CIRJE-F-333

Action Timing as a Collusive Common Good

Nobuo Akai
University of Hyogo

Dan Sasaki
University of Tokyo

March 2005

Action timing as a collusive common good

Nobuo Akai
School of Business Administration
University of Hyogo
8-2-1 Gakuen-Nishi-machi, Nishi-ku
Kobe 651-2197 Japan
akai@biz.u-hyogo.ac.jp
akai@kobeuc.ac.jp

Dan Sasaki
Institute of Social Science
University of Tokyo
7-3-1 Hongo, Bunkyo-ku
Tokyo 113-0033 Japan
dsasaki@iss.u-tokyo.ac.jp

March 2005

Acknowledgements : This research has benefited from seminar presentations in the University of Tokyo and in the University of Hyogo, and been financially assisted in part by the Grant-in-Aid in Scientific Research No. 15730090 (2003-2004). The usual disclaimer applies.

Action timing as a collusive common good

Abstract : Frequent revision of firms' actions facilitates to sustain tacit collusion. Even when some, not all, firms revise their actions with enhanced frequency, the change contributes positively to collusive sustainability, i.e., lowering the critical discount factor. In this sense, the added frequency in revising actions can be viewed as a common good shared among oligopolists. Particularly noteworthy is the fact that, in a large class of environments, a firm's deviation can be deterred by no more than one punisher, implying that at most two firms need to invest in frequent revision in order to sustain collusion.

Keywords : repeated game, Bertrand, subgame perfect equilibrium, mid-period decision.

JEL classification : L13, D81, M14.

1 Introduction

ONE OF THE STEREOTYPICAL FEATURES in inflexible business management and conservative corporate culture is the characteristic tardiness in adjusting the firm's actions according to various changes in the economic environment. In economic terms, swiftness in adjusting actions is an ability to invest in. Namely, the aforesaid inflexibility can be viewed as abstinence from such an investment.

In this paper, we analyse swiftness or tardiness of firms' actions in the contexts of their repeated, intertemporal oligopolistic interactions.

We start from a standard duopoly supergame where, by default, if neither firm invests in structural effort, each firm can review its action only once every period. However, if a firm invests in managerial self-improving effort, its internal decisions can be made at an enhanced frequency, which enables the firm to review its action every half period. This increases the effective discount factor *per decision interval* from δ to $\sqrt{\delta}$, enhancing collusive sustainability.

A more interesting question here is what if only one duopolist invests in frequent action revision whilst the other remains uninvested. We discover that this asymmetric game accommodates collusive sustainability which is in between the aforementioned two symmetric supergames, that is, more sustainable than in the slow (default) supergame but less sustainable than in the fast supergame. In this sense, the investment undertaken by one of the duopolists acts as a common good between the two firms. It is also noteworthy that collusive sustainability is optimised when the invested firm is to claim a lion's share in collusive equilibrium sales and profits.

By extension, when there are more firms in repeated oligopolistic interactions, the presence of a small number of swift "non-conservative" oligopolists can substantially enhance collusive sustainability in spite of the rest of the industry consisting of slow "conservative" firms. In such a case, the non-conservative firms serve as potential quick punishers to guard the tacitly collusive scheme, which provides collusive security to the whole industry in return for disproportionately large shares in collusive sales and supernormal profits.

In microtheory literature, quickness and tardiness in strategic actions have mostly been analysed in the contexts of non-repeated games. The well-known Stackelberg model is a static framework which nevertheless models a delayed action and its consequences. Endogenisation of such delays owes to the seminal contribution by Gal-Or (1983), and more explicitly to the *extended game* modelled by Hamilton and Slutsky (1990).[1] In this stream of literature, a variety of theoretical explanations have been attempted in regard to firms' incentives to move asynchronously, especially a follower's incentive to wait. Some studies consider costs. Albæk (1990) takes into account *cost* uncertainty. Robson (1990) and Matsumura (1997) impose costs associated with an early action. Some others take into account the effects of informational heterogeneity between firms, as in Mailath (1993) and Normann (1997). On the other hand, a few recent contributions focus on *demand uncertainty*, which makes *a priori identical firms* choose different action timings. In Sadanand and Sadanand (1996) and Maggi (1996), the trade-off between commitment and demand information gives rise to the endogenous leader-follower relation. Namely, earlier production can utilise less information in exchange for the strategic advantage of commitment, whereas later production does the converse. Saloner (1987) and Pal (1991; 1996) construct models consisting of two feasible production periods, where a firm can earn the status of a Stackelberg leader by choosing to produce in the former period.[2]

In spite of their intendedly dynamic framework, many of these preceding theoretical studies consider a *static* market that opens *only once*. As we discover, it turns out mostly because of this static structure that a quick action becomes unequivocally and

[1]The extended game has two versions, one called the extended game with observable delay, the other called the extended game with action commitment. The former is the formulation most relevant to our purposes in this paper.

[2]The strategic purpose of advance production is not necessarily to earn Stackelberg leadership. Even in the absence of the leader-follower relation, firms may still have a pre-emptive incentives for advance production. Note that advance production cannot automatically preclude the possibility that the firm may still choose not to sell all the quantity produced. In other words, advance production *alone* cannot guarantee any quantity commitment.

monotonically less attractive and thus less likely to be endogenously opted for as its costs grow dearer, a result which might appear as if obvious to our spontaneous intuition.

Realism of the static framework can also be limited. There may not always be cogent reason to believe that the market needs to wait idly until the second mover makes up its mind. More often than not, the market repeats over multiple periods, where two qualitative differences from the static market arise. First, the swiftness or the tardiness of actions should be measured by the *frequency* of actions rather than the timing of a one-shot action. Second, the prospect of *tacit collusion* as a subgame perfect path should also be taken into consideration.

The remainder of the paper is organised as follows. In section 2 we conduct a comparative statics exercise with respect to the frequency of each firm's action revision in a simple linear Bertrand duopoly. In section 3 we extend the game by adding a pre-play stage where each firm can either invest in frequent revision or not. We then solve the extended game backward to establish the existence of collusive equilibria where only one firm invests. Those ranges of the initial investment costs and the discount factor where such asymmetric collusive equilibria materialise are identified in section 4. Generality of our findings is probed in section 5. In a large class of dynamic oligopoly games including our linear Bertrand example, a firm's unilateral deviation can be effectively deterred by no more than one punisher, implying that at most two firms investing in frequent action revision can suffice in order to attain utmost collusive sustainability. Finally we discuss economic implications of our analysis in section 6 to conclude the paper.

2 Repeated linear Bertrand duopoly

2.1 The stage game

Bertrand duopolists, labelled 1 and 2 henceforth, face a non-stochastic inverse demand $Q[P]$ where $P = \min\{p_1, p_2\}$ and the aggregate quantity of transaction $Q[P]$ is supplied by whichever firm who charges the lower price between the two firms. If and only if the two firms set the same price $P = p_1 = p_2$, the aggregate supply Q can be shared between them. Marginal production costs are nil.

In the following, we consider a supergame wherein the aforesaid stage game recurs stationarily along continuous time $t \in [0, \infty)$. Let δ denote the discount factor per unit period.

2.2 The default supergame

Suppose first that each firm can revise its action, that is the sales price, every *integer period* $t = 0, 1, 2, 3, \cdots$. In effect, this is equivalent to a simple Bertrand oligopoly supergame with discrete time $t = 0, 1, 2, 3, \cdots$ where symmetric collusion at any positive price $P = p_1 = p_2 > 0$, $q_1 = q_2 = \dfrac{Q[P]}{2}$ is sustainable with subgame perfection if and only if $\delta \geq \dfrac{1}{2}$. It is easily verified that no collusion, at any positive price, may be sustained with subgame perfection when $\delta < \dfrac{1}{2}$.[3]

The collusive profits discounted to the beginning of the supergame are hereby

$$\int_{t=0}^{\infty} \frac{PQ[P]}{2}\,\delta^t\,dt = -\,\frac{PQ[P]}{2\ln\delta} = \frac{PQ[P]}{2r}$$

for either firm, where we define the discount rate $r = -\ln\delta$, that is $\delta = e^{-r}$.

2.3 The expedited supergame

We now assume that each firm can revise its action every *half period* $t = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \cdots$. This is equivalent to a discrete-time linear Bertrand oligopoly supergame with half-integer time $t = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \cdots$. The only difference from the previous default supergame is the time interval, which has now been halved. This translates to the discount factor being $\sqrt{\delta}$ per every *half period*. Collusive sustainability hereby stipulates $\sqrt{\delta} \geq \dfrac{1}{2}$, that is $\delta \geq \dfrac{1}{4}$.

The discounted collusive profits per firm remain unchanged from the previous default supergame.

2.4 The asymmetric game

Finally, if only one of the two firms can revise its action every half period whilst the other firm can revise its action only every full period, the existing results established in preceding literature can no longer directly apply.

Note first that, if these two firms were to share their collusive profits evenly, the firm with swifter revision would have higher incentives to deviate from collusion. The intuition

[3] Lambson (1987) shows that when the threat point entails zero discounted profits (which Lambson refers to as the "security level"), the so-called optimal punishment *à la* Abreu (1986, 1988), Abreu, Pearce and Stacchetti (1986), and Häckner (1996), yields the same critical discount factor as the trigger strategy with static Nash reversion *à la* Friedman (1971).

is straightforward: the swifter firm's deviation would not be punished until a full period later, whereas the slower firm's deviation could be punished only a half period later.

In general, if the swifter firm supplies q_i and the slower firm q_u , their incentives against deviation are

$$\left(1+\sqrt{\delta}\right) PQ[P] \le Pq_i \sum_{t=0}^{\infty} \sqrt{\delta}^{\,t} \tag{2.4.1}$$

and

$$PQ[P] \le Pq_u \sum_{t=0}^{\infty} \sqrt{\delta}^{\,t} \tag{2.4.2}$$

where

$$q_i + q_u = Q[P]\,. \tag{2.4.3}$$

Any quantity profile $\{q_i, q_u\}$ in accordance with constraints (2.4.1) through (2.4.3) can be sustained with subgame perfection as a collusive outcome. The discounted collusive profits for the swifter firm are

$$\int_{t=0}^{\infty} Pq_i\delta^{\,t}dt = -\frac{Pq_i}{\ln\delta} = \frac{Pq_i}{r}$$

and those for the slower firm

$$\int_{t=0}^{\infty} Pq_u\delta^{\,t}dt = -\frac{Pq_u}{\ln\delta} = \frac{Pq_u}{r}\,.$$

In particular, (2.4.1) through (2.4.3) can be solved as a system of equations by

$$\delta = \frac{3-\sqrt{5}}{2}\,, \qquad q_i = \frac{\sqrt{5}-1}{2}Q[P]\,, \qquad q_u = \frac{3-\sqrt{5}}{2}Q[P]\,. \tag{2.4.4}$$

Now that collusion requisites only $\delta \ge \dfrac{3-\sqrt{5}}{2} = 0.381966\cdots < \dfrac{1}{2}$, this solution implies that collusive sustainability can be enhanced even when only one of the duopolists acquires swiftness in revising its action.

3 The extended game

We now envision an extended game at the beginning of which each of the two duopolists chooses whether to invest in enhancing its action frequency. A firm who has made the initial investment can thereafter revise its action, the sales price, every *half period* $t = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \cdots$, whilst a firm who has made no investment can revise its action only every period $t = 0, 1, 2, 3, \cdots$. The initial investment costs k.

Whenever practicable, firms voluntarily endeavour to sustain price collusion, foreseeing which they choose their initial investment. From our foregoing analysis in section 2, the following subgame perfect equilibrium results obtain, where P denotes the collusive price and $Q[P]$ the collusive *aggregate* supply quantity.

3.1 High discount factor - weak time preferences

In the range $\frac{1}{2} \leq \delta < 1$, tacit collusion is sustainable after any initial investment profile. Game-theoretically speaking, Nash equilibria may depend upon the shares q_i and q_u because, especially for a very high δ, a vast range of shares $\{q_i, q_u\}$ can be sustained as collusive profiles. Obviously, however, the most profit-efficient equilibrium is for neither firm to invest, with such an off-equilibrium profile $\{q_i, q_u\}$ that discourages both firms from the initial investment, according to the following discounted profit matrix.

	invested	uninvested
invested	$-k - \frac{PQ[P]}{2\ln\delta}$, $-k - \frac{PQ[P]}{2\ln\delta}$	$-k - \frac{Pq_i}{\ln\delta}$, $-\frac{Pq_u}{\ln\delta}$
uninvested	$-\frac{Pq_u}{\ln\delta}$, $-k - \frac{Pq_i}{\ln\delta}$	$-\frac{PQ[P]}{2\ln\delta}$, $-\frac{PQ[P]}{2\ln\delta}$

3.2 Medium discount factor - moderate time preferences

In the range $\frac{3-\sqrt{5}}{2} \leq \delta < \frac{1}{2}$, tacit collusion is sustainable when at least one of the duopolists has invested initially. Clearly from the matrix below, the most aggregate-profit efficient is for only one firm to invest.

	invested	uninvested
invested	$-k - \frac{PQ[P]}{2\ln\delta}$, $-k - \frac{PQ[P]}{2\ln\delta}$	$-k - \frac{Pq_i}{\ln\delta}$, $-\frac{Pq_u}{\ln\delta}$
uninvested	$-\frac{Pq_u}{\ln\delta}$, $-k - \frac{Pq_i}{\ln\delta}$	0, 0

For one of those efficient profiles (off-diagonal cells in the table above) to equilibrate, the collusive quantities q_i and q_u must satisfy both the collusive incentives (2.4.1) and (2.4.2) which can be respectively rearranged into

$$q_i \geq (1-\delta)Q[P] \tag{3.2.1}$$

$$q_u \geq \left(1-\sqrt{\delta}\right) Q[P] \tag{3.2.2}$$

and also the incentive compatibility in the extended game:

$$-k - \frac{Pq_i}{\ln\delta} \geq 0 \quad \text{for the invested firm,}$$

$$-\frac{Pq_u}{\ln\delta} \geq -k - \frac{PQ[P]}{2\ln\delta} \qquad \text{for the uninvested firm,}$$

which can be rewritten as

$$q_i \geq -\frac{k}{P}\ln\delta\,, \tag{3.2.3}$$

$$q_u \geq \frac{Q[P]}{2} + \frac{k}{P}\ln\delta\,. \tag{3.2.4}$$

Our qualitative conclusions can be categorised into four subcases depending upon the cost of the initial investment k in its proportion to the collusive profits. The exact subcases are listed in the appendix.

3.3 Lower discount factor - stronger time preferences

In the range $\frac{1}{4} \leq \delta < \frac{3-\sqrt{5}}{2}$, both firms need to invest in order to sustain collusion. If the initial investment proves prohibitively costly, that is when $k > -\frac{PQ[P]}{2\ln\delta}$, collusion will not materialise in equilibrium.

	invested	uninvested
invested	$-k-\frac{PQ[P]}{2\ln\delta}$, $-k-\frac{PQ[P]}{2\ln\delta}$	$-k$, 0
uninvested	0, $-k$	0, 0

3.4 Lowest discount factor - strongest time preferences

Finally, when $0 \leq \delta < \frac{1}{4}$, there is no hope to sustain collusion. Therefore, neither firm invests.

	invested	uninvested
invested	$-k$, $-k$	$-k$, 0
uninvested	0, $-k$	0, 0

4 Equilibrium investment in the common good

Our equilibrium analysis in the foregoing two sections entail the following.

Proposition 1 : The subgame perfect equilibria in the extended game are such that:

[1-c] neither firm invests, yet collusion is sustained, if $\delta \geq \frac{1}{2}$;

[1-o] only one firm invests to sustain collusion if

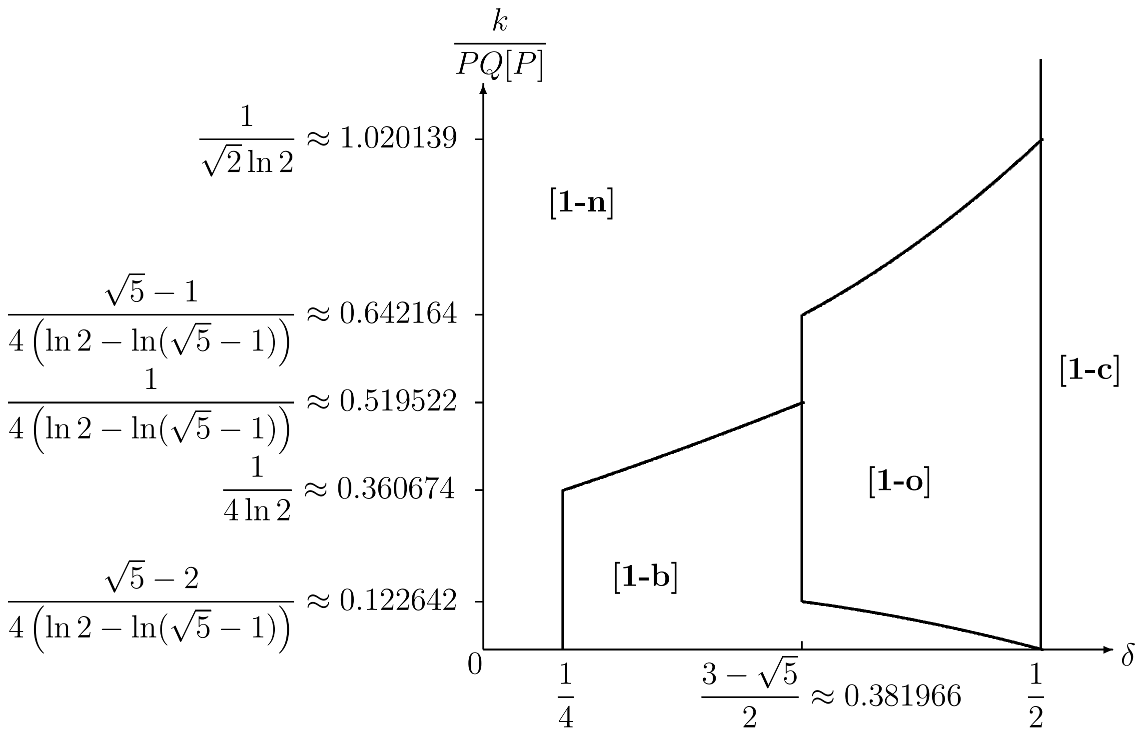
$$\max\left\{\underset{d}{\arg}\left\{d = \frac{1}{2} + \frac{k}{PQ[p]}\ln d\right\},\ \frac{3-\sqrt{5}}{2},\ \underset{d}{\arg}\left\{\sqrt{d} = -\frac{k}{PQ[P]}\ln d\right\}\right\} \leq \delta < \frac{1}{2};$$

[1-b] both firms invest to sustain collusion if

$$\max\left\{\frac{1}{4},\ \exp\left[-\frac{PQ[P]}{2k}\right]\right\} \leq \delta < \max\left\{\underset{d}{\arg}\left\{d = \frac{1}{2} + \frac{k}{PQ[p]}\ln d\right\},\ \frac{3-\sqrt{5}}{2}\right\};$$

[1-n] neither firm invests, no collusion is sustained, if

$$\delta < \min\left\{\frac{1}{2},\max\left\{\frac{3-\sqrt{5}}{2},\ \underset{d}{\arg}\left\{\sqrt{d} = -\frac{k}{PQ[P]}\ln d\right\}\right\},\max\left\{\frac{1}{4},\ \exp\left[-\frac{PQ[P]}{2k}\right]\right\}\right\}.$$



Proposition 2 : Only one firm's initial investment enables the most efficient equilibrium in the two firms' *joint* collusive profits when, and only when

- $\underset{d}{\arg}\left\{d = \frac{1}{2} + \frac{k}{PQ[p]}\ln d\right\} \leq \delta < \frac{1}{2}$ if $\frac{k}{PQ[P]} < \frac{\sqrt{5}-2}{4\left(\ln 2 - \ln(\sqrt{5}-1)\right)} \approx 0.122642$

- $\frac{3-\sqrt{5}}{2} \leq \delta < \frac{1}{2}$

  if $\frac{\sqrt{5}-2}{4\left(\ln 2 - \ln(\sqrt{5}-1)\right)} \approx 0.122642 \leq \frac{k}{PQ[P]} \leq \frac{\sqrt{5}-1}{4\left(\ln 2 - \ln(\sqrt{5}-1)\right)} \approx 0.642164$

- $\arg_d \left\{ \sqrt{d} = -\dfrac{k}{PQ[P]} \ln d \right\} \leq \delta < \dfrac{1}{2}$

  if $\quad \dfrac{\sqrt{5}-1}{4\left(\ln 2 - \ln(\sqrt{5}-1)\right)} \approx 0.642164 < \dfrac{k}{PQ[P]} < \dfrac{1}{\sqrt{2}\ln 2} \approx 1.020139$

In each of these cases, one firm's investment suffices to sustain the collusive outcome, hence serves as an effective *common good* between the two firms. In price collusion, the invested firm claims a larger share than that of the uninvested firm, whereby the *rent* for its initial investment in the common good is recovered.

Detailed proofs of these two propositions are substantiated in the appendix.

5 Generalisations

In this section we briefly discuss how those findings from our simple linear Bertrand duopoly model could be applied to more general settings.

5.1 More firms

Consider the same extended repeated Bertrand game as in sections 2 through 4 except that we now have $n \geq 3$ firms. The following observations can be established without detailed computations.

- If *only one* firm has invested, when it deviates by undercutting the collusive price, it can sweep the entirety of the market demand for one full period.
- If *at least two* firms have invested, when one of them deviates, another invested firm will re-undercut in a half period.

Hence, there is *no added collusive enforceability gained by three or more firms investing* in frequent revisions of actions.

This implies that, in repeated Bertrand oligopoly games involving three or more firms, there exist vast ranges of (relatively inexpensive) initial investment costs where only some (no more than two) not all of the oligopolists revise their actions more frequently than others, to sustain tacitly collusive subgame perfect equilibria.

5.2 Nonlinear Bertrand oligopoly

Our findings from sections 2 through 4 are independent of functional specificities of the demand function insofar as usual regularity conditions (such as the existence of an interior joint-revenue maximum) are met. Also, when marginal cost structures involve slight economies or diseconomies of scale, or when products are slightly but not strongly differentiated between the firms, our foregoing findings shall approximately apply, where the precision of such applicability varies reflecting the nonlinearity of the modified model.

5.3 Cournot oligopoly

In Cournot, deviation profits depend specifically on the functional form of the inverse demand. It is therefore less straightforward to quantify the exact conditions for subgame perfect equilibrium configurations than in Bertrand. Qualitatively, however, the following analogy to the foregoing Bertrand case can be contemplated.

Consider first the asymmetric duopoly, involving one invested firm and one uninvested firm. Assuming that the firms split the market evenly when colluding, the invested firm's deviation profits is the same as when neither firm invests, as its static best response affords the firm a maximum deviation profit during one full uninterrupted period. The uninvested firm, on the other hand, can gain less from a unilateral deviation because its opponent can intercept the deviant attempt after half a period. This implies the following, analogously to our Bertrand observations.

- If the two firms are to share the market evenly whenever colluding, then only one firm's initial investment contributes little to collusive sustainability.
- If the two firms adjust their collusive market shares commensurate to their initial investment profiles, then only one firm's initial investment can enhance collusive sustainability.

With $n \geq 3$ firms, the possible analogy to our Bertrand observations in 5.1 hinges critically upon an invested firm's ability to inflict as severe a punishment on an uninvested deviator as more than one invested firms together. Whilst such ability was guaranteed in the case of perfect Bertrand, it is no longer guaranteed in Cournot, implying that in the latter there generally remains a prospect for more than two invested firms to enhance collusive sustainability.

6 Conclusion and economic implications

Firms' ability to revise their actions frequently might, to a spontaneous mind, sound as if positively contributing to social welfare. Such an innocuous intuition may well turn out factually accurate if the economy, or more specifically the industry in question, faces a series of stochastic shocks. Non-probabilistically, on the other hand, it should not be overlooked that frequent revision of oligopolistic firms' actions facilitates tacit collusion and thereby, in general, hinders social welfare.

In this paper we have shown that such collusive impacts can result from only *some, not all* firms being fast in revising their actions. Especially in an environment where collusive excess profits are overwhelmingly higher than non-collusive normal profits, such as repeated perfect Bertrand oligopoly, a small minority among the oligopolists being able to revise their actions much faster than the remainder of them, may substantially contribute to the sustainability of tacit collusion in the entirety of the oligopolistic industry.

Presumably, a firm's capacity to revise its action with an enhanced frequency is a fruit of costly organisational efforts. The reward for such efforts is the enhanced sustainability of collusion and the lion's share which the firm can claim in collusive sales in relation to other colluding oligopolists who have made less of such efforts. This can be viewed as if the firm is serving as a collusive watchdog delegated by the remainder of the industry for a "fee" or a "rent" in the form of an extra entitlement in the market share.

This scenario indicates that an industry consisting of a few "progressive" firms who are quick in revising their actions and a large majority of "conservative" firms who are slow in decision making, where the former firms are profiting considerably more than the latter, makes a potential antitrust suspect.

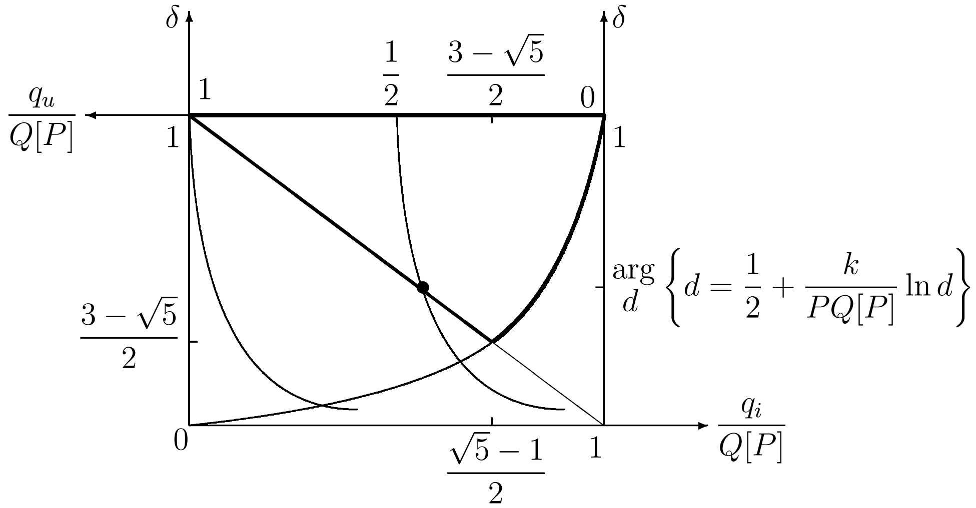
Appendix

In the parametrical range $\frac{3-\sqrt{5}}{2} \leq \delta < \frac{1}{2}$, equilibrium conditions for only one firm's initial investment are specified separately in the following four ranges of investment costs.

- $\frac{k}{PQ[P]} < \frac{\sqrt{5}-2}{4\left(\ln 2 - \ln(\sqrt{5}-1)\right)} \approx 0.122642$

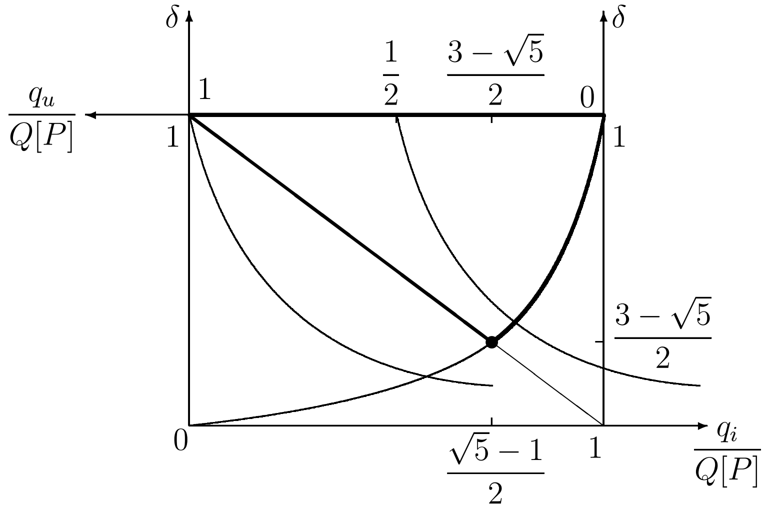
  When k is sufficiently low, (2.4.4) violates (3.2.4). Therefore the lowest admissible discount factor δ to sustain tacit collusion with only one firm investing initially, is determined by the two binding constraints (3.2.1) and (3.2.4). If δ is below this lower bound (but $\frac{1}{4}$ or above), both firms invest to sustain collusion.

The range of δ and q_i , q_u satisfying (3.2.1) and (3.2.2) is surrounded by the thickened loci in the diagramme. In addition, (3.2.3) and (3.2.4) stipulate that the collusive quantities q_i , q_u lie between the two parallel down sloping curves. The slope of these curves vary depending upon k , whereby the minimum admissible δ consistent with these conditions also vary as indicated by the thickened dots in the diagramme.



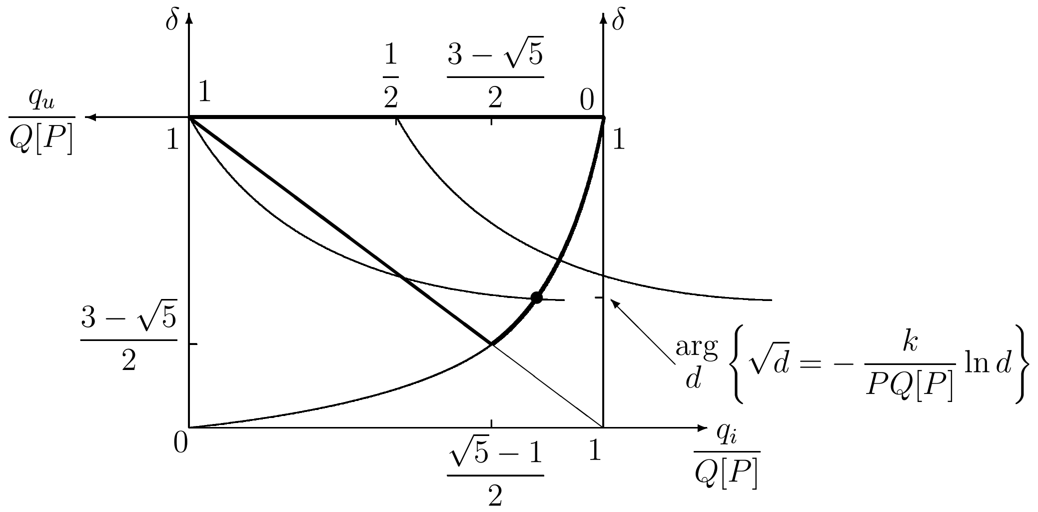
- $$\frac{\sqrt{5}-2}{4\left(\ln 2 - \ln(\sqrt{5}-1)\right)} \approx 0.122642 \leq \frac{k}{PQ[P]} \leq \frac{\sqrt{5}-1}{4\left(\ln 2 - \ln(\sqrt{5}-1)\right)} \approx 0.642164$$

When k is moderate, (2.4.4) complies with both (3.2.3) and (3.2.4). The lowest discount factor δ sustaining tacit collusion with only one invested firm, is therefore $\frac{3-\sqrt{5}}{2}$ as in (2.4.4).



- $$\frac{\sqrt{5}-1}{4\left(\ln 2 - \ln(\sqrt{5}-1)\right)} \approx 0.642164 < \frac{k}{PQ[P]} < \frac{1}{\sqrt{2}\ln 2} \approx 1.020139$$

When k is higher, (2.4.4) violates (3.2.3). The lowest discount factor δ admitting price collusion between one invested firm and one uninvested firm, is therefore determined by the two binding constraints (3.2.2) and (3.2.3). If δ is below this lower bound, neither firms invests, and collusion is no longer sustainable.



- $\frac{k}{PQ[P]} \geq \frac{1}{\sqrt{2}\ln 2} \approx 1.020139$

  When k is prohibitively high, the aforementioned lowest discount factor sustaining collusion between an invested and an uninvested firms determined by (3.2.2) and (3.2.3) becomes higher than $\frac{1}{2}$, the range where no initial investment can be nett profit efficient. Collusion is thereby unsustainable.

References

Abreu, D.J., "Extremal Equilibria of Oligopolistic Supergames", *Journal of Economic Theory*, Vol. 39 (1986), pp. 191 - 225.

Abreu, D.J., "On the Theory of Infinitely Repeated Games with Discounting", *Econometrica*, Vol. 56 (1988), pp. 383 - 96.

Abreu, D.J., Pearce, D., and Stacchetti, E., "Optimal Cartel Equilibria with Imperfect Monitoring", *Journal of Economic Theory*, Vol. 39 (1986), pp. 251 - 69.

Albæk, S., "Stackelberg Leadership as a Natural Solution under Cost Uncertainty", *Journal of Industrial Economics*, Vol. 38 (1990), pp. 335 - 47.

Friedman, J.W., "A Noncooperative Equilibrium for Supergames", *Review of Economic Studies*, Vol. 38 (1971), pp. 1 - 12.

Gal-Or, E., "First Mover and Second Mover Advantage", *International Economic Review*, Vol. 26 (1985), pp. 649 - 52.

Häckner, J., "Optimal Symmetric Punishments in a Bertrand Differentiated Product Duopoly", *International Journal of Industrial Organization*, Vol. 14 (1996), pp. 611 - 30.

Hamilton, J.H. and Slutsky, S.M., "Endogenous Timing in Duopoly Games: Stackelberg or Cournot Equilibria", *Games and Economic Behavior*, Vol. 2 (1990), pp. 29 - 46.

Lambson, V.E., "Optimal Penal Codes in Price-Setting Supergames with Capacity Constraints", *Review of Economic Studies*, Vol. 54 (1987), pp. 385 - 97.

Maggi, G., "Endogenous Leadership in a New Market", *RAND Journal of Economics*, Vol. 27 (1996), pp. 641 - 59.

Mailath, G.J., "Endogenous Sequencing of Firm Decisions", *Journal of Economic Theory*, Vol. 59 (1993), pp. 169 - 82.

Matsumura, T., "A Two-Stage Cournot Duopoly with Inventory Costs", *Japanese Economic Review*, Vol. 48 (1997), pp. 81 - 89.

Normann, H.T., "Endogenous Stackelberg Equilibria with Incomplete Information", *Journal of Economics* (Zeitschrift für Nationalökonomie), Vol. 66 (1997), pp. 177 - 87.

Pal, D., "Cournot Duopoly with Two Production Periods and Cost Differentials", *Journal of Economic Theory*, Vol. 55 (1991), pp. 441 - 48.

Pal, D., "Endogenous Stackelberg Equilibria with Identical Firms", *Games and Economic Behavior*, Vol. 12 (1996), pp. 81 - 94.

Robson, A.J., "Duopoly with Endogenous Strategic Timing: Stackelberg Regained", *International Economic Review*, Vol. 31 (1990), pp. 263 - 74.

Sadanand, A. and Sadanand, V., "Firm Scale and Endogenous Timing of Entry: a Choice between Commitment and Flexibility," *Journal of Economic Theory*, Vol. 70 (1996), pp. 516 - 30.

Saloner, G., "Cournot Duopoly with Two Production Periods," *Journal of Economic Theory*, Vol. 42 (1987), pp. 183 - 87.