CIRJE-F-289

Accounting for Human Capital and Weak Identification in Evaluating the Epstein-Zin-Weil Non-Expected Utility Model of Asset Pricing

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July 2004

Accounting for Human Capital and Weak Identification in Evaluating the Epstein-Zin-Weil Non-Expected Utility Model of Asset Pricing[1]

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This Version: June 20, 2004

Abstract

In this paper, I first develop a new approach to estimating the return on the aggregate wealth portfolio that accounts for human capital and financial assets other than stocks. Using the estimated return on the wealth portfolio and the quarterly U.S. aggregate data on consumption and asset returns from 1959 to 2001, I then test the asset pricing and consumption implications of the Epstein and Zin (1991) and Weil (1990) model by employing the weak-identification robust tests of Stock and Wright's (2000) in the context of continuous updating generalized method of moments. In contrast with previous studies that ignored human capital and weak identification in evaluating this model, I find that its asset pricing implications cannot be rejected at conventional significance levels for reasonable parameter values. For example, the 95% confidence sets for unknown parameters include values of the relative risk aversion around 2 or lower, values of the elasticity of intertemporal substitution for consumption closely around 1, and the time discount factor around 0.987. Some of these parameter value combinations are able to simultaneously match the average equity premium and the average riskfree rate in the data. Furthermore, they imply that the dominant determinant of the equity premium is, surprisingly, the volatility of stock returns, the risk factor in the traditional capital asset pricing model.

Key Words: Asset Pricing, Non-Expected Utility Preferences, Human Capital, Weak Identification

[1] I thank John Conlon, Bill Smith, the seminar audience at University of Memphis, Ole Miss, and University of Tokyo, and the participants of the 2004 Midwest Macroeconomics Meetings held at Iowa State University for helpful comments on different versions of this paper. Bill Smith also corrected the stylistic errors in the paper, for which I am grateful. I thank Jonathan Wright for sharing with me some of the GAUSS and Matlab programs used in his paper with James Stock, and Sydney Ludvigson for providing the price deflator data used in a joint paper of hers with Martin Lettau. All remaining errors are mine.

1. Introduction

The consensus view in the asset pricing literature on the Epstein and Zin (1991) and Weil (1990) non-expected utility model (henceforth the EZW model) is that it cannot resolve the equity premium puzzle (Mehra and Prescott (1985)) and the related risk-free rate puzzle. See e.g. Weil (1989), Kocherlakota (1996), and Campbell (2003).[2] The goal of this paper is to fully evaluate the EZW model to see if this consensus still holds. A full evaluation that incorporates the major relevant developments in the asset pricing literature and the econometric literature should include the following three elements: to account for the presence in the wealth portfolio of human capital and financial assets that are usually ignored in an empirical analysis, such as private (i.e. noncorporate) equity, housing, and consumer durable goods; to account for weak identification in the estimation and testing; and to estimate all the parameters and test both the consumption and asset pricing implications of this model. Viewed in this light, the previous studies of the EZW model all lack at least one of these three elements, although they are very useful in fleshing out the properties and implications of the model. See, among others, Attanasio and Weber (1989), Weil (1989), Bufman and Leiderman (1990), Kocherlakota (1990a, 1996), Epstein and Zin (1991), Kandel and Stambaugh (1991), Cochrane and Hansen (1992), Cecchetti, Lam and Mark (1994), Campbell (1996), Koskievic (1999), Smith (1999), Stock and Wright (2000), Otrok, Ravikumar and Whiteman (2002), and Vissing-Jørgensen and Attanasio (2003).

Epstein and Zin (1991), in tests of their own model, had to assume that a return on an aggregate stock index was adequate to proxy the return on the optimal portfolio of the representative agent.[3] However, the return on the optimal portfolio should theoretically contain

[2] See also Cochrane (2001) and Constantinides (2002) for recent surveys and references therein for notable contributions to this literature.

[3] In equilibrium asset pricing models featuring a representative agent, the wealth portfolio, or market portfolio, is interpreted as the optimal portfolio of the representative agent in an empirical test. So the three terms, wealth portfolio, market portfolio and optimal portfolio, are interchangeable in this context.

the returns on all assets, despite that some of them are not observable (Roll (1977)). Epstein and Zin were, of course, aware of this problem. They wrote:

> "… Roll (1977)'s critique of CAPM is relevant here. If stochastic wages are a large factor in the wealth constraint of the typical agent, then, …, the return on the optimal portfolio of the agent should reflect the shadow return of the agent's human capital."

Researchers have since then made important progress on accounting for human capital in empirically evaluating asset pricing models. An influential approach pioneered by Campbell (1996) assumes that the conditional mean of (the logarithm of) the gross return on human capital is equal to the conditional mean of (the logarithm of) the gross return on financial assets.[4] He used a version of the EZW model as a starting point to motivate a multifactor asset pricing model. Vissing-Jørgensen and Attanasio (2003) extended Campbell's approach to allow the expected return on human capital to depend explicitly on the expected returns on bonds and stocks.[5] Another influential study by Jagannathan and Wang (1996) incorporated human capital by linking the return on human capital to the growth of labor income in evaluating the conditional capital asset pricing model (CAPM). More recently, Palacios-Huerta (2003) expanded the determinants of human capital return to include work effort and a skill premium in testing the conditional CAPM. Since available estimates indicate that human capital is the largest component of the U.S. wealth portfolio, accounting for it is the most important step in dealing with the Roll critique. But to account for other forms of financial assets such as real estate, private equity, and durable goods is also important in order to respond to Roll's critique sufficiently. Hence a full evaluation of the EZW model cannot ignore human capital and other assets that are usually ignored in an empirical analysis.

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[4] As noted by Campbell (1996), his approach includes as special cases the earlier methods of dealing with human capital in studying asset pricing, such as Fama and Schwert (1977), Shiller (1993) and Jagannathan and Wang (1996). Other earlier efforts include Mayers (1972) and Williams (1978).

[5] Braun and Shioji (2003) extended the Campbell model to identify and measure the risk in Japanese equity market.

To account for weak identification is necessary because there is now considerable evidence that the nonlinear asset pricing models are unlikely to be well identified. One major source of weak identification is the presence of weak instruments, i.e. instruments that are weakly correlated with the endogenous variables in a model. A recent survey by Stock, Wright and Yogo (2002) emphasized that weak identification invalidates statistical inference based on the conventional generalized method of moments (GMM) asymptotic theory of Hansen (1982). Neely, Roy and Whiteman (2001) examined the standard expected utility consumption-CAPM (C-CAPM) in detail, and found that the curvature parameter capturing the relative risk aversion coefficient (RRA) or the elasticity of intertemporal substitution for consumption (EIS) was near non-identification due to the weakness of the usual instruments, the lagged consumption growth and asset returns. To deal with the weak identification caused by weak instruments in testing nonlinear models, Stock and Wright (2000) developed an alternative asymptotic theory that is robust to the presence of weak instruments for the continuous updating GMM estimator of Hansen, Heaton and Yaron (1996). They also provided evidence for weak identification in the major C-CAPM models including the EZW model. An earlier paper by Smith (1999) also reported Monte Carlo evidence for poor identification in estimating the EZW model.

The EZW model has testable implications for both consumption and asset pricing, and includes three major preference parameters that are important for many purposes: the time discount factor, the RRA, and the EIS. A full evaluation of this model should naturally test both implications and estimate all the three parameters. It is useful to test the consumption implication, even in an asset-pricing context, because the test result determines how the riskfree rate should be calculated in this model. Suppose the Euler equation for the asset returns are not rejected by the data. If the test result on the consumption Euler equation is positive, it is then reasonable to combine the consumption Euler equation and the Euler equation for the riskfree rate to solve the riskfree rate by using the joint lognormality assumption for consumption growth and asset returns. See e.g. Campbell's (2003) eq. (23) for such a solution. But if the test result on the consumption Euler equation is negative, it is necessary to calculate the riskfree rate from the second Euler equation alone, provided that the return on the optimal portfolio is known. Therefore, knowing if the consumption Euler equation holds will be helpful in assessing whether the EZW model can resolve the riskfree rate puzzle.[6]

A full evaluation of this model should also estimate all the three parameters above. Many authors have estimated them in various contexts, but so far few of them have estimated them altogether, and none of them have accounted for human capital and weak identification at the same time. It is therefore of great interest to see how estimates of the three parameters above will be affected when these two important factors are both taken into account at the same time. This may help to address issues created by puzzling empirical findings on preference parameters. For example, most economists are more comfortable with a time discount factor smaller than 1 than a discount factor that is larger than 1, and there have been lots of empirical evidence for it. But in the empirical asset pricing literature, estimates of discount factor larger than 1 are not rare (See e.g. Hansen and Singleton (1983) and Epstein and Zin (1991)). Even though it is theoretically possible for the discount factor to be larger than 1 (Kocherlakota (1990b)), there is still the issue of how to bridge the gap between the empirical estimates in the asset pricing literature and those in other areas. For another example, Jones, Manuelli and Siu (2000) found that the relevant EIS values emerging from calibrations of growth models to match macroeconomic facts are usually around 1.[7] But the EIS estimates based on aggregate consumption data are usually close to zero (Hall (1988)). Furthermore, the estimates based on microeconomic data and those based on

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[6] If all assets are tradable and the asset Euler equation cannot be rejected for any of their returns, the consumption Euler equation cannot be rejected, either, because it is just a linear combination of all the asset return Euler equations. However, since not all assets are tradable and the returns for non-traded assets do not satisfy the asset Euler equations, the consumption Euler equation cannot really be written as a linear combination of all the asset return Euler equations. Therefore, it is possible for the test results on the consumption Euler equation and the asset Euler equation to differ.

[7] Lucas (1990) pointed out that to understand the cross-country differences in interest rates and consumption growth rates, the EIS should at least be larger than 0.5.

aggregate data typically contradict each other. Guvenen (2003a) addressed these EIS issues by appealing to limited participation in asset markets and the heterogeneity in EIS across individuals. My approach can be viewed as an alternative to his: i.e. I maintain the representative agent framework but account for elements that have been known to be important, to see if these issues can be addressed differently.

In this paper, I first propose a new and simple approach to estimate the return on the optimal portfolio that incorporates human capital and other forms of financial assets aforementioned. I start from the original (i.e. not log-linearized) dynamic budget constraint of the representative agent, and utilize Lettau and Ludvigson's (2004) *cay* variable to estimate the consumption wealth ratio series and the wealth growth series in order to estimate the return on the optimal portfolio. My approach produces with ease an explicit estimate for the return on the optimal portfolio that reflects the returns on human capital and the other financial assets than stocks and bonds. This is useful because it allows for a direct comparison between my estimate and the usual proxy for the return on the optimal portfolio used in the previous papers. For example, it will be easy to see how much the statistical properties of these two differ. More importantly, it allows me to fully evaluate the EZW model without assuming lognormality of, and conditional homoskedasticity for, asset returns and consumption growth, and without having to use the log-linearized Euler equation. [8] It also allows me to avoid the assumptions that were necessary in Vissing-Jørgensen and Attanasio (2003) for identifying the EIS and other structural parameters, e.g. the assumption on the share of bonds in households' financial assets.[9] Last but not least, it will become straightforward to pin down the major determinant of the equity premium once such a measure of the return on the optimal portfolio and the model parameter estimates are available. In other woods, it helps me to address a fundamental question in financial economics:

[8] Using the log-linearized Euler equation means that the time discount factor cannot be identified.

[9] They assumed that bonds accounted for half of household financial assets.

what makes stocks risky, the comovement of stock returns with consumption, the volatility of stock prices, or something else?

I then apply Stock and Wright's (2000) weak-identification robust asymptotic theory to the testing of the asset pricing and consumption implications of the EZW model. When parameters are weakly identified, it is usually impossible to obtain precise point estimates for them, although the estimates obtained by the conventional econometric methods ignoring weak identification may look precise. A researcher can, however, construct confidence sets for these parameters that account for weak identification. Stock and Wright's (2000) χ^2 tests are developed for this purpose. The use of their testing approach and my return estimates on the optimal portfolio produces remarkable results. In sharp constrast with the empirical results in Epstein and Zin (1991) and Stock and Wright (2000), I find that the model cannot be rejected at conventional significance levels for reasonable parameter values. For example, the 95% confidence sets for unknown parameters includes values of RRA around 2 or even lower, values of EIS closely around (but exclusive of) 1, and the time discount factor around 0.987. Some of these parameter value combinations are able to match simultaneously the average equity premium and average riskfree rate in the data. They also imply that the dominant determinant of the equity premium is, surprisingly, the volatility of stock returns, the risk factor in the traditional CAPM. The favorable results for the EZW model are obtained even though aggregate consumption data is used, and no market friction or incompleteness is introduced into the model. Therefore the consensus view on the usefulness of the EZW model in understanding asset pricing puzzles no longer holds, once human capital and weak identification are accounted for.

In addition to using a different approach to account for human capital in estimating the return on the optimal portfolio, the present paper differs from Campbell (1996) in several other aspects. First, I directly estimate or test the entire EZW model, while Campbell (1996) mainly examined the usefulness of his multifactor pricing model when human capital was taken into account. Therefore he did not (need to) estimate and test the entire EZW model. Indeed, the only parameter that matters in his pricing model is RRA. In contrast, I estimate, or test hypotheses on, all the parameters of the EZW model. Second, I account for weak identification of parameters caused by weak instruments. And third, I test the original EZW model with consumption growth remaining in the stochastic discount factor (SDF), whereas in Campbell (1996) consumption is substituted out. The first two points are also the distinctions between Vissing-Jørgensen and Attanasio (2003) and the present paper. In Section 6.2, I compare my results with theirs and Campbell's (1996) in greater detail.

The rest of the paper is organized as follows. In Section 2, I present my approach of addressing the Roll critique, and compare the optimal portfolio return constructed by this approach with the usual proxy used in the literature, the value-weighted return on New York Stock Exchange (NYSE) stocks. In Section 3, I briefly review the EZW model. I present the econometric methods used in this study in Section 4. I then describe the instruments used in my econometric analysis along with the data in Section 5. I report the estimation and test results in Section 6. In Section 7, I discuss related papers and offer some suggestions for future research.

2. Estimating the Optimal Portfolio Return

In this section, I show that the return on the optimal portfolio can be computed from the budget constraint once the ratio of optimal consumption to wealth and the growth of consumption for the representative agent are known. Then I present a figure that compares and contrasts the estimated return on the optimal portfolio with the value-weighted return on NYSE stocks. I also present a figure on the relationship between the estimated optimal portfolio return and the labor income growth to facilitate the discussion on the differences between my approach and those of others in accounting for human capital.

Consider the following constraint to the representative agent's utility maximization problem:

$$W_{t+1} = (W_t - C_t)R_{t+1}^m, \tag{1}$$

where W_t is the agent's total wealth, including all financial assets and human wealth, at period *t*, C_t is his consumption for the same period, and R_{t+1}^m is the stochastic gross return on his portfolio from period *t* to $t+1$. Since W_t includes human wealth, R_{t+1}^m is the weighted average of the return on financial assets and the return on human wealth.

It should be noted that whether human capital is tradable or not, and whether labor income is stochastic or not, (1) is the relevant budget constraint for the agent. See Epstein (1988) and Epstein and Zin (1991, p. 267, footnote 3) for an explanation on why the constraint (1) accommodates non-tradable human capital and stochastic labor income. To recapitulate, a shadow value can be calculated for human capital when it is not tradable and the shadow value can be included in W_t. They also wrote, "The problem of stochastic labor income is, therefore, a problem in the measurement of the return on the wealth portfolio." On the other hand, Campbell (1996), Lettau and Ludvigson (2001, 2004), among others, assumed that human capital is tradable and used (1) as the budget constraint. It should also be noted that the optimal consumption and portfolio choices of the representative agent, both of which affect wealth level, must satisfy this constraint. For this reason, and with a little abuse of notation, I henceforth use C_t in the rest of the paper as the optimal consumption, and W_t as the resulting wealth level from the representative agent's optimal decisions. Naturally, R_{t+1}^m from this point on should be interpreted as the return on his optimal portfolio.

To understand why knowing the series C_t/W_t is adequate for calculating the series R_{t+1}^m, first rewrite (1) as

$$\frac{W_{t+1}}{W_t} = \left(1 - \frac{C_t}{W_t}\right) R_{t+1}^m \ . \tag{2}$$

Then note that

$$\frac{W_{t+1}}{W_t} = \frac{C_{t+1}}{C_t} \cdot \frac{C_t / W_t}{C_{t+1} / W_{t+1}} \ . \tag{3}$$

Since the first term on the right-hand side of (3), consumption growth, is readily available, I only need the series C_t / W_t to calculate the wealth growth series. Using the W_{t+1} / W_t estimates along with the C_t / W_t series, I can then obtain the return series R_{t+1}^m using (2). Under special assumptions, such as independently and identically distributed (I.I.D.) asset returns and nonstochastic labor income, some consumption/portfolio choice models (including the model examined below) imply that the optimal C_t / W_t is constant over time and therefore $W_{t+1} / W_t = C_{t+1} / C_t$ in the equilibrium. But the I.I.D. assumption does not hold in the data. With non-I.I.D. asset returns, the EZW model in general implies time varying C_t / W_t unless the EIS is 1. See e.g. Giovannini and Weil (1989). Therefore it is useful and important to consider time varying C_t / W_t .

I now explain how to construct the C_t / W_t series from Lettau and Ludvigson (2004)'s *cay* variable. For completeness I provide a brief introduction to their approach and use their notation here. The reader may refer to Lettau and Ludvigson (2001, 2004) for more details. The investor's wealth W_t is the sum of his financial and human wealth: $W_t = A_t + H_t$, where A_t stands for financial assets and H_t stands for human wealth. Suppose that the steady-state share of H_t in W_t is *v*, a constant. A loglinear approximation yields (ignoring a linearization constant)

$$w_t \approx (1 - v) a_t + v h_t \ . \tag{4}$$

The lower-case variables here are the natural logarithms of the corresponding upper-case variables and henceforth, unless otherwise noted. A key insight in Lettau and Ludvigson (2001, 2004) is that the non-stationary component of h_t is captured by y_t, the logarithm of labor income. This motivates the following representation for h_t

$$h_t = \kappa + y_t + z_t \ , \tag{5}$$

where κ is a constant and $z_t = E_t \sum_{j=1}^{\infty} \rho_h^j \left(\Delta y_{t+j} - r_{t+j}^h \right)$ is a zero-mean stationary term. In the expression for z_t, E_t denotes the conditional expectation based on information up to time *t*, $\rho_h = 1/[1+\exp(E(y_t - h_t))]$, Δ is the first difference operator, and r_{t+j}^h is the logarithm of the gross return on human capital. Substituting (4) and then (5) into $c_t - w_t$ yields

$$c_t - w_t \approx c_t - (1-v)\,a_t - vh_t = c_t - (1-v)\,a_t - vy_t - v\kappa - vz_t \equiv cay_t - v\kappa - vz_t \,, \tag{6}$$

where cay_t is defined to equal $c_t - (1-v)\,a_t - vy_t$. Now assume that r_{t+j}^h is equal to labor income growth (Δy_{t+j}) plus a constant (α) and an additive zero-mean random disturbance (ζ_{t+j}). Then z_t becomes a constant. Denote it as *z*. This assumption, which I call the human capital return assumption for the ease of exposition in the rest of this section, is different from the assumptions used in Campbell (1996), Palacios-Huerta (2003), and Vissing-Jørgensen and Attanasio (2003), who did not directly associate the return on human capital with labor income growth. But it is somewhat similar to that of Jagannathan and Wang (1996) who employed labor income growth to capture the return on human capital. However, since I do not need to assume that labor income growth is unforecastable as in Jagannathan and Wang (1996), my method is also different from theirs. In addition, the human capital return assumption implies that labor income is a constant fraction of human capital over time, which can be seen when one moves y_t

in (5) to the left-hand side.[10] Therefore it can be argued that this assumption abstracts from other determinants of labor income such as effort. As a result of this assumption, the consumption-wealth ratio is approximately proportional to $\exp(cay_t)$, i.e.

$$\frac{C_t}{W_t} \approx \exp(cay_t) \cdot \exp(-v(\kappa - z)) \equiv k \exp(cay_t), \tag{7}$$

where k is defined to be $\exp(-v(\kappa + z))$. Hence the question now is how to pin down k, the constant of proportionality in (7).

If the U.S. economy has been in a steady state, as many economists believe, C_t/W_t should fluctuate around its steady-state value. Since $C/W = (C/A)(A/W)$, the steady-state value of C/W can be obtained by plugging the steady-state values of the two constituent ratios. I assume that the steady state value of C/A is equal to the long-term average of C_t/A_t. Using the consumption and the household net worth data in Lettau and Ludvigson (2004), the average of C_t/A_t in annual terms is 0.1899 for the period of 1959 to 2001. The household net worth is from the *Flow of Funds Accounts of the United States* compiled by the Federal Reserve System. It includes essentially all forms of financial assets: various forms of deposits, stocks, bonds, real estate, private equity, and durable goods. According to Lettau and Ludvigson's (2004) cointegration regression estimates, the steady state ratio of A/W is $1 - v = 0.3022$. Therefore I obtain the steady-state value of C/W as 0.1899 × 0.3022 = 0.05739 in annual terms, or 0.01436 in quarterly terms. Next I assume that the long-term average of the quarterly C_t/W_t series is equal to its steady state value 0.01436. This means that the time average of the right-hand side of (7) is 0.01436. Given the series cay_t for the period mentioned above, k then can be solved as 0.00697 for quarterly C_t/W_t series.

[10] This assumption implies that the dividend yield on human capital is constant. But the return on human capital, which includes capital gains on human capital, is not constant.

With the C_t/W_t series available, the estimation of wealth growth using (3) is straightforward. I then use (2) to estimate the return on the optimal portfolio. It is important to note that the R_{t+1}^m series calculated by my approach has incorporated human capital, almost all financial assets, or their returns. In other words, it is practically all-inclusive. Although approximations have to be used in my approach, and hence the estimates cannot be expected to be perfectly accurate, they are nonetheless likely to be much more accurate than those obtained by using financial assets or their returns alone. In this sense, the Roll critique has been adequately addressed in the present paper.

For a visual inspection of the estimated return on the optimal portfolio in comparison with the value-weighted real return on the NYSE stocks, refer to Figure 1. The (quarterly) means of the two series are very close to each other: 1.986% for NYSE stocks, and 1.976% for the estimated R_{t+1}^m. What is striking is how much the volatilities of these two series differ. The standard deviation of the real quarterly return on the NYSE stocks is almost ten times of that of R_{t+1}^m: 7.976% v.s. 0.838%. The small volatility of the estimated R_{t+1}^m should have implications for the estimation of the EZW model, because its SDF includes R_{t+1}^m, and a consumption-based asset pricing model relies on sufficiently volatile SDF to be relevant. But it does not necessarily imply that the EZW model is now more likely to be rejected by the data than when the usual proxy for R_{t+1}^m is used, as the reader will see in Section 6 of this paper. One reason for the non-rejection of the EZW model is that the volatility of the estimated R_{t+1}^m is still much larger than that of the consumption growth, and they are positively correlated, making the overall volatility of the SDF implied by the EZW model larger than the standard C-CAPM.[11] The major reason,

[11] The real per person growth of the consumption measure used in this paper has a standard deviation of 0.46% at the quarterly frequency. See Section 5 for data description.

though, is that R_{t+1}^{m} is raised to a certain power in the SDF, and it is the overall volatility of this whole term that matters. See Section 3 for more details.

The substantial volatility difference between the estimated R_{t+1}^{m} and that of the value-weighted return on NYSE stocks is due to the fact that the major component of the estimated R_{t+1}^{m} is the return on human capital, and the volatility of the latter is much smaller than that of the stock returns. To understand this point, note that

$$R_{t+1}^{m} = (1 - v_{t+1})R_{t+1}^{a} + v_{t+1}R_{t+1}^{h}, \tag{8}$$

where R_{t+1}^{a} and R_{t+1}^{h} are the gross returns on financial assets and human capital, respectively, and v_{t+1} is the share of human capital in the total wealth. Since v is about 70% as mentioned earlier, v_{t+1} should fluctuate around it. This gives the return on human capital a weight of about 70%. In addition, the human capital return assumption implies $R_{t+1}^{h} = \exp(\alpha + \Delta y_{t+1} + \zeta_{t+1})$. So the volatility of R_{t+1}^{h}, and hence the volatility of R_{t+1}^{m}, depends on the volatility of the labor income growth (and that of ζ_t). For the sample period, the standard deviation of the real per person after-tax labor income growth is 0.865%, very close to that of for the estimated R_{t+1}^{m}. I present in Figure 2 the estimated R_{t+1}^{m} and the labor income growth. It is clear that the two series move together most of the time and share the same pattern of fluctuations.

3. The Epstein-Zin-Weil Model

In the EZW model, the representative agent is endowed with a recursive utility function that isolates his risk preference from his willingness to substitute consumption over time. He chooses consumption and portfolio weights on *N* assets to maximize

$$U_t = \left\{ (1-\beta) C_t^{\rho} + \beta \left(E_t U_{t+1}^{\alpha} \right)^{\rho/\alpha} \right\}^{1/\rho}, \tag{9}$$

subject to the budget constraint (1) and the constraint that the portfolio weights add up to 1. One of the *N* assets is human capital, though it may not be tradable. In this formulation, β is the time discount factor for the deterministic consumption path, $\alpha = 1 - \text{RRA}$, $\rho = 1 - 1/\text{EIS}$ and $\rho \neq 0$. Following Epstein and Zin (1991) and others, I define $\lambda = \alpha/\rho$ and $\gamma = 1/\text{EIS}$ to write the Euler equations for asset returns as follows

$$E_t \left[\beta^{\lambda} \left(\frac{C_{t+1}}{C_t} \right)^{-\lambda\gamma} \left(R_{t+1}^{m} \right)^{\lambda - 1} R_{j,t+1} \right] = 1, \qquad j = 1, \ldots, N. \tag{10}$$

Here $R_{j,t+1}$ is the gross return on asset *j* from period *t* to *t* + 1. The consumption Euler equation is

$$E_t \left[\beta^{\lambda} \left(\frac{C_{t+1}}{C_t} \right)^{-\lambda\gamma} \left(R_{t+1}^{m} \right)^{\lambda} \right] = 1. \tag{11}$$

Clearly, the presence of R_{t+1}^{m} in the Euler equations (10) and (11) means that accounting for human capital and other financial assets than stocks in measuring the return on optimal portfolio is very important. Given the substantial difference between the volatilities of the estimated R_{t+1}^{m} and its proxy the aggregate returns on NYSE stocks, it is imperative that a test of the EZW model uses the correct measure of the optimal portfolio return. This is because such a large difference should affect the λ estimate substantially, which in turn influences the estimates of EIS and RRA. The reason that the λ estimate should be heavily affected is that a small volatility in R_{t+1}^{m} requires a large λ (in absolute terms) to make the SDF sufficiently volatile. Since $\lambda = (1 - \text{RRA})/(1 - 1/\text{EIS})$, a combination of low RRA and EIS around 1 can produce a large λ

value. This is essentially the major reason for the non-rejection of the EZW model for reasonable values of β and RRA, and plausible values of EIS.

The unconditional version of (10) can be log-linearized using the joint lognormality assumption for consumption growth and asset returns to show that the following decomposition holds for the average equity premium over the return on Treasury bill (T bill henceforth). Both the stock and T bill returns below are in logarithmic terms:

$$E(r_s - r_b) = \frac{\sigma_{r_b}^2 - \sigma_{r_s}^2}{2} + (1-\lambda)\left(\sigma_{r^m, r_s} - \sigma_{r^m, r_b}\right) + \lambda\gamma\left(\sigma_{\Delta c, r_s} - \sigma_{\Delta c, r_b}\right), \tag{12}$$

where the subscript s and b denote stocks and T bills, respectively, a σ^2 denotes variance, a σ with double subscripts is for covariance, and Δc is for consumption growth. As noted in Campbell (1996), the first term on the right-hand side of (12) is due to Jensen's inequality. The term σ_{r^m, r_s} captures the "market risk" of holding stocks, i.e. the covariation of (log) stock returns with the (log) return on the entire wealth portfolio, and $\sigma_{\Delta c, r_s}$ captures the consumption risk of holding stocks.

In addition, the value of the real gross risk-free rate R_f when the R_{t+1}^m series and the values of the three parameters are known can be computed from (10) by setting $R_{j,t+1} = R_f$ as follows

$$R_f = \frac{1}{E\left[\beta^{\lambda}(C_{t+1}/C_t)^{-\lambda\gamma}\left(R_{t+1}^m\right)^{\lambda-1}\right]}. \tag{13}$$

In the next section, I discuss how to estimate or test the EZW model using (10) and (11) with the R_{t+1}^m series calculated from (2). In Section 6, I use the sampling counterparts to (12) and (13) to help validate a confidence set of unknown parameters constructed using Stock and Wright (2000)'s method: given the estimates of the mean, variances, and covariances in (12) and a reasonable estimate of the riskfree rate, is there any combination of reasonable λ and γ values (along with β for the riskfree rate) in a confidence set that is able to match the mean equity premium and the risk-free rate at the same time? For this purpose, it is helpful to note that the sample variances and covariances in (12) are as follows:

$$\hat{\sigma}_{r_b}^2 = 2.760 \times 10^{-5},\ \hat{\sigma}_{r_s}^2 = 6.528 \times 10^{-3},\ \hat{\sigma}_{r^m, r_s} = 3.979 \times 10^{-4},$$

$$\hat{\sigma}_{r^m, r_b} = 5.312 \times 10^{-6},\ \hat{\sigma}_{\Delta c, r_s} = 4.907 \times 10^{-5},\ \hat{\sigma}_{\Delta c, r_b} = 2.215 \times 10^{-6}.$$

Which of the two risks captured by the EZW model is more important in determining the equity premium obviously depends on the sizes of λ and γ. First, if $\lambda = 1$ the "market risk" does not matter, only the consumption risk does. This, of course, is the major implication of the standard expected utility C-CAPM that has been studied extensively in the literature. Second, if $\lambda = 0$, only the "market risk" matters. Under this condition, (12) becomes a version of the CAPM extended to incorporate the human capital and financial assets other than stocks. Third, since the "market risk" σ_{r^m, r_s} is one order of magnitude larger than the consumption risk $\sigma_{\Delta c, r_s}$ as shown above, if the magnitudes of the two coefficients $1-\lambda$ and $\lambda\gamma$ in (12) are close to each other, then the "market risk" dominates the consumption risk in determining the equity premium. Furthermore, the "market risk" itself can be decomposed into the following three components

$$\sigma_{r^m, r_s} = v\sigma_{\Delta y, r_s} + (1-v)(1-\phi)\sigma_{r_s, r_o} + (1-v)\phi\sigma_{r_s}^2,$$

where ϕ is the share of financial wealth invested in stocks, and r_o stands for the return on other financial assets than stocks. To understand what drives the "market risk," it is useful to note that $\hat{\sigma}_{\Delta y, r_s} = 1.927 \times 10^{-5}$, and $\hat{\sigma}_{r_s}^2 = 6.528 \times 10^{-3}$. After taking into account the weights, the first component of the "market risk" in the last equation is still on the order of 10^{-5}, and the third component is on the order of 10^{-4} for possible values of ϕ. The second component of the "market risk" is hard to pin down because no estimate for the return on other financial assets than stocks is available. It is, however, possible to show that σ_{r_s, r_o} can be of orders of magnitude from 10^{-4} to 10^{-6} for possible values of ϕ.[12] Therefore, as long as the magnitudes of the two coefficients $1-\lambda$ and $\lambda\gamma$ in (12) are close to each other, the volatility of stock returns ($\sigma^2_{r_s}$), and possibly, the covariation of stock returns with returns on other financial assets (σ_{r_s, r_o}), will be the driving force(s) of the "market risk." Since I have shown that under the same conditions on these coefficients, the "market risk" dwarfs the consumption risk in determining the size of the equity premium, this result implies that the volatility of stock returns alone can be the dominating determinant of the equity premium.

4. Econometric Methods

I now substitute the estimated R^m_{t+1} as described in Section 2 into (10). This yields the following Euler equations for asset returns:

$$E_t\left[\beta^{\lambda}\left(\frac{C_{t+1}}{C_t}\right)^{\lambda(1-\gamma)-1}\left(\frac{\exp(cay_t - cay_{t+1})}{1-k\cdot\exp(cay_t)}\right)^{\lambda-1} R_{j,t+1} - 1\right] = 0, \qquad j = 1, \ldots, N. \tag{14}$$

Recall $k = 0.00697$. Now define $\boldsymbol{\mu} = (\beta, \lambda, \gamma)$, and denote the true value of $\boldsymbol{\mu}$ by $\boldsymbol{\mu}_0$. Let $\varepsilon_{j,t+1}(\boldsymbol{\mu})$ be the bracketed term in (14), and $\boldsymbol{\varepsilon}_{t+1}(\boldsymbol{\mu}) = (\varepsilon_{1,t+1}(\boldsymbol{\mu}), \ldots, \varepsilon_{m,t+1}(\boldsymbol{\mu}))'$, where $m \leq N$ is the number of assets used in a test. Let the p-vector $\mathbf{Z}_t$ be a subset of the representative

[12] For example, given $\hat{\sigma}_{r^m, r_s} = 3.979 \times 10^{-4}$, for $\phi = 12.5\%$, 15%, or 20%, the second component of the market risk is 1.38×10^{-4}, 8.872×10^{-5}, or -9.844×10^{-6}, respectively. All the three ϕ ratios used here have been observed in the data.

agent's information set up to time t. Define $\boldsymbol{\varphi}_{t+1}(\boldsymbol{\mu}) = \boldsymbol{\varepsilon}_{t+1}(\boldsymbol{\mu}) \otimes \mathbf{Z}_t$, where $\otimes$ is the Kronecker product. Then $E(\boldsymbol{\varphi}_{t+1}(\boldsymbol{\mu}_0)) = \mathbf{0}$ are the $m \times p$ orthogonality conditions that can be employed in estimating and testing asset pricing implications of the model. Let $\overline{\boldsymbol{\varphi}}(\boldsymbol{\mu}) = (1/T)\sum_{t=1}^{T} \boldsymbol{\varphi}_t(\boldsymbol{\mu})$. The GMM criterion function is a quadratic form in $\overline{\boldsymbol{\varphi}}(\boldsymbol{\mu})$,

$$S_T(\boldsymbol{\mu};\boldsymbol{\mu}) \equiv T\, \overline{\boldsymbol{\varphi}}(\boldsymbol{\mu})' \mathbf{W}_T(\boldsymbol{\mu})\, \overline{\boldsymbol{\varphi}}(\boldsymbol{\mu}),$$

where T is the sample size. The efficient weighting matrix is written as $\mathbf{W}_T(\boldsymbol{\mu})$ to accommodate the case in which it continuously updates with $\boldsymbol{\mu}$ in estimation. The minimizer $\hat{\boldsymbol{\mu}}$ obtained with such a weighting matrix is known as the continuous updating GMM estimator. See Hansen, Heaton, and Yaron (1996). The conventional two-step GMM estimator, on the other hand, uses weighting matrices that do not update with $\boldsymbol{\mu}$. There is no reason to think that conditional homoskedasticity holds for $\boldsymbol{\varphi}(\boldsymbol{\mu}_0)$. So I will use a heteroskedasticity-robust weighting matrix.

It is well known that the small sample properties of the conventional two-step GMM estimator and the associated test statistics are not satisfactory when they are used to test C-CAPM models.[13] For example, the minimum χ^2 test (i.e. Hansen's J test) tends to over-reject in testing the time- and state- separable C-CAPM. On the other hand, Hansen et al. (1996) showed that the minimum χ^2 test based on the continuous updating GMM estimator has smaller size distortions in the finite sample than those based on the two-step and iterative GMM estimators. Stock and Wright (2000) developed an alternative asymptotic theory for the continuous updating GMM estimator that is robust to the presence of weak instruments, i.e. instruments that are weakly correlated with the bracketed term in (10). Weak instruments cause at least some parameters to be weakly identified. Stock and Wright (2000) documented that models of the C-CAPM are

[13] See e.g. Ferson and Foerster (1994).

usually weakly identified. Since I am not aware of any other paper in asset pricing that uses their approach, I provide a brief, non-technical, introduction to it here.

They propose two methods to test *jointly* the model specification and the null hypothesis $\boldsymbol{\mu} = \boldsymbol{\mu}_0$ via the construction of a confidence set for the unknown parameters. The first method uses the following property of the criterion function of the continuous updating GMM (i.e. Theorem 2 in their paper):

$$S_T(\boldsymbol{\mu}_0; \boldsymbol{\mu}_0) \xrightarrow{d} \chi^2_{m \times p},$$

where $m \times p$ is the degree of freedom of the χ^2 distribution. This result holds without any additional assumption on instrument validity except that $E(\boldsymbol{\varphi}(\boldsymbol{\mu}_0)) = \mathbf{0}$. This is how the test above can accommodate weak instruments and therefore weakly identified models. The set of parameter values $\boldsymbol{\mu}_0$ that do no generate a large $S_T(\boldsymbol{\mu}_0; \boldsymbol{\mu}_0)$ relative to the $\alpha\%$ critical value of the $\chi^2_{m \times p}$ distribution is called the $(1-\alpha)\%$ joint *S* set for all the parameters in Stock and Wright (2000).

On the other hand, it is possible that some parameters in the model are well identified while others are not. In such a case, a different confidence set for the weakly identified model parameters can be constructed according to Theorem 3 in Stock and Wright (2000). The construction of this confidence set involves two steps. First, estimate the well-identified parameters for various values of the weakly identified parameters using the continuous updating GMM. Second, evaluate the continuous updating GMM criterion function using various values of the weakly identified parameters and the corresponding estimates for the well-identified parameters. The continuous updating GMM criterion function so evaluated converges in distribution to a $\chi^2_{m \times p - w}$ statistic, where *w* is the number of well-identified parameters. The collection of values of the weakly identified parameters that enable a model to pass the $\chi^2_{m \times p - w}$

test at the significance level $\alpha\%$ is called a $(1-\alpha)\%$ concentrated *S* set, because the well-identified parameters are concentrated out in constructing the *S* set in this case. This result relies on stronger assumptions than those for the $\chi^2_{m \times p}$ test above, and I refer the reader to Stock and Wright (2000) for technical details. Importantly, the size distortion of their χ^2 tests of model validity described above is much smaller than that of the conventional GMM asymptotics in Hansen (1982).

If a model is correctly specified, when it is run through the entire parameter space at a certain significance level α, the $(1-\alpha)\%$ joint *S* set, or a $(1-\alpha)\%$ concentrated *S* set should not be null. A null *S* set indicates the rejection of the over-identifying restrictions and therefore the rejection of the model being tested. A small *S* set causes some ambiguity: it could indicate that the model is not rejected, and the parameters are precisely estimated, or that the data is too weak to reject the model completely. How to formally handle the ambiguity associated with a small *S* set seems to be a gap in the literature. But intuitively speaking, it may be sufficient to use the sampling counterparts to (12) and (13) to validate an *S* set as explained at the end of Section 3. The idea is that if no single element of a small *S* set can nearly produce the average equity premium and the risk-free rate (as approximated by the T bill rate) observed in the U.S. data, this *S* set is considered invalid. Another measure that I adopt is to check if the *S* set elements can pass the boundary conditions for the EZW model. See Section 6.1.C below.

Stock and Wright (2000) described symptoms of weak identification in GMM estimation. These include, but are not limited to, the following: the parameter estimates from asymptotically equivalent GMM estimators are very different from each other, the estimates are not robust to the addition of instruments, inferences on model specification are sensitive to the particular GMM estimator used, and a confidence set for the 2-step GMM estimates has substantial areas of disagreement with a comparable *S* set. For the purpose of determining the existence of weak identification, and for comparing the test results associated with different GMM estimators, I will report estimation and test results based on 2-step GMM, continuous updating GMM using the conventional Hansen (1982) asymptotics, and results using Stock and Wright's (2000) weak-identification robust asymptotic theory. I will start with the Euler equations for asset returns, i.e. eqs. (10) or (13), and then move to estimate and test the consumption Euler equation (11).

5. Instruments and Data

Since the goal of this paper is to investigate if the EZW model can solve the equity premium puzzle and the risk-free rate puzzle, I use two quarterly returns to test the asset return Euler equations (10): the value-weighted real return on NYSE stocks (rvwrq henceforth) and the real return on U.S. Treasury bills (rtbillq henceforth).[14] The consumption measure that I use is real per person nondurable goods and services expenditure excluding clothing and shoes, seasonally adjusted and in 1996 chain-weighted dollars.[15] Other measures of consumption are not used because the *cay* estimates are based on this particular definition of consumption in Lettau and Ludvigson (2004). The real after-tax labor income per person, used as an instrument, is also the same as in their paper. The rvwrq and rtbillq, along with the real dividend yield (rdivq henceforth) and the bond default premium used below as instruments, are compounded from monthly counterparts. They are taken from Ibbotson Associates (2002). The term premium is the

[14] Epstein and Zin (1991) cautioned that when the return on the optimal portfolio is proxied by rvwrq, it is usually not adequate to use just these two returns to test their model. This is not a problem here because I do not use this proxy for the return on the optimal portfolio.

[15] The use of this measure of consumption means that the wealth portfolio in Section 2, W, should include the stock of clothing and shoes, which are not included in the household net worth measure reported by the Fed. This is not problematic because their share in A, and therefore W, is very small. For example, let's assume that the stock of clothing and shoes per person were \$5,000 by the end of 2001. It would be about 4.15% of A by then. Since A is only 30% of W, the stock of clothing and shoes would only be 1.25% of W.

difference between the rates of return on U.S. Treasury bonds and bills. The sample period is from the first quarter of 1959 to the fourth quarter of 2001.

I use four sets of instruments in testing the asset return Euler equations. Set 1 consists of four instruments: a constant and the first lags of real quarterly consumption growth, rdivq, and the term premium. The real dividend yield and the term premium have been used in other studies as instruments of stock returns; see e.g. Stock and Wright (2000). Instruments set 1 is very close to the two sets of instruments used in their paper, which used monthly data. The only difference is that here I have dropped their first-lagged "MR" (return on stock market portfolio) because at quarterly frequency the first order serial correlation of rvwrq is too weak for it to be a relevant instrument. The second set of instruments consists of Set 1, and the first lags of rtbillq and the quarterly after-tax per capita labor income growth in real terms. The first order serial correlation coefficient for the rtbillq series is somewhat high (0.35), and it could therefore be a valid instrument for the T bill return. In addition, I find that the labor income growth forecasts stock returns with a large coefficient at the 5% level of significance. This is why labor income growth is included in Set 2. The third set of instruments includes seven instruments: those in the second set and the bond default premium. This premium is very close to being significant at 10% level in explaining rvwrq in a multiple regression, and it is significant at 10% level in explaining R_{t+1}^{m}. These seven instruments, except the (lagged) consumption growth, are also significant in forecasting the estimated R_{t+1}^{m} at the 10% level. So they also serve as the instruments for R_{t+1}^{m} in my tests.

The fourth set of instruments is the third set augmented by Lettau and Ludvigson's (2004) *cay*. I include *cay* because it has been demonstrated to predict stock returns by Lettau and Ludvigson (2001), and it has been used as an instrument in a few papers. I am, however, somewhat skeptical about its use as an instrument in testing the *conditional* version of the Euler equations (10). This is because in such a test, the instruments should be the variables that are included in the information set of the representative agent up to period t, i.e. information that is *publicly* available up to t. Despite the predictive power of cay_t, it seems very difficult to argue that it has actually been in the information set of a typical investor. It had not at least before Lettau and Ludvigson's study on this issue was published. Investors might have already used components of *cay*, i.e. consumption, household net worth, and after-tax labor income to forecast stock returns. But they certainly did not know of the particular way of organizing these data that Lettau and Ludvigson uncovered, i.e. the cointegrating regression of c on a and y. Even if some of them did, it is still difficult to argue that the "representative" agent's information set included this knowledge. However, using instrument set 4 as described above in my empirical analysis facilitates the comparison between my results and those in the literature that used *cay*.

In estimating or testing the consumption Euler equation of the EZW model, eq. (11), I use the following eight instruments: those in instrument set 3 and the second lag of real consumption growth. They are called instrument set 5. Among these instruments, the first lag of consumption growth is to instrument the consumption growth in (11), and the other seven are to instrument R^m_{t+1}. The selection of these seven instruments is based on regressions of R^m_{t+1} estimates on various variables. Lastly, following the standard practice in the literature in dealing with possible time aggregation bias, I also lag each set of instruments by one more quarter in testing.

6. Test Results

I report the estimation and test results for Euler equations (10) for asset returns in the first subsection below. Then I compare my results with those in the literature in the second subsection. I present the empirical results for the consumption Euler equation (11) in the third part of this section.

6.1. Results for Euler Equations for Asset Returns

6.1.A. Estimation by Conventional GMM and Evidence on Weak Identification

Table 1 collects the estimation and test results for the asset return Euler equations produced by two conventional GMM approaches for the four sets of instruments and their lags defined in Section 5. Comparing the 2-step GMM results with those of the continuous updating GMM in this table, I find that two patterns emerge and they are consistent with the findings in Hansen et al. (1996) and Stock and Wright (2000). First, the estimates of λ vary substantially across the two GMM estimators employed and across different instrument sets in the continuous updating GMM. For example, in panel 1, the λ estimates are –33.19 and –100.6 for the 2-step and continuous updating estimators, respectively. On the other hand, the λ estimate produced by the continuous updating GMM changes from –100.6 in panel 1 to –59.94 in panel 2 when two additional instruments, lagged labor income growth and lagged rtbillq, are added to instrument set 1. The high sensitivity of parameter estimates to the GMM estimator used and to the addition of instruments is a sign of weak identification. Second, the minimum χ^2 tests associated with the two GMM estimators portray different pictures about the overall fit of the EZW model at the 5% significance level in four of the eight cases considered, even though these two estimators are asymptotically equivalent. For example, in panel 1, the χ^2 statistic from the 2-step GMM suggests that the model is rejected at the 5% level. But the same statistic from the continuous updating GMM, with a p-value of 11.1%, indicates that the model is not rejected at conventional significance levels. Such disagreement also occurs in panels 2, 3 and 8. Even at the 10% significance level, there are still disagreements in test results in three cases (See panels 1, 3, and 8). This is another symptom of weak identification I alluded to in Section 4. In addition, the γ estimates also vary a lot overall, though in the top four panels they seem to be around 0.96.[16] See

[16] These estimates are centered on 1, suggesting that the EIS could just be 1. It is, however, difficult to test the restriction $\gamma = 1$, because an assumption underlying the Euler equations (8) is that $\gamma \neq 1$ (so that

e.g. the γ estimate of 0.52, 0.59, and 0.68 at the bottom two panels. Moreover, given that γ is a part of the exponent of consumption growth in the SDF, and the variation in consumption growth is small, it seems difficult to see why it can be well identified. It is therefore more appropriate to treat γ as weakly identified, along with λ. Stock and Wright (2000), however, treated β and λ as well identified, and γ as weakly identified. The difference between their treatment and my treatment of these parameters, given that we use similar instruments, can be attributed to the fact that my estimate of R^m is much less volatile than the value-weighted return on NYSE stocks that they used. In other words, the small variability in R^m makes the identification of λ difficult in my context, since λ is the exponent of R^m in the Euler equations. The large volatility of the aggregate stock return used as the proxy for the optimal portfolio return in their paper, on the other hand, may have made λ well identified in their context.

The discussions above on results in Table 1 suggest that the conventional GMM asymptotics are not adequate for assessing if the EZW model fits data well due to the weak identification problem. To further verify this point, I follow Stock and Wright (2000) to compare the confidence ellipses for the 2-step GMM estimates with the *S* sets to see if there are substantial areas of disagreement. To implement their approach, I need to run the model through the entire parameter space to search out the combinations of parameter values that are not rejected by the data. Table 2 presents the parameter ranges and increments used in this search. Note that $\lambda = 1$

$\rho \neq 0$). Imposing $\gamma = 1$ changes the Euler equation to a form that includes an unknown function of the state of the economy. See Giovannini and Weil (1989). They also showed that with a Markovian and lognormal return on optimal portfolio, it is possible to derive the explicit Euler equation for the case of unitary EIS. But even in that case, the RRA cannot be identified without very strong assumptions. Furthermore, the Markovian assumption is not satisfied for my estimates of the R^m series. An AR(4) regression for this series indicates that only the third lag of R^m is significant (at 5% level); the first lag has a slope coefficient of 0.056 with a *t* statistic of 0.78.

is included in the search to test for an alternative to the EZW model, the expected utility C-CAPM.

I now present in Figure 3 more definitive evidence on weak identification. This figure plots for different sets of instruments the 95% confidence ellipses for the 2-step GMM estimates of λ and γ and the 95% concentrated *S* sets for these two parameters.[17] See the explanations at the end of Fig. 3. There is not a graph for the case of instrument set 4 because the corresponding 95% concentrated *S* set is empty. For two of these seven cases, i.e. parts (c) and (g), there is no overlap between confidence region and *S* set. For each of the remaining five cases, there is substantial area of disagreement between confidence region and *S* set. Non-overlapping and substantial area of disagreement are both important signs of weak identification that Stock and Wright (2000) emphasized. Therefore it is necessary that weak identification be taken into account in the empirical analysis.

6.1.B. Results of *S* Set Analysis

I summarize the results of *S* set analysis based on instrument sets 1, 2, and 4 in Table 3. The results based on instrument set 3 are similar to those based on instrument sets 2 and 1, and are not reported to conserve space.[18] I will, however, present some results for each of the four instrument sets in Table 4.

In Table 3, the reader can see that when instrument sets 1 and 2 or their first lags are used, the *S* set analysis overall presents favorable evidence for the EZW model at the 5% level of significance. Out of twelve *S* sets, only one is null. This is the concentrated *S* set for λ for instrument set 2. It is obtained by assuming that β and γ are both well identified. But as mentioned in Section 6.1.A., the relatively large variation in γ estimates in Table 2 indicated that

[17] The result of the comparison between the non-empty 90% *S* sets and the 90% confidence ellipses is very similar.

[18] These results and other results not reported to preserve space are available upon request.

it is difficult to treat γ as well identified. Therefore the nullity of this *S* set is more likely to indicate the inappropriateness of the assumption that γ is well identified than to indicate the rejection of the EZW model. Such a conjecture is consistent with the fact that the range of γ is much wider than the range of β in the 95% joint *S* sets for (β, λ, γ) reported in the upper panel of Table 3. While β values are very tightly around 0.986 and therefore very close to the estimates of β in Table 2, γ values change from 0.4 to slightly larger than 1 when instrument sets 1 and 2 are used. The range of λ values in the 95% joint *S* sets is even wider than that of γ, mirroring the wide range of γ estimates in Table 1. These large variations reflect the weak identification of these two parameters. Due to the wide ranges of λ and γ, RRA values in all the *S* sets reported in this table also swing widely because $\mathrm{RRA} = 1 - \lambda(1-\gamma)$ in the EZW model. Some of the RRA values are far away from a typical economist's prior. For example, several *S* sets include RRA values as low as 0.0025. Such values will be examined using (12) and (13) later in this subsection: if they, along with the corresponding γ values, cannot produce reasonable equity premia and risk-free rate, they should have been rejected by the *S* set testing in the first place. What is more important here, though, is the fact that these ranges all include values that imply what economists believe to be the reasonable values of RRA and EIS. They indicate that the EZW model is not rejected for these values of RRA and EIS (along with reasonable values of β). See the two columns labeled "RRA" and "EIS" in each panel of Table 3. The EIS values in this table are higher than many estimates in the literature that are smaller than 1. [19, 20] But it should be noted that estimates larger than 1 are not at all unusual. See e.g.

[19] See Hall (1988), Campbell and Mankiw (1989), Patterson and Pesaran (1992), Atkeson and Ogaki (1996), Ogaki and Reinhart (1998), Evans (2000), Basu and Kimball (2002), Vissing-Jørgensen (2002), Yogo (2004), among others, for estimates of EIS smaller than 1. Vissing-Jørgensen (2002) also had some EIS estimates larger than 1. Attanasio and Weber (1993) reported EIS estimates based on cohort data that were larger than 0.7 and statistically not different from 1.

Vissing-Jørgensen and Attanasio (2003), Koskievic (1999), Beaudry and van Wincoop (1996), Bufman and Leiderman (1990), and Attanasio and Weber (1989) for EIS estimates that are much larger than 1, or even multiples of 1. The smaller EIS estimates that the other authors found could be due to two reasons. First, they usually assume conditional homoskedasticity of consumption growth. Bansal and Yaron (forthcoming) and Guvenen (2003a) showed that such an assumption leads to a serious downward bias in the EIS estimates. Bansal and Yaron also demonstrated that an EIS value of 1.5 (and a RRA value of 10) in their model helps to explain several asset pricing puzzles. Second, these other authors usually avoid the use of R^m and hence use log-linearized Euler equations. For example, Yogo (2004) also accounted for weak identification in estimating the EIS and found small estimates, but he used the log-linearized version of (10).

The *S* set analysis using the fourth set of instruments and its lag, however, delivers mixed results. First, the 95% *S* sets, concentrated or not, are all empty when the fourth set of instruments is used. See the two rows labeled "Set 4" in Table 3. This is evidence that the EZW model is rejected at the 5% level of significance. Second, when the fourth set of instruments is lagged one more quarter, the *S* sets are not null any more. Two of them, the joint *S* set for (β, λ, γ) and the concentrated *S* set for (λ, γ), imply similar values of RRA and EIS to those in *S* sets based on instrument set 1, 2, and 3. They are favorable evidence for the EZW model. The concentrated *S* set when β and γ are both treated as well identified is not null and the implied RRA values are all above 26. However, our discussions above indicate that γ should not be treated as well identified. So this *S* set does not carry much weight either way.

[20] The relevant EIS values for resolving the equity premium puzzle and the riskfree rate puzzle are around 1. See Table 4 below.

So far the results that I have reported are for the 5% significance level. At the 10% level, when the instrument sets 1 to 4 are used, the 90% joint *S* sets for (β, λ, γ) for instrument sets 1 and 3 and the 90% concentrated *S* set for (λ, γ) for instrument set 1 remain non-empty. But the joint *S* set for (β, λ, γ) for instrument set 2 and the concentrated *S* sets for (λ, γ) for instrument sets 2 and 3 become empty. When the instrument sets 1 to 4 are lagged one more quarter, the joint *S* sets for (β, λ, γ) and the concentrated *S* sets for (λ, γ) are all non-empty. To summarize, there is strong evidence for the EZW model even at the 10% level.

6.1.C. The Resolution of the Two Puzzles and the Major Determinant of Equity Premium

All the non-empty *S* sets include reasonable values for β and combinations of λ and γ values that imply RRA values around 2 or smaller, and EIS value around 1. See Table 4.[21] (Similar combinations can be found in 90% *S* sets obtained with twice-lagged instruments and are not reported to conserve space.) For example, the joint *S* set for (β, λ, γ) for instrument set 1 in panel 1 of this table includes λ and γ combinations that imply RRA values of 0.95 and 1.98 for EIS values of 0.999 and 1.019, respectively. The corresponding β values imply reasonable time discount rates around 5.2% in annual terms. The last two columns of this table report the quarterly equity premium implied by the λ and γ values of each row using the sampling counterpart to (12), and, for the joint *S* sets in the upper panel, the quarterly riskfree rate calculated using the sampling counterpart to (13) by plugging the β, λ, and γ values of each row. For the 1959-2001 period, the average quarterly equity premium is 1.5%, and the average quarterly T bill rate is 0.46%. It is clear that in five of the six *S* sets, the combinations of λ and γ values reported in this table are able to match the exact average equity premium in the data. These five *S* sets are the joint *S* sets for (β, λ, γ) for instrument set 1 and 3, and the

[21] For this table, to further pin down the values of β, RRA and EIS, I use smaller increments of 10^{-4} for β to rerun the *S* set analysis throughout the parameter space specified by the *S* sets presented in Table 3.

concentrated S sets for (λ, γ) for instrument sets 1, 2, and 3. For instance, in the joint S set for instrument set 1, a combination of $\lambda = -51.5$ and $\gamma = 0.981$ generates the right size of the equity premium. The remaining one of the six S sets, the joint S set for (β, λ, γ) obtained with instrument set 2, has parameter value combinations that produce quarterly equity premia around 1.4%. In terms of matching the riskfree rate, all the three joint S sets contain parameter value combinations that produce the right size of the rate at the same time that they match (or almost match, in the case of using instrument set 2) the average equity premium. On the other hand, the RRA value 0.0025 and the like usually produce negative, or positive and very small (relative to 0.46%), riskfree rates (not reported in Table 4), although they show up in a couple of S sets in Table 3 as mentioned earlier. For example, in the concentrated S set for (λ, γ) for instrument set 3, the RRA of 0.0025 corresponds to a riskfree rate of –2%. Hence these RRA values should not have been part of an S set.

Furthermore, as yet another check on the validity of these results, I also examine if the parameter value combinations in the upper panel of Table 4 can satisfy the two boundary conditions in Smith (1996) for the consumption and portfolio choice model in Svensson (1989) that features the EZW preferences. The solutions in Svensson (1989) require a constant riskfree rate and still hold with deterministic labor income and tradable human wealth. The Euler equatoins (10) in the present paper can accommodate these three variations. Therefore, I can take the parameter values in the joint S sets reported in Table 4, if they are interpreted as the evidence for this particular version of (10), to the boundary conditions in Smith (1996) and see if the two conditions hold. It will be reassuring if they hold. Let δ denote the time preference rate. Let M denote $1/2$ times the square of the Sharpe ratio. The feasibility condition says that the consumption-wealth ratio must be positive, i.e. $\mathrm{EIS}\left[\delta - (1 - 1/\mathrm{EIS})(r_f + M/\mathrm{RRA})\right] > 0$. The transversality condition is $-\delta(1 + \mathrm{EIS}) + r_f \mathrm{EIS} + (\mathrm{EIS} - 1)M/\mathrm{RRA} < 0$. This restriction ensures that the value function implied by the EZW utility function converges. The results of this exercise are in Table 5. Fortunately, the parameter value combinations that produce the right equity premium and riskfree rate satisfy both the feasibility and the transversality conditions.

Therefore, once the role of human capital and financial assets other than stocks is appropriately accounted for, and the impact of weak identification is correctly reflected in statistical inference, the EZW model can resolve the twin puzzles at the same time. The parameter-value combinations reported in Table 4 demonstrate that in this model, not only the equity premium of 6% per year can be consistent with the low RRA's, but also the low risk-free rate in the data does not require the time discount factor β to be larger than 1. As is well known, in the standard expected utility C-CAPM with the power utility function, a very high RRA is necessary to explain the six percent equity premium and, at the same time, a discount factor larger than 1 is needed to accommodate a low riskfree rate in the equilibrium. The RRA estimate based on evidence on many observed economic decisions is, however, around 2. It is also hard, intuitively speaking, to accept a time discount factor larger than 1. These tensions were exactly what gave rise to the twin puzzles of equity premium (Mehra and Prescott (1985)) and risk-free rate (Weil (1989)). Since none of the *S* sets above contain $\lambda = 1$, which is required by the standard expected utility C-CAPM, the *S* set analysis unequivocally rejects the standard expected utility model and favors the EZW non-expected utility model.

So what explains the size of the equity premium in this model? Recall that in (12) the second covariance term for consumption risk is one order of magnitude smaller than the first for "market risk." Since the λ and γ values reported in Table 4 imply that the absolute values of $1-\lambda$ and $\lambda\gamma$ are close to each other, the major determinant of the equity premium is the "market risk." This in turn implies that the dominating determinant of the equity premium is the volatility of stock returns (and possibly, the covariance of stock returns and the returns on other financial assets) as I explained at the end of Section 3. Therefore, the success of the EZW model in resolving the equity premium puzzle is ultimately linked to the risk factor that the traditional CAPM emphasizes.[22] Of course, if the traditional CAPM were true, $\lambda = 0$ must hold in (12). But none of the *S* sets reported in Table 3 contains $\lambda = 0$. Therefore the traditional CAPM is still formally rejected.

I also bring to the reader's attention that the β estimates are all smaller than 1 in statistically significant terms in Table 1. Remarkably, it still holds in *S* set analysis, as can be seen in Tables 4 and 5. This could be due to the incorporation of the human capital return in my estimates of the optimal portfolio return.

6.2. Comparison with the Related Empirical Results in the Literature

The empirical analysis in Epstein and Zin (1991), which assumed away the return on human capital and was conducted before weak-identification in the sense of Stock and Wright (2000) was recognized as a problem, rejected their own model for the most part. Because of the volatile proxy that they used for R^m_{t+1}, their λ estimates fall between –0.412 and 0.141, and their EIS estimates were always somewhat below 1. As a result, the RRA estimates in their paper were centered on 1. Their results can be compared with those in Table 1 of the present paper for us to understand the impact of accounting for human capital but not weak identification. For example, thirteen of the sixteen RRA estimates $1-\hat{\lambda}(1-\hat{\gamma})$ implied by the results reported in Table 1 are larger than 1. These higher RRA estimates are consistent with the finding in Campbell (1996) that incorporating human capital raises RRA estimates in the conventional GMM framework. The EIS estimates $1/\hat{\gamma}$ are larger than 1 except in three cases, though it is difficult to judge if they are statistically different from 1 as explained in Footnote 16 in Section 6.1.A. But in terms

[22] This finding is similar in spirit to the result in Campbell (1996) that the cross-sectional variation in asset returns is mainly explained by the market risk, though the market risk in his model is the covariance between the return of a portfolio and the aggregate stock return.

of statistical inference on model specification, the results in Table 1 are largely the same as those in Epstein and Zin (1991). The results in Epstein and Zin (1991) can also be compared with those in Tables 3 and 4 of the present paper for the purpose of understanding the overall impact of accounting for both human capital and weak identification.

Vissing-Jørgensen and Attanasio (2003) accounted for human wealth in a different way in testing the EZW model by extending Campbell's (1996) approach. Therefore, it is interesting to compare my results with those in these two papers. Campbell (1996) substituted out consumption in his model. His estimate of RRA was 5.5 in annual data, and 23 in monthly data when the share of human capital in total wealth was assumed to be $2/3$ (which is close to the 0.698 mentioned in Section 2 of this paper). See Table 6 in his paper. Under the same assumption on human capital share, Vissing-Jørgensen and Attanasio's (2003) RRA estimates were 10.2 and 6.3 for all the stockholders in their Consumer Expenditure Survey sample when consumption is not substituted out, and 11.6 when consumption is substituted out; their EIS estimate is 1.17. See Tables 1 (case 3) and 2 (cases 3 and 4 of panel A) of their paper. These RRA and EIS estimates imply λ estimates of –60.00, –37.06 and –68.24, respectively, for their three RRA estimates above. These estimates, especially the EIS estimate, and what I have reported in this paper are somewhat close to each other.[23] But my results are obtained using aggregate data on consumption and asset returns. It is usually more difficult to obtain small RRA estimates and at the same time to find the model not rejected in the aggregate data. I have estimated or tested the full EZW model, while Vissing-Jørgensen and Attanasio (2003) focused on the estimation of two parameters of this model. Campbell's (1996) results can be viewed as an indirect test of the EZW model and inform us of the size of the RRA needed to explain the asset returns in the cross section when the human capital return is modeled in his particular way.

[23] It should be noted that the comparison here is between their point estimates and the *values* of RRA and EIS implied by my *S* sets, because *S* set analysis does not provide point estimates of unknown parameters.

Neither of these two papers tested the full specification of the EZW model.[24] I also account for weak-identification, but they did (or could) not. It will be interesting to see how their results will change when the weak identification of model parameters is taken into account in their respective contexts.

Stock and Wright's (2000) conventional GMM estimates of λ are between 0 and 1. When they accounted for weak identification (but not human capital) in testing the EZW model, their joint *S* set for (β, λ, γ) and their concentrated *S* set for (λ, γ) produced with twice-lagged instruments contain $\lambda = 1$, implying that the standard C-CAPM based on the expected utility preferences cannot be rejected. Furthermore, when using instruments that are lagged only once, they obtained empty *S* sets for the EZW model for both sets of instruments that they considered, thereby rejecting it. My results as described in Section 6.1 are in sharp contrast with theirs. The difference between their results and mine attest to the importance of accounting for human capital and financial assets other than stocks in evaluating models that involve the return on market portfolio and wealth growth.

6.3. Results for the Consumption Euler Equation

In estimating the consumption Euler equation using the conventional GMM, both the 2-step and the continuous updating GMM estimates of the unknown parameters are very sensitive to the initial parameter values used. So are the results for model specification testing associated with the continuous updating estimator (not reported to conserve space). These sensitivities suggest that the conventional estimation and testing methods are especially not appropriate for estimating this equation when R_{t+1}^{m} incorporates human capital and has very low volatility. Therefore, it is natural to use the weak-identification robust asymptotic theory of Stock and Wright (2000) to conduct the tests of the consumption Euler equation.

[24] Vissing-Jørgensen and Attanasio (2003) only tested the consumption Euler equation.

I present the results of *S* set analysis for the consumption Euler equation (11) in Table 6. There are both similarities and differences between this set of results and those in Table 3. The similarities are that all the six 95% *S* sets contain parameter values that imply RRA around 2 that correspond to values of EIS closely around 1 and β values around 0.987, and β values are all smaller than 1. However, there are three major differences between the two sets of the results. First, four of the six *S* sets in Table 5 contain $\lambda = 1$ for reasonable values of β and RRA, a sign that the Euler equation of the standard expected utility C-CAPM cannot be rejected at 5% level. This result is robust to variations in instruments. For example, dropping the default premium, the term premium, and the household net worth growth from the instruments list in the testing still yields $\lambda = 1$ for reasonable values of β and RRA. In one particular sense, these results are not surprising, nor are they puzzling. This is because when (11) holds, it should be difficult to reject the standard expected utility C-CAPM, as explained by Epstein and Zin (1991) and Kocherlakota (1990a). However, when using a 10% critical value in the test, the hypothesis $\lambda = 1$ can be rejected for both sets of instruments that do not lag an additional quarter. Therefore some of the evidence for the standard expected utility C-CAPM disappears at the 10% level of significance. Second, the range of λ is now the whole range tested, from –100 to 2. This is a sign that the empirical results here for the consumption Euler equation may not be as informative as those based on (10) about the validity of the EZW model. Therefore the results for the consumption part of the model should be interpreted with more caution.

To summarize, the test results based on Stock and Wright's (2000) weak-identification robust asymptotic theory for three of the four sets of instruments do not reject at conventional significant levels the asset pricing implications of the EZW model for reasonable values of β, RRA, and EIS when the human capital return is taken into account. The fourth instrument set includes a variable that was not publicly available, and the test results based on this set of instruments may therefore be invalid. The non-empty *S* sets obtained with these instrument sets contain combinations of parameter values that produce the right equity premia and riskfree rates that match the observed averages of these two quantities in the data. These parameter values also satisfy the feasibility and transversality conditions for one version of the EZW model. However, there seems to be more uncertainty about the consumption Euler equation of the model, because to test it is difficult in both the conventional GMM framework and the Stock and Wright (2000) framework when the volatility of R_{t+1}^{m} is very small. In addition, the fact that the Euler equation of the standard expected utility model cannot be rejected at 5% level also clouds the interpretation of the test results on the consumption Euler equation.

7. Discussions and Conclusions

In this paper, I propose a new and simple approach that accounts for the return on human capital (whether it is tradable or non-tradable) and other financial assets than stocks in constructing the return on the market portfolio. My approach produces an explicit estimate of the return on the market portfolio that is much less volatile than the usual proxy used in the literature. Since there are signs of weak identification for at least two parameters in the model, I proceed to apply Stock and Wright's (2000) weak-identification robust asymptotic theory for the continuous updating GMM to the testing of the model by forming *S* sets, the confidence space of unknown parameters. The overall estimation and test results that I obtain are much different from those in the literature. I find that for three sets of instruments, the asset pricing implication of the EZW non-expected utility model cannot be rejected for values of the RRA coefficient around 2 or lower, values of the time discount factor around 0.987, and values of the EIS closely around 1. I further demonstrate that some of these parameter value combinations can simultaneously match the average equity premium and the average riskfree rate in the real world. Therefore the use of the correctly measured return on the optimal portfolio, along with the proper econometric method to take into account weak identification, is indeed very important in testing models of this nature.

The empirical results in this paper are based on U.S. aggregate data that do not differentiate between stockholders and non-stockholders. However, my results on RRA and EIS are both consistent with those based on micro data. My RRA values are in line with the estimates obtained from U.K. household level data in a model that stresses limited participation in asset markets in Attanasio, Banks, and Tanner (2002). The EIS values in my paper are consistent with the estimates obtained in cohort and household level data, such as Attanasio and Weber (1993) and Vissing-Jorgenson and Attanasio (2003). In addition, these EIS values closely around 1 in my results are notable for three other reasons. First, they are consistent with the finding in Jones, Manuelli and Siu (2000) that I alluded to in the introduction that was based on U.S. data. Second, they imply that if Guvenen's (2003a) conjecture holds (i.e. the EIS estimation based on aggregate consumption data mainly reflects the EIS's of the low-income individuals), then the low-income individuals' EIS's should be very close to 1, or the EIS's for the rich should be around 5 given their consumption share of 20%. Otherwise, my EIS results do not support his conjecture. Third, previous high EIS estimates were obtained either with household or U.S. state level data, or with aggregate data from economies that have more volatile consumption than the U.S.

Several other recent papers have also stressed the role of limited participation in stock market in solving the two asset pricing puzzles studied in this paper, see e.g. Brav, Constantinides, and Geczy (2002) and Guvenen (2003b), following the lead of Mankiw and Zeldes (1991). My results, however, suggest that both puzzles can be resolved without appealing to limited participation or other market frictions when human capital is taken into account.[25]

The finding that the volatility of the stock returns drives the equity premium in the EZW model is a surprising one, because as shown in Friend and Blume (1975), this is an implication of the traditional CAPM of Sharpe (1964) and Lintner (1965). It is an implication that the proponents of the intertemporal CAPM, especially the C-CAPM, have strongly questioned. Since

[25] Heaton and Lucas (1996) showed that transaction cost cannot explain the size of the actual equity premium.

my empirical results are obtained in a model that defines the risk of stocks as the covariance between stock returns and the marginal utility of investors that Merton (1973) and others have stressed, they are strong evidence that the risk factor implied by the traditional CAPM is more relevant and more powerful than the C-CAPM in explaining the tradeoff between risk and return of financial assets. They are also consistent with the finding in Mankiw and Shapiro (1989) that the traditional CAPM explains portfolio returns much better than the C-CAPM. However, it should be noted that although my results corroborate Friend and Blume's (1975) finding of RRA estimates around 2 obtained in the context of static CAPM, and the CAPM implication that the risk of investing in stocks is the uncertainty of stock returns, there are two differences between my results and theirs. First, in Friend and Blume (1975), the market price of risk is just the aggregate RRA of all the investors. This is no longer the case in (12), although quantitatively they are still somewhat close to each other. Second, in Friend and Blume's (1975) version of the CAPM, the only risk that matters in determining stock returns is the volatility of stock returns. But in the EZW model, the covariance between the stock returns and the returns on other financial assets may also be a significant risk, as explained at the end of Section 3.

The results in this paper contradict the conclusion in Weil (1989) that disentangling risk aversion from the aversion to intertemporal substitution of consumption cannot explain the equity premium puzzle and the risk-free rate puzzle at the same time. It also contradicts the claim in Kocherlakota (1996) that the EZW model cannot resolve the equity premium puzzle. Their views are echoed in Campbell (2003). Weil (1989) reached a negative conclusion on the EZW model because he did not take into account human capital. The reasons that Kocherlakota (1996) reached a negative conclusion are two-fold. First, in his own formulation of the Euler equations for asset returns, human capital and the EIS do not play any role. As a result, his implied Euler equation for the equity premium is exactly the same as that for the standard C-CAPM with the power utility function. This in turn implies that a high RRA is still needed to explain the equity premium in the data. Second, he conjectured that when human capital return is taken into account in Epstein and Zin's (1991) formulation of Euler equations, the covariability of the representative agent's marginal rate of substitution with asset returns would decrease by so much that the RRA would have to be high in order to explain the equity premium. My results show that this does not have to be the case, because an EIS value close to 1, through increasing the (absolute) value of λ, is sufficient to keep this covariability high and the RRA low. Campbell's (2003) reasoning assumed that the EIS was low, though he did mention that the EIS estimates may become higher under some conditions. He also assumed that using other measures of optimal portfolio return than that implied by an aggregate stock index would lower the volatility of the SDF of the EZW model so much that a rejection would be hard to avoid. Again, this does not have to happen once the EIS is in the neighborhood of 1.

Two ideas suggest themselves for future research. Given that the values of EIS in the *S* sets and GMM estimation are often around 1, it is of great interest to find how the hypothesis of unitary EIS can be tested, and how the RRA and the time discount factor estimates will change with the imposition of unitary EIS. The first step towards solving this problem is to spell out testable Euler equations that are empirically relevant. Another idea that is also of great interest is to study if the EZW model coupled with the return on the optimal portfolio estimated in this paper can explain other puzzles in asset pricing.

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TABLE 1

RESULTS OF TWO-STEP AND CONTINUOUS UPDATING GMM ESTIMATION OF EULER EQUATIONS FOR STOCK AND T BILL RETURNS USING CONVENTIONAL ASYMPTOTICS

Instruments	Method	β	λ	γ	χ^2
Set 1	2-step	0.9861	-33.19	0.96	11.85
		(0.0010)	(8.92)	(0.18)	(0.037)
	continuous	0.9863	-100.6	0.90	8.94
	updating	(0.0012)	(17.9)	(0.20)	(0.111)
Set 2	2-step	0.9859	-26.50	0.91	14.96
		(0.0009)	(4.51)	(0.17)	(0.092)
	continuous	0.9859	-59.94	0.95	19.07
	updating	(0.0009)	(8.49)	(0.18)	(0.025)
Set 3	2-step	0.9863	-25.95	1.00	15.54
		(0.0008)	(4.47)	(0.16)	(0.159)
	continuous	0.9862	-60.15	1.00	20.08
	updating	(0.0009)	(8.52)	(0.17)	(0.044)
Set 4	2-step	0.9861	-24.48	1.01	28.43
		(0.0008)	(3.92)	(0.14)	(0.008)
	continuous	0.9858	-98.03	0.96	27.96
	updating	(0.0008)	(11.66)	(0.16)	(0.009)
Set 1	2-step	0.9864	-45.14	0.97	10.46
Lagged		(0.0016)	(18.16)	(0.27)	(0.063)
	continuous	0.9860	-56.45	0.88	10.02
	updating	(0.0016)	(19.41)	(0.27)	(0.075)
Set 2	2-step	0.9857	-37.31	0.84	12.79
Lagged		(0.0014)	(14.62)	(0.25)	(0.172)
	continuous	0.9856	-60.55	0.80	11.97
	updating	(0.0014)	(18.79)	(0.23)	(0.215)
Set 3	2-step	0.9843	-29.11	0.59	15.51
Lagged		(0.0013)	(9.90)	(0.23)	(0.160)
	continuous	0.9850	-77.49	0.68	13.83
	updating	(0.0013)	(19.92)	(0.19)	(0.243)
Set 4	2-step	0.9847	-27.82	0.68	26.37
Lagged		(0.0009)	(4.84)	(0.17)	(0.015)
	continuous	0.9842	-131.5	0.52	17.57
	updating	(0.0009)	(16.9)	(0.16)	(0.175)

(continued from Table 1 on the last page)

Note. - Set 1 includes four instruments: 1, and first lags of consumption growth, term premium, and real dividend yield. Set 2 includes Set 1 and the first lags of real T Bill rate and labor income growth. Set 3 is Set 2 and the first lag of the default premium. Set 4 is Set 3 and Lettau and Ludvigson (2004)'s *cay*. Standard errors are in parentheses, except for the last column, which reports the minimum χ^2 test of over-identifying restrictions and the corresponding *p*-values in parentheses. The degree of freedom for the χ^2 test is the number of orthogonality conditions (i.e. the number of instruments$\times$2) subtracted by 3, the number of parameters estimated.

TABLE 2
PARAMETER VALUE RANGES AND INCREMENTS IN S SET ANALYSIS

	β	λ	γ
Range	[0.983, 1.01]	[-150, 2]	[0.401, 2.521]
Increment	0.001	0.25	0.02

Note.-The range of β only affects the joint S set for all the three parameters.

TABLE 3

RESULTS OF S SET ANALYSIS OF THE EULER EQUATIONS (10) FOR STOCK AND TREASURY BILL RETURNS

Instruments	95% Joint S Set for (β, λ, γ)				
	β	λ	γ	RRA	EIS
Set 1	[0.984, 0.988]	[-150, -17.25]	[0.401, 1.041]	[0.0025, 90.85]	[0.961, 2.494]
Set 1 Lagged	[0.983, 0.987]	[-150, -10.25]	[0.401, 1.081]	[0.0025, 90.85]	[0.925, 2.494]
Set 2	[0.985, 0.986]	[-87.25, -37.5]	[0.73, 1.01]	[0.17, 19.94]	[0.99, 1.37]
Set 2 Lagged	[0.983, 0.988]	[-150, -3.75]	[0.401, 1.221]	[0.002, 74.5]	[0.819, 2.49]
Set 4			Null		
Set 4 Lagged	[0.983, 0.987]	[-150, -46.25]	[0.401, 1.001]	[0.85, 90.85]	[0.999, 2.49]

Instruments	95% Concentrated S Set for (λ, γ)				95% Concentrated S Set for λ		
	λ	γ	RRA	EIS	λ	RRA	EIS
Set 1	[-150, -26.25]	[0.401, 1.021]	[0.0025, 90.85]	[0.979, 2.494]	[-107.75, -48]	[4.18, 13.66]	[1.067, 1.13]
Set 1 Lagged	[-150, -15.75]	[0.401, 1.041]	[0.0025, 90.85]	[0.961, 2.494]	[-110.5, -48]	[6.28, 27.83]	[1.123, 1.32]
Set 2	[-74, -47.25]	[0.841, 1.001]	[0.93, 11.8]	[0.999, 1.189]		Null	
Set 2 Lagged	[-150, -15]	[0.401, 1.061]	[0.0025, 84.86]	[0.943, 2.494]	[-150, -20.25]	[1.68, 77.28]	[1.035, 2.43]
Set 4		Null				Null	
Set 4 Lagged	[-150, -52.25]	[0.401, 1.001]	[0.85, 91]	[0.999, 2.494]	[-150, -58.75]	[26.2, 74.5]	[1.75, 1.96]

Note.—This table presents the ranges of each parameter in a 95% S set and the ranges of implied RRA and EIS values. These ranges are, however, not the 95% confidence intervals for individual parameters. The results when instrument set 3 is used are similar to those for instrument set 2 and are not reported to conserve space.

TABLE 4
SELECTED ELEMENTS OF 95% S SETS, IMPLIED VALUES OF RRA AND EIS, QUARTERLY EQUITY PREMIUM, AND RISKFREE RATE

β	λ	γ	RRA	EIS	E.P.	r_f
		Joint S Set for (β, λ, γ), Instrument Set 1				
0.9866	-51.25	0.981	1.974	1.019	0.0149	0.0045
0.9867	-51.25	1.001	0.949	0.999	0.0149	0.0044
0.9866	-51.50	0.981	1.979	1.019	0.0150	0.0047
0.9868	-51.75	1.001	0.948	0.999	0.0150	0.0046
		Joint S Set for (β, λ, γ), Instrument Set 2				
0.9865	-48.75	0.971	2.414	1.030	0.0141	0.0046
0.9866	-48.75	0.991	1.439	1.009	0.0140	0.0045
0.9865	-49.00	0.971	2.421	1.030	0.0142	0.0042
0.9866	-49.00	0.991	1.441	1.009	0.0141	0.0041
		Joint S Set for (β, λ, γ), Instrument Set 3				
0.9867	-51.50	1.001	0.949	0.999	0.0149	0.0045
0.9866	-51.50	0.981	1.979	1.019	0.0150	0.0047
0.9868	-51.75	1.001	0.948	0.999	0.0150	0.0046
0.9867	-52.00	0.981	1.988	1.019	0.0152	0.0043
		Concentrated S Set for (λ, γ), Instrument Set 1				
–	-51.25	0.981	1.978	1.019	0.0149	–
–	-51.50	0.981	1.979	1.019	0.0150	–
–	-51.75	0.981	1.983	1.019	0.0151	–
		Concentrated S Set for (λ, γ), Instrument Set 2				
–	-51.50	1.001	0.949	0.999	0.0149	–
–	-51.50	0.981	2.002	1.019	0.0150	–
–	-51.75	0.981	1.983	1.019	0.0151	–
		Concentrated S Set for (λ, γ), Instrument Set 3				
–	-51.50	0.981	1.979	1.019	0.0150	–
–	-51.75	0.981	1.983	1.019	0.0151	–
–	-51.75	1.001	0.948	0.999	0.0150	–

Note.—The first three columns report selected elements in the 95% joint S sets or concentrated S sets. The next two columns present the implied RRA and EIS values implied by the λ and γ values of each row. The column labeled "E.P." shows the unconditional quarterly equity premium implied by the values of RRA and EIS in each row under the assumption of lognormality for consumption growth and asset returns. The last column reports the quarterly risk-free rate implied by the values of β, λ, γ of each row in the upper panel.

TABLE 5

FEASIBILITY CONDITION AND TRANSVERSALITY CONDITION

β	δ	EIS	RRA	Feasibility	Transversality
		Joint *S* Set for (β, λ, γ), Instrument Set 1			
0.9866	0.01358	1.019	1.974	0.0137	-0.0225
0.9866	0.01358	1.019	1.979	0.0137	-0.0226
0.9867	0.01348	0.999	0.949	0.0135	-0.0224
0.9868	0.01338	0.999	0.948	0.0134	-0.0222
		Joint *S* Set for (β, λ, γ), Instrument Set 2			
0.9865	0.01368	1.030	2.414	0.0139	-0.0228
0.9865	0.01368	1.030	2.421	0.0139	-0.0228
0.9866	0.01358	1.009	1.439	0.0137	-0.0225
0.9866	0.01358	1.009	1.441	0.0137	-0.0225
		Joint *S* Set for (β, λ, γ), Instrument Set 3			
0.9866	0.01358	1.019	1.979	0.0137	-0.0226
0.9867	0.01348	0.999	0.949	0.0135	-0.0224
0.9867	0.01348	1.019	1.988	0.0136	-0.0224
0.9868	0.01338	0.999	0.948	0.0134	-0.0221

Note.—This table demonstrates that the joint *S* set elements in Table 4 that simultaneously match the equity premium and the riskfree rate satisfy the feasibility condition and the transversality condition in Smith (1996) for the EZW model.

TABLE 6
RESULTS OF S SET ANALYSIS OF THE CONSUMPTION EULER EQUATION

Instruments	95% Joint S Set for (β, λ, γ)				
	β	λ	γ	RRA	EIS
Set 5	[0.983, 0.988]	[-100, 2]	[0.401, 1.541]	[0.0018, 60.9]	[0.649, 2.494]
Set 5 Lagged	[0.983, 0.992]	[-100, 2]	[0.401, 2.261]	[0.0018, 60.9]	[0.442, 2.494]

Instruments	95% Concentrated S Set for (λ, γ)				95% Concentrated S Set for λ		
	λ	γ	RRA	EIS	λ	RRA	EIS
Set 5	[-100, 2]	[0.401, 1.361]	[0.0018, 60.9]	[0.735, 2.494]	-86.0 and −93.0	1.812 and 6.738	1.009 and 1.072
Set 5 Lagged	[-100, 2]	[0.401, 2.101]	[0.0018, 60.9]	[0.476, 2.494]	0.75	0.972	1.039

Note.—The instrument set 5 includes 1, the first and second lags of real consumption growth, the first lag of household net worth growth, and the first lags of the real after-tax labor income growth, the term premium, the real dividend yield, and the default premium.

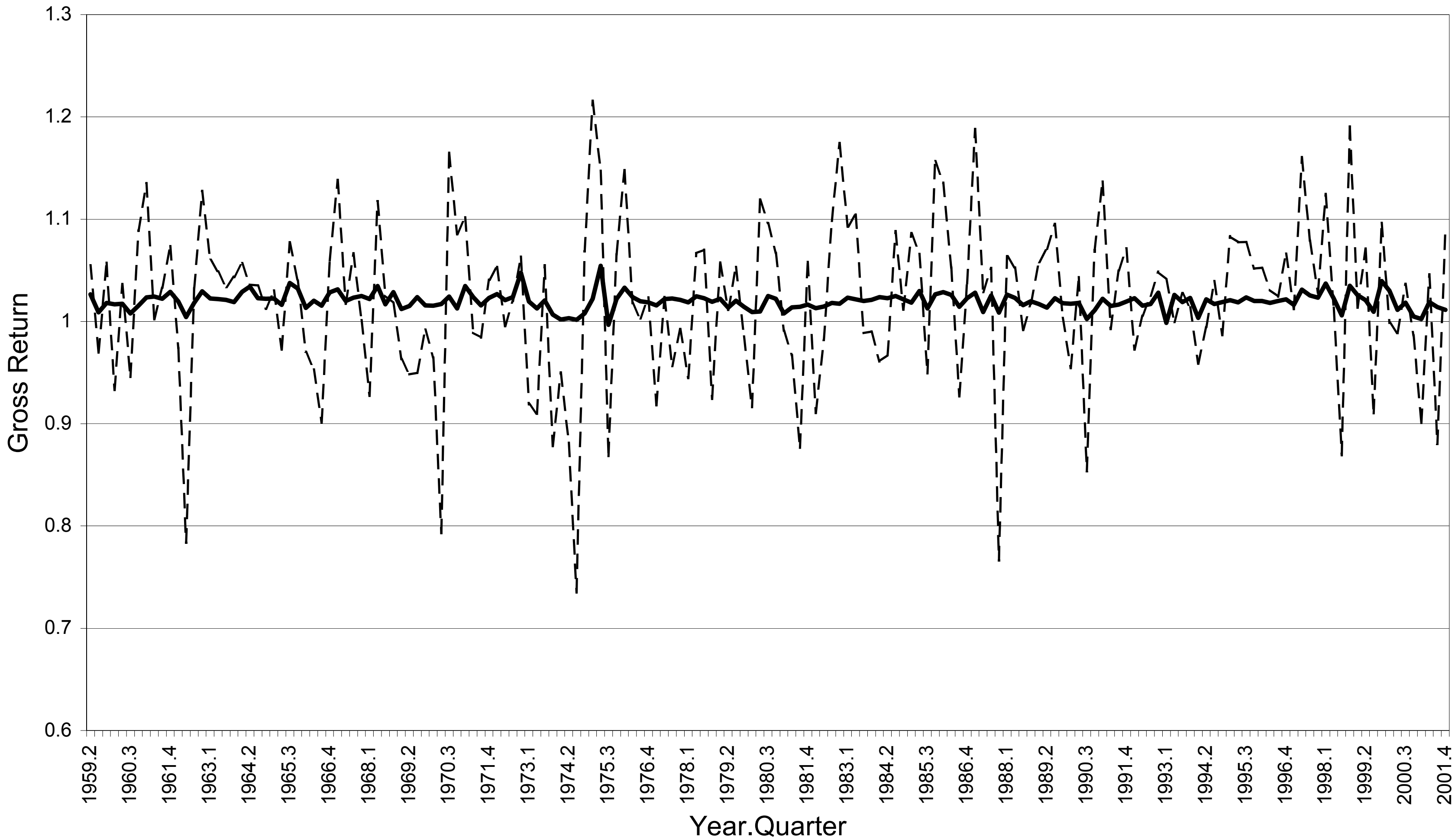


Fig. 1.—The real value-weighted return of NYSE stocks (dashed line) and R^m_{t+1}, the real return on optimal portfolio incorporating human capital (solid line). Both are quarterly rates.

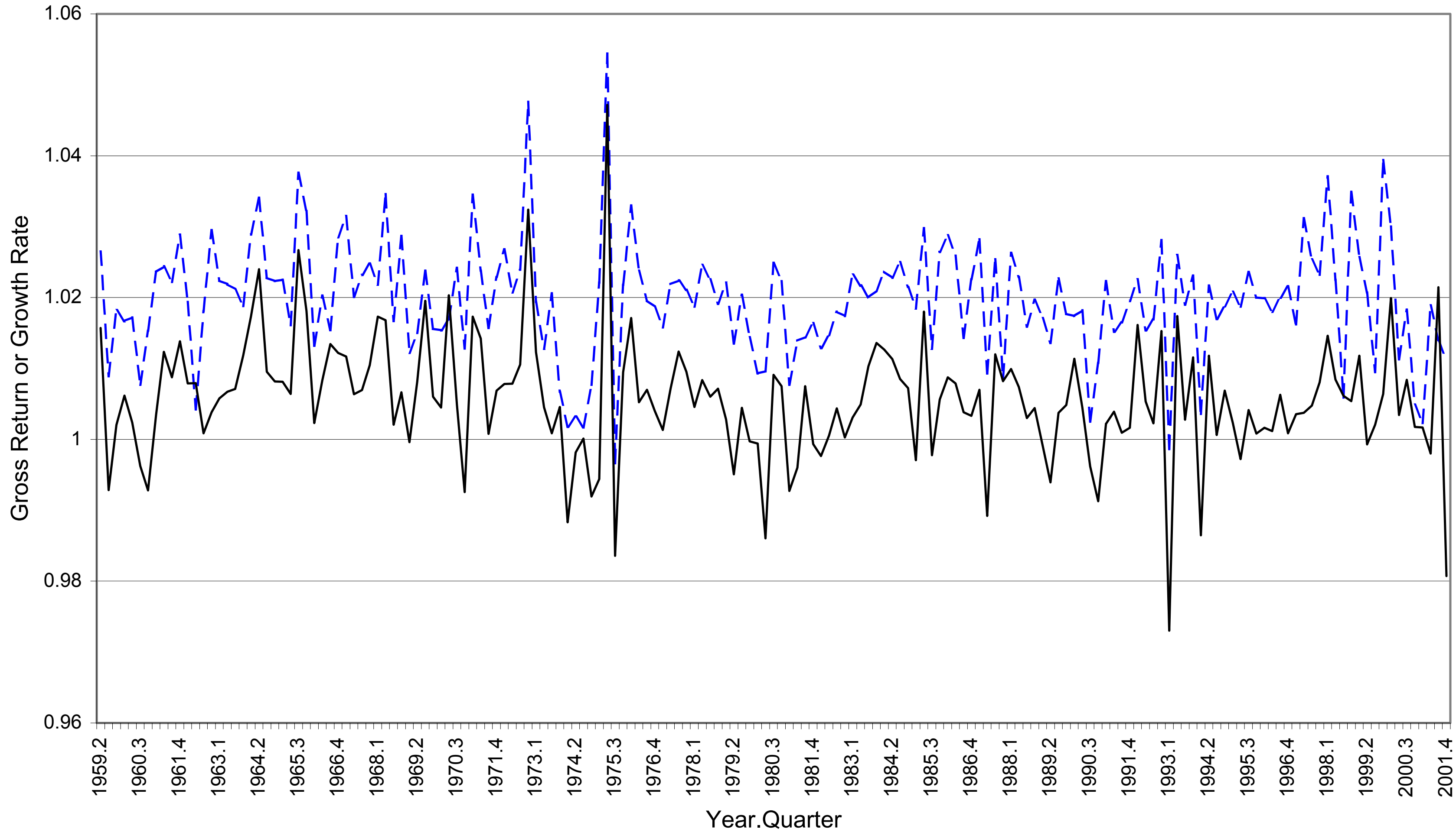
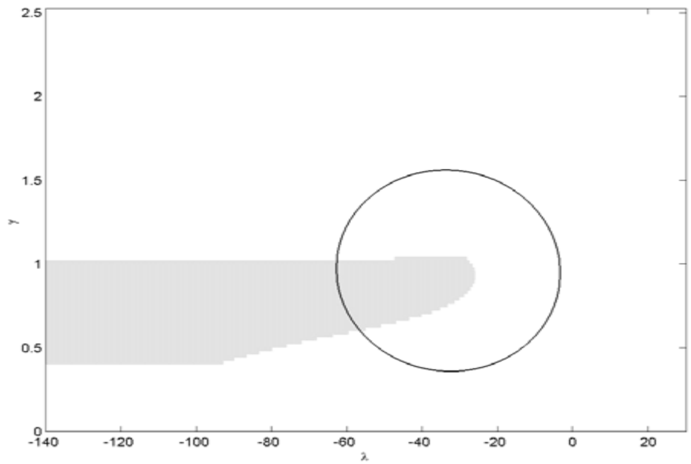
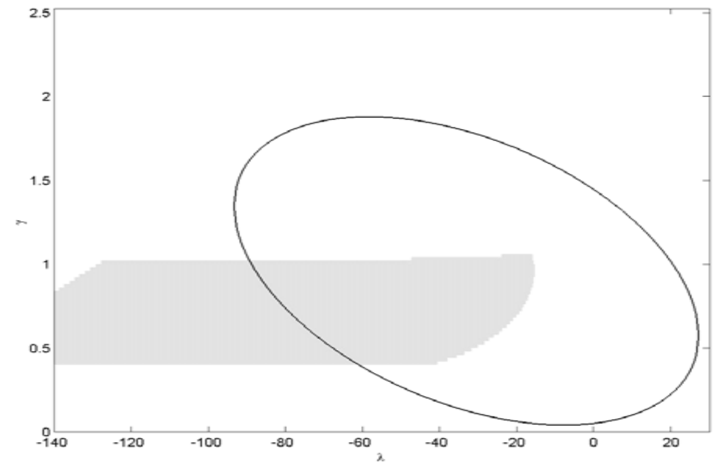


Fig. 2.—The real return on optimal portfolio incorporating human capital R_{t+1}^{m} (dashed line) and the growth of real after-tax labor income (solid line). Both are in quarterly terms



(a)



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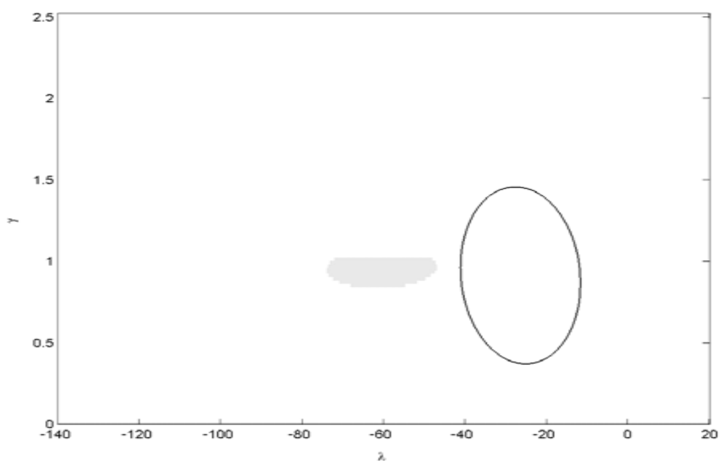

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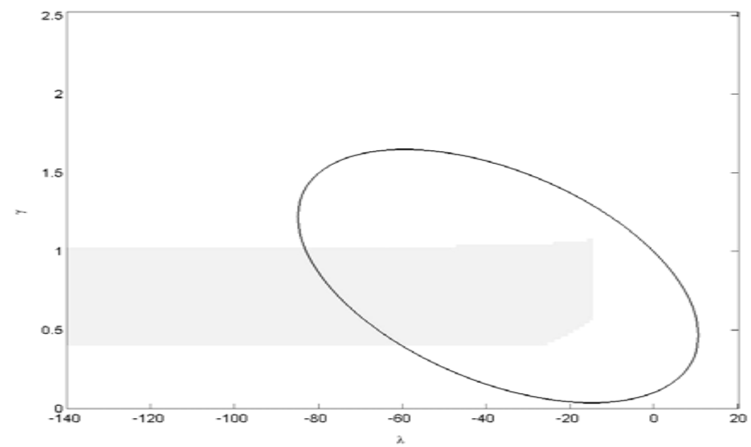
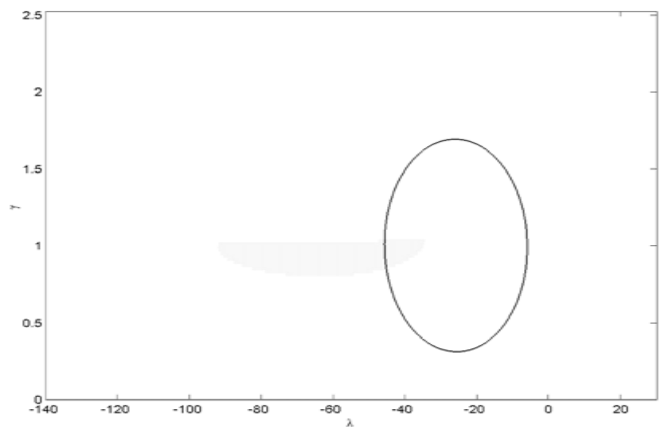


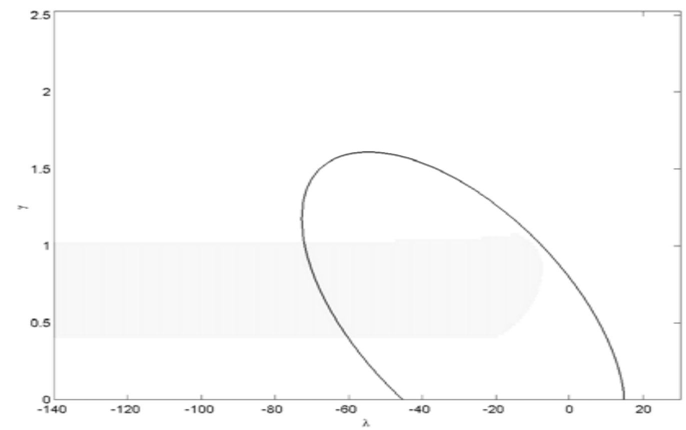
(d)

(Fig. 3 to be continued)

(Fig. 3 continued from last page)



(e)



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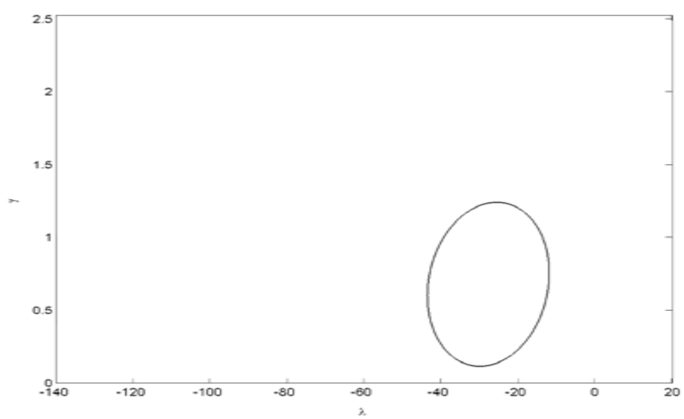

(g)

Fig. 3. —The 95% Concentrated S Set (Shaded) for λ and γ and the 95% Confidence Ellipse for the 2-Step GMM Estimates of λ and γ Based on (a). Instrument Set 1; (b). Instrument Set 1 Lagged; (c). Instrument Set 2; (d). Instrument Set 2 Lagged; (e). Instrument Set 3; (f). Instrument Set 3 Lagged; and (g). Instrument Set 4 Lagged.