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Nobuo Akai  
University of Hyogo

Dan Sasaki  
University of Tokyo

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# Inflexibility as a stabilisation device

**Nobuo Akai**

School of Business Administration

University of Hyogo

8-2-1 Gakuen-Nishi-machi, Nishi-ku

Kobe 651-2197 Japan

akai@biz.u-hyogo.ac.jp

akai@kobeuc.ac.jp

**Dan Sasaki**

Institute of Social Science

University of Tokyo

7-3-1 Hongo, Bunkyo-ku

Tokyo 113-0033 Japan

dsasaki@iss.u-tokyo.ac.jp

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# Inflexibility as a stabilisation device

**Abstract :** A possible rationale for institutional conservatism, i.e., reluctance to adjust actions in accordance with external environmental changes, may be found in the payoff stabilisation effect it strategically affords. Suppose, for example, that one of the duopolists is capable of adjusting its action, either price or quantity, in response to unexpected demand fluctuations. Then the other duopolist, if incapable of such adjustments, recuperates some of the meager opportunities when the shock is negative whilst forgoing lucrative profit opportunities when the demand shock is positive, thereby “smooths” its profits across varying states of demand in exchange for a small loss in *expected* profits, as opposed to when being as adjustable as its competitor. Similar qualitative results hold true both in Cournot and in Bertrand, and by extension, in a larger class of situations where economic decision makers interact through either strategic substitution or strategic complementarity.

**Keywords :** uncertainty, risk aversion, profit smoothing.

**JEL classification :** L13, D81, M14.

## 1 Introduction

ONE OF THE PARADIGMATIC FEATURES of inflexible business management and conservative corporate culture is the characteristic reluctance to adjust the firm’s actions in accordance with various changes in the economic environment. Our most spontaneous economic intuition might deem the lack of such adjustability economically disadvantageous, such that any institution harbouring unadjustability is considered economically irrational. Broadly speaking, there are two alternative ways to counter this intuition. One is to take into account the costs needed to make adjustments. The other, more intriguing alternative involves the question of whether unadjustability may entail any genuine economic advantage in a certain class of circumstances.

In this paper, we answer affirmatively the aforesaid question. Our basic intuition can be summatively previewed as follows.

An isolated institution, in the sense of not interacting strategically with other economic players, is unlikely to benefit from its own inflexibility. For example, a monopolist failing to adjust its production level according to demand fluctuations, will produce the same quantity whether the demand is high or low, underproducing relative to high demand in a

boom overproducing relative to low demand in a recession, resulting in profit reductions in both states of demand.

Once the institution becomes involved in strategic interactions, its inflexibility may induce those reactions from opponent institutions which can turn out beneficial in some, if not all, states of nature. For instance, if a Cournot oligopolist fails to adjust its production according to the state of demand, its underproduction relative to booming demand will encourage its flexible competitors' production, further hindering the inflexible oligopolist's profits, whilst its overproduction relative to recessing demand will pre-empt its flexible opponents' production through strategic substitution, boosting the inflexible firm's profits. Thereby if these strategic effects outweigh those non-strategic losses from inflexibility which were also present in the aforementioned monopoly case, then the oligopolist will net lose during the boom but net gain during the recession by opting for inflexibility.

Therefore, if an institution [i] has any incentive to smooth its payoffs across different states of nature, and [ii] interacts strategically with others who are capable of adjusting their actions according to the states, then the inability to adjust its own actions may serve the institution's interests.

Another crucial proviso for the above intuition is that it can materialise only if [iii] the institution's lack of adjustability is observable to its opponent players. For, it is their reactions to the said unadjustability that may entail the aforementioned payoff smoothing effect. In plain words, the institution must not only lack adjustability, but also establish the reputation that it is unable to adjust, in order to reap the possible benefit of stabilisation.

Notably, the presence or the absence of the above-mentioned strategic stabilisation effect depends qualitatively upon the form of strategic interactions. It is present, however, in a broad class of strategic interactions which are of practical economic relevance. In fact, adjustability of institutional actions can be, and has indeed been, analysed in a number of well established microtheory models which, as aforesaid, can be divided into those on adjustment costs, and those on genuine merits of unadjustability without recourse to the presence of adjustment costs. In particular, models in the latter category feature some of those very key attributes entailing the aforesaid stabilisation effect.

One of such attributes is that an institution's lack of adjustability, when observable to other players, can serve as a strategic commitment device. Such commitments can be sorted into two dimensions: to commit before information about others' actions becomes perfect, which allows for the scope that the committed player's action need not necessarily make a best response against others' actions, and to commit before information about the state of nature completes and thus to play an *ex ante* but not *ex post* best response. In

this paper, we discuss the latter in the absence of the former.

Curiously, in literature, strategic commitments has mainly been discussed in the light of informational perfection, whilst its other aspect, informational completion, has often been treated in somewhat auxiliary ways. As to the former, namely committing before others' moves, the value of such a commitment in supermodular games and submodular games has first been discussed in the well-known Stackelberg model. The seminal contribution by Gal-Or (1983) contrasts the advantage of Stackelberg leadership against that of Stackelberg followership, i.e., the gains from reacting optimally to the leader's actions. Explicit endogenisation of leader/followership owes to the "extended game" modelled by Hamilton and Slutsky (1990), a game augmented with a pre-play stage where each player can choose either to act or to wait.<sup>1</sup> Clearly, the essence of Stackelberg leader-follower relations in these seminal models concerns only informational perfection, not informational completion, as these are models without state-dependent uncertainty. Subsequently when the extended game is further extended to encompass demand uncertainty (e.g., by ) the aspect of informational completion comes to be added as part of the Stackelberg follower's advantage. However, even then, the added informational advantage of the Stackelberg follower is mostly considered as a "non-strategic advantage" and thus the overall leader-follower relation tends to be treated as the balance between the follower's non-strategic advantage of informational completion versus the leader's strategic pre-emptive advantage.

On the other hand, apparently for no obvious reason, strategic commitment in the light of informational completion has attracted relatively scarce research attention. Amongst those few studies which did not overlook this aspect, Sakai (1985, 1990, 1991) analyses how the value of information about the states of demand can be affected via duopolistic interactions. In general, the value a strategic commitment in this sense, i.e., making a move or at least committing with an action without waiting for the state of nature to unfold, is the negative of the value of information about the state of nature, and hence its *ex ante* expectation is generally negative. However, its *ex post* realisation may turn out positive in some states and negative in some other states. Hansen, Møllgaard, Overgaard and Sørensen (1996) touches this issue indirectly, in showing that an oligopolist has a strategic incentive to adjust its action asymmetrically between unexpected demand surge and unexpected demand slump, due to the monotone comparative statics regarding the strategic interaction among the oligopolists. More generally, if there are realistically conceivable situations such that a player has reason to regard those states where precommitment proves *ex post* profitable more importantly than other states where precommitment turns out

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<sup>1</sup>The extended game has two versions, one called the extended game with observable delay, the other called the extended game with action commitment. The distinction between these two categories is not directly relevant to our purposes in this paper.

*ex post* unprofitable, then it makes sense to forfeit the flexibility of adjusting actions according to realised states and thus to precommit, even when such precommitment does not entail Stackelberg leadership.

One such situation can be found in institutional risk aversion where, by definition, low income states are more of institutional concerns than high income states. Therefore, if the *ex post* value of precommitment is positive in the lower income states and is negative in the higher income states, then a risk-averse institution might rightfully opt to precommit, in spite of its negative *expected* value as aforementioned.

Presumably, a possible source of institutional risk aversion may be located in managerial incentives. Namely, a managerial job can be risked if the firm's profits implodes precipitously. A manager in such a position may therefore opt for a risk averse pattern of decision making even if the firm's authentic objective remains in risk-neutral expected profit maximisation.

In fact, even without regard to the issue of managerial incentives, a firm's *post-tax* profits may well be made a concave function of its pre-tax profits, due to the progressivity of corporate profit taxation. For instance, profit tax rates are generally positive when the firm's taxable profits are positive, but zero if its taxable profits are negative, inducing the firm to be loss averse rather than truly risk neutral.

To develop intuition, in the present paper we use familiar linear-quadratic oligopoly models to exemplify those situations where institutional risk aversion as aforesaid may indeed find it agreeable not to resolve informational incompleteness. As we observe in sections 2 and 3, an oligopolist who fails to adjust either its output quantity (Cournot) or price (Bertrand) according to stochastic demand fluctuations, earns lower expected profits both than its opponent who is swifter in adjusting either prices or quantities, and also than in the case where the firm itself is as flexible as its opponents. However, in an unexpectedly low state of demand, the inflexible firm secures a higher profit than when the firm is adjustable, in exchange for a sacrifice in profit when the demand is high. Therefore if, for some reason such as managerial risk aversion, the firm wants to smooth its profits throughout different states of demand, the said inadjustment may indeed serve the purpose.

In section 2 we model a simple linear-quadratic Cournot duopoly *à la* Dixit (1983) and Singh and Vives (1984), which allows for imperfect demand substitution (in other words, product differentiation) between the two firms' products to maintain generality and also to be consistent in treatment with our Bertrand analysis in section 3. Section 4 provides economic explanations why and how the aforesaid profit smoothing effect arises through either strategic complementarity or strategic substitution. Section 5 separates

welfare issues from our main analysis on firms' strategic incentives and explain briefly why such separation is due. Section 6 concludes the paper by summarising our findings interconnecting a firm's external behavioural conservatism and its internal remunerative conservatism.

## 2 Cournot duopoly with stochastic demand

Cournot duopolists, labelled 1 and 2 henceforth, face the system of inverse demand

$$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} a - q_1 - \gamma q_2 \\ a - \gamma q_1 - q_2 \end{bmatrix} \quad (2.0.1)$$

where  $\gamma$  is a commonly known parameter indicating the demand substitutability between the two firms' products, and  $a$  is a random variable with a unit mean. Assume for simplicity that production costs are insubstantial, and that  $\gamma \in [1 - \sqrt{3}, 1]$  for algebraic simplicity. Note that  $\gamma > 0$  entails a *strategic substitution* (*submodular*) game whereas  $\gamma < 0$  a *strategic complementarity* (*supermodular*) game.

### 2.1 Symmetric information games

As benchmarks, we define the following two alternative informational structures, both of which are symmetric between the duopolists.

**Benchmark game AA** : Both oligopolists are "adjustable," i.e., capable of choosing their respective supply quantities  $q_1$  and  $q_2$  in accordance with the state of demand  $a$ .

The Cournot-Nash equilibrium of this game is  $q_1 = q_2 = \frac{a}{2 + \gamma}$ , with the resulting profit per firm

$$\pi^{AA}[a] = \left( \frac{a}{2 + \gamma} \right)^2. \quad (2.1.1)$$

**Benchmark game UU** : Both firms are "unadjustable," i.e., supply a constant quantity irrespective of the state of demand.

The Cournot-Nash equilibrium is  $q_1 = q_2 = \frac{1}{2 + \gamma}$  entailing each firm's profit

$$\pi^{UU}[a] = \left( a - \frac{1 + \gamma}{2 + \gamma} \right) \frac{1}{2 + \gamma}. \quad (2.1.2)$$

## 2.2 Asymmetric information game

We now consider the case where only one duopolist can utilise the information about the state of demand before deciding its production. Our model expressly precludes situations where either firm can make a positive commitment to afford a Stackelberg leadership. Therefore in our model, neither firm is a leader nor a follower. The duopolists thus compete as simultaneous movers. However, only one of them is, and is well reputed to be, receptive of information relevant to the state of demand, e.g., its customers' opinions. The other firm is, and is known to be, deaf to such information.

The Cournot-Nash of this asymmetric information duopoly is for the unadjustable firm to produce  $q^U = \frac{1}{2+\gamma}$  whilst the other firm adjusts its supply level to  $q^A[a] = \frac{a}{2} - \frac{\gamma}{2(2+\gamma)}$ . The resulting equilibrium profits are

$$\pi^{AU}[a] = \left( \frac{a}{2} - \frac{\gamma}{2(2+\gamma)} \right)^2 \quad (2.2.1)$$

for the adjustable firm,

$$\pi^{UA}[a] = \left( (2-\gamma)a - \frac{2-\gamma^2}{2+\gamma} \right) \frac{1}{2(2+\gamma)} \quad (2.2.2)$$

for the unadjustable firm.

## 2.3 Value of adjustability and commitments

Value of adjustability for each firm should be evaluated as the equilibrium profit differential between when the firm is adjustable and when it is unadjustable, given its competitor's (un)adjustability.

**Lemma i :**  $E[\pi^{AA}[a]] > E[\pi^{UA}[a]]$  and  $E[\pi^{AU}[a]] > E[\pi^{UU}[a]]$ .

In words, it is *expectedly* profitable for a firm to utilise information about the state of demand and thus to adjust its production, whether the opponent does likewise or otherwise.

**Lemma ii :**  $E[\pi^{AA}[a]] - E[\pi^{UA}[a]] \lesseqgtr E[\pi^{AU}[a]] - E[\pi^{UU}[a]]$  if and only if  $\gamma \gtrless 0$ .

That is, a firm's expected marginal gain from adjustability is higher (resp. lower) when the opponent is unadjustable, than when the opponent is adjustable, when these duopolists supply substitutional (resp. complementary) products. In other words, the benefits from adjustability are substitutional or complementary between the Cournot oligopolists when, and only when, the game is submodular or supermodular, respectively.



**Proof :** From (2.1.1) and (2.2.2),

$$E[\pi^{UA}[a]] = E[\pi^{AA}[a]] - \frac{\text{Var}[a]}{(2 + \gamma)^2} \quad (2.3.1)$$

whereas

$$E[\pi^{UU}[a]] = E[\pi^{AU}[a]] - \frac{\text{Var}[a]}{4} \quad (2.3.2)$$

from (2.2.1) and (2.1.2).

**Proposition 1 :**  $\pi^{AA}[1] = \pi^{UA}[1]$  and  $\pi^{AA}[1] > \pi^{UA}[1] > 0$ .

Namely, in the neighbourhood  $a \approx 1$ ,  $\pi^{UA}[a] \lesseqgtr \pi^{AA}[a] \iff a \gtrless 1$ .

**Proof :** (2.1.1) and (2.2.2) imply  $\pi^{AA}[1] = \pi^{UA}[1] = \frac{1}{(2 + \gamma)^2}$ , which immediately proves the former half of Proposition 1. Also follows from (2.1.1) and (2.2.2) the relation

$$\left. \frac{\partial \pi^{UA}[a]}{\partial a} \right|_{a=1} = \frac{2 - \gamma}{2(2 + \gamma)} < \left. \frac{\partial \pi^{AA}[a]}{\partial a} \right|_{a=1} = \frac{2}{(2 + \gamma)^2}$$

which proves the latter half of Proposition 1.

**Proposition 2 :**  $\pi^{AU}[a] \geq \pi^{UU}[a]$  for all  $a$ , and  $\pi^{AU}[a] > \pi^{UU}[a]$  for all  $a \neq 1$ .

**Proof** follows directly from (2.2.1) and (2.1.2).

Proposition 2 ensures that  $\pi^{AU}[a]$  *first-order stochastic dominates*  $\pi^{UU}[a]$ , whereas Proposition 1 reveals that  $\pi^{AA}[a]$  generally (i.e., for a general distribution of  $a$ ) *neither first- nor second-order stochastic dominates*  $\pi^{UA}[a]$ .

## 2.4 Choices of adjustability

We now hypothesise an environment where firms may choose whether to be able to adjust production according to the state of demand. That is, we imply an extended duopoly game unfolding through the following two stages.

1. In the beginning, each of the duopolists chooses to be either “adjustable” being capable of deciding its supply quantity *after observing* the state of demand  $a$ , or “unadjustable” having to decide its output *without regard to* the state.

Adjustability of each firm then becomes commonly observable.

2. The two firms then simultaneously and independently decide how much to produce, whereby the Cournot duopoly materialises.

Lemma i shows that, if firms act genuinely as expected profit maximisers, they should invariably opt for full adjustability. When we treat a firm as a single entity in, say, a game-theoretic oligopoly model, as seems the case in standard microeconomic theory to this date, it is usually assumed as a risk-neutral profit maximiser whose internal decision making procedures are blackboxed and thus unquestioned. Thus in such a traditional textbook-like framework, institutional conservatism could not easily be endogenised.

In reality, however, the complexity of intra-organisational incentives may bend institutional decisions away from pure profit maximisation. If, for instance, managerial compensations are proportionate to the firm's profits except when the profits take a precipitous nose-dive in which case the managers in charge risks their positions, then it is foreseen that the managerial board members behave *as if risk-averse*. Corporate conservatism may also arise due to progressive profit taxation, such as positive tax rates for positive taxable profits whilst zero tax rates for negative profits.<sup>2</sup>

Proposition 2 implies that a firm should unequivocally opt for full adjustability irrespectively of its risk attitude insofar as its opponent lacks such adjustability. However, Proposition 1 shows that, given firm 1 being adjustable, firm 2's profits are less volatile across the states of demand when firm 2 is unadjustable than when adjustable. Therefore if, for example, the managerial remuneration scheme in firm 2 is profit dependent in the aforementioned way, the (as if) risk averse managerial decision might agree to accept the loss in expected profits,  $\frac{\text{Var}[a]}{(2 + \gamma)^2}$  as calculated in (2.3.1), as an affordable "insurance fee."

In such an environment, one of the duopolists opting for adjustability whilst the other being unadjustable may materialise as the profile of endogenous choices.<sup>3</sup> Otherwise, if firms act sufficiently risk-neutrally, then both firms shall opt for adjustability. As ascertained by Proposition 2, both firms opting for unadjustability can never be chosen endogenously.

## 2.5 A numerical illustration

For the sake of concreteness, let  $a$  be either 0.95 in recession or 1.05 in boom with probability one half each. Varying the inter-firm demand substitutability  $\gamma$ , we obtain Table 1 (see Appendix).

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<sup>2</sup>Typically, even when a firm's nett profit is positive, not its entirety is taxable. Legal taxable profits can be zero even when the firm is earning modest positive profits. Thereby the said progressivity takes its effect in the interior of the "positive profits" range in our definition.

<sup>3</sup>It has been assumed throughout 2.2 that an unadjustable firm still maximises its expected profits when deciding its supply level. This can be interpreted, for example, as the production decision being operative, i.e., an employee decision rather than managerial, whilst the structural choice between adjustability and unadjustability being a matter of top management.

When the substitutability parameter  $\gamma$  is close to zero, the aforesaid profit insurance effect of unadjustability is insubstantial, and thus Proposition 1 is operative only in a very small neighbourhood around  $a \approx 1$ . This reflects the fact that the insurance effect is generated by the *strategic interaction* between the firms (as shall be expounded in section 4), and that such interaction is weak when  $|\gamma|$  is low. When  $|\gamma|$  is as low as 0.3, even a moderate demand fluctuation such as  $a \in \{0.95, 1.05\}$  already lies beyond the scope of Proposition 1, so that  $\pi^{AA}[a]$  dominates  $\pi^{UA}[a]$  in both states, rendering the profit insurance effect inoperative. However, as soon as  $|\gamma|$  becomes high enough, the range where Proposition 1 is relevant enlarges, so that the insurance effect grows rapidly. Clearly from the table,  $\pi^{UA}[a]$  is not even *second-order* stochastic dominated by  $\pi^{AA}[a]$ . When  $\gamma = 1$ , that is when the duopolists supply perfect substitutes, if both firms are adjustable, the recession profit  $\pi^{AA}[0.95] = 0.1002\dot{7}$  is 18.14% below the boom profit  $\pi^{AA}[1.05] = 0.1225$ , whereas if one firm switches to unadjustability, its recession profit  $\pi^{UA}[0.95] = 0.102\dot{7}$  is only 13.95% below its boom profit  $\pi^{UA}[1.05] = 0.119\dot{4}$ . Hence if the latter secures the managerial jobs and remunerations better than the former, the managers may well be willing to sacrifice the expected profit difference  $E[\pi^{AA}[a]] - E[\pi^{UA}[a]] = 0.0002\dot{7}$  as an affordable insurance fee.

Meanwhile,  $\pi^{AU}[a]$  always first-order stochastic dominates  $\pi^{UU}[a]$ , indicating that at least one firm must retain adjustability regardless of its risk attitude.<sup>4</sup>

Similar tendencies are present in the complementarity range  $\gamma < 0$  as summarised in Table 2 (in Appendix). The only qualitative difference is that  $\pi^{AU}[a]$  is even better insured than  $\pi^{UA}[a]$  (that is,  $\pi^{UA}[0.95] < \pi^{AU}[0.95] < \pi^{AU}[1.05] < \pi^{UA}[1.05]$ ), whereas  $\pi^{AU}[a]$  suffered a very sizeable spread between the recession and the boom profits in the previous substitution range  $\gamma > 0$ . In other words, under strategic complementarity, the profit insurance effect resulting from a firm's non-adjustment spills more heavily over insuring its *opponent's profit* than insuring the firm's own profit.

### 3 Bertrand duopoly with stochastic demand

We now contemplate a two-stage game analogous to that defined in 2.1 except that the firms are now Bertrand duopolists. Inverting the system (2.0.1), we obtain the system of demand

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<sup>4</sup>If, however, managerial contracts are susceptible to punitive dismissal based upon a *serial decline* in profits, then the relative ratio (or spread) between  $\pi^{AU}[0.95]$  and  $\pi^{AU}[1.05]$  or that between  $\pi^{UU}[0.95]$  and  $\pi^{UU}[1.05]$  shall be more material than the absolute profit levels, whereby profit smoothing incentives may possibly supercede even the *first-order* stochastic dominance relation in profit distributions.

$$\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \frac{1}{1 - \gamma^2} \begin{bmatrix} (1 - \gamma)a - p_1 + \gamma p_2 \\ (1 - \gamma)a + \gamma p_1 - p_2 \end{bmatrix}. \quad (3.0.1)$$

For technical simplicity, we limit our attention to the range  $\gamma \in (-1, \sqrt{3} - 1]$  whereby preclusion of the so-called Bertrand paradox facilitates our analysis hereinafter.

### 3.1 Symmetric information games

As in 2.1, we define the following two alternative informational structures as benchmarks, both of which are symmetric between the duopolists.

**Benchmark game aa** : Both oligopolists are “adjustable,” i.e., capable of charging their respective sales prices  $p_1$  and  $p_2$  according to the state of demand  $a$ .

The Bertrand-Nash equilibrium materialises as  $p_1 = p_2 = \frac{1 - \gamma}{2 + \gamma} a$ , whereby each firm earns

$$y^{aa} = \frac{1 - \gamma}{(1 + \gamma)(2 - \gamma)^2} a^2. \quad (3.1.1)$$

**Benchmark game uu** : Both firms are “unadjustable,” i.e., charge a constant price irrespective of the state of demand.

The Bertrand-Nash equilibrium is  $p_1 = p_2 = \frac{1 - \gamma}{2 - \gamma}$ , for each firm to earn

$$y^{uu} = \frac{1 - \gamma}{(1 + \gamma)(2 - \gamma)^2} \left( (2 - \gamma)a - (1 - \gamma) \right). \quad (3.1.2)$$

### 3.2 Asymmetric information game

When one firm is adjustable whilst the other is unadjustable, the latter charges  $p_i = \frac{1 - \gamma}{2 - \gamma}$

whilst the former charges  $p_f = \frac{1 - \gamma}{2 - \gamma} \cdot \frac{(2 - \gamma)a + \gamma}{2}$ . The associated equilibrium profits are

$$y^{au} = \frac{1 - \gamma}{(1 + \gamma)(2 - \gamma)^2} \left( \frac{(2 - \gamma)a + \gamma}{2} \right)^2 \quad (3.2.1)$$

for the adjustable firm, whilst

$$y^{ua} = \frac{1 - \gamma}{(1 + \gamma)(2 - \gamma)^2} \cdot \frac{(4 - \gamma^2)a - (2 - \gamma^2)}{2} \quad (3.2.2)$$

for the unadjustable firm, assuming that variations in  $a$  are small enough for either firm to avail of an interior optimum.

### 3.3 Value of adjustability and commitments

Bertrand analogues of Lemmata i, ii and Propositions 1, 2 are as follows.

**Lemma iii :**  $E[y^{aa}[a]] > E[y^{ua}[a]]$  and  $E[y^{au}[a]] > E[y^{uu}[a]]$ .

In words, it is *expectedly* more profitable for a firm to be adjustable than to be unadjustable, whatever the opponent's adjustability.

**Lemma iv :**  $E[y^{aa}[a]] - E[y^{ua}[a]] \gtrless E[y^{au}[a]] - E[y^{uu}[a]]$  if and only if  $\gamma \gtrless 0$ .

Contrary to Lemma ii in the Cournot case, a firm's expected marginal gain from adjustability is higher (resp., lower) when the opponent is also adjustable, than when the opponent is unadjustable. Namely, the benefits from adjustability are either complementary or substitutional between the Bertrand oligopolists depending upon whether their pricing actions are strategic complements or substitutes, respectively.

**Proof :** From (3.1.1) and (3.2.2),

$$E[y^{ua}] = E[y^{aa}] - \frac{(1-\gamma)\text{Var}[a]}{(1+\gamma)(2-\gamma)^2} \quad (3.3.1)$$

and from (3.1.2) and (3.1.4),

$$E[y^{uu}] = E[y^{au}] - \frac{(1-\gamma)\text{Var}[a]}{4(1+\gamma)}. \quad (3.3.2)$$

**Proposition 3 :**  $y^{aa}[1] = y^{ua}[1]$  and  $y^{aa'}[1] > y^{ua'}[1]$ .

Namely, in the neighbourhood  $a \approx 1$ ,  $y^{ua}[a] \lesseqgtr y^{aa}[a] \iff a \gtrless 1$ .

**Proof :** (3.1.1) and (3.2.2) imply  $y^{aa}[1] = y^{ua}[1] = \frac{1-\gamma}{(1+\gamma)(2-\gamma)^2}$ , and also the relation

$$\left. \frac{\partial y^{ua}[a]}{\partial a} \right|_{a=1} = \frac{1-\gamma}{(1+\gamma)(2-\gamma)^2} \left( 2 - \frac{\gamma^2}{2} \right) < \left. \frac{\partial y^{aa}[a]}{\partial a} \right|_{a=1} = \frac{2(1-\gamma)}{(1+\gamma)(2-\gamma)^2}.$$

**Proposition 4 :**  $y^{au}[a] \geq y^{uu}[a]$  for all  $a$ , and  $y^{au}[a] > y^{uu}[a]$  for all  $a \neq 1$ .

**Proof** follows directly from (3.2.1) and (3.1.2).

Proposition 4 ensures that  $y^{au}[a]$  *first-order stochastically dominates*  $y^{uu}[a]$ , whereas Proposition 5 provides that, for general distributions of  $a$ ,  $y^{aa}[a]$  *neither first- nor second-order stochastically dominates*  $y^{ua}[a]$ .

### 3.4 A numerical illustration

Analogously to 2.5 in our foregoing Cournot model, we let  $a$  be either 0.95 or 1.05 with probability one half each, to tabulate in Tables 3 and 4 (see Appendix) Bertrand profits as functions of  $\gamma$ .

Similarly to 2.4 and 2.5 in the Cournot game,  $y^{aa}[a]$  first-order stochastically dominates  $y^{uu}[a]$ , whilst  $y^{aa}[a]$  generally does not  $y^{ua}$ . Therefore given firm 1's adjustability, firm 2 can earn profits that are less volatile in  $a$  by being unadjustable, with a sacrifice of  $\frac{(1-\gamma)\text{Var}[a]}{(1+\gamma)(2-\gamma)^2}$  as shown in (3.3.1). Note on the other hand that  $y^{aa}[a]$  dominates  $y^{uu}[a]$  for all  $a$ . Adjustability thus unambiguously profits a firm against its unadjustable opponent.

Opposite from our previous Cournot example, here in Bertrand  $y^{ua}[a]$  is better insured than any other profits in that range where  $\gamma < 0$  (Table 4), whereas it results in insuring  $y^{aa}$  when  $\gamma > 0$  (Table 3). It is analogous to the Cournot case, however, that the former corresponds to strategic substitution (submodularity) whilst the latter corresponds to strategic complementarity (supermodularity).

## 4 Monotone comparative statics

In this section we develop economic explanations pertaining to our foregoing qualitative observations. In the following, by default we consider the most familiar cases where the duopolists supply products which are substitutes perceived from the demand side.

### 4.1 Why unadjustability smoothes profits

An unadjustable Cournot oligopolist underproduces when the state of demand  $a$  is unexpectedly high, and overproduces when the state is low. Through *strategic substitution*, this induces its adjustable competitor to expand in the high state, to downsize in the low state. These reactions deliver feedback effects to the unadjustable firm, so as to reduce the profit of the unadjustable firm in the high state, whilst enhancing it in the low state. Altogether, these feedback effects serve to smooth the profits of the unadjustable firm throughout various states of demand. A similar effect shall be present in a Bertrand oligopoly where the firms supply demand complements, and also in a broader class of games with strategic substitution.

On the other hand, an unadjustable Bertrand oligopolist underprices when the state  $a$  is above average, overprices when the state is below average. Now via *strategic complementarity* (assuming again that these firms supply demand substitutes), this encourages its adjustable competitor to follow, albeit to a lesser extent, the underpricing when the state is high, and also to follow the overpricing when the state is low. Feedback from these reactions to the unadjustable firm's profits is to curb the latter's profit in the high state, but to rescue it in the low state. Altogether, the feedback once again helps smooth the unadjustable firm's profits across different states  $a$ . An analogous effect shall emerge in a Cournot oligopoly where the firms supply demand complements, and also in a more general class of games with strategic complementarity.

## 4.2 Why profits are easier to smooth under submodularity than under supermodularity

When an unadjustable Cournot oligopolist overproduces in a recession, its output quantity is naturally higher than that by its adjustable competitor. This affords the former a strictly higher profit at the expense of the latter. When the unadjustable firm underproduces in a boom, its competitor claims a higher market share both in quantities and thus in profits. Overall, the unadjustable firm smoothes its own profits across the states, all the while magnifying its competitor's profit fluctuations. Effects are similar under strategic substitution in general, such as in a Bertrand game with complementary products.

On the other hand, when an unadjustable Bertrand oligopolist overprices in a recession, its price is higher than its adjustable competitor's price, the latter earning a higher profit than the former. In a boom, the unadjustable firm underprices at a price below its competitor's, earning a higher profit than the competitor. Thereby the unadjustable firm serves to smooth its competitor's profits more than its own profits. Whilst the unadjustable Bertrand firm's own profits are smoothed by virtue of its unadjustability, most of the profit smoothing effects spills over to its competitor. Effects are qualitatively the same under strategic complementarity in general, including a Cournot game with complementary products.

## 4.3 When unadjustability fails to smooth profits

Now, seeking a counter-case where unadjustability fails to serve as a profit smoothing device, may help us develop a further insight. As a thought experiment, contemplate an unusual kind of duopoly where two *a priori* symmetrical duopolists nevertheless adopt asymmetrical strategy spaces, one firm choosing its output quantity whilst the other

setting its supply price. Suppose first that the quantity-setting firm is unadjustable. If the state of demand is high, the said unadjustability forces the firm to underproduce relative to the high demand, leaving a bountiful residual demand the other firm who thus overprices. This feeds back to the unadjustable quantity-setter as a *positive* contributor to its profits. In a low demand state, the unadjustable firm overproduces relative to the demand, which oppresses the other firm who is then forced to underprice. This in turn puts the quantity-setter in a competitive disadvantage, hindering its profits. Unadjustability hereby serves to amplify the volatility in the firm's profits.

## 5 Welfare and socioeconomic implications

Thus far, we have concentrated our attention on incentives to be or not to be adjustable from the firms' points of view. We now complete our analysis by investigating the social implications of firms' adjusting or not adjusting to the state of economy.

### 5.1 Consumers surplus and welfare in the Cournot market

In 2.3 and 2.4 we have shown that incentive structures fostering institutional risk aversion may encourage some, not all, of the oligopolists to opt away from adjustability.

A natural question here is whether this is good news for consumers, and ultimately for the industry-wide total surplus.

**Lemma v :** The *expected* consumers surplus is higher when both firms are adjustable than when only one of them is, and when one firm is adjustable than when neither is.

**Proposition 5 :** Between any two different adjustability profiles, the *probability distributions* of consumers surpluses have no first- or second-order stochastic dominance relation guaranteed for all distributions of  $a$ .

**Proofs** follow from equilibrium supply quantities in each adjustability profile computed in 2.1 and 2.2. The consumers surplus

$$\begin{aligned}
& \frac{1}{q_2} \int_{x_2=0}^{q_2} \int_{x_1=0}^{q_1} (1 - x_1 - \gamma x_2 - p_1) dx_1 dx_2 + \frac{1}{q_1} \int_{x_1=0}^{q_1} \int_{x_2=0}^{q_2} (1 - \gamma x_1 - x_2 - p_2) dx_2 dx_1 = \\
& = \left( q_1 - \frac{q_1^2}{2} - \frac{\gamma q_1 q_2}{2} - p_1 q_1 \right) + \left( q_2 - \frac{q_2^2}{2} - \frac{\gamma q_1 q_2}{2} - p_2 q_2 \right) = \\
& = q_1 - \frac{q_1^2}{2} - \frac{\gamma q_1 q_2}{2} - (1 - q_1 - \gamma q_2) q_1 + q_2 - \frac{q_2^2}{2} - \frac{\gamma q_1 q_2}{2} - (1 - \gamma q_1 - q_2) q_2 = \\
& = \frac{q_1^2}{2} + \gamma q_1 q_2 + \frac{q_2^2}{2}
\end{aligned}$$



is equal to

$$\begin{aligned} \frac{(1 + \gamma)a^2}{(2 + \gamma)^2} & \quad \text{in benchmark game AA (in 2.1)} \\ \frac{4(1 - \gamma^2) + (\gamma + (2 + \gamma)a)^2}{8(2 + \gamma)^2} & \quad \text{in asymmetric game (in 2.1)} \\ \frac{1 + \gamma}{(2 + \gamma)^2} & \quad \text{in benchmark game UU (in 2.1)} \end{aligned}$$

of which the means are

$$\frac{(1 + \gamma)(1 + \text{Var}[a])}{(2 + \gamma)^2} > \frac{1 + \gamma}{(2 + \gamma)^2} + \frac{\text{Var}[a]}{8} > \frac{1 + \gamma}{(2 + \gamma)^2}$$

respectively, whilst there is no first-order or even second-order stochastic dominance among these three.

It is noteworthy that these properties are *non-strategic*. Namely, when a firm adjusts its production, it supplies a large quantity when the demand booms, which enables a surge in the consumers surplus. This increment outweighs the decrement in the consumers surplus incurred by the reduced production when demand recesses. Adjustments in production thereby enhance the expected consumers surplus. This effect is present whether the firm is a monopolist or an oligopolist.

An analogue applies to the total surplus.

**Lemma vi :** Both firms being adjustable entails a higher *expected* total surplus than when only one firm being adjustable, and one firm being adjustable than both firms being unadjustable.

**Proposition 6 :** There is no welfare ranking in the sense of first- or second-order stochastic dominance among adjustability profiles that hold universally under any distribution of  $a$ .

**Welfare implications :** According to Lemmata v and vi, adjustability enhances *expected* consumers surplus and welfare. However, Propositions 5 and 6 counterclaim that, for some distribution of the state  $a$ , firms' unadjustability may serve to stabilise the resulting consumers surplus and total surplus. This implies that if representative consumers are risk averse, they may not necessarily want both firms to be adjustable.

## 5.2 Consumers surplus and welfare in the Bertrand market

Curiously, *regardless of strategic substitution or strategic complementarity*, qualitative assertions on the Cournot game in Lemmata v and vi are to be reversed in the Bertrand game, whilst Propositions 5 and 6 remain intact.

**Lemma vii :** The *expected* consumers surplus is higher when both firms are unadjustable than when one firm is adjustable, and when only one firm is adjustable than when both are adjustable.

**Lemma viii :** Both firms being unadjustable entails a higher *expected* total surplus than when one firm being adjustable, and only one firm being adjustable than both firms being adjustable.

**Proofs** follow from equilibrium supply quantities in each adjustability profile computed in 3.1 and 3.2.

**Welfare implications :** Lemmata vii and viii provide that unadjustability enhances *expected* consumers surplus and welfare. This implies that managerial risk aversion and resulting unadjustability in the Bertrand firm contribute positively to the total surplus in the industry.

### 5.3 Summary on adjustability and welfare

Firms' incentives for or against adjusting their actions in accordance with the state of economy is, unfortunately, not directly linked with whether the adjustments in question are socially desirable. This mismatch reflects the fact that their incentives are strategic whilst welfare impacts of adjustments are not always strategic. In fact, in our oligopoly examples, the recipients of these welfare impacts are consumers who, by the very definition of oligopoly, are not strategic players in the game theoretic sense. This contributes to the non-strategic nature of these welfare impacts.

## 6 Concluding remarks

Among the most prominently pervasive features characterising institutional rigidity and managerial unadjustability, is the excessive tendency to heed precedence in conjunction with relative lack of receptivity towards newly emerging environmental changes. Conservatism in this sense may affect firm behaviour in at least two ways.

On one hand, it makes the firm slow, or reluctant, in changing its *status quo* actions. Slow processes in corporate decision making, lengthy procedures for a new proposal to obtain consensus within the firm, pre-existing agreements allowing little room for discretionary flexibility, and oversized in-firm bureaucracy, are all too typical in conservative, uneconomical organisational structures. In mainstream microeconomic theory, this aspect

tends to be dealt with by conceptualising *adjustment costs* required to alter the firm's actions. Such costs may well be present and substantial in the case of menu costs, for example, where an active action is required if and only if the *status quo* is to be altered. However, many other kinds of corporate decisions are essentially a matter of whether to follow the precedent or not. The decision of production quantity, for example, may not be substantially cost-savvy even when replicating what happened in the past. In such a case, the rigidity in decision making, rather than the actual transaction costs, seems to play the central role. It is also conceivable that the said inflexibility in production decisions may be less likely to serve as a device to establish a Stackelberg leadership than other kinds of rigidity such as menu costs which are easier to verify and substantiate externally, e.g., from the viewpoint of competing economic players.

On the other hand, conservatism tends to make the management of the firm defensively risk averse. Namely, in a conservative system, a manager tends to be punished more heavily when the firm's profits decrease than to be rewarded when the profits increase. Such remunerative asymmetry incites managerial risk aversion, which materialises observationally as if the firm itself were averting risk.

In this paper we have modelled a simple duopoly game to exemplify that institutional conservatism may be endogenously sustained in the presence of inter-institutional strategic interactions. The central irony here is that an institution may be encouraged to become, or to remain, conservative precisely by the presence of a non-conservative competitor. We have also observed that such endogenously induced conservatism may contribute either positively or negatively to welfare, and that it is not always likely to entail the best welfare-efficient outcome.

## Appendix

Numerical tables pertaining to our Cournot and Bertrand duopoly examples in sections 2 and 3, respectively, are taxonomically laid out on the following two pages.

**Table 1 :** Cournot firms supplying substitutable products.

$\gamma$	0.3	0.3	0.5	0.6	1
$\pi^{AA}[0.95]$	0.170604...	0.165765...	0.14447	0.126914...	0.10027
$\pi^{AA}[1.05]$	0.208412...	0.2025	0.1764	0.155039...	0.1225
$\frac{\pi^{AA}[1.05]}{\pi^{AA}[0.95]}$	0.818594...	0.818594...	0.818594...	0.818594...	0.818594...
$\pi^{UA}[0.95]$	0.170557...	0.165816...	0.145	0.128125	0.1027
$\pi^{UA}[1.05]$	0.207514...	0.201530...	0.175	0.153125	0.1194
$\frac{\pi^{UA}[1.05]}{\pi^{UA}[0.95]}$	0.821908...	0.822784...	0.8285714	0.836734...	0.860465...
$\frac{\pi^{UA}[1.05] - \pi^{UA}[0.95]}{\pi^{AA}[1.05] - \pi^{AA}[0.95]}$	0.9775	0.972	0.9375	0.8	0.75
$\pi^{AU}[0.95]$	0.167921...	0.162869...	0.140625	0.1225	0.0950694
$\pi^{AU}[1.05]$	0.211400...	0.205727...	0.180625	0.16	0.1284027
$\frac{\pi^{AU}[1.05]}{\pi^{AU}[0.95]}$	0.794331...	0.791679...	0.778546...	0.765625	0.740400...
$\frac{\pi^{AU}[1.05] - \pi^{AU}[0.95]}{\pi^{AA}[1.05] - \pi^{AA}[0.95]}$	1.15	1.16	1.25	1.3	1.5
$\pi^{UU}[0.95]$	0.167296...	0.162244...	0.14	0.121875	0.094
$\pi^{UU}[1.05]$	0.210775...	0.205102...	0.18	0.159375	0.127

**Table 2 :** Cournot firms supplying complementary products.

$\gamma$	0	-0.3	-0.3	-0.5	-0.6
$\pi^{AA}[0.95]$	0.225625	0.312283...	0.3249	0.401	0.50765625
$\pi^{AA}[1.05]$	0.275625	0.381487889...	0.3969	0.49	0.62015625
$\frac{\pi^{AA}[1.05]}{\pi^{AA}[0.95]}$	0.818594...	0.818594...	0.818594...	0.818594...	0.818594...
$\pi^{UA}[0.95]$	0.225	0.312197...	0.325	0.4027	0.5125
$\pi^{UA}[1.05]$	0.275	0.379844...	0.395	0.4861	0.6125
$\frac{\pi^{UA}[1.05]}{\pi^{UA}[0.95]}$	0.81	0.821908...	0.822784...	0.8285714	0.836734...
$\frac{\pi^{UA}[1.05] - \pi^{UA}[0.95]}{\pi^{AA}[1.05] - \pi^{AA}[0.95]}$	1	0.9775	0.972	0.9375	0.8
$\pi^{AU}[0.95]$	0.225625	0.317233...	0.330625	0.4117361	0.525625
$\pi^{AU}[1.05]$	0.275625	0.376057...	0.390625	0.4784027	0.600625
$\frac{\pi^{AU}[1.05]}{\pi^{AU}[0.95]}$	0.818594...	0.843578...	0.8464	0.840647...	0.875130...
$\frac{\pi^{AU}[1.05] - \pi^{AU}[0.95]}{\pi^{AA}[1.05] - \pi^{AA}[0.95]}$	1	0.85	0.83	0.75	0.6
$\pi^{UU}[0.95]$	0.225	0.316608...	0.33	0.41	0.525
$\pi^{UU}[1.05]$	0.275	0.375432...	0.39	0.47	0.6

**Table 3 :** Bertrand firms supplying substitutable products.

$\gamma$	0	0.3	0.3	0.5	0.6
$y^{aa}[0.95]$	0.225625	0.168152...	0.16245	0.13370	0.10153125
$y^{aa}[1.05]$	0.275625	0.205416...	0.19845	0.163	0.12403125
$\frac{y^{aa}[0.95]}{y^{aa}[1.05]}$	0.818594...	0.818594...	0.818594...	0.818594...	0.818594...
$y^{ua}[0.95]$	0.225	0.168106...	0.1625	0.134259	0.1025
$y^{ua}[1.05]$	0.275	0.204531...	0.1975	0.162037	0.1225
$\frac{y^{ua}[0.95]}{y^{ua}[1.05]}$	0.81	0.821908...	0.822784...	0.8285714	0.836734...
$\frac{y^{ua}[1.05] - y^{ua}[0.95]}{y^{aa}[1.05] - y^{aa}[0.95]}$	1	0.9775	0.972	0.9375	0.8
$y^{au}[0.95]$	0.225625	0.170818...	0.1653125	0.137245370	0.105125
$y^{au}[1.05]$	0.275625	0.202492...	0.1953125	0.159467592	0.120125
$\frac{y^{au}[0.95]}{y^{au}[1.05]}$	0.818594...	0.843578...	0.8464	0.840647...	0.875130...
$\frac{y^{au}[1.05] - y^{au}[0.95]}{y^{aa}[1.05] - y^{aa}[0.95]}$	1	0.85	0.83	0.75	0.6
$y^{uu}[0.95]$	0.225	0.170481...	0.165	0.1370	0.105
$y^{uu}[1.05]$	0.275	0.202155...	0.195	0.1592	0.12

**Table 4 :** Bertrand firms supplying complementary products.

$\gamma$	-0.3	-0.3	-0.5	-0.6	$\varepsilon - 1$
$y^{aa}[0.95]$	0.316837...	0.331530...	0.4332	0.634570...	0.2005/ $\varepsilon$
$y^{aa}[1.05]$	0.387051...	0.405	0.5292	0.775195...	0.245/ $\varepsilon$
$\frac{y^{aa}[0.95]}{y^{aa}[1.05]}$	0.818594...	0.818594...	0.818594...	0.818594...	0.818594...
$y^{ua}[0.95]$	0.316749...	0.331632...	0.435	0.640625	0.205/ $\varepsilon$
$y^{ua}[1.05]$	0.385383...	0.403061...	0.525	0.765625	0.238/ $\varepsilon$
$\frac{y^{ua}[0.95]}{y^{ua}[1.05]}$	0.821908...	0.822784...	0.8285714	0.836734...	0.860465...
$\frac{y^{ua}[1.05] - y^{ua}[0.95]}{y^{aa}[1.05] - y^{aa}[0.95]}$	0.9775	0.972	0.9375	0.8	0.75
$y^{au}[0.95]$	0.311854...	0.325739...	0.421875	0.6125	0.190125/ $\varepsilon$
$y^{au}[1.05]$	0.392600...	0.411454...	0.541875	0.8	0.256725/ $\varepsilon$
$\frac{y^{au}[0.95]}{y^{au}[1.05]}$	0.794331...	0.791679...	0.778546...	0.765625	0.740400...
$\frac{y^{au}[1.05] - y^{au}[0.95]}{y^{aa}[1.05] - y^{aa}[0.95]}$	1.15	1.16	1.25	1.3	1.5
$y^{uu}[0.95]$	0.310694...	0.324489...	0.42	0.609375	0.18/ $\varepsilon$
$y^{uu}[1.05]$	0.391439...	0.410204...	0.54	0.796875	0.24/ $\varepsilon$

## References

- Dixit, A.K., "A Model of Duopoly Suggesting a Theory of Entry Barriers", *Bell Journal of Economics*, Vol. 10 (1979), pp. 20 - 32.
- Gal-Or, E., "First Mover and Second Mover Advantage", *International Economic Review*, Vol. 26 (1985), pp. 649 - 52.
- Hamilton, J.H. and Slutsky, S.M., "Endogenous Timing in Duopoly Games: Stackelberg or Cournot Equilibria", *Games and Economic Behavior*, Vol. 2 (1990), pp. 29 - 46.
- Hansen, P.S., Møllgaard, H.P., Overgaard, P.B., and Sørensen, J.R., "Asymmetric Adjustment in Symmetric Duopoly", *Economics Letters*, Vol. 53 (1996), pp. 183 - 88.
- Sakai, Y., "The Value of Information in a Simple Duopoly Model", *Journal of Economic Theory*, Vol. 36 (1985), pp. 36 - 54.
- Sakai, Y., "Information Sharing and Oligopoly: Overview and Evaluation - Part I. Alternative Models with a Common Risk", *Keio Economic Studies*, Vol. 27 (1990), pp. 17 - 41.
- Sakai, Y., "Information Sharing and Oligopoly: Overview and Evaluation - Part II. Private Risks and Oligopoly Models", *Keio Economic Studies*, Vol. 28 (1991), pp. 51 - 71.
- Singh, N. and X. Vives, "Price and Quantity Competition in a Differentiated Duopoly", *RAND Journal of Economics*, Vol. 15 (1984), pp. 546 - 54.