properties as the excess demand function used in the ITE.[17]

Note that Equations (9) and (10) also hold in this economy.[18] Then, replacing the reduced form

---

[17] This function is (i) differentiable, (ii) homogenous of degree zero, and (iii) concave in $\boldsymbol{q}$. For the detail, see Dixit and Norman (1982, p. 91).

[18] The budget equation of the private sector in this economy can be written as $\boldsymbol{q}'\boldsymbol{x} + q_n x_n = \boldsymbol{q}'\boldsymbol{y} + q_n y_n$. However, this equation is reduced to (8) by (20).

$z_i^*(\boldsymbol{q},\ u)$ for the excess demand function $z_i(\boldsymbol{q},\ u)$ in (9) and (10), we find that the equilibrium equations in this economy can be expressed as:

$$\boldsymbol{p}'\boldsymbol{z}^*(\boldsymbol{q},\ u) + r = 0\,,$$

$$\boldsymbol{q}'\boldsymbol{z}^*(\boldsymbol{q},\ u) = 0\,.$$

Solving the maximizing problem in the same way as in the proof of the Lemma, we obtain the Harberger expression for the presence of a non-tradable good:

$$\frac{\tau_1}{\tau_2} = \frac{\eta_{20}^* + \eta_{12}^* + \eta_{21}^*}{\eta_{10}^* + \eta_{12}^* + \eta_{21}^*}\,, \tag{22}$$

which is the same as Formula (ii) except that each η_{ij}^* is replaced by $\eta_{ij}^* = (q_j / z_i^*) \cdot (\partial z_i^* / \partial q_j)$. Formula (22) has implications similar to those of Propositions 1 and 2. Note that $(\partial z_i^* / \partial q_j)$ involves the indirect effect through the price change of the non-tradable good.

6. Other Extensions.

6-1. An Export Tax or Subsidy

In this section, we analyze the optimal tariff problem in the GTE, where a tax or subsidy is imposed on the export good.

In the GTE, if all the tariffs $(t_0,\ t_1,\ t_2)$ are simultaneously adjusted, the tariff vector that maximizes the utility level is not unique.[19] Fixing a tariff level on one of the three tradable goods, we can find the optimal

---

[19] Since $\boldsymbol{x}(\boldsymbol{q},\ u)$ and $\boldsymbol{y}(\boldsymbol{q})$ are homogenous of degree zero with respect to $\boldsymbol{q}$, a proportional increase in $\boldsymbol{q}$ dose not affect the value of $\boldsymbol{x}(\boldsymbol{q},\ u)$ and $\boldsymbol{y}(\boldsymbol{q})$, keeping the level of utility and the tariff revenue of the GTE intact. If $\boldsymbol{q}^*$

tariff levels on the two remaining goods. Here, we will fix the export tax (or subsidy) at a given level. In this case, we obtain the following:

$$\frac{\tau_1 - \tau_0}{\tau_2 - \tau_0} = \frac{\eta_{20} + \eta_{12} + \eta_{21}}{\eta_{10} + \eta_{12} + \eta_{21}}. \tag{23}$$

This formula is derived from Lemma 4 in Hatta and Ogawa (2002, p. 12).[20] Assume that $\tau_i - \tau_0 > 0$ (for $i = 1,2$) and $\eta_{i0} + \eta_{12} + \eta_{21} > 0$ (for $i = 1,2$). Noting that the export tax (or subsidy) rate τ_0 is fixed, we find that the cross-substitution rule is satisfied since the cross-elasticity of imports $\eta_{12} + \eta_{21}$ is common in both the numerator and the denominator of (23). We also obtain the Corlett-Hague rule: if and only if $\eta_{10} < (>) \eta_{20}$ then $\tau_1 > (<) \tau_2$ holds.

6-2. An Imported Input

We can analyze an imported input in our model simply by assuming that the consumer neither consumes nor supplies one of the imported goods. Then all the optimal tariff rules for the ITE hold as such.

---

is an equilibrium domestic price vector of the GTE, then the vector $\kappa \boldsymbol{q}^*$ is also an equilibrium price vector of the same model for any positive scalar κ. The vectors of $\boldsymbol{q}^*$ and $\kappa \boldsymbol{q}^*$ have the identical equilibrium allocation and hence attain the same levels of utility and tariff revenue. Let $\boldsymbol{t}^* = \boldsymbol{q}^* - \boldsymbol{p}$, and $\boldsymbol{t}^{**} = \kappa \boldsymbol{q}^* - \boldsymbol{p}$. It is readily found that $\boldsymbol{t}^*$ and $\boldsymbol{t}^{**}$ have an identical equilibrium allocation. Since κ is any positive scalar, $\boldsymbol{t}^{**}$ is not unique. This result is not affected even if $\boldsymbol{q}^*$ is the equilibrium price vector that maximizes the utility level. Therefore, the optimal tariff vector in the GTE is not unique. Hence, the optimal tariff rate vector is also not unique.

20 Equation (19) in Hatta and Ogawa (2002, p. 12) is:

$$\frac{\tau_1}{\tau_2} = \left(\frac{\eta_{20} + \eta_{12} + \eta_{21}}{\eta_{10} + \eta_{12} + \eta_{21}}\right) \cdot \left(1 - \frac{\tau_0}{\tau_2}\right) + 1 \cdot \frac{\tau_0}{\tau_2}.$$

Multiplying both sides by $\tau_2 / (\tau_2 - \tau_0)$ and subtracting $\tau_0 / (\tau_2 - \tau_0)$ from both sides, we obtain (23).

It is generally considered that cross-substitution dominates in consumption. Thus, it is natural to assume that two imports are substitutable if they are consumption goods. However, the assumption of substitutable imports may not hold when one of the imports is an imported input. As Lopez and Panagariya (1992) pointed out, an imported input is necessarily complementary with a produced good in certain models. The first model of Lopez and Panagariya had a fixed coefficient technology with respect to the imported input, and their second model was the Heckscher-Ohlin model, where the number of primary factors is equal to that of produced goods. In both models, an increase in the price of an imported input necessarily increases the output level of the good that uses this input less intensively, through the Rybczynski effect.

In models where the Rybczynski effect does not work, the imported input can be substitutable for all goods. As was pointed out by Jones and Scheinkman (1977), the Rybczynski effect does not work even in a Heckscher-Ohlin model when the number of inputs is more than the number of outputs. In addition, in the case where each sector employs a specific factor and an imported input, for example, all goods are necessarily substitutable for the imported input. The cross-substitutability rule can then be applicable even in the case with the pure imported input.

7. Conclusion

In the present paper, we derived an explicit formula for the revenue-constrained optimal import tariff in a general trade model. This formula is expressed in terms of the elasticities of supply as well as those of demand.

This formula indicates that the optimal tariff is close to uniformity when imported goods are close substitutes either in consumption or in production. This theoretically explains why empirical studies show that the efficiency loss from a uniform tariff structure is negligible.

Our formula reveals that the existence of close substitutes in production makes the optimal tariff structure more uniform than otherwise. Our explicit formula was generalized to complex trade settings that incorporate a non-tradable good and an imported input. In the case of the imported input, some elasticities in our formula become purely supply elasticities. Even in these cases, our basic formula reveals the conditions on the structure of production under which the optimal tariff structure is close to uniformity.

Acknowledgments

We are grateful to Kenzo Abe, Takashi Fukushima, Michael Keen, Jenny E. Ligthart, Yoshiyasu Ono, Koji Shimomura, Katsuhiko Suzuki, David Tarr and Makoto Tawada for helpful comments and suggestions. We would like to thank Shantayanan Devarajan and Arvind Panagariya for introducing us to the topic of the optimal tariff problem under a revenue constraint.

References

Auerbach, A. J., (1985) "The Theory of Excess Burden and Optimal Taxation" in: A. J. Auerbach and M. Feldstein (eds), *Handbook of Public Economics I* (Amsterdam: North Holland).

Corlett, W. C. and D. C. Hague, (1953) "Complementarity and the Excess Burden Taxation" *Review of Economic Studies*, 21, pp. 21-30.

Dahl, H., S. Devarajan and S. van Wijnbergen, (1986) "Revenue-Neutral Tariff Reform: Theory and an Application to Cameroon" *Country Policy Department Discussion Paper*, 1986-25, The World Bank.

__________, (1994) "Revenue-Neutral Tariff Reform: Theory and an Application to Cameroon" *Economic Studies Quarterly*, 45, pp. 213-226.

Dasgupta, P. and J. Stiglitz, (1974) "Benefit-Cost Analysis and Trade Policies" *Journal of Political Economy*, 82, pp. 1-33.

Diamond, P. A. and P. A. Mirrlees, (1971) "Optimal Taxation and Public Production I and II" *American Economic Review*, 61, pp. 8-27 and pp. 261-278.

Dixit, A. and V. Norman, (1980) *A Theory of International Trade* (Cambridge: Cambridge University Press).

Harberger, A. C., (1964) "Taxation, Resource Allocation and Welfare", in: J. Due. ed., *The Role of Direct and Indirect Taxes in the Federal Reserve System*, (Princeton: Princeton University Press).

__________, (1974) *Taxation and Welfare* (Chicago: The University of Chicago Press).

Hatta, T., (1986) "Welfare Effect of Changing Commodity Tax Rates toward Uniformity" *Journal of Public Economics*, 29, pp. 99-112.

__________, (1994) "Why Not Set Tariffs Uniformly Rather Than Optimally" *Economic Studies Quarterly*, 45, pp. 196-212.

__________, (1993) "Four Basic Rules of Optimal Commodity Taxation", in: A. M. El-Agraa. ed., *Public and International Economics*, (New York: St. Martin) pp. 125-147.

Hatta, T. and Y, Ogawa, (2003) "Export Subsidy, Duty Drawback and Revenue-Constrained Optimal Tariff" *mimeo*.

Heady, C. J. and P. K. Mitra, (1987) "Distributional and Resource Raising Arguments for Tariffs" *Journal of*

Development Economics, 20, pp. 77-101.

Johnson, H. G., (1954-55) "Optimal Tariff and Retaliation" *Review of Economic Studies*, 22, pp. 142-153.

Jones, R. W. and J. Scheinkman, (1977) "The Relevance of Two-Sector Production Model in Trade Theory" *Journal of Political Economy*, 73, pp. 909-935.

Kaldor, N., (1940) "A Note on Tariffs and Terms of Trade" *Economica,* 7, pp. 377-380.

Keen, M. and J. M. Ligthart, (2002) "Coordinating Tariff Reduction and Domestic Tax Reform" *Journal of International Economics*, 56, pp. 489-507.

Lopez, R. and A. Panagariya, (1992) "On The Theory of Piecemeal Tariff Reform: The Case of Pure Imported Intermediate Input" *American Economic Review*, 82, pp. 615-625.

Mitra, P. K., (1992) "Tariff Design and Reform in a Revenue-Constrained Economy: Theory and an Illustration from India" *Journal of Public Economics*, 30, pp. 293-316.

__________, (1994) "Protective and Revenue Raising Trade Taxes: Theory and an Application to India" *Economic Studies Quarterly*, 45, pp. 265-287.

Mirrlees, J., (1976) "Optimal Tax Theory: A Synthesis" *Journal of Public Economics*, 6, pp. 327-358.

Panagariya, A., (1992) "Input Tariffs, Duty Drawbacks and Tariff Reform" *Journal of International Economics*, 32, pp. 132-147.

__________, (1994) "Why and Why-Not of Uniform Tariffs" *Economic Studies Quarterly*, 45, pp. 227-245.

Rajaram, A., (1994) "Tariff and Tax Reforms – Do World Bank Recommendations Integrate Revenue and Protection Objectives?" *Economic Studies Quarterly*, 45, pp. 321-338.

Ramsey, E., (1927) "A Contribution to the Theory of Taxation" *Economic Journal*, 37, pp. 47-61.

Sandmo, A. (1974) "A Note on the Structure of Optimal Taxation" *American Economic Review*, 64, pp. 701-706