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Locational Disadvantage and Losses from Trade: Three Regions in Economic Geography

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Abstract

We show how spatial evolution is different between the two representative models of economic geography: Krugman (1991 *JPE*) and Ottaviano *et al.* (2002 *IER*). We analyze the impacts of falling transport costs on the spatial distribution of economic activities and welfare for a network economy consisting of three regions located on a line. In the former model, the central region always has locational advantage and manufacturing workers gain from trade. In the latter model, however, the opposite is true when markets are opened up to trade. This is because the price competition is so keen in the central region that manufacturing sector moves to the peripheral regions, which aggravates the social welfare. We then show that when goods are close substitutes and share of manufacturing is of an intermediate level, the manufacturing activities completely disappears from the central region leading to a full agglomeration in one peripheral region.

Keywords: second nature, market hierarchy, role of migration, agglomeration, autarky to trade, coordination failure.

J.E.L. Classification: F12, R12, R13.

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1 Introduction

Most general equilibrium models on interregional and international economies deal with two regions for analytical solvability. There are indeed a few exceptions that analytically consider multiple regions, such as Cremer, de Kerchove and Thisse (1985), Economides and Siow (1988), Krugman (1993), Alesina and Spolaore (1997), Casella (2001), Tabuchi, Thisse and Zeng (2002), and Furusawa and Konishi (2002). But, since geographical space in these studies is perfectly symmetric, there as a result exist no locational advantage and disadvantage among regions. For the sake of mathematical tractability, analysis is usually confined to find symmetry breaking and agglomeration sustain thresholds, where there is no room for a hierarchical system of cities.

In this paper, we tackle the analysis of asymmetric equilibria under *asymmetric locations of regions*. Specifically, we consider an asymmetric network economy, where a central region is linked to two axisymmetrically located peripheral regions. Under this setting, one may expect that the central region has locational advantages of home market effects, accruing from accessibility for firms to large market demand and for consumers to large supply of diverse goods. However, as argued by Venables and Limão (2002), the central region may have locational disadvantages. If competition between firms is fierce, then firms would leave the central region for the peripheries in order to avoid competition and seek local monopoly. This competition reduces the wage level in the central region and decreases the utility level of consumers (although they enjoy low prices of goods and a variety of goods). In general, whether the central region is locationally advantageous or not is uncertain, and hence this is one motivation for considering asymmetric location of regions.

According to the core-periphery model typified by Krugman (1991), the spatial configuration of economic activities is dispersed for sufficiently large transport costs because the demand effect by immobile farmers is dominant. In the case of two regions, dispersion means the manufacturing share of each region is 1/2, in which the price competition would be minimized and as the farmers' demand is well met. However, the notion of full dispersion is not so obvious in the case of the asymmetric locations of three regions. In autarky, price competition is relaxed most when each share is 1/3. In the case of trade, however, price competition in the central region would be intensified because all varieties of goods are available there while some of them are not available in the peripheral regions. It must be that competition is softer when the share in the central region is smaller than the peripheries, implying that the equal share of 1/3 does not minimize price competition after trade openings. This is another motivation for considering the asymmetric location of regions.

In this paper, we study the impacts of falling transport costs and the effects of locational differences on the size and welfare of regions. In order to depict the long-run evolutionary process of regional development, we start from the position of autarky with an even distribution of economic activities. The decrease in the transport costs enables firms to trade between regions, which alters equilibrium prices, wages and profits, and generates migration of firms together with workers.

We utilize two representative core-periphery models of economic geography under monopolistically competitive and perfectly competitive markets, namely that of Krugman (1991) and Ottaviano, Tabuchi and Thisse (2002). Since the former assumes a CES utility function and iceberg transport costs, we call it the CES model; and since the latter assumes a quadratic utility function and linear transport costs, we call it the quadratic model hereafter. It has been shown in the literature that both models with two regions yield very similar results in terms of distributions of manufacturing activities: dispersion for large transport costs and agglomeration for small transport costs.

In the case of three asymmetric regions, one may conjecture the following evolutionary process. When the transport costs are prohibitively large, each region is in autarky with an equal share of manufacturing activities in either model. In accordance with the decrease in transport costs, the central region would steadily gain manufacturing share from the peripheries. When the transport costs fall down sufficiently, all the manufacturing activities would agglomerate in the central region due to its locational advantage. This scenario is confirmed numerically in the CES model.[1]

However, this is not true in the quadratic model. It will be shown that outcome is the complete reverse in the initial stages following the opening up of markets to interregional trade. According to the quadratic model, since the number of varieties suddenly increases in the central region owing to imports from other regions, price competition intensifies leading to a decrease in the central region's profits. Furthermore, the CES model yields gains from

[1] In a different setting, De Fraja and Norman (1993) analytically show that duopolistic firms always cluster at the market center, where consumers are uniformly distributed over a line segment.

trade, whereas the quadratic model yields losses from trade. Our model is in agreement with international trade models of Brander and Krugman (1983) and Anderson, Schmitt and Thisse (1995). Such a contrast is not due to the difference between mill pricing and discriminatory pricing, but due to a difference between constant elasticity and variable elasticity.

This difference in the demand functions also exerts an influence on welfare. In the CES model, firms gain from trade, which is augmented by free migration to the central region due to its locational advantages. However, the reverse is true in the quadratic model, where firms lose from trade. These losses are aggravated by the freeing up of migration from the central region due to its locational disadvantages. This would suggest that the prohibition of free migration is welfare-enhancing in this early stages of development. These findings comprise the primary result of this paper: *the CES model forms a striking contrast to the quadratic model.*

By making the most use of its analytical tractability, we completely characterize the equilibrium paths of the quadratic model . It will be shown that despite its apparent locational advantages, the central region may not experience growth throughout the evolutionary process. In fact, economic activities in the center will relocate to the peripheries and may become totally empty because firms want to avoid intense price competition. Note that such behavior cannot occur in the case of two symmetric regions with trade since there are no peripheries for firms to migrate to in order to relax price competition.

The remaining of the paper is organized as follows. The general setting of three regions is described in the next section. The CES model is analyzed when transport costs are large enough and numerical simulations are conducted in Section 3. The results of the CES model are contrasted with those of the quadratic model in Section 4. The stable interior equilibrium paths of the quadratic model are examined and disappearance of the central region is shown. Section 5 concludes.

2 Three regions

We consider a network economy made of three regions $r = 1, 2, 3$. They are located equidistantly on a line, where region 2 is the central region and the distance between regions 1 and 2 is equal to that between regions 2 and 3.

There are two factors, denoted A and L. Factor A is immobile and distributed evenly, while factor L is mobile and its share is $\lambda_r \in [0,1]$. For expositional purposes, we call the first sector "agriculture" and the second sector "manufacturing" so that A is "farmers" and L is "workers". Thus, there are $A/3$ immobile farmers and $\lambda_r L$ mobile workers in region r. Denote the share of manufacturing workers by $\mu \equiv L/(A+L)$ and normalize $A+L = 1$.

The first good is the numéraire, which is homogeneous and is produced in the agricultural sector using factor A as the only input assuming constant returns to scale and perfect competition. Technology in agriculture requires one unit of A in order to produce one unit of the homogeneous good. Consumers also have a positive initial endowment of this good. We assume that this good can be traded costlessly between regions so that its price is identical across regions. Hence, farmers' income is equal to 1 in each region.

The second good is horizontally differentiated and is produced in the manufacturing sector using factor L as the only input under increasing returns to scale and monopolistic competition. Technology in manufacturing is such that producing $q(i)$ units of variety i requires l units of L given by

$$l = F + cq(i)$$

where F and c are the fixed and marginal costs respectively. We assume that there is a continuum of potential firms so that the impact of each firm on the market outcome is negligible. Due to increasing returns to scale in production, each firm produces a variety of differentiated goods and the total number of firms in the whole economy is given by $n = L/l$.

Following a established tradition in economic geography, it is assumed that markets for goods adjust instantaneously, while interregional migration of firms and workers is relatively slow. After markets for goods are cleared, the equilibrium distribution is given by $\boldsymbol{\lambda}^* = (\lambda_1^*, \lambda_2^*, \lambda_3^*)$. Following Ginsburgh, Papageorgiou and Thisse (1985), $\boldsymbol{\lambda}^*$ is a *spatial equilibrium* when no individual is able to get a higher utility level by moving to another region. Mathematically, $\boldsymbol{\lambda}^*$ is a spatial equilibrium if V^* exists such that

$$\begin{array}{lll} V_r(\boldsymbol{\lambda}^*) = V^* & \text{if} & \lambda_r^* > 0 \\ V_r(\boldsymbol{\lambda}^*) \leq V^* & \text{if} & \lambda_r^* = 0 \end{array}$$

where $V_r(\boldsymbol{\lambda}^*)$ is the indirect utility in region r. If there is no empty region, the above second condition is unnecessary. In this case, the interior equilibrium condition is simply given by

$$V_1(\boldsymbol{\lambda}^*) = V_2(\boldsymbol{\lambda}^*) = V_3(\boldsymbol{\lambda}^*) \tag{1}$$

Assuming that regions experience in-migration (resp., out-migration) if its utility is higher (resp., lower) than the weighted average utility, we employ the replicator dynamics as:

$$\begin{cases} \dot{\lambda}_1 = \lambda_1 \left[V_1(\boldsymbol{\lambda}^*) - \sum_{r=1}^{3} \lambda_r V_r(\boldsymbol{\lambda}^*) \right] \equiv y_1 \\ \dot{\lambda}_3 = \lambda_3 \left[V_3(\boldsymbol{\lambda}^*) - \sum_{r=1}^{3} \lambda_r V_r(\boldsymbol{\lambda}^*) \right] \equiv y_3 \end{cases} \tag{2}$$

where the dot denotes the derivative with respect to time, and the redundant equation for $\dot{\lambda}_2$ is omitted since $\lambda_1 + \lambda_2 + \lambda_3 = 1$. Equilibrium $\boldsymbol{\lambda}^*$ is *asymptotically stable* if any sufficiently small change in the distribution results in a movement back toward the equilibrium. Mathematically, it is stable if all the real parts of eigenvalues of the Jacobian of (2) are negative. Since there are two variables, the system is asymptotically stable (resp. unstable) if the trace of the Jacobian is negative and (resp. positive or) the determinant of the Jacobian is positive (resp. negative).

3 CES model

This section is based on Krugman (1991). A representative worker maximizes a CES utility:

$$U = \left[\int_0^n q(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\mu\sigma}{\sigma-1}} q_A^{1-\mu} \tag{3}$$

given the budget constraint

$$\int_0^{n_r} p(i)q(i)di + q_A = w_r + \overline{q}_A \tag{4}$$

where $p(i)$ and $q(i)$ are the price and quantity of variety $i \in [0, n]$, q_A is the quantity of the numéraire, w is the nominal wage, and $\mu(> 0)$ and $\sigma(> 1)$ are parameters. The endowment of the numéraire $\overline{q}_A$ is zero in this section.

A firm producing variety i maximizes its profits:

$$\pi(i) = p(i)Q(i) - w(i)l$$

where $Q(i)$ is the perceived aggregate demand with elasticity of σ. This yields

$$p(i) = \frac{\sigma}{\sigma - 1} cw(i)$$

Assuming the free entry of firms, $\pi_r^*(i) = 0$ holds, which leads to the market equilibrium:

$$q^*(i) = \frac{F(\sigma - 1)}{c} \qquad l^* = F\sigma \qquad n^* = \frac{L}{F\sigma}$$

The iceberg type of transport technology is assumed: if a unit of good i is shipped from one region to the next region, only a fraction $1/T$ of the original unit arrives. Therefore, if it is shipped from regions 1 to 3, $1/T^2$ arrives. Let $p_{rs}(i)$ be the price of variety i produced in region r and sold in region s. Then, we have

$$p_{rs}^*(i) = p_{rr}^*(i)T^{|r-s|} \tag{5}$$

which implies mill pricing. Even if each firm wanted to price discriminate between regions, competition under the CES utility with the iceberg transport costs would force firms to use mill pricing (Fujita and Thisse, 2002, pp.327-328). To ease the burden of notation, we drop i hereafter, but add subscripts $r, s = 1, 2, 3$for regions. Note that trade takes place between any regions for all $T < \infty$, which differs from the next section with a linear demand.

With normalization of $c = (\sigma - 1)/\sigma$, $F = \mu/\sigma$ and $\lambda_1 + \lambda_2 + \lambda_3 = 1$, we have the following simultaneous equations for instantaneous equilibrium for $r = 1, 2, 3$,

$$f_r \equiv \sum_{s=1}^{3} Y_s \left(G_s/T^{|r-s|}\right)^{\sigma-1} - w_r^\sigma = 0 \tag{6}$$

with

$$\begin{aligned} Y_r &\equiv \mu\lambda_r w_r + \frac{1-\mu}{3} \\ G_r &\equiv \left[\sum_{s=1}^{3} \lambda_s \left(w_s T^{|r-s|}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \end{aligned}$$

where Y_r is the income of region r and G_r is the price index in region r. The indirect utility in region r is given by

$$V_r = w_r/G_r^\mu$$

Thus, the interior equilibria are obtained by simultaneously solving the five equations of (6) and (1) with respect to three short-run variables w_1, w_2, w_3 and two long-run variables λ_1, λ_3.

In order for the "no-black-hole" condition to hold, the autarky equilibrium $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$ with $T \to \infty$ and $w_r^* = 1$ should be stable in dynamics (2).Performing comparative statics as shown in Appendix A(i), we have

$$\begin{aligned} \frac{\partial V_1}{\partial \lambda_1} &= \frac{\partial V_2}{\partial \lambda_2} = \frac{3^{\frac{1-\sigma+\mu}{1-\sigma}}}{\sigma - 1}(1 - \sigma + \mu\sigma) \\ \frac{\partial V_1}{\partial \lambda_2} &= \frac{\partial V_2}{\partial \lambda_1} = 0 \end{aligned}$$

where each value is evaluated at $T \to \infty$, $\lambda_r^* = 1/3$ and $w_r^* = 1$ for all r. Hence, the no-black-hole condition of the three regions is given by

$$\frac{\sigma - 1}{\sigma} > \mu \tag{7}$$

which is identical to that of two regions.

3.1 Gains from trade and locational advantage

Next, consider the marginal change in the manufacturing distribution and welfare when markets open up to trade at the symmetric equilibrium. In the former case, we compute $\partial\lambda_r/\partial T$ for all r evaluated at $T \to \infty$, where $\lambda_r^* = 1/3$ and $w_r^* = 1$ holds for all r.[2]For the sake of mathematical simplicity, multiplying $\partial\lambda_r/\partial T$ by T^σ, we obtain comparative statics in Appendix A2 as

$$\begin{aligned} T^\sigma \frac{\partial \lambda_1}{\partial T} &= \frac{\mu(\sigma - 1)}{9(\sigma - 1 - \mu\sigma)} > 0 \\ T^\sigma \frac{\partial \lambda_2}{\partial T} &= -\frac{2\mu(\sigma - 1)}{9(\sigma - 1 - \mu\sigma)} < 0 \end{aligned}$$

where the inequalities are due to the no-black-hole condition (7). Since $T^\sigma > 0$, we get $\partial\lambda_1/\partial T > 0$ and $\partial\lambda_2/\partial T < 0$. Therefore, when the markets open up to trade, the center 2 experiences an in-migration while the peripheries 1 and 3 out-migration, implying that *the center has a locational advantage* in the vicinity of autarky.

[2] Since the equation system (6) is differentiable, the solution $\boldsymbol{\lambda}^*$ is also continuous insofar as it is stable in the vicinity of $T \to \infty$.

The welfare changes at $T \to \infty$ are computed as

$$\begin{aligned} T^{\sigma}\frac{dV_r}{dT} &= T^{\sigma}\left(\frac{\partial V_r}{\partial T}+\sum_{s=1}^{3}\frac{\partial V_r}{\partial w_s}\frac{\partial w_s}{\partial T}+\frac{\partial V_r}{\partial \lambda_1}\frac{\partial \lambda_1}{\partial T}+\frac{\partial V_r}{\partial \lambda_3}\frac{\partial \lambda_3}{\partial T}\right) \\ &= -4\,(3^{\frac{\sigma-1+\mu}{1-\sigma}})\mu < 0 \end{aligned}$$

when evaluated at the equilibrium values. This shows that falling transport costs raises the welfare, i.e., *gains from trade* in the vicinity of autarky. Likewise, the changes of farmers' utility $V_{Ar} = G_r^{-\mu}$ $(r = 1, 2, 3)$ are also calculated as

$$\begin{aligned} T^{\sigma}\frac{dV_{A1}}{dT} &= T^{\sigma}\frac{dV_{A3}}{dT} = -3^{\frac{\sigma-1+\mu}{1-\sigma}}\mu\frac{3-3\sigma+4\mu\sigma}{1-\sigma+\mu\sigma} \\ T^{\sigma}\frac{dV_{A2}}{dT} &= -2\,(3^{\frac{\sigma-1+\mu}{1-\sigma}})\mu\frac{3-3\sigma+2\mu\sigma}{1-\sigma+\mu\sigma} < 0 \end{aligned}$$

where the inequality is from (7). Hence, farmers in the center gain from trade, while net welfare for farmers in the peripheries is indeterminate. Comparing all the welfare changes, we have

$$\frac{dV_{A2}}{dT} < \frac{dV_r}{dT} < \frac{dV_{A1}}{dT} = \frac{dV_{A3}}{dT}$$

That is, the gains for farmers in the center are the largest, followed by the gains of workers.

What if migration of workers is not allowed as is often the case in the international economy. This is to compute the welfare changes of workers at $T \to \infty$ with the fixed distribution $\lambda_r = 1/3$ as

$$\begin{aligned} T^{\sigma}\frac{d\tilde{V}_1}{dT} &= T^{\sigma}\frac{d\tilde{V}_3}{dT} = T^{\sigma}\left(\frac{\partial V_1}{\partial T}+\sum_{s=1}^{3}\frac{\partial V_1}{\partial w_s}\frac{\partial w_s}{\partial T}\right) = -3^{\frac{\mu}{1-\sigma}}\mu < 0 \\ T^{\sigma}\frac{d\tilde{V}_2}{dT} &= T^{\sigma}\left(\frac{\partial V_2}{\partial T}+\sum_{s=1}^{3}\frac{\partial V_2}{\partial w_s}\frac{\partial w_s}{\partial T}\right) = -2\,(3^{\frac{\mu}{1-\sigma}})\mu < 0 \end{aligned}$$

In this case, the welfare changes of farmers are the same as above, thus showing gains from trade for everyone. However, the gains are larger in the center, which is consistent with the locational advantage mentioned above.

The effects of migration on the welfare can be revealed by the comparison between the above welfare changes. Since

$$\frac{dV_r}{dT} < \frac{d\tilde{V}_r}{dT} < 0$$

holds, migration plays a role in enhancing welfare increases, especially in the center. Summarizing the above, we have the following.[3]

Proposition 1 *Assume CES utility with iceberg transport costs. When markets open up to trade, each worker gains from trade. If migration is allowed, the gains from trade increase further and the center attracts workers from the peripheries due to its locational advantage.*

In the case of two symmetric regions, we can similarly show the gains from trade. It should be noted however that locational advantage does not arise in two symmetric regions, which have no locational differences by definition.

3.2 Stable equilibrium path

The next question is how the distribution of workers evolve as transport costs are steadily decreasing. Unfortunately, however, the CES model is not analytically solvable due to its high non-linearity, and hence we conduct simulations given some parameter values. We start from the infinite transport costs $T = \infty$, where the initial equilibrium distribution is given by $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$, which is stable under the no-black-hole condition (7).

With $\sigma = 2$ and $\mu = 0.1$, we computed an equilibrium path numerically and depicted it in Figure 1. As expected, *the central region expands continuously from full dispersion to full agglomeration* $(0, 1, 0)$. By simulations with different values of the parameters, we found intuitive results that the agglomeration is likely to occur if the varieties are sufficiently differentiated (small σ) and if the share of the manufacturing sector is large (large μ). However, it will be shown that the property of this equilibrium path is substantially different from that in the quadratic model.

[3] The same proposition is shown to hold by using a similar model to Forslid and Ottaviano (2003). This would imply that these results are attributable to CES utility with iceberg transport costs.

4 Quadratic model

In this section, replacing the CES model with the quadratic model of Ottaviano *et al.* (2002), we investigate whether or not we can reproduce the same results of Proposition 1. The individual utility in region r is given by

$$U_r = \alpha \int_0^{n_r} q(i)di - \frac{\beta - \gamma}{2} \int_0^{n_r} [q(i)]^2 di - \frac{\gamma}{2} \left[\int_0^{n_r} q(i)di \right]^2 + q_A \tag{8}$$

where n_r is the number of available varieties in region r, $\alpha > 0$, and $\beta > \gamma > 0$. Any individual is endowed with $\overline{q}_A$ units of the numéraire, and maximizes the utility (8) given the budget constraint:

$$\int_0^{n_r} p(i)q(i)di + q_A = w_r + \overline{q}_A$$

The first-order conditions are

$$\alpha - (\beta - \gamma)q(i) - \gamma \int_0^{n_r} q(j)dj = p(i)$$

which lead to the linear demand

$$q(i) = \frac{\sigma\left(\alpha n + \sigma G_r\right)}{\gamma n\left(n + \sigma n_r\right)} - \frac{\sigma}{\gamma n} p(i)$$

where $G_r \equiv \int_0^{n_r} p(j)dj$, $\sigma \equiv \gamma n / \left(\beta - \gamma\right)$ and $n(\geq n_r)$ is the total number of firms (varieties) in the economy, which are similar definitions as in the CES model. Substituting this and the budget constraint into (8) yields the indirect utility:

$$V_r = \frac{\alpha^2 \sigma n_r}{2\gamma\left(n + \sigma n_r\right)} - \frac{\alpha \sigma G_r}{\gamma\left(n + \sigma n_r\right)} + \frac{\sigma H_r}{2\gamma n} - \frac{\sigma^2 \left(G_r\right)^2}{2\gamma n\left(n + \sigma n_r\right)} + w_r + \overline{q}_A$$

where $H_r \equiv \int_0^{n_r} \left[p(j)\right]^2 dj$. As in the previous section, we drop i hereafter.

The production sector is the same as before except that $c = 0$ is assumed for analytical simplicity. Still increasing returns to scale prevail in the manufacturing sector because of $F > 0$. Labor market clearing implies that $n = L/F = L/l$ is constant. The transport technology is linear: $\tau_{rr} = 0$ for

$r = 1, 2, 3$, $\tau_{12} = \tau_{23} = \tau$, $\tau_{13} = 2\tau$, where τ is the transport costs per unit shipped per unit distance.[4]

Each firm is able to price discriminate between spatially separated markets. Let p_{rs} be the price of a food produced in region r and sold in region s, and $q_{rs}(p_{rs})$ is the demand in region s for a good produced in region r. Let

$$\hat{q}_{rs}(p_{rs}) = \max\{q_{rs}(p_{rs}), 0\}$$

If trade does not take place from regions r to s, then $\hat{q}_{rs}(p_{rs}) = 0$. Since the regional sizes differ in general, *one-way trade* from regions s to r may arise in equilibrium, which is $\hat{q}_{rs}(p_{rs}) = 0$ and $\hat{q}_{sr}(p_{sr}) > 0$. Such a one-way trade does not arise in the CES model because the CES demand is positive for all positive prices of import. We call *autarky* if $\hat{q}_{rs}(p_{rs}) = 0$ for all $r \neq s$; *neighboring trade* if $\hat{q}_{13}(p_{13}) = \hat{q}_{31}(p_{31}) = 0$ and $\hat{q}_{rs}(p_{rs}) > 0$ otherwise; *full trade* if $\hat{q}_{rs}(p_{rs}) > 0$ for all r, s.

Each price-discriminating firm incurs the transport costs, the profits of a firm in region r are given by:

$$\max_{p_{r1}, p_{r2}, p_{r3}} \pi_r = \sum_{s=1}^{3} (p_{rs} - \tau_{rs}) \hat{q}_{rs}(p_{rs}) \left(\mu\lambda_s + \frac{1-\mu}{3} \right) - w_r l \tag{9}$$

Maximizing (9) yields k equations of the first-order conditions with k unknown prices. For example, $k = 3$ for autarky since $\hat{q}_{rs}(p_{rs}) = 0$ for all $r \neq s$; $k = 7$ for neighboring trade since $\hat{q}_{13}(p_{13}) = \hat{q}_{31}(p_{31}) = 0$; and $k = 9$ for full trade. Solving them simultaneously with

$$G_s = \sum_{r \in \mathcal{A}_s} \lambda_r n p_{rs}$$

where $\mathcal{A}_s = \{r \mid q^*_{rs} > 0\}$ is the set of firms active in region s, we have

$$\begin{aligned} p^*_{ss} &= \frac{2\alpha + \sigma \sum_{r \in \mathcal{A}_s} \tau_{rs}\lambda_r}{2\left(\sigma \sum_{r \in \mathcal{A}_s} \lambda_r + 2\right)} \\ p^*_{rs} &= p^*_{ss} + \frac{\tau_{rs}}{2} \qquad \text{for } r \in \mathcal{A}_s \end{aligned} \tag{10}$$

[4]Alternatively, τ may be regarded as the costs of loading and unloading a unit good between neighboring regions. In the network economy, each good should be unloaded and loaded at the center of a port town when transporting it from one periphery to the other.

Thus, the equilibrium price depends not only upon produced and sold regions, but also upon the whole trade pattern. For example, p^*_{11} stands for the autarky price in region 1 when $\mathcal{A}_1 = \{1\}$, and the price of a home good when $\mathcal{A}_1 = \{1,2\}$ or $\{1,2,3\}$; while p^*_{21} is the import price of a foreign good when $\mathcal{A}_1 = \{1,2\}$ or $\{1,2,3\}$.

Likewise, given the trade pattern, the gross and net profits of a firm located in region r and shipped to regions $s_1 \cdots s_2$ are

$$\begin{aligned} R^*_r &= \frac{\sigma}{\gamma n} \sum_{s \,|\, r \in \mathcal{A}_s} (p^*_{rs} - \tau_{rs})^2 \left(\mu\lambda_s + \frac{1-\mu}{3} \right) \\ \pi^*_r &= R^*_r - w^*_r l \end{aligned}$$

Assuming the free entry of firms, $\pi^*_r = 0$ holds, and hence the equilibrium wage and the indirect utility are given by

$$\begin{aligned} w^*_r &= R^*_r / l \\ V_r &= V_r(w^*_r; p^*_{ss}, p^*_{rs}) \end{aligned}$$

When the transport costs are sufficiently high, firms are not concerned about demand in other regions, and set an autarky price. Therefore, (9) is reduced to

$$\max_{p_{rr}} \pi_r = p_{rr}\, \hat{q}_{rr}(p_{rr}) \left(\mu\lambda_r + \frac{1-\mu}{3} \right) - w_r l$$

Solving it with $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$,[5] the autarky price in region r is

$$p^*_{rr} = \frac{3\alpha}{\sigma + 6} \tag{11}$$

which is common for all regions.

Consider next the trade-opening condition. Trade opens up if the export price is exceeds the transport costs: $p^*_{rs} > \tau_{rs}$, which is $p^*_{ss} > \tau_{rs}/2$ from (10).Solving this yields

$$\lambda_s < \frac{2\,(\alpha - \tau)}{\sigma\tau} \tag{12}$$

This means that smaller regions are more likely to import goods because of weak price competition. Substituting the symmetric autarky equilibrium

[5] We exclude possibility of asymmetric autarky equilibria since they may become unstable once trade opens up as shown by Behrens (2003) in the case of two regions.

$\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$ into the trade-opening condition (12) with equality, we have the threshold of transport costs:

$$\tau_{trade} \equiv \frac{6\alpha}{\sigma + 6}$$

at which point trade begins.

Since τ_{trade} is increasing in α and decreasing in σ, trade is likely to open up when the demand is high (high α), each variety is differentiated (low γ), there are few firms and workers (small n and L), and/or transporting technology is improved (low τ).

Computing eigenvalues of the Jacobian of (2) and evaluating them at $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$, the autarky is shown to be stable for $\tau > \tau_{trade}$ only if[6]

$$\mu < \mu_3^* \equiv \frac{4\sigma}{5\sigma + 18} \tag{13}$$

which corresponds to the "no-black-hole" condition in the quadratic model.

4.1 Losses from trade and locational disadvantage

When $\tau = \tau_{trade}$, some firms are able to get positive demand $q_{rs}^* > 0$ from neighboring regions if the autarky equilibrium is perturbed. Specifically, firms in region r earn positive (resp. no) profits by exporting to neighboring regions s if λ_s of (12) is smaller (resp. equal to or greater) than 1/3 for $|r - s| = 1$. It is shown in Appendix B that there are four types of one-way trade patterns (C1)-(C4) generated by perturbations near the equilibrium $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$. In addition, when τ becomes just below τ_{trade}, the fifth type of neighboring trade pattern (C5) emerges in the vicinity of $\boldsymbol{\lambda}^*$, which is examined in Appendix C. In these patterns,trade between the peripheral regions does not occur.

In summary, we show below that there are three possible equilibrium paths for decreasing transport costs depending upon the parameter values. Let thresholds be

$$\begin{aligned} \hat{\mu}_2 &\equiv \frac{4\sigma(5\sigma + 18)}{37\sigma^2 + 252\sigma + 324} \qquad \bar{\mu}_2 \equiv \frac{2\sigma}{4\sigma + 9} \\ \mu_2^* &\equiv \min\{\hat{\mu}_2, \bar{\mu}_2\} \end{aligned}$$

[6] See Behrens (2003) for equilibrium stability of autarky and one-way trade in the case of two regions.

where $\max\{\hat{\mu}_2, \bar{\mu}_2\} < \mu_3^*$ holds. The three cases are dealt with the following three lemmas respectively.

Lemma 1 *If $\mu < \hat{\mu}_2$, the symmetric equilibrium $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$ is stable at $\tau = \tau_{trade}$.*

In proving Lemma 1, we should consider perturbations of $\boldsymbol{\lambda}$ generating 4 kinds of one-way trade patterns, which are contained in Appendix B. It assures stability for $\mu < \hat{\mu}_2$, which is subdivided into two cases: Lemmas 2 and 3 below. Note that when $\hat{\mu}_2 < \mu < \mu_3^*$, trade opening breaks the symmetric equilibrium and triggers an agglomeration of manufacturing activities. In this case, a unique equilibrium may not arise: for example both $(1/2, 0, 1/2)$ and $(0, 1, 0)$ are stable equilibria depending on the initial distribution.

Lemma 2 *If $\mu < \mu_2^*$, there exists a unique stable equilibrium path, which passes through $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$ at $\tau = \tau_{trade}$ with the property:*

$$\frac{\partial \lambda_2}{\partial \tau} > 0 > \frac{\partial \lambda_1}{\partial \tau} = \frac{\partial \lambda_3}{\partial \tau} \tag{14}$$

The proof is contained in Appendix C. Lemma 1 shows that whenever there exists a unique stable equilibrium path, the central region necessarily shrinks continuously. However, such uniqueness does not hold in the following case (the proof is in Appendix D).

Lemma 3 *If $\bar{\mu}_2 < \mu < \hat{\mu}_2$, the autarky equilibrium $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$ trifurcates stable equilibrium paths when trade opens at $\tau = \tau_{trade}$. One of the path has the same property as (14).*

In sum, given the "no-black-hole" condition (13), *at least one of the paths or trajectories moves toward the decrease in the size of the central region* (Lemmas 1 and 3) in the vicinity of $\tau = \tau_{trade}$. In order to continue neat analysis, we will focus on the most interesting case of

$$\mu < \mu_2^*$$

in which there is a unique stable equilibrium path corresponding to falling transport costs (Lemma 2). Note that all varieties are available in the central region, whereas some of them are not in the peripheries, which exhibits *market hierarchy* in the central place theory of Christaller (1933). Nevertheless, by use of Lemma 2, we can show that the central region is smaller than the peripheries and that its welfare is lower than the peripheries as follows.

Proposition 2 *Assume the quadratic utility with linear transport costs. For $\mu < \mu_2^*$, although there are more varieties in the central region, it necessarily shrinks and all workers are worse off when trade opens up.*

Proposition 2 states that in spite of its geographical advantage, the size of the central region becomes smaller than that of the peripheral regions.[7] This is explained by the firms' behavior to relax keen price competition generated by the trade opening. At the very initial stages of trade, firms in the central region can export to two peripheral regions, while firms in the peripheral regions can export to the central region only. However, the central region's firms compete with firms from all regions, whereas firms in the peripheral regions only compete with those in the central region. The net outcome is that the price competition effect dominates the market area effect, which induces movement to the peripheries. Comparing Proposition 2 with Proposition 1, we know that *the quadratic model is shown to be opposite to the CES model.*

Next, consider the change in the welfare when τ gets just below τ_{trade}, at which the neighboring trade or one-way trade to the center is about to take place. This is to evaluate $dV_r/d\tau$ at $\tau = \tau_{trade}$ and $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$, which is

$$\begin{aligned}
\frac{dV_r}{d\tau} &= \frac{\partial V_r}{\partial \tau} + \sum_{s=1}^{3} \frac{\partial V_r}{\partial \lambda_s} \frac{\partial \lambda_s}{\partial \tau} \\
&= \frac{\alpha\sigma^2 (2-\mu) \left[(19\sigma + 36)\mu - 8\sigma\right]}{2(\sigma+6)\left[(34\sigma^2 + 153\sigma + 162)\mu - 2\sigma(7\sigma+18)\right]\gamma n} > 0
\end{aligned}$$

under $\mu < \mu_2^*$. Therefore, workers lose from trade, i.e. falling transport costs τ decrease the welfare of all workers. Applying similar calculations for farmers' utility V_{Ar} with $w_A^* = 1$, we obtain

$$\frac{dV_{Ar}}{d\tau} < 0 \qquad r = 1, 3$$

Hence, falling transport costs τ raises the welfare of peripheral farmers. This is because peripheral farmers are better off due to the in-migration of firms.

[7]Fujita and Mori (1997) conducted simulations and obtained a similar result of locational disadvantage in the center. However, it seems to be attributed to the fact that there are unevenly more farmers in the peripheries in their model. On the other hand, farmers are equally distributed in our model. Nevertheless, we have locational disadvantage in the center, which is due to price competition under non-constant elastic demand.

In order to see the net effects of the opening of markets to trade on social welfare, consider the sum of individual utilities:

$$W \equiv \sum_{r=1}^{3} (V_r \lambda_r L + V_{Ar} A/3)$$

where V_{Ar} is the farmer's utility in region $r = 1, 2, 3$. Such a summation is possible since the quasi-linear utility is transferable. It can be readily shown that $dW/d\tau > 0$ in the neighborhood of $\tau = \tau_{trade}$. In sum, we have the following.

Proposition 3 *Assume the quadratic utility with linear transport costs. For $\mu < \mu_2^*$, all manufacturing workers are worse off, whereas farmers in the peripheries are better off when markets are open to trade. In net, the social welfare decreases.*

Although more varieties of goods become available due to the trade openings, all manufacturing workers are worse off in the quadratic model. Therefore, we also confirm that *the quadratic and CES models are shown to give reverse outcomes* by comparing Proposition 3 with Proposition 1. This welfare loss from trade in the manufacturing sector is also observed in a Cournot duopoly model of two countries developed by Brander and Krugman (1983).

What if migration is not allowed as in the context of international economy? Let the upper bar stand for no migration, then we have

$$\frac{d\tilde{V}_r}{d\tau} = \frac{\partial V_r}{\partial \tau}$$

evaluated at $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$ and $\tau = \tau_{trade}$. Applying similar calculations for $\tilde{V}_r$ and $\tilde{W}$, we can derive

$$\frac{d\tilde{W}}{d\tau} < \frac{dW}{d\tau}$$

That is, the welfare loss is greater when migration is free, which implies *interregional migration of firms and workers aggravates the social welfare.* Again, this is opposite to the result of Proposition 1.

It can be shown that prohibition of migration is socially suboptimal. The first-order conditions for the optimum allocation of manufacturing activities are $\partial W/\partial\lambda_1 = \partial W/\partial\lambda_2 = 0$ after plugging $\lambda_3 = 1 - \lambda_1 - \lambda_2$ into W.[8] Evaluating these conditions at $\tau = \tau_{trade}$ and $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$, we have $\partial W/\partial\lambda_1 = 0$ and $\partial W/\partial\lambda_2 > 0$. This means λ_2 should be larger than 1/3 in optimum. Hence, when markets are about to open to trade, the optimum size of the central region is larger than that under prohibited migration, but smaller than that where free migration is allowed.

4.2 Stable equilibrium path

Consider the steady decrease in τ as before. Given the stability condition $\mu < \mu_2^*$, the equilibrium path $\boldsymbol{\lambda}^*(\tau)$ passing through $(1/3, 1/3, 1/3)$ continues to be asymptotically stable for τ just below τ_{trade}. However, there is no guarantee that the stable path is always continuous for falling transport costs. Nevertheless, we can show the following strong result (the proof is given in Appendix E).

Lemma 4 *For $\mu < \mu_2^*$, the stable equilibrium path $\boldsymbol{\lambda}^*(\tau)$ passing through $\boldsymbol{\lambda} = (1/3, 1/3, 1/3)$ at $\tau = \tau_{trade}$ is always axisymmetric for all $\lambda_1^* = \lambda_3^* \in (0, 1/2)$.*

Lemma 4 says that any trajectory near the axisymmetric configuration $\lambda_1 = \lambda_3$ is directed toward the line $\lambda_1 = \lambda_3$. It follows that even if the stable equilibrium $\boldsymbol{\lambda}^*$ becomes unstable or disappears, the trajectory starting from the old equilibrium should go in the direction of the new stable equilibrium which is axisymmetric. This is because any small perturbation $(\pm\varepsilon_1, \mp\varepsilon_3)$ near an axisymmetric equilibrium results in a movement back to the equilibrium. Suppose an equilibrium path suddenly disappears or becomes unstable. Since the axisymmetry property holds from Lemma 4, the two-dimensional dynamics with $(\lambda_1, 1 - \lambda_1 - \lambda_3, \lambda_3)$ behaves similar to one-dimensional dynamics with $(\lambda_1, 1 - 2\lambda_1, \lambda_1)$. As a result, the direction toward a new equilibrium can be uniquely determined. When the stable equilibrium $\boldsymbol{\lambda}^*$ becomes unstable or disappears, the smaller (resp. larger) regions get smaller (resp. larger) regions with a sudden jump. This implies that there exists a unique stable equilibrium path $\boldsymbol{\lambda}^*(\tau)$ for the whole domain of $\mu < \mu_2^*$.

Next, we show that there are two types of interior stable equilibrium path $\boldsymbol{\lambda}^*(\tau)$ according to the parameter values of μ and σ. Since $V_1 = V_3$ always

[8] This optimum is called the second-best optimum in Ottaviano and Thisse (2002).

holds due to the axisymmetry property of $\lambda_1^* = \lambda_3^*$, we focus on the curve

$$\Delta V_{12}(\lambda_1, \tau) \equiv V_1 - V_2|_{\lambda_2 = 1-2\lambda_1, \lambda_3 = \lambda_1} = 0$$

which passes through $(\lambda_1, \tau) = (1/3, \tau_{trade})$ and intersects with $\lambda_1 = 1/2$ if $f(\lambda_1) > 0$ holds for all $\lambda_1 \in [1/3, 1/2]$ (where $f(\lambda_1)$ is defined by (26) in Appendix E). It can be shown that $f(\lambda_1) > 0$ for all $\lambda_1 \in [1/3, 1/2]$ is equivalent to $g(\mu, \sigma) \equiv f(\tilde{\lambda}_1) > 0$, where $\tilde{\lambda}_1$ is the smallest solution of $f'(\lambda_1) = 0$. Let $\hat{\mu}_1 = \mu(\sigma)$ be the function derived from the implicit function $g(\mu, \sigma) = 0$. Then, we have the following (the proof is given in Appendix F).

Lemma 5 *Assume the quadratic utility with linear transport costs. The interior stable equilibrium path passing through* $(\lambda_1, \tau) = (1/3, \tau_{trade})$ *is either one of the following:*

If $\hat{\mu}_1 < \mu < \mu_2^*$, *then* λ_2^* *decreases monotonically from* $1/3$ *to* 0 *at positive* τ*;*

if $\mu < \hat{\mu}_1$, *then* λ_2^* *decreases from* $1/3$ *and then increases to* 1.

In either case of Lemma 5, the central region becomes small at trade openings in spite of more available varieties there. In the former case, when λ_2^* reaches 0, the corner equilibrium $(1/2, 0, 1/2)$ is realized. Since the manufacturing is empty in region 2, one-way trade from the peripheral to the central regions takes place. Computing stability and equilibrium conditions for this equilibrium, we can find a positive threshold of τ such that this equilibrium will cease to exist or become unstable. We do not know the new stable equilibrium at the threshold, but it is certain that for a sufficient decrease in τ, the new stable equilibrium always involves full agglomeration: $(1, 0, 0)$, $(0, 1, 0)$ or $(0, 0, 1)$. In the latter case, once λ_2^* reaches 1, the central agglomeration $(0, 1, 0)$ continues to be a stable equilibrium all the time.

Examining both interior and corner stable paths, we establish the following main result during the whole evolutionary process with

$$\mu_1^* \equiv \max\left\{\frac{2}{11}, \min\left\{\frac{(5\sigma+2)(\sigma+2)}{2(7\sigma^2+30\sigma+29)}, \frac{(5\sigma+2)(\sigma+2)}{8\sigma^2+25\sigma+18+3\sqrt{32\sigma^4+240\sigma^3+489\sigma^2+324\sigma+36}}\right\}\right\}$$

where $\mu_1^* > \hat{\mu}_1$ (the proof is contained in Appendix G).

Proposition 4 *Assume the quadratic utility with linear transport costs* $\tau \leq \tau_{trade}$.

(i) If $\mu_1^* < \mu < \mu_2^*$, the share of the central region shrinks continuously and reaches 0 *realizing full agglomeration in one of the peripheries.*

(ii) If $\mu < \min\{\mu_1^*, \mu_2^*\}$, then the share of the central region first decreases from $1/3$ *and then increases to* 1*.*

The domain of the two cases (i) and (ii) in the (σ, μ) coordinates is given in Figure 2, and the stable paths in the (τ, λ_2^*) coordinates are drawn in Figure 3. Observe in both cases (i) and (ii) that the full dispersion during the initial autarky is due to the *demand effect* of immobile farmers, while the partial agglomeration in the peripheries is ascribed to the *price competition effect* for intermediate transport costs. Thus, dominance of these two dispersion forces depends on the stages of development in the case of asymmetric location, whereas the two forces are not separated in the case of symmetric location models.

In case (ii) of small manufacturing share μ, the central region becomes smaller when τ is just below τ_{trade} as shown in Proposition 2, but it will be larger for small τ, and full agglomeration at the center ($\lambda_2^* = 1$) is realized in the end,[9] which is consistent with Christaller's (1933) central place theory. Thus, as transport costs steadily decrease, the equilibrium configuration involves, first, full dispersion, then partial agglomeration in the peripheries and, last agglomeration in the central region.[10]

On the other hand, in case (i) of intermediate manufacturing share μ and high substitutability σ, the stable equilibrium path is totally different. When trade opens up, λ_2^* decreases monotonically with axisymmetry $(1/2 - \lambda_2^*/2, \lambda_2^*, 1/2 - \lambda_2^*/2)$, which reaches $(1/2, 0, 1/2)$. For a further decrease in τ, $(1/2, 0, 1/2)$ breaks and *full agglomeration at one periphery is realized*, but not at the center, thus necessarily exhibiting undesirable agglomeration in the end. That is, *the central region steadily loses manufacturing activities and becomes empty after all in spite of its seemingly locational advantage.*

Such a counterintuitive result may be explained by the two opposite forces. First, the force of dispersion is associated with the high degree of substitutability among varieties, which means keen price competition. As

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[9] To be more precise, there are two subcases of (ii) for intermediate τ. If $\hat{\mu}_1 < \mu < \min\{\mu_1^*, \mu_2^*\}$, then $\lambda_2^* = 0$; but if $\mu < \hat{\mu}_1$, then $\lambda_2^* > 0$. In either case, however, $\lambda_2^* = 1$ for sufficiently small τ.

[10] Such a bell-shaped relationship between the transport costs and spatial distribution is somewhat related to the Hotelling's model with logit (Anderson, de Palma and Thisse, 1992, chapter 9.4).

previously mentioned, the dispersion force does not lead to full dispersion, but migration toward peripheries, leading to the configuration $(1/2, 0, 1/2)$ at the intermediate stage of development (medium τ). Second, the force driving full agglomeration is the large share of manufacturing activities μ, which often upsets a desirable location of manufacturing activities determined by the "first nature" (Cronon, 1991). In case (i), *it is the "second nature"* $(1/2, 0, 1/2)$ *at the intermediate stage that selects full agglomeration in one periphery at the later stage.* This is typical coordination failure since the peripheral agglomeration is worse than the central agglomeration.

5 Conclusion

What is important for regional growth is not the "first nature" of geographically convenient locations, as much as the "second nature" of market interactions among firms and consumers. We have shown that the roles of these natures are different between the two seemingly similar models: the CES model with iceberg transport technology and the quadratic model with the linear transport technology. They yield the same market outcome in that large transport costs lead to dispersion, and small transport costs lead to agglomeration. However, we have shown that the results are opposite in the case of three regions when the markets opens up to trade. Whereas the CES model always exhibits the locational advantage in the central region and gains from trade (Proposition 1), the quadratic model shows the locational disadvantage (Proposition 2) as well as losses from trade (Proposition 3).

Such a sharp contrast is ascribed to the differences in demand elasticity. While the degree of price competition among monopolistically competitive firms is moderate in the CES model, it is high at the opening up of markets to trade in the quadratic model. Fierce price competition leads to the so-called prisoners' dilemma. Each firm is worse off due to a price war, resulting in the decrease in the workers' wages and utility levels. Furthermore, the welfare losses are aggravated in the presence of interregional migration. In order to avoid intense levels of price competition, mobile firms and workers move away from the central region, which generates wasteful transport costs and hence lowers social welfare.

This coordination failure persists and affects the long-run growth of regions during a gradual decrease in transport costs. As shown in Proposition 4 (i), when goods are close substitutes and the manufacturing share is intermediate, the manufacturing activities in the central region becomes completely vacant. In the end, bad agglomeration is realized at either of the peripheries, exhibiting that the regional evolution is subject to the lock-in effect together with the "second nature".

Appendix A: Comparative statics

(i) Applying comparative statics to (6), $f_4 \equiv V_1 - w_1/G_1^{\mu} = 0$ and $f_5 \equiv V_3 - w_3/G_3^{\mu} = 0$, we have

$$\begin{pmatrix} \frac{\partial w_1}{\partial \lambda_1} & \frac{\partial w_1}{\partial \lambda_2} \\ \frac{\partial w_2}{\partial \lambda_1} & \frac{\partial w_2}{\partial \lambda_2} \\ \frac{\partial w_3}{\partial \lambda_1} & \frac{\partial w_3}{\partial \lambda_2} \\ \frac{\partial V_1}{\partial \lambda_1} & \frac{\partial V_1}{\partial \lambda_2} \\ \frac{\partial V_2}{\partial \lambda_1} & \frac{\partial V_2}{\partial \lambda_2} \end{pmatrix} = \begin{pmatrix} \frac{\partial f_1}{\partial w_1} & \frac{\partial f_1}{\partial w_2} & \frac{\partial f_1}{\partial w_3} & \frac{\partial f_1}{\partial V_1} & \frac{\partial f_1}{\partial V_3} \\ \frac{\partial f_2}{\partial w_1} & \frac{\partial f_2}{\partial w_2} & \frac{\partial f_2}{\partial w_3} & \frac{\partial f_2}{\partial V_1} & \frac{\partial f_2}{\partial V_3} \\ \frac{\partial f_3}{\partial w_1} & \frac{\partial f_3}{\partial w_2} & \frac{\partial f_3}{\partial w_3} & \frac{\partial f_3}{\partial V_1} & \frac{\partial f_3}{\partial V_3} \\ \frac{\partial f_4}{\partial w_1} & \frac{\partial f_4}{\partial w_2} & \frac{\partial f_4}{\partial w_3} & \frac{\partial f_4}{\partial V_1} & \frac{\partial f_4}{\partial V_3} \\ \frac{\partial f_5}{\partial w_1} & \frac{\partial f_5}{\partial w_2} & \frac{\partial f_5}{\partial w_3} & \frac{\partial f_5}{\partial V_1} & \frac{\partial f_5}{\partial V_3} \end{pmatrix}^{-1} \begin{pmatrix} \frac{\partial f_1}{\partial \lambda_1} & \frac{\partial f_1}{\partial \lambda_2} \\ \frac{\partial f_2}{\partial \lambda_1} & \frac{\partial f_2}{\partial \lambda_2} \\ \frac{\partial f_3}{\partial \lambda_1} & \frac{\partial f_3}{\partial \lambda_2} \\ \frac{\partial f_4}{\partial \lambda_1} & \frac{\partial f_4}{\partial \lambda_2} \\ \frac{\partial f_5}{\partial \lambda_1} & \frac{\partial f_5}{\partial \lambda_2} \end{pmatrix}$$

$$= \begin{pmatrix} \mu - 1 & 0 & 0 & 0 & 0 \\ 0 & \mu - 1 & 0 & 0 & 0 \\ 0 & 0 & \mu - 1 & 0 & 0 \\ 3^{\frac{\mu}{1-\sigma}}(\mu - 1) & 0 & 0 & 1 & 0 \\ 0 & 3^{\frac{\mu}{1-\sigma}}(\mu - 1) & 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 3(\mu - 1) & 0 \\ 0 & 3(\mu - 1) \\ 3(1 - \mu) & 3(1 - \mu) \\ \frac{\mu}{1-\sigma} 3^{\frac{\sigma - 1 - \mu}{\sigma - 1}} & 0 \\ 0 & \frac{\mu}{1-\sigma} 3^{\frac{\sigma - 1 - \mu}{\sigma - 1}} \end{pmatrix}$$

By rearranging it, we get

$$\begin{aligned} \frac{\partial V_1}{\partial \lambda_1} &= \frac{\partial V_2}{\partial \lambda_2} = \frac{3^{\frac{1-\sigma+\mu}{1-\sigma}}}{\sigma - 1}(1 - \sigma + \mu\sigma) \\ \frac{\partial V_1}{\partial \lambda_2} &= \frac{\partial V_2}{\partial \lambda_1} = 0 \end{aligned}$$

(ii) From (1), let $f_6 \equiv V_1 - V_2$ and $f_7 \equiv V_1 - V_3$. Then, we similarly have

$$T^\sigma \begin{pmatrix} \frac{\partial w_1}{\partial T} \\ \frac{\partial w_2}{\partial T} \\ \frac{\partial w_3}{\partial T} \\ \frac{\partial \lambda_1}{\partial T} \\ \frac{\partial \lambda_2}{\partial T} \end{pmatrix} = T^\sigma \begin{pmatrix} \frac{\partial f_1}{\partial w_1} & \frac{\partial f_1}{\partial w_2} & \frac{\partial f_1}{\partial w_3} & \frac{\partial f_1}{\partial \lambda_1} & \frac{\partial f_1}{\partial \lambda_2} \\ \frac{\partial f_2}{\partial w_1} & \frac{\partial f_2}{\partial w_2} & \frac{\partial f_2}{\partial w_3} & \frac{\partial f_2}{\partial \lambda_1} & \frac{\partial f_2}{\partial \lambda_2} \\ \frac{\partial f_3}{\partial w_1} & \frac{\partial f_3}{\partial w_2} & \frac{\partial f_3}{\partial w_3} & \frac{\partial f_3}{\partial \lambda_1} & \frac{\partial f_3}{\partial \lambda_2} \\ \frac{\partial f_6}{\partial w_1} & \frac{\partial f_6}{\partial w_2} & \frac{\partial f_6}{\partial w_3} & \frac{\partial f_6}{\partial \lambda_1} & \frac{\partial f_6}{\partial \lambda_2} \\ \frac{\partial f_7}{\partial w_1} & \frac{\partial f_7}{\partial w_2} & \frac{\partial f_7}{\partial w_3} & \frac{\partial f_7}{\partial \lambda_1} & \frac{\partial f_7}{\partial \lambda_2} \end{pmatrix}^{-1} \begin{pmatrix} \frac{\partial f_1}{\partial T} \\ \frac{\partial f_2}{\partial T} \\ \frac{\partial f_3}{\partial T} \\ \frac{\partial f_6}{\partial T} \\ \frac{\partial f_7}{\partial T} \end{pmatrix}$$

$$= \begin{pmatrix} \mu - 1 & 0 & 0 & 3(\mu-1) & 0 \\ 0 & \mu - 1 & 0 & 0 & 3(\mu-1) \\ 0 & 0 & \mu - 1 & 3(1-\mu) & 3(1-\mu) \\ 3^{\frac{\mu}{1-\sigma}}(\mu-1) & 3^{\frac{\mu}{1-\sigma}}(1-\mu) & 0 & \frac{\mu}{1-\sigma}3^{\frac{\sigma-1-\mu}{\sigma-1}} & \frac{\mu}{1-\sigma}3^{\frac{\sigma-1-\mu}{\sigma-1}} \\ 3^{\frac{\mu}{1-\sigma}}(\mu-1) & 0 & 3^{\frac{\mu}{1-\sigma}}(1-\mu) & \frac{2\mu}{1-\sigma}3^{\frac{\sigma-1-\mu}{\sigma-1}} & \frac{\mu}{1-\sigma}3^{\frac{\sigma-1-\mu}{\sigma-1}} \end{pmatrix}^{-1} \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ 3^{\frac{\mu}{1-\sigma}}\mu \\ 0 \end{pmatrix}$$

where each element is evaluated at $T \to \infty$, $\lambda_r^* = 1/3$ and $w_r^* = 1$ for all r. By rearranging it, we have

$$\begin{aligned} T^\sigma \frac{\partial \lambda_1}{\partial T} &= \frac{\mu(\sigma-1)}{9(\sigma - 1 - \mu\sigma)} > 0 \\ T^\sigma \frac{\partial \lambda_2}{\partial T} &= -\frac{2\mu(\sigma-1)}{9(\sigma - 1 - \mu\sigma)} < 0 \end{aligned}$$

Appendix B: Proof of Lemma 1

From (12), the boundary conditions are $\lambda_r = 1/3$ ($r = 1, 2, 3$), which are $\lambda_1 = 1/3$, $\lambda_1 + \lambda_3 = 2/3$ and $\lambda_3 = 1/3$ in the (λ_1, λ_3) coordinates. In considering perturbations in the vicinity of $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$, we analyze only four domains (C1)-(C4) in Figure 4due to the axisymmetry between regions 1 and 3.

(C1) One-way trade from 1 and 3 to 2:

Computing the Jacobian of (2) and evaluating it at $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$ with $\tau = \tau_{trade}$, we have the stability condition: $\mu < \hat{\mu}_2$. Applying the implicit function theorem to $y_1 = 0$ and $y_3 = 0$ at $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$ with $\tau = \tau_{trade}$, the slopes of $y_1 = 0$ and $y_3 = 0$ are

$$\left.\frac{d\lambda_3^*}{d\lambda_1^*}\right|_{y_1=0} = \frac{4\sigma(7\sigma+18) - \mu\left(41\sigma^2 + 252\sigma + 324\right)}{4\left(2-\mu\right)\sigma^2} < 0$$

$$\left.\frac{d\lambda_3^*}{d\lambda_1^*}\right|_{y_3=0} = \frac{4\left(2-\mu\right)\sigma^2}{4\sigma(7\sigma+18) - \mu\left(41\sigma^2 + 252\sigma + 324\right)} < 0$$

Hence the equilibrium is stable within domain (C1) for $\mu < \hat{\mu}_2$.

(C2) One-way trade from 1 to 2 and neighboring trade between 2 and 3:

Suppose the equilibrium moves to $(\lambda_1, \lambda_3) = (1/3 + \varepsilon_1, 1/3 - \varepsilon_3)$ for sufficiently small ε_1 and ε_3 with $\varepsilon_1 > \varepsilon_3 > 0$. Then, ignoring the second or higher order terms, we have

$$dy_1 \approx \frac{\alpha^2\sigma\left[\mu\left(41\sigma^2 + 252\sigma + 324\right) - (7\sigma+18)\right]}{6\gamma\mu\left(\sigma+2\right)\left(\sigma+6\right)^3}\varepsilon_1 - \frac{\alpha^2\sigma^3\left(2-\mu\right)}{6\gamma\mu\left(\sigma+2\right)\left(\sigma+3\right)\left(\sigma+6\right)^2}\varepsilon_3 < 0$$

for all $\varepsilon_1 > \varepsilon_3 > 0$ and $\mu < \hat{\mu}_2$. Likewise, we have $dy_3 > 0$. Hence, any deviation within domain (C2) will necessarily come back to the original equilibrium $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$.

(C3) Autarky in 1 and one-way trade from 2 to 3:

From the Jacobian of (2) evaluated at $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$ with $\tau = \tau_{trade}$, we have the stability condition:

$$\mu < \breve{\mu} \equiv \frac{4\sigma(2\sigma+9)}{13\sigma^2 + 99\sigma + 162}$$

The slopes of $y_1 = 0$ and $y_3 = 0$ are

$$\left.\frac{d\lambda_3^*}{d\lambda_1^*}\right|_{y_1=0} = \frac{3\left(\sigma+3\right)\left[\mu(5\sigma+18) - 4\sigma\right]}{\left(2-\mu\right)\sigma^2}$$

$$\left.\frac{d\lambda_3^*}{d\lambda_1^*}\right|_{y_3=0} = 0$$

Since the former is less than the slope of the boundary $\lambda_3 = 2/3 - \lambda_1$, which is -1, for all $\mu < \hat{\mu}_2 < \breve{\mu}$, it passes through domain (C3). Thus, both are nonpositive for all $\mu < \breve{\mu}$, which means the equilibrium is stable in domain (C3).

(C4) One-way trade from 2 to 1 and 3:

Suppose the equilibrium moves to $(\lambda_1, \lambda_3) = (1/3 - \varepsilon_1, 1/3 - \varepsilon_3)$ for sufficiently small $\varepsilon_1, \varepsilon_3 > 0$. Then, ignoring the second or higher order terms, we have

$$
\begin{aligned}
dy_1 &\approx \frac{\alpha^2\sigma\left[4\sigma(2\sigma+9) - \mu\left(13\sigma^2 + 99\sigma + 162\right)\right]}{2\gamma\mu\left(\sigma+3\right)\left(\sigma+6\right)^3}\varepsilon_1 + \frac{\alpha^2\sigma^3\left(2-\mu\right)}{2\gamma\mu\left(\sigma+3\right)\left(\sigma+6\right)^3}\varepsilon_3 > 0 \\
dy_3 &\approx \frac{\alpha^2\sigma^3\left(2-\mu\right)}{2\gamma\mu\left(\sigma+3\right)\left(\sigma+6\right)^3}\varepsilon_1 + \frac{\alpha^2\sigma\left[4\sigma(2\sigma+9) - \mu\left(13\sigma^2 + 99\sigma + 162\right)\right]}{2\gamma\mu\left(\sigma+3\right)\left(\sigma+6\right)^3}\varepsilon_3 > 0
\end{aligned}
$$

for all $\varepsilon_1, \varepsilon_3 > 0$ and $\mu < \hat{\mu}_2$. Hence, any deviation within domain (C4) will necessarily come back to the original equilibrium.

Appendix C: Proof of Lemma 2

Let

$$\breve{\mu} \equiv \frac{4\sigma}{11\sigma + 18} < \hat{\mu}_2$$

We will show that when for τ is just below τ_{trade}, the stable path enters the domain of neighboring trade between 1 and 2 and between 2 and 3 (C5) in Figure 5 for $\mu < \breve{\mu}$ (Lemma 6) and the domain of one-way trade from 1 and 3 to 2 (C1) for $\breve{\mu} < \mu < \hat{\mu}_2$ (Lemma 7), but that it does not enter the domains of (C2), (C3) and (C4) (Lemmas 8-10). The shaded triangular domain of (C5) is given by the three boundary conditions (12), where λ_r is no longer $1/3$ since τ is slightly less than τ_{trade}. The vertexes of the triangle in Figure 5 are computed as:

$$\left(\frac{2(\alpha-\tau)}{\sigma\tau}, \frac{2(\alpha-\tau)}{\sigma\tau}\right), \left(\frac{2(\alpha-\tau)}{\sigma\tau}, \frac{(\sigma+4)\tau - 4\alpha}{\sigma\tau}\right), \left(\frac{(\sigma+4)\tau - 4\alpha}{\sigma\tau}, \frac{2(\alpha-\tau)}{\sigma\tau}\right) \quad (15)$$

Lemma 6 *If $\mu < \breve{\mu}$, the stable equilibrium path $\boldsymbol{\lambda}^*(\tau)$ passing through $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$ at $\tau = \tau_{trade}$ enters the domain of neighboring trade (C5) with*

$$\frac{\partial \lambda_1}{\partial \tau} = \frac{\partial \lambda_3}{\partial \tau} < 0$$

Proof. In the case of neighboring trade,we compute the Jacobian of (2) and evaluate it at $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$ with $\tau = \tau_{trade}$. Then, the stability condition is given by

$$\mu < \tilde{\mu} \equiv \frac{2\sigma\,(7\sigma + 18)}{34\sigma^2 + 153\sigma + 162} \tag{16}$$

Applying the implicit function theorem to $y_1 = y_3 = 0$ at $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$ with $\tau = \tau_{trade}$, we get

$$\frac{\partial \lambda_1}{\partial \tau} = \frac{\partial \lambda_3}{\partial \tau} = \frac{(2-\mu)\,\sigma\,(\sigma+6)^2}{18\alpha\left[\mu\,(34\sigma^2 + 153\sigma + 162) - 2\sigma\,(7\sigma + 18)\right]} < 0 \tag{17}$$

where the inequality is due to (16). Therefore, the slope of the stable equilibrium path is 1.

On the other hand, the marginal change in the first vertex of (15) at $\tau = \tau_{trade}$ is

$$\frac{\partial \lambda_1}{\partial \tau} = \frac{\partial \lambda_3}{\partial \tau} = -\frac{(\sigma+6)^2}{18\alpha\sigma} < 0 \tag{18}$$

Hence, the vertex moves toward the same direction as the stable equilibrium path for a marginal decrease in τ. In order for the stable equilibrium path $\boldsymbol{\lambda}^*(\tau)$ to enter the domain of neighboring trade (C5), the change in the stable path (17) is smaller then the latter (18) in absolute value, i.e.

$$-\frac{(\sigma+6)^2}{18\alpha\sigma} < \frac{(2-\mu)\,\sigma\,(\sigma+6)^2}{18\alpha\left[\mu\,(34\sigma^2 + 153\sigma + 162) - 2\sigma\,(7\sigma + 18)\right]} < 0$$

The first inequality is equivalent to $\mu < \check{\mu}$. However, since $\check{\mu} < \tilde{\mu}$, this lemma holds for $\mu < \check{\mu}$. ■

Lemma 7 *If $\check{\mu} < \mu < \hat{\mu}_2$, the stable equilibrium path $\boldsymbol{\lambda}^*(\tau)$ passing through $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$ at $\tau = \tau_{trade}$ enters the domain (C1) with*

$$\frac{\partial \lambda_1}{\partial \tau} = \frac{\partial \lambda_3}{\partial \tau} < 0$$

Proof. Computing and evaluating the Jacobian of (2) at $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$ with $\tau = \tau_{trade}$ for one-way trade to the center,we have the stability condition

$$\mu < \hat{\mu}_2 \tag{19}$$

The comparative statics at $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$ with $\tau = \tau_{trade}$ is

$$\frac{\partial \lambda_1}{\partial \tau} = \frac{\partial \lambda_3}{\partial \tau} = \frac{2\sigma(\sigma+6)^2(2-\mu)}{9\alpha\left[\mu\left(37\sigma^2+252\sigma+324\right)-4\sigma\left(5\sigma+18\right)\right]} < 0$$

where the inequality is due to (19).

In order for the equilibrium path to enter the domain of the one-way trade to the center (C1), it is necessary that

$$\frac{2\sigma(\sigma+6)^2(2-\mu)}{9\alpha\left[\mu\left(37\sigma^2+252\sigma+324\right)-4\sigma\left(5\sigma+18\right)\right]} < -\frac{(\sigma+6)^2}{18\alpha\sigma} < 0$$

which is equivalent to $\breve{\mu} < \mu$. Hence, this lemma is true for $\breve{\mu} < \mu < \hat{\mu}_2$. ■

Lemma 8 *There is no stable equilibrium path $\boldsymbol{\lambda}^*(\tau)$ passing through $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$ at $\tau = \tau_{trade}$ that enters domain (C2).*

Proof. Computing and evaluating the Jacobian of (2) at $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$ with $\tau = \tau_{trade}$ for one-way trade from 1 to 2 and neighboring trade between 2 and 3,the stability condition requires

$$(53\sigma^3+453\sigma^2+1188\sigma+972)\mu^2-12\sigma(5\sigma^2+28\sigma+36)\mu+16\sigma^2\left(\sigma+3\right) > 0 \quad (20)$$

The comparative statics at $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$ with $\tau = \tau_{trade}$ is

$$\frac{\partial \lambda_3}{\partial \tau} = \frac{(\mu-2)\sigma(\sigma+6)^2}{9\alpha[(53\sigma^3+453\sigma^2+1188\sigma+972)\mu^2-12\sigma(5\sigma^2+28\sigma+36)\mu+16\sigma^2\left(\sigma+3\right)]} < 0$$

where inequality holds from (20). Thus, any slight decrease in τ leads to $\lambda_3 > 1/3$, which does not pass through domain (C2). ■

Lemma 9 *If $\mu < \bar{\mu}_2$, then there is no stable equilibrium path $\boldsymbol{\lambda}^*(\tau)$ passing through $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$ at $\tau = \tau_{trade}$ that enters domain (C3).*

Proof. Computing and evaluating the Jacobian of (2) at $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$ with $\tau = \tau_{trade}$ autarky in 1 and one-way trade from 2 to 3,we have the stability condition

$$\mu < \breve{\mu} \quad (21)$$

Since $\hat{\mu}_2 < \breve{\mu}$, stability condition (21) is met. The comparative statics at $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$ with $\tau = \tau_{trade}$ is

$$\frac{\partial \lambda_3}{\partial \tau} = \frac{(\mu-2)\sigma(\sigma+6)^2}{9\alpha[(13\sigma^2+99\sigma+162)\mu-4\sigma\left(2\sigma+9\right)]} > 0 \quad (22)$$

where the inequality holds from (21). We also have $d\lambda_3/d\lambda_1 = -2$ along the equilibrium path $\boldsymbol{\lambda}^*(\tau)$ at $\tau = \tau_{trade}$.

In order for the equilibrium path to enter domain (C3), $\boldsymbol{\lambda}^*(\tau)$ should not enter the neighboring trade triangle (C5). This is shown to be equivalent that (22) is smaller than the partial derivative of the vertical axis of the southeast vertex $(2(\alpha - \tau)/\sigma\tau, [(\sigma + 4)\tau - 4\alpha]/\sigma\tau)$ in absolute value. A straightforward computation yields $\mu > \bar{\mu}_2$. ∎

Lemma 10 *There is no stable equilibrium path $\boldsymbol{\lambda}^*(\tau)$ passing through $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$ at $\tau = \tau_{trade}$ that enters domain (C4).*

Proof. Computing and evaluating the Jacobian of (2) at $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$ with $\tau = \tau_{trade}$ for one-way trade from the center, we have the stability condition

$$\mu < \bar{\mu}_2 \tag{23}$$

The comparative statics at $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$ with $\tau = \tau_{trade}$ is

$$\frac{\partial\lambda_1}{\partial\tau} = \frac{\partial\lambda_3}{\partial\tau} = \frac{2(\mu - 2)\sigma(\sigma + 6)^2}{18\alpha[(14\sigma^2 + 99\sigma + 162)\mu - 2\sigma(5\sigma + 18)]} > 0$$

where the inequality holds from (23). In order for the equilibrium path to enter the domain of the one-way trade from the center (C4), it is necessary that

$$\frac{(\mu - 2)\sigma(\sigma + 6)^2}{18\alpha[(14\sigma^2 + 99\sigma + 162)\mu - 2\sigma(5\sigma + 18)]} > \frac{(\sigma + 6)^2}{36\alpha\sigma}$$

or equivalently $\mu > \bar{\mu}_2$, which contradicts (23). ∎

Appendix D: Proof of Lemma 3

We show that when $\bar{\mu}_2 < \mu < \hat{\mu}_2$, there are 3 stable equilibrium paths passing through $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$ at $\tau = \tau_{trade}$.

Since $\mu < \hat{\mu}_2$, Lemmas 6 and 7 guarantee existence of a stable path such that

$$\frac{\partial\lambda_1}{\partial\tau} = \frac{\partial\lambda_3}{\partial\tau} < 0$$

in the vicinity of $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$ with $\tau = \tau_{trade}$. This property is the same as (14).

Since $\bar{\mu}_2 < \mu < \hat{\mu}_2 < \breve{\mu}$, two other stable paths are shown to exist from the proof of Lemma 9: one is in (C3) and the other is in (C3'). When the stable path enters (C3), from $\mu < \hat{\mu}_2 < \breve{\mu}$, the stability condition (21) is satisfied with the property

$$\frac{\partial \lambda_1}{\partial \tau} = \frac{\partial \lambda_2}{\partial \tau} < 0 < \frac{\partial \lambda_3}{\partial \tau}$$

which is due to (22) and $d\lambda_3/d\lambda_1 = -2$. Similarly, we can show that when the other path which enters (C3') has the property

$$\frac{\partial \lambda_3}{\partial \tau} = \frac{\partial \lambda_2}{\partial \tau} < 0 < \frac{\partial \lambda_1}{\partial \tau}$$

Appendix E: Proof of Lemma 4

Define $y_{rs} \equiv \partial y_r/\partial \lambda_s|_{\lambda_1^*=\lambda_3^*}$. Since $y_{11} = y_{33}$ and $y_{13} = y_{31}$, the two eigenvalues are real and given by $y_{11} \pm y_{13}$ in an axisymmetric equilibrium $\lambda_1^* = \lambda_3^*$. Let $X \equiv y_{13}(\lambda_1 - \lambda_3)$, then

$$\dot{X} = y_{13}(\dot{\lambda}_1 - \dot{\lambda}_3) = (y_{11} - y_{13})\, X$$

in the vicinity of an axisymmetric equilibrium. We show $y_{11} - y_{13} < 0$ below so that X goes to zero, which necessarily results in the axisymmetric configuration $\lambda_1 = \lambda_3$. Let

$$\bar{\tau}_{trade} \equiv \frac{2\sigma}{\sigma + 4} < \tau_{trade}$$

at which full trade starts.

(i) Neighboring trade: $0 < \mu \leq \breve{\mu}$ *and* $\bar{\tau}_{trade} \leq \tau \leq \tau_{trade}$.

Noting that $y_{1r} = \lambda_1 \partial \left(V_1 - \sum_{s=1}^{3} \lambda_s V_s\right)/\partial \lambda_r$ about an axisymmetric equilibrium because $V_1 - \sum_{s=1}^{3} \lambda_s V_s = 0$ holds, we have

$$\operatorname{sgn}(y_{11} - y_{13}) = \operatorname{sgn}(\tau(\lambda_1) - \tau) \tag{24}$$

where

$$\tau(\lambda_1) = \frac{2\alpha(4\sigma - 18\mu - 19\mu\sigma + 27\mu\sigma\lambda_1)}{\sigma(2\lambda_1 - 1)(4\sigma - 6\mu - 13\mu\sigma + 21\mu\sigma\lambda_1)} \tag{25}$$

It is readily shown that $\tau'(\lambda_1) < 0$ for all $\mu < \hat{\mu}_2$. Four cases may arise.

(i-a) For $0 < \mu \leq \hat{\mu}_1$ *and* $\bar{\tau}_{trade} \leq \tau \leq \tau_{trade}$.

Solving $\tau(0) = \bar{\tau}_{trade}$ yields $\mathrm{sgn}(\bar{\tau}_{trade} - \tau(0)) = \mathrm{sgn}(2\sigma/(8\sigma+9) - \mu)$. If $\mu < 2\sigma/(8\sigma+9)$, then we have $\tau \geq \bar{\tau}_{trade} > \tau(0) > \tau(\lambda_1)$, where the last inequality is due to $\tau'(\lambda_1) < 0$. That is, if $\mu < 2\sigma/(8\sigma+9)$, then $y_{11} < y_{13}$ holds from (24). Therefore, we only have to show $\hat{\mu}_1 < 2\sigma/(8\sigma+9)$.

Let

$$\begin{aligned} f(\lambda_1) \equiv\ & 36\mu^2\sigma^2(\lambda_1)^4 - 12\mu\sigma(108\mu + 2\sigma + 73\mu\sigma)(\lambda_1)^3 \\ & +(1981\mu^2\sigma^2 + 5580\mu^2\sigma + 3888\mu^2 + 202\mu\sigma^2 + 360\mu\sigma + 4\sigma^2)(\lambda_1)^2 \\ & -6\left(164\mu^2\sigma^2 + 552\mu^2\sigma + 414\mu^2 + 9\mu\sigma^2 - 8\mu\sigma - 36\mu + 4\sigma^2 + 8\sigma\right)\lambda_1 \\ & +4\left(35\mu^2\sigma^2 + 118\mu^2\sigma + 99\mu^2 - \mu\sigma^2 - 14\mu\sigma - 18\mu + 2\sigma^2 + 4\sigma\right) \end{aligned} \quad (26)$$

Tedious computations yield a discriminant of the fourth-order polynomial $f(\lambda_1)$as

$$\begin{aligned} & J(\mu,\sigma) \equiv (47990870424\sigma^{10} + 818858377608\sigma^9 + 6233866885776\sigma^8 + 27871575543648\sigma^7 \\ & +81006445037160\sigma^6 + 159828536439576\sigma^5 + 216649318299408\sigma^4 + 199048380545664\sigma^3 \\ & +118496258578656\sigma^2 + 41214652554240\sigma + 6347497291776)\mu^9 + (244773597756\sigma^{10} \\ & +4033200123992\sigma^9 + 29638387205080\sigma^8 + 127933562389632\sigma^7 + 359305187569956\sigma^6 \\ & +686338813527120\sigma^5 + 903560758379424\sigma^4 + 810184042378368\sigma^3 + 474066051432192\sigma^2 \\ & +163712534317056\sigma + 25389989167104)\mu^8 + (139706960442\sigma^{10} + 2489442201882\sigma^9 \\ & +19667440053936\sigma^8 + 90885419798064\sigma^7 + 272376170796816\sigma^6 + 553486369904064\sigma^5 \\ & +772370491431744\sigma^4 + 730506530264832\sigma^3 + 447694061333760\sigma^2 + 160274306617344\sigma \\ & +25389989167104)\mu^7 - (67559427555\sigma^{10} + 933013798244\sigma^9 + 5688082731088\sigma^8 \\ & +20240570983968\sigma^7 + 46734537234528\sigma^6 + 73257998984064\sigma^5 + 78523014339072\sigma^4 \\ & +55696311783936\sigma^3 + 23696996705280\sigma^2 + 4584303599616\sigma)\mu^6 - (28157518950\sigma^{10} \\ & +402032785240\sigma^9 + 2471744214224\sigma^8 + 8541639937344\sigma^7 + 18133097662656\sigma^6 \\ & +24191622839808\sigma^5 + 19779778914048\sigma^4 + 9044268585984\sigma^3 + 1765642572288\sigma^2)\mu^5 \\ & +(5390512692\sigma^{10} + 70243998096\sigma^9 + 389050575360\sigma^8 + 1187171367936\sigma^7 \\ & +2155191936768\sigma^6 + 2327029088256\sigma^5 + 1383238798848\sigma^4 + 349091389440\sigma^3)\mu^4 \\ & +(1208928\sigma^{10} - 35357408\sigma^9 - 499871872\sigma^8 - 2310391296\sigma^7 - 4954556160\sigma^6 \\ & -5052658176\sigma^5 - 1981946880\sigma^4)\mu^3 + (-3847056\sigma^{10} - 30310336\sigma^9 - 86482304\sigma^8 \\ & -102706176\sigma^7 - 35659008\sigma^6 + 9704448\sigma^5)\mu^2 - (114816\sigma^{10} + 1102080\sigma^9 + 3784704\sigma^8 \\ & +5514240\sigma^7 + 2875392\sigma^6)\mu + 5376\sigma^{10} + 31744\sigma^9 + 60416\sigma^8 + 36864\sigma^7 \end{aligned}$$

While $J(\hat{\mu}_1, \sigma) = 0$ by definition, we have $J(2\sigma/(8\sigma+9), \sigma) < 0$, implying that $\hat{\mu}_1 < 2\sigma/(8\sigma+9)$.

(i-b) neighboring trade: $\hat{\mu}_1 < \mu < \check{\mu}$ *and* $\bar{\tau}_{trade} \leq \tau \leq \tau_{trade}$.

In this case, λ_1 lies in the interval of $[1/3, 1/2]$ from Lemma 5 (i).Since $\tau'(\lambda_1) < 0$, $\tau(1/3) > \tau(\lambda_1)$ for all $\lambda_1 \in [1/3, 1/2]$. Furthermore, $\bar{\tau}_{trade} > \tau(1/3)$ for $\mu = \check{\mu}$, and hence $\tau > \tau(\lambda_1)$, which means $y_{11} < y_{13}$ at $\lambda_1 = \lambda_3$.

(ii) One-way trade to the center: $\check{\mu} < \mu < \hat{\mu}_2$ and $\bar{\tau}_{trade} \leq \tau \leq \tau_{trade}$.

A direct computation yields

$$y_{11} - y_{13}|_{\lambda_1=\lambda_3} = \frac{\alpha^2\sigma(18\mu - 4\sigma + 4\mu\sigma + 3\mu\sigma\lambda_1)}{\gamma\mu(\sigma\lambda_1 + 2)^3}$$

Since the RHS is increasing in μ and λ_1, substituting $\mu = \hat{\mu}_2$ and $\lambda_1 = 1/2$, we have

$$y_{11} - y_{13}|_{\lambda_1=\lambda_3} < -\frac{2\alpha^2\sigma^2(19\sigma + 126)}{3\gamma(5\sigma + 18)(\sigma + 4)^3} < 0$$

(iii) Full trade: $\mu < \acute{\mu} \equiv (\sigma^2 - 4)/(10\sigma^2 + 48\sigma + 50)$ and $\tau < \bar{\tau}_{trade}$.

The full trade condition $\mu < \acute{\mu}$ is equivalent to at $\lambda_1^*(\bar{\tau}_{trade}) > 0$, which the interior equilibrium condition at $\tau = \bar{\tau}_{trade}$.

Since the stable equilibrium path $\boldsymbol{\lambda}^*(\tau)$ and its inverse function $\tau_c(\lambda_1)$ are increasing functions, we get

$$\tau_c(\lambda_1) > \tau_c(0) = \frac{4\alpha(2 + 7\mu + \sigma + 5\mu\sigma)}{4 + 4\sigma + \sigma^2 + 2\mu + 2\mu\sigma} > \bar{\tau}_{trade} \tag{27}$$

On the other hand, the definition (25) is revised by

$$\tau(\lambda_1) = \frac{3\alpha\mu(2\sigma + 3)}{\mu + 2\sigma + \sigma^2 + 94\mu\sigma - \mu\sigma^2 + 3\mu\sigma(\sigma + 2)\lambda_1}$$

Since $\tau'(\lambda_1) < 0$ holds, we have

$$\begin{aligned}
\tau(\lambda_1) &< \tau(0) = \frac{3\alpha\mu(2\sigma + 3)}{\mu + 2\sigma + \sigma^2 + 94\mu\sigma - \mu\sigma^2} \\
&< \frac{3\alpha\acute{\mu}(2\sigma + 3)}{\acute{\mu} + 2\sigma + \sigma^2 + 94\acute{\mu}\sigma - \acute{\mu}\sigma^2} \\
&= \frac{\alpha(\sigma - 2)(2\sigma + 3)}{3\sigma^3 + 18\sigma^2 + 17\sigma - 6} \\
&< \bar{\tau}_{trade}
\end{aligned} \tag{28}$$

where the second inequality is due to $\partial\tau(0)/\partial\mu > 0$ and the last inequality is from $\sigma > 2$ which is from $\mu < \acute{\mu}$. Putting (27) and (28) together, we have $\tau(\lambda_1) < \bar{\tau}_{trade} < \tau_c(\lambda_1)$, which is $y_{11} < y_{13}$ at $\lambda_1 = \lambda_3$.

It is known in the two-dimensional dynamics that the equilibrium point is a node (resp. saddle point) if $|y_{11}| > |y_{13}|$ (resp. $|y_{11}| < |y_{13}|$).

Appendix F: Proof of Lemma 5

(C6) stands for full trade between any pair of regions. Since the denominator of $\Delta V_{12}(\lambda_1, \tau)$ is positive except $\lambda_1 = (\sigma + 2)/\sigma$, define the numerator of $\Delta V_{12}(\lambda_1, \tau)$ by $\Delta V_{12}^n(\lambda_1, \tau)$ for all $\lambda_1 \neq (\sigma + 2)/\sigma$. Because the implicit function $\Delta V_{12}^n(\lambda_1, \tau) = 0$ is quadratic in τ in all cases of (C1), (C4), (C5) and (C6), it has two explicit solutions: $\tau_a(\lambda_1)$ and $\tau_b(\lambda_1)$, which are inverse functions of $\lambda_1^*(\tau)$.

Lemma 11 *The function $\lambda_1^*(\tau)$ is continuous for all domain of $0 \leq \lambda_1 \leq 1/2$ and $0 \leq \tau \leq \tau_{trade}$ irrespective of the trade patterns.*

Proof. From the implicit functional theorem, $\lambda_1^*(\tau)$ is continuous inside the cases of (C1), (C4), (C5) and (C6). Therefore, we only have to check the continuity at the boundaries. However, say, at the boundaries of cases (C1) and (C4), if $\Delta V_{12}(\lambda_1, \tau) = 0$ holds in case (C1), then this also holds in case (C4). The same is true for all other boundaries. ■

Lemma 12 *The functions $\tau_a(\lambda_1)$ and $\tau_b(\lambda_1)$ are real for all $\lambda_1 \in [1/2, (\sigma + 2)/\sigma)$ except that there are at most 2 discontinuous points in the interval.*

Proof. First, we consider the number of discontinuous points. The denominator of $\tau_a(\lambda_1)$ and $\tau_b(\lambda_1)$ is common and quartic in λ_1 with the positive coefficient of $(\lambda_1)^4$. Since, the denominator is shown to be negative at $\lambda_1 = 1/2$, $(\sigma + 2)/\sigma$, there exist at most two discontinuous points.

Second, we show that $\tau_a(\lambda_1)$ and $\tau_b(\lambda_1)$ are real. The common terms in the square root of $\tau_a(\lambda_1)$ and $\tau_b(\lambda_1)$ are $f(\lambda_1)$, which is defined by (26). $f(\lambda_1)$ is quartic in λ_1 with a positive coefficient of $(\lambda_1)^4$. It is readily shown that

$$f'(1/2) > 0 \qquad \lim_{\lambda_1 \to \frac{\sigma+2}{\sigma} - 0} f'\left(\frac{\sigma + 2}{\sigma}\right) > 0$$

and the larger inflexion point is outside the interval of $[1/2, (\sigma + 2)/\sigma)$. These imply that both $\lambda_1 = 1/2$ and $\lambda_1 = (\sigma + 2)/\sigma - 0$ are on the left-hand side of the quartic curve with a positive slope. Since $f(1/2) > 0$, we have $f(\lambda_1) > 0$ for all $\lambda_1 \in [1/2, (\sigma + 2)/\sigma)$, which means real $\tau_a(\lambda_1)$ and $\tau_b(\lambda_1)$. ■

Lemma 13 *When a stable equilibrium lying on $\tau_a(\lambda_1)$ (resp. $\tau_b(\lambda_1)$) disappears, then it will jump to the same $\tau_a(\lambda_1)$ (resp. $\tau_b(\lambda_1)$) or the corners $\lambda_1 = 0, 1/2$.*

Proof. It is readily shown that $\Delta V_{12}(\lambda_1, \tau) = 0$ is quartic in λ_1 in cases (C1) and quadratic in λ_1 in case (C6). Since $\lim_{\lambda_1 \to \pm\infty} \tau_a(\lambda_1) = \lim_{\lambda_1 \to \pm\infty} \tau_b(\lambda_1) = 0$ holds for all cases of (C1), (C4), (C5) and (C6), $\tau_a(\lambda_1)$ and $\tau_b(\lambda_1)$ have at most one trough $(\check{\lambda}_1, \check{\tau})$, at which a jump may occur.

Case (C1). Suppose $\tau_a(\lambda_1)$ has a peak $(\hat{\lambda}_1, \hat{\tau})$, such that is $\check{\tau} < \hat{\tau}$ in the interval of $[0, 1/2]$, then a stable equilibrium path $\tau_b(\lambda_1)$ may jump at the trough $(\check{\lambda}_1, \check{\tau}_b)$ to the different curve $\tau_a(\lambda_1)$. However, such a trough-peak combination violates the property in that there are more than 4 solutions of λ_1 in $\Delta V_{12}(\lambda_1, (\check{\tau} + \hat{\tau})/2) = 0$ as shown below.

From Lemma 12, $\tau_b(\lambda_1)$ is discontinuous at most 2 points.

(i) If $\tau_b(\lambda_1)$ is continuous in $\lambda_1 \in [1/2, (\sigma+2)/\sigma)$, then $\tau_b(\lambda_1)$ necessarily crosses the horizontal line $\tau = (\check{\tau} + \hat{\tau})/2$ in $(\max\{\check{\lambda}_1, \hat{\lambda}_1\}, (\sigma+2)/\sigma)$ in addition to 4 points of intersection in $[\min\{\check{\lambda}_1, \hat{\lambda}_1\}, \max\{\check{\lambda}_1, \hat{\lambda}_1\}]$, thus violating the property.

(ii) If $\tau_b(\lambda_1)$ is discontinuous only at $\bar{\lambda}_1 \in (1/2, (\sigma+2)/\sigma)$, then $\tau_b(\lambda_1)$ goes to either $-\infty$ or $+\infty$. However, the sign of $\tau_b(\lambda_1)$ does not change in the vicinity of $\bar{\lambda}_1$ $\tau_b(\lambda_1)$ since $\bar{\lambda}_1$ must be a repeated root of the quartic denominator of $\tau_b(\lambda_1)$. Hence, the above case (i) applies.

(iii) If $\tau_b(\lambda_1)$ is discontinuous at two distinct $\bar{\lambda}_1, \tilde{\lambda}_1$ with $1/2 < \bar{\lambda}_1 < \tilde{\lambda}_1 < (\sigma+2)/\sigma$, then $\tau_b(\lambda_1)$ changes its sign in the vicinity of $\bar{\lambda}_1, \tilde{\lambda}_1$. If $\lim_{\lambda_1 \to \bar{\lambda}_1 - 0} \tau_b(\lambda_1) = -\infty$, then $\tau_b(\lambda_1)$ crosses $\tau = (\check{\tau} + \hat{\tau})/2$ in $(\check{\lambda}_1, \bar{\lambda}_1)$, and hence the number of intersection points with $\tau = (\check{\tau} + \hat{\tau})/2$ exceeds 4.

On the other hand, if $\lim_{\lambda_1 \to \bar{\lambda}_1 - 0} \tau_b(\lambda_1) = +\infty$, then it must be that $\lim_{\lambda_1 \to \bar{\lambda}_1 + 0} \tau_b(\lambda_1) = -\infty$ $\lim_{\lambda_1 \to \tilde{\lambda}_1 - 0} \tau_b(\lambda_1) = -\infty$ and $\lim_{\lambda_1 \to \tilde{\lambda}_1 + 0} \tau_b(\lambda_1) = +\infty$. Thus, $\tau_b(\lambda_1)$ crosses the horizontal line $\tau = (\check{\tau} + \hat{\tau})/2$ in $(\tilde{\lambda}_1, (\sigma+2)/\sigma)$, and hence the number of intersection points with $\tau = (\check{\tau} + \hat{\tau})/2$ exceeds 4.

Cases (C4) and (C5). It is easily shown that the equilibrium path crosses at most once at the boundary between cases (C1) and (C4) and at the boundary between cases (C1) and (C5). Therefore, once a stable equilibrium path crosses one of the boundaries, it will never cross again, implying that the above trough-peak combination does not arise. Hence, it will be sure to hit the corner $\lambda_1 = 1/2$ (resp. 0) in case (C1) (resp. (C4)).

Case (C6). Since $\Delta V_{12}(\lambda_1, \tau) = 0$ is quadratic in λ_1, the above trough-peak combination cannot arise. ■

Lemma 13 together with Lemma 11 ensures that the stable equilibrium path starting from $\boldsymbol{\lambda}^* = (1/3, 1/3, 1/3)$ at $\tau = \tau_{trade}$ will keep on the same curve or hit the corners. The only exception occurs when $d\tau_a(\lambda_1)/d\lambda_1 = d\tau_b(\lambda_1)/d\lambda_1 = \infty$, which corresponds to $d\lambda_1^*(\tau)/d\tau = 0$. At this point, an equilibrium does not disappear, and the two stable curves $\tau_a(\lambda_1)$ and $\tau_b(\lambda_1)$ are connected. This is case (ii) with $\mu < \hat{\mu}_1$, where the path first moves toward $\lambda_1 = 1/2$ and then moves back to $\lambda_1 = 0$, resulting in $(0, 1, 0)$. Otherwise, the path moves according to case (i) with $\mu > \hat{\mu}_1$ and reaches $(1/2, 0, 1/2)$ monotonically. Finally, since there is no stable equilibrium path approaching an interior solution of λ_1 at $\tau = 0$, Lemma 5 is proven.

Appendix G: Proof of Proposition 4

(i) In the case of neighboring trade, when $\lambda_2 = 0$, we have

$$\left.\frac{\partial \left(V_1 - V_3\right)|_{\lambda_2=0}}{\partial \lambda_1}\right|_{\lambda_1=\lambda_3=1/2} = \frac{4\alpha^2\sigma\left[\mu\left(11\sigma + 36\right) - 8\sigma\right]}{3\gamma\mu\left(\sigma+4\right)^3}$$

which is negative for all $\mu \leq \mu_2^*$, implying that $\boldsymbol{\lambda}^* = (1/2, 0, 1/2)$ is a stable during the neighboring trade of $\tau > \bar{\tau}_{trade}$. This equilibrium continues during $\tau > \bar{\tau}_{trade}$ if $V_1 - V_2|_{\lambda_1=\lambda_3=1/2, \lambda_2=0, \tau=\bar{\tau}_{trade}} \geq 0$, which is equivalent to

$$\mu > \frac{2}{11} \tag{29}$$

Otherwise, $(1/2, 0, 1/2)$ ceases to be an equilibrium, leading to the new equilibrium $(0, 1, 0)$ for a sufficiently large period of time.

(ii) In the case of full trade, when $\lambda_2 = 0$, we get

$$\left.\frac{\partial \left(V_1 - V_3\right)|_{\lambda_2=0}}{\partial \lambda_1}\right|_{\lambda_1=\lambda_3=1/2} = \frac{2\sigma\tau\left[\mu\left(\sigma^2 + 14\sigma + 18\right) + 2\sigma(\sigma+2)\right]}{3\gamma\mu\left(\sigma+2\right)^2}\left(\tau_{break} - \tau\right)$$

where

$$\tau_{break} \equiv \frac{6\alpha\mu(2\sigma+3)}{\mu\left(\sigma^2 + 14\sigma + 18\right) + 2\sigma(\sigma+2)}$$

That is, $(1/2, 0, 1/2)$ is a stable equilibrium for $\tau > \tau_{break}$. Since

$$\frac{\partial \left.\dot{\lambda}_2/\lambda_2\right|_{\lambda_3=1-\lambda_1}}{\partial \lambda_1} = \frac{18\alpha^2\mu^2\sigma^3(2\sigma+3)^2\left(2\lambda_1 - 1\right)}{\gamma(\sigma+2)^2\left(\mu\sigma^2 + 2\sigma^2 + 14\mu\sigma + 18\mu + 4\sigma\right)^2}$$

$\left.\dot{\lambda}_2/\lambda_2\right|_{\lambda_3=1-\lambda_1}$ is maximized at $\lambda_1 = 0, 1$. Solving $\dot{\lambda}_2/\lambda_2 = 0$ at $\lambda_1 = 1$ yields $\operatorname{sgn}\left(\dot{\lambda}_2\right) = \operatorname{sgn}\left(\breve{\mu}_1 - \mu\right)$, where

$$\breve{\mu}_1 \equiv \frac{(5\sigma+2)(\sigma+2)}{8\sigma^2+25\sigma+18+3\sqrt{32\sigma^4+240\sigma^3+489\sigma^2+324\sigma+36}}$$

The stability condition is therefore

$$\mu > \breve{\mu}_1 \tag{30}$$

This equilibrium continues during $\tau > \bar{\tau}_{trade}$ if $V_1 - V_2|_{\lambda_1=\lambda_3=1/2,\lambda_2=0} \geq 0$, which is equivalent to

$$\bar{\tau}_{break} \equiv \frac{8\alpha(1-\mu)}{7\mu\sigma+6\mu+2\sigma+12} < \tau \leq \bar{\tau}_{trade}$$

where $\bar{\tau}_{break} < \bar{\tau}_{trade}$ holds for $\mu > 2/11$. Solving $\dot{\lambda}_2/\lambda_2 = 0$ at $\lambda_1 = 1$ and $\tau = \bar{\tau}_{break}$ yields

$$\bar{\mu}_1 \equiv \frac{3\sqrt{193\sigma^4+1044\sigma^3+1932\sigma^2+1296\sigma+144}-13\sigma^2-50\sigma-36}{98\sigma^2+310\sigma+252}$$

However, it can be shown that $\breve{\mu}_1 > \bar{\mu}_1$ for all $\mu > 2/11$.

(iii) Finally, we consider the case that $(1/2, 0, 1/2)$ becomes unstable when full trade starts at $\tau = \bar{\tau}_{trade}$. Since

$$\left.\frac{\partial \left.\dot{\lambda}_2/\lambda_2\right|_{\lambda_3=1-\lambda_1}}{\partial\lambda_1}\right|_{\tau=\bar{\tau}_{trade}} = \frac{2\sigma\alpha^2\left(8\sigma^2-17\mu\sigma^2-19\mu\sigma-72\mu+16\sigma\right)(2\lambda_1-1)}{3\gamma\mu(\sigma+4)^2(\sigma+2)^2}$$

$\left.\dot{\lambda}_2/\lambda_2\right|_{\lambda_3=1-\lambda_1,\tau=\bar{\tau}_{trade}}$ is maximized at $\lambda_1 = 0, 1/2$ or 1. Solving $\dot{\lambda}_2/\lambda_2 = 0$ at $\lambda_1 = 1/2$ and 1 yields the stability conditions

$$\mu > \tilde{\mu}_1 \qquad \text{and} \qquad \mu > \frac{2}{11} \tag{31}$$

where

$$\tilde{\mu}_1 \equiv \frac{(5\sigma + 2)(\sigma + 2)}{2(7\sigma^2 + 30\sigma + 29)}$$

Hence, the conditions for the stable path reaching $(1, 0, 0)$ or $(0, 0, 1)$ are summarized as

$$\mu_1^* \equiv \max\left\{\frac{2}{11}, \min\{\tilde{\mu}_1, \check{\mu}_1\}\right\}$$

References

[1] Alesina, A. and E. Spolaore (1997) On the number and size of nations, *Quarterly Journal of Economics* 112, 1027-1056.

[2] Anderson, S.P., A. de Palma and J.-F. Thisse (1992) *Discrete Choice Theory of Product Differentiation*, Cambridge (Mass.), MIT Press.

[3] Anderson, S.P., N. Schmitt and J.-F. Thisse (1995) Who benefits from antidumping legislation?, *Journal of* International *Economics* 38, 321-337.

[4] Behrens, K. (2003) Asymmetric trade and agglomeration, LATEC, Université de Bourgogne, mimeographed.

[5] Brander, J. and P. Krugman (1983) A 'reciprocal dumping' model of international trade, *Journal of International Economics* 15, 313-321.

[6] Casella, A. (2001) The role of market size in the formation of jurisdictions, *Review of Economic Studies* 68, 83-108.

[7] Christaller, W. (1933) *Die Zentralen Orte in Süddeutschland*, Jena, Gustav Fischer Verlag. English translation: *The Central Places of Southern Germany*. Englewood Cliffs, Prentice-Hall, 1966.

[8] Cremer, H., de Kerchove A.M. and J.-F. Thisse (1985) An economic-theory of public facilities in space *Mathematical Social Sciences* 9, 249-262.

[9] Cronon, W. (1991) *Nature's Megalopolis: Chicago and the Great West*, New York, Norton.

[10] De Fraja, G. and G. Norman (1993) Product differentiation, pricing policy and equilibrium, *Journal of Regional Science* 33, 343-363.

[11] Economides, N. and A. Siow (1988) The division of markets is limited by the extent of liquidity (spatial competition with externalities), *American Economic Review* 78, 108-121.

[12] Forslid, R. and G.I.P. Ottaviano (2003) An analytically solvable core-periphery model, *Journal of Economic Geography*, forthcoming.

[13] Fujita, M. and T. Mori (1997) Structural stability and evolution of urban systems. *Regional Science and Urban Economics* 27, 399-442.

[14] Furusawa, T. and H. Konishi (2002) Free trade networks, Working Paper 548, Boston College.

[15] Ginsburgh, V., Y.Y. Papageorgiou and J.-F. Thisse (1985) On existence and stability of spatial equilibria and steady-states. *Regional Science and Urban Economics* 15, 149-158.

[16] Krugman, P. (1991) Increasing returns and economic geography, *Journal of Political Economy* 99, 483-499.

[17] Krugman, P. (1993) On the number and location of cities, *European Economic Review* 37, 293-298.

[18] Ottaviano, G., T. Tabuchi and J.-F. Thisse (2002) Agglomeration and trade revisited, *International Economic Review* 43, 409-436.

[19] Ottaviano, G. and J.-F. Thisse (2002) Integration, agglomeration and the political economics of factor mobility, *Journal of Public Economics* 83, 429-456.

[20] Tabuchi, T., J.-F. Thisse and D.-Z. Zeng (2002) On the number and size of cities, CEPR Discussion Paper No.3386, http://www.e.u-tokyo.ac.jp/~ttabuchi.

[21] Venables, A.J. and N. Limão (2002) geographical disadvantage: a Heckscher-Ohlin-von Thünen model of international specialisation, *Journal of International Economics* 58, 239-263.

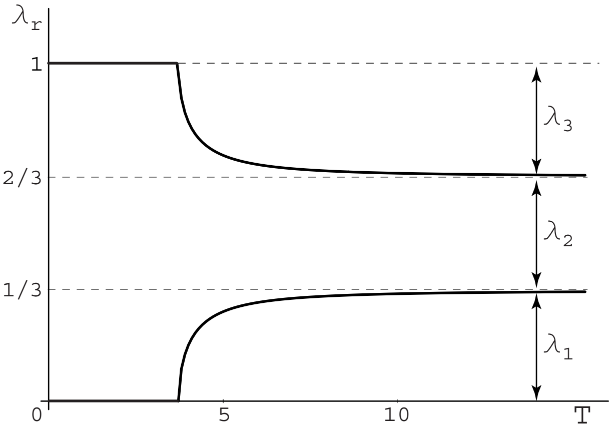

Figure 1: Equilibrium path in Proposition 1

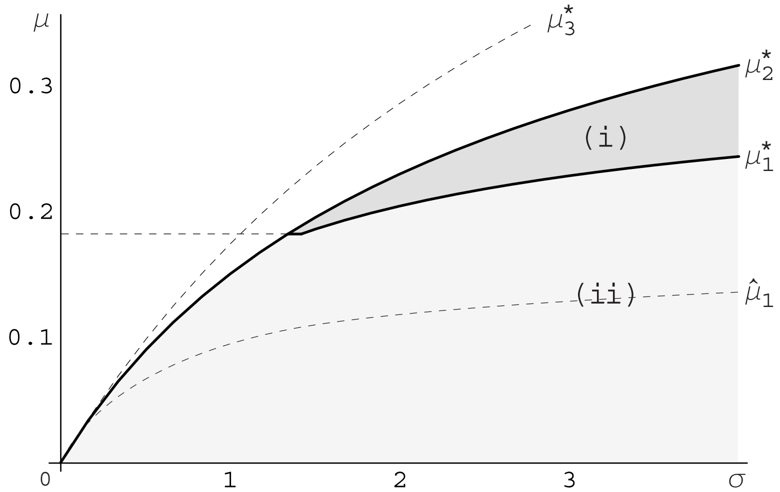

Figure 2: The domain of equilibrium paths

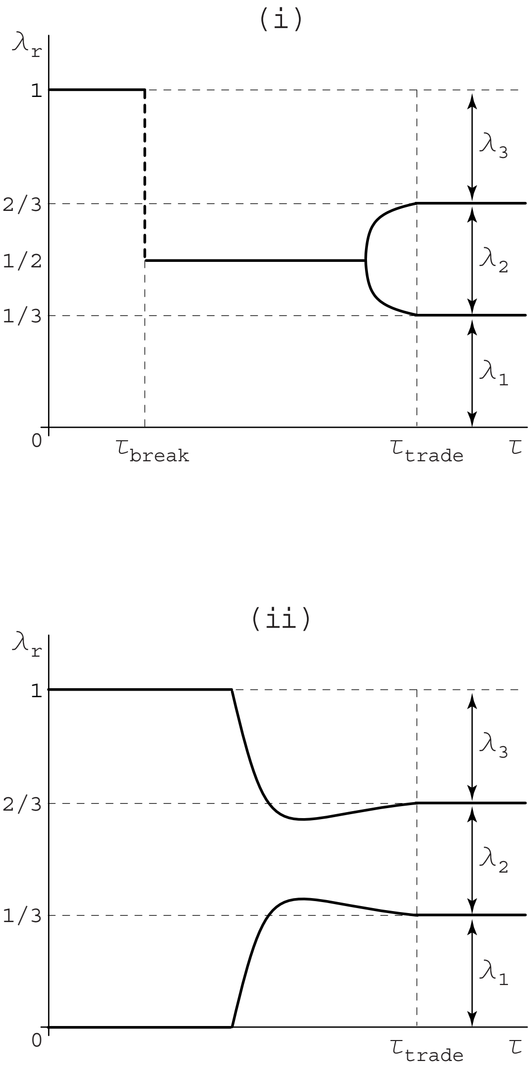

Figure 3: Two stable equilibrium paths in Proposition 4: (i) the central region shrinks and then disappears; (ii) the central region shrinks, but expands and becomes fully agglomerated in the end.

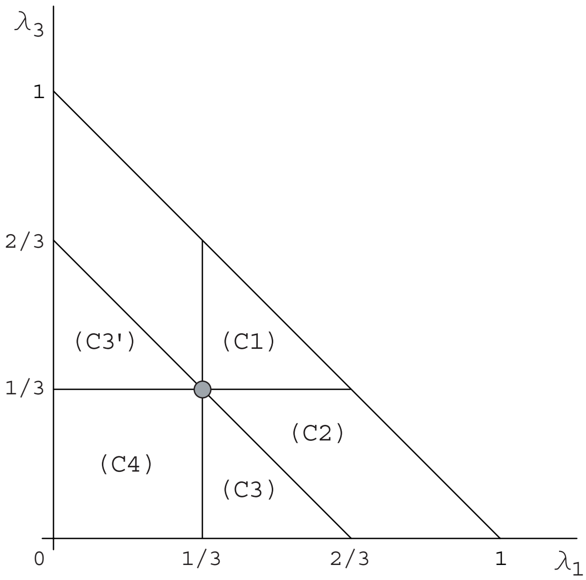

Figure 4: (C1) one-way trade from 1 and 3 to 2, (C2) one-way trade from 1 to 2 and neighboring trade between 2 and 3, (C3) autarky in 1 and one-way trade from 2 to 3, (C3') autarky in 3 and one-way trade from 2 to 1, (C4) one-way trade from 2 to 1 and 3

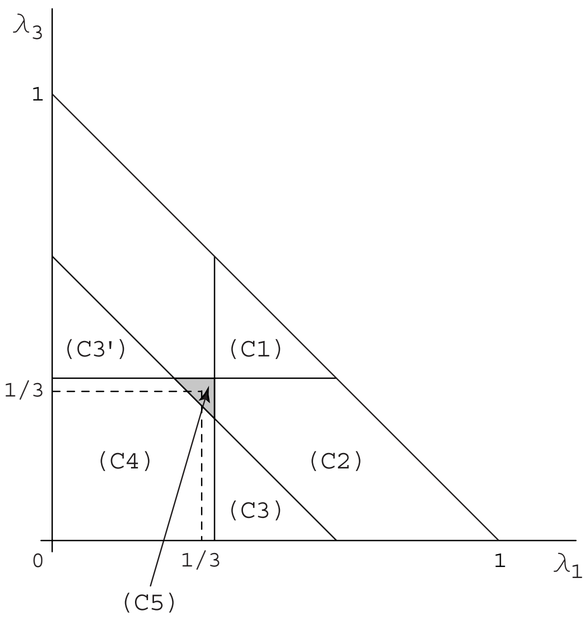

Figure 5: (C1)-(C4) same as Figure 4, (C5) neighboring trade