CIRJE-F-343

# Monetary Policy during Japan's Lost Decade

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May 2005

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June 16, 2005

**Abstract**

We develop a quantitative costly price adjustment model with capital formation for the Japanese Economy. The model respects the zero interest rate bound and is calibrated to reproduce the nominal and real facts from the 1990s. We use the model to investigate the properties of alternative monetary policies during this period. The setting of the long-run nominal interest rate in a Taylor rule is much more important for avoiding the zero bound than the setting of the reaction coefficients. A long-run interest rate target of 2.3 percent during the 1990s avoids the zero bound and enhances welfare.

**JEL Classification Number:** E30, E50.

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## 1 Introduction

Since 1990 Japan has experienced over a decade of slow growth in real economic activity. Between 1990 and 2000 per capita output grew at an annual rate of 0.68 percent, per capita investment declined at the rate of 1.4 percent per annum and weekly hours per adult worker declined by 1.18 percent per annum. This period has come to be referred to as "the lost decade." During the same period the inflation rate, as measured by the growth rate of the GDP deflator, fell from 2.3 percent to -1.8 percent and the nominal interest rate fell from 7.4 percent to 0.1 percent. Japan's recent experience of slow growth accompanied by deflation and zero nominal interest rates raises questions about the role of monetary policy in times of deflation. Should monetary policy take actions to avoid the zero nominal interest rate bound and if so, what policies can avoid it and/or ameliorate its negative effects?

This paper develops a model that accounts for the real and nominal facts from the 1990s and uses this model to answer the two questions posed above.

We consider a costly price adjustment model along the lines of Rotemberg (1996) and extend it to allow for capital accumulation. In this economy monopolistically competitive firms face convex costs of adjusting prices. Households own the capital stock and are subject to convex costs of adjustment. The economy experiences exogenous shocks to technology and government purchases and the monetary authority follows an interest rate targeting rule that assigns weight to current output deviations from trend and current deviations of inflation from its target level.

Solving for the equilibrium is complicated by the possibility of a zero nominal interest rate constraint. We develop an algorithm for computing perfect foresight equilibria in situations where the nominal interest rate is zero over some interval of time. The model is then solved and simulated using a parameterization that is calibrated to Japanese data.

An impulse response analysis is used to answer the first question. We find that the dynamic response of the economy to shocks in technology and government purchases is very different depending on whether the zero nominal interest rate constraint binds. When the constraint is not binding output and investment increase in response to improvements in technology under the interest rate targeting rule we consider. However, when the constraint binds, monetary policy cannot respond and output and investment all fall in response to positive technology shocks. A binding constraint also exacerbates the contractionary effects of negative government purchase shocks on these same variables.

To answer the second question we need a baseline specification that reproduces the nominal and real facts from Japan's lost decade. This is found by simulating the model under the assumption of perfect foresight using the actual realizations of TFP and government purchases from Japanese data. The long-run output share of government purchases and the level of the long-run nominal interest rate are then adjusted to produce a specification that accounts for the facts from the 1990s. A very low level of of the long-run nominal interest rate target (about 0.3 percent) and a high long run value of output share of government purchases (about 0.2) are needed to account for the nominal facts.

Having found a specification that reproduces Japan's experience in the 1990s we turn to consider whether alternative monetary policy rules could have avoided the zero nominal interest rate bound and/or ameliorated its effects. We vary three aspects of monetary policy - the magnitude of the output and inflation reaction coefficients in the Taylor rule, the setting of the long-run nominal interest rate and the duration of the time interval that the nominal interest rate is zero. Varying the reaction coefficients produces monetary policies that differ significantly in terms of their implications for e.g. the growth rate of output during the 1990s and welfare, but not in terms of their implications for the zero nominal interest rate bound. In all cases, the nominal interest rate falls to zero.

A carry tax on money as proposed by Goodfriend (2000) allows the effective nominal interest rate to be negative and thereby relaxes the zero nominal interest rate constraint. Our simulations indicate that this is a good policy that increases output growth by 0.6 percent per annum in the last half of the 1990s and improves welfare by about 0.07 percent relative to the baseline specification.

Another monetary policy that achieves welfare gains of a similar magnitude is a commitment to set the long-run nominal interest rate at a level that is consistent with price stability. This policy avoids the bound for plausible values of the shock processes and is easier to administer than a tax on money. We argue that this policy is also likely to be a good policy in a stochastic environment.

Jung, Teranishi and Watanabe (2003) and Eggertsson and Woodford (2003) have found that optimal monetary policy calls for keeping the nominal interest rate at zero for a number of periods after the constraint ceases to bind. We refer to this characteristic of monetary policy as *policy duration*. In our model policy duration also ameliorates the negative effects of a binding zero nominal interest rate constraint and produces welfare gains that increase with the number of periods that the interest rate is kept at zero.

Our work is related to previous work by Orphanides and Wieland (2000), Jung, Teranishi and Watanabe (2003) and Eggertsson and Woodford (2003).

Our work extends this previous research in four respects. First, our economy has endogenous capital formation. This generalization allows for aggregate saving, which fundamentally alters the response of households to shocks and also allows us to link our model's implications to data on the Japanese national income and product accounts. Second, we formally calibrate our model to Japanese data and empirically assess the quality of our model's fit to Japanese data from the 1990s. Third, the time zero shock that produces a binding zero nominal interest rate constraint in our model is empirically relevant. For the 1990s we assume that TFP and government purchases follow the same trajectories that the Japanese economy experienced. In these other papers the time zero shocks considered are arbitrary. Fourth, the previous literature considers optimal policy under the assumption that a long-run objective of the central bank is price stability. We fail to find a Taylor rule that is both consistent with the facts from Japan in the 1990s and a long-run inflation target of zero.

The remainder of the paper is organized in the following way. Section 2 describes the model. Section 3 describes how the model is calibrated, solved and simulated. Section 4 reports the results and we conclude in Section 5.

# 2 The Model

## 2.1 Household Problem

Consider an economy with a large number of identical infinitely-lived households with utility function:

$$U = \sum_{t=0}^{\infty} \beta^t \{\log c_t + \Upsilon(M_{t+1}/P_t) + \nu \log(1 - h_t)\}, \tag{1}$$

where $c_t$ is consumption of the composite good, $M_{t+1}$ is per capita holdings of money at the end of the period $t$, and $h_t$ is hours worked expressed as a fraction of a time endowment of one. We assume satiation of utility from real balances, i.e. there exists $\overline{m}$ such that $\Upsilon'(m) > 0$ for all $m < \overline{m}$ and $\Upsilon'(m) = 0$ for all $m \geq \overline{m}$.[1]

The household's period $t$ budget constraint is

$$\begin{aligned} c_t + x_t + M_{t+1}/P_t + B_{t+1}/P_t & \\ = M_t/P_t + (1 + R_{t-1})B_t/P_t + \textstyle\int_0^1 (\Xi_t(i)/P_t)di & \\ + T_t + (1-\tau)r_t k_t + w_t h_t + \tau\delta k_t, & \quad (2) \end{aligned}$$

where $P_t$ is the price level, $B_{t+1}$ is the household's holdings of nominal debt at the end of the period $t$, $k_t$ is capital and $x_t$ is investment. Households hold equal amounts of shares in each intermediate goods firm so that $\Xi_t(i)$ is per capita nominal profits from intermediate firm index $i$. Finally, households pay a proportional tax $\tau$ on capital income and receive lump-sum transfers of size $T_t$ from the government.

Capital is subject to adjustment costs with the following form:

$$x_t = \Phi\Big(k_{t+1}/k_t\Big)k_t. \quad (3)$$

The function $\Phi$ is assumed to have the following properties: $\Phi(e^\mu) = e^\mu - 1 + \delta$, $\Phi'(e^\mu) = 1$ and $\Phi''(e^\mu) = \phi$ $(> 0)$ where $e^\mu$ is the steady-state growth rate of

[1] If $\Upsilon'(m) > 0$ for all $m$, then the zero interest rate bound never binds. Since we want to analyze monetary policy under a binding zero nominal interest rate constraint, this assumption is needed.

TFP.[2] The first property implies that adjustment costs in the steady-state are zero, i.e. the capital accumulation equation becomes linear in the steady-state. This form of adjustment costs is the same as specifications previously considered by Woodford (2003) and Christiano (2004).

## 2.2 Final Good Firm Problem

The final goods sector is perfectly competitive. Firms combine intermediate goods to produce output which can either be consumed or used for investment. The production technology for this sector is:

$$y_t = \left( \int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \tag{4}$$

Profit maximization yields the following input demand functions for intermediate firms

$$y_t(i)^d = (p_t(i)/P_t)^{-\theta} y_t \tag{5}$$

where $p_t(i)$ is the price of good $i$.

## 2.3 The Intermediate Goods Firm Problem

Each intermediate good producing firm has access to the following production function

$$y_t(i) = k_t(i)^{\alpha} \big( A_t h_t(i) \big)^{1-\alpha} \tag{6}$$

[2] We log-linearize the model when solving it and these assumptions are the only restrictions on adjustment costs needed to perform the log-linearization.

where $A$ is a shock to the technology for goods production. Each firm faces a demand function for its good given by (5) and maximizes the discounted sum of future profits. It is convenient to solve the intermediate goods firm's problem in two steps. Cost minimization yields the following relations governing input demands and marginal cost:

$$r_t = \alpha \chi_t k_t(i)^{\alpha-1}\big(A_t h_t(i)\big)^{1-\alpha} \tag{7}$$

$$w_t = (1-\alpha)\chi_t A_t^{1-\alpha} k_t(i)^{\alpha} h_t(i)^{-\alpha} \tag{8}$$

$$\chi_t = r_t^{\alpha} w_t^{1-\alpha}/\{\alpha^{\alpha}(1-\alpha)^{1-\alpha}A_t^{1-\alpha}\}. \tag{9}$$

Then total costs for firm $i$ have the following form

$$r_t k_t(i) + w_t h_t(i) = \chi_t\big\{\alpha y_t(i) + (1-\alpha)y_t(i)\big\} = \chi_t y_t(i) \tag{10}$$

The second step has each intermediate goods firm choose its price to maximize:

$$\textstyle\sum_{t=0}^{\infty} \beta^t (c_0/c_t)\big[p_t(i)y_t(i) - P_t\chi_t y_t(i) - P_t\Gamma\Big(\Pi_t(i)/\Pi\Big)y_t\big]/P_t \tag{11}$$

subject to the demand function given by (5) where $\Pi$ denotes the steady-state *gross* inflation rate, which is defined as the gross growth rate of the overall price level, $\Pi_t(i)$ denotes $p_t(i)/p_{t-1}(i)$, the gross growth rate of the price of intermediate good $i$, and the function $\Gamma$ represents convex costs of price adjustment. Price adjustment costs for firm $i$ are proportionate to final goods output and depend on the current gross growth rate of the firm's prices relative to the steady-state gross growth rate of the overall price level. This specification of price adjustment costs is also used in e.g. Ireland (2004). We will assume that

$\Gamma$ satisfies $\Gamma(1) = 0$, $\Gamma'(1) = 0$, and $\Gamma''(1) = \gamma > 0$. These assumptions are sufficient to log-linearize the first order condition for the optimization problem given by (11). The resulting log-linearized representation is the same as log-linearized representations for specifications where the costs are proportional to a firm's gross profits and specifications where firms are subject to Calvo price-setting rules instead of convex adjustment costs.[3]

## 2.4 Government and feasibility

The government budget constraint is:

$$T_t + g_t = \tau(r_t - \delta)k_t + (M_{t+1} - M_t)/P_t + \{B_{t+1} - (1 + R_{t-1})B_t\}/P_t \qquad (12)$$

where $g_t$ denotes government purchases in period $t$. Since our economy has lump-sum taxation, Ricardian equivalence applies and the time paths of government bonds and lump-sum taxation don't affect prices or allocations.

From the previous definitions it follows that the aggregate resource constraint is:

$$c_t + x_t + g_t = \big\{1 - \Gamma\big(\Pi_t/\Pi\big)\big\}y_t. \qquad (13)$$

## 2.5 Monetary Policy

We consider interest rate targeting rules (see e.g. Taylor(1993)) of the following form:

$$R_t = \max[R(y_t, \pi_t), 0], \qquad (14)$$

[3]This point is discussed in more detail Braun and Waki (2005).

where $\pi_t \equiv P_t/P_{t-1} - 1$ is the net inflation rate and the function $R$ represents a feedback mechanism. When the economy is log-linearized the Taylor rule becomes:

$$\begin{aligned}\hat{R}_t \equiv R_t - R &= \max[R + \rho_y \hat{y}_t + \rho_\pi \hat{\pi}_t, 0] - R \\ &= \max[\rho_y \hat{y}_t + \rho_\pi \hat{\pi}_t, -R] \qquad (15)\end{aligned}$$

where $\hat{y}_t$ denotes a percentage deviation of detrended $y_t$ from its steady-state value, $\hat{\pi}_t$ a deviation of $\pi_t$ from its steady-state value and $R$ is the steady-state value of $R(y_t, \pi_t)$.[4] We refer to $R$ as the *long-run nominal interest rate.*

## 2.6 Equilibrium

**Definition (Symmetric Monopolistically Competitive Equilibrium)** Given ( $P_{-1}$, $R_{-1}$, $k_0$, $M_0$, $B_0$, $\{g_t, A_t\}_{t=0}^{\infty}$ ) and a monetary policy $R_t = \max[R(y_t, \pi_t), 0]$, a *monopolistically competitive symmetric equilibrium* is a factor price sequence $\{r_t, w_t, \chi_t\}_{t=0}^{\infty}$, a final good price sequence $\{P_t\}_{t=0}^{\infty}$, a nominal interest rate sequence $\{R_t\}_{t=0}^{\infty}$, an allocation $\{c_t, k_{t+1}, h_t, M_{t+1}\}_{t=0}^{\infty}$ and a finite set of integers $I_B$ which satisfies the following conditions;

- Given all prices, households maximize their utility.
- Given factor prices, the price of the final good and (5), profits are maximized for each intermediate good firm at $(k_t(i), h_t(i), p_t(i)) = (k_t, h_t, P_t)$ for all $t$ and $i$.
- Monetary policy
  - The zero interest rate constraint binds for all $t \in I_B$ and $R_t = R(y_t, \pi_t)$ for other $t \geq 0$.
  - When $R_t > 0$, the monetary authority supplies $M_{t+1}$ which satisfies households' demand for money. Otherwise, the monetary authority supplies $\underline{\mathrm{M}}_{t+1}$ which is the minimal amount of money that satisfies households' demand for money.
- The government budget constraint is satisfied.
- Markets clear.

This completes the description of the model.

[4] We detrend variables by scaling them by the level of TFP, which is assumed to grow at an exponential rate. More details on how the variables are detrended can be found in Braun and Waki (2005).

# 3 Calibration and Simulation

## 3.1 Simulation Method

Computing the equilibrium consists of the following two steps: (1) First we log-linearize the equilibrium conditions about a balanced growth path and (2) we solve the resulting log-linearized system. This section provides an overview of the solution method. (Complete details of the steps involved in linearizing and solving the model can be found in the Appendix to Braun and Waki (2005).)

The presence of the zero nominal interest rate bound on monetary policy creates two difficulties. First it complicates the solution of the model since the policy function is not well approximated by a linear function. The second difficulty is that the zero nominal interest rate bound alters the stability properties of the model as pointed out by Benhabib, Schmitt-Grohe and Uribe (2001). They find that there are two steady-states; one where the nominal interest rate is zero and one with a positive nominal interest rate. There are infinitely many equilibria that converge to the former steady-state and a unique convergent path to the latter one.

We confront these two issues by approximating the Taylor rule (14) with the piece-wise linear function (15) and focusing on a particular class of equilibria. Attention is restricted to equilibria in which the zero nominal interest rate constraint binds once for a finite number of periods. Other equilibria in which the zero constraint might bind for a while, cease to bind and then start to bind again are ruled out. These assumptions imply that there exists some period $T$ such that the nominal interest rate is zero in period $T-1$ and for all $t \geq T$ the nominal interest rate is strictly positive. Similarly, there is a period $S$ such that the nominal interest rate is positive in period $S-1$ and zero in period $S$. Then the nominal interest is zero from $S$ to $T-1$. As of period $T$, any equilibrium in this class has the property that under standard regularity conditions there is a unique convergent path to a steady-state with a positive nominal interest rate. Consider next the remaining two subintervals, $S \leq t < T$ and $0 \leq t < S$. Since each of these two intervals has a finite number of periods, there is a unique equilibrium sequence for given $S$ and $T$.

For given $S$ and $T$ the equilibrium is computed in the following way. Given a level of the capital stock in period $T$, $k_T$, calculate the equilibrium path for all $t \geq T$. Next use the equilibrium values of the variables in period $T$ to solve the system backward for $k_0$. Repeat for different choices of $k_T$ until the implied initial capital stock $k_0$ is equal to its value in Japanese data.

The conditions described so far produce a unique equilibrium for given choices of $S$ and $T$. However, once these two parameters are allowed to vary multiple equilibria can and *do* arise in calibrated versions of our model. Multiple combinations of $S$ and $T$ produce bona fide equilibria. Imposing the further restrictions that $S$ occurs in 1997 and then choosing $T$ to be the earliest year where the constraint ceases to bind is sufficient to rule out all equilibria but one.

### 3.2 Calibration

Table 1 reports the parameterization of the model. Most of the parameters of our model are calibrated along the lines of Hayashi and Prescott (2002). This includes the capital share parameter, $\alpha$, the rate of depreciation on capital, $\delta$, the preference discount rate, $\beta$, and the tax rate on capital income, $\tau$.[5]

The elasticity of substitution for intermediate goods in final goods production is chosen to produce a markup of 15 percent. We set the leisure weight parameter, $\nu$, so that steady-state hours in the model is 31.6 hours per week. This number is calibrated in the following way. Multiply the average work-week for workers in each year from 1990 to 2000 times employment in the same year. Then divide the resulting values by the working age population in each year and finally compute the sample average.

We set the *baseline* Taylor rule reaction coefficients to 0.4 for output and 1.7 for inflation. These values are found to be optimal by Fujiwara et al. (2004) using the Bank of Japan's Japanese Economic Model when the weight on the output gap in the monetary authority's loss function is 0.08. This choice of reaction coefficients may or may not be optimal in the present model. The calibration of the long-run nominal interest rate is described below in Section 4.2 and Section 4.3 conducts a sensitivity analysis of this aspect of the calibration.

We choose the adjustment cost parameter on prices, $\gamma$, so that the model reproduces the level of the inflation rate in 1990. The resulting value for $\gamma$ is 101.4. The adjustment cost parameter on capital, $\phi$, was set to two. Our choice is somewhat lower than the value assumed by Christiano (2004) who sets the same parameter to three.

In order to conduct simulations, we still need to specify the value of the initial capital stock and the entire path of exogenous variables (technology and government purchases). A description of how this is done is deferred to Section 4.2.

[5] Interested readers are referred to Hayashi and Prescott (2002) for more details.

# 4 Results

## 4.1 Impulse response analysis

Before reporting simulation results it is useful to first describe the dynamic responses of our model economy to shocks in government purchases and technology. The principal objective for conducting this analysis is to ascertain whether a binding zero nominal interest rate constraint matters and if so how it matters. A binding zero nominal interest rate constraint gives rise to a liquidity trap with falling output and prices in models without endogenous investment such as those considered by Auerbach and Obstfeld (2003) and Eggertsson and Woodford (2003). Christiano (2004), however, finds that this property relies on the assumption that investment is exogenous. Since investment is endogenous in our model it is important to understand how a binding zero nominal interest rate constraint affects the dynamics of the model. Impulse response functions also provide a way to assess the calibration of the model parameters. Assessing the calibration is particularly important in costly price adjustment models with capital formation. Basu and Kimball (2003), for instance, find that the response of output to a positive shock in government purchases is negative in a similar model to ours when there are no adjustment costs on capital or investment.

In costly price adjustment models the dynamic responses of the economy to shocks in technology and government purchases can also vary considerably depending on the details of the monetary policy rule. Braun and Waki (2005) provide a detailed analysis of these characteristics of the model. Here we limit attention to the baseline specification described above in Section 3.1.

Figure 1 reports model impulse response functions to AR 1 shocks in technology and government purchases under the baseline Taylor rule assuming a long-run nominal interest rate target of 0.1 percent, a steady-state growth rate of technology of 2 percent per annum and a steady-state government share of output of 0.20.[6] For a government purchases shock we simulate (A23) setting the $\hat{A}$'s to zero and use the following sequence $(\hat{g}_0, \hat{g}_1, \hat{g}_2, ...) = (-0.1, -0.08, -0.064, ...)$. This choice corresponds to an AR 1 rule with an autoregressive coefficient of 0.8. A shock to technology is simulated in an analogous way. Each plot contains two lines. The dashed line reports results for the case where the zero nominal interest rate constraint is ignored and the solid line shows the responses when the zero bound on the nominal interest rate is modeled.

Consider first the results for government purchases that abstract from the zero bound constraint reported in Panel A. Lower government purchases lower output and crowd in private consumption and investment. The intuition for these responses is as follows. Lower government purchases mean lower (lump-sum) taxes and consumption rises on impact as emphasized in the analyses of Hall (1980) and Barro (1981). In subsequent periods consumption monotonically declines back towards its steady-state level. From the household first order condition (A1) this implies a lower real interest rate. Lower government purchases also lower the markup which from (A6) and (A7) acts to increase firm demand for labor and capital. However, these markup effects are small relative to the intertemporal substitution effects associated with a lower real interest rate and hours fall and investment increases.

Imposing the zero nominal interest rate constraint has negative effects on the

[6] The value of the long-run nominal interest rate used here is specifically scaled to insure that the zero constraint binds for 10 percent impulses to either exogenous variable. In Section 4.2 we will find that a somewhat higher setting of this parameter can reproduce the facts from the 1990s.

responses of most variables. Output and hours fall by nearly twice as much and the positive responses of investment and consumption are now much smaller. In our model the binding zero nominal interest constraint is limiting the ability of the monetary authority to counteract the negative effects of a decline in government purchases.

Consider next an improvement in technology as reported in Panel B of Figure 1. For the case where the zero bound on the nominal interest rate is ignored, the impulse responses broadly resemble those that would arise in a real business cycle model. Output, consumption and investment all increase. Altig et al. (2002) and Kahn et al. (2002) have previously found that an optimal monetary policy in sticky price models will seek to undo the constraints that costly price adjustment imposes on allocations. To the extent that monetary policy is successful in undoing these constraints, the resulting dynamic responses will reproduce the dynamic real responses that would arise in a real economy without price distortions. This effect of monetary policy is operating here but it is not completely successful as can be seen by the responses of the markup and hours.[7] The higher markup lowers the wage rate and this in turn induces a decline in hours. [8]

The solid lines in Panel B of Figure 1 report impulse responses to a 10 percent improvement in technology for the case where a zero nominal interest rate bound is imposed. These results are also consistent with the widely held view that the zero nominal interest rate bound ties the hands of the monetary authority. The

[7] Under alternative monetary policies the responses can be very different from the responses reported in Panel B of Figure 1. If, for instance, monetary policy is assumed to follow an exogenous $k$ percent rule instead, output and investment both fall on impact (see Braun and Waki (2005) for details).

[8] Gali and Rabanal (2004) argue that a decline in hours in response to an improvement in technology is a robust property of costly price adjustment models and provide empirical evidence that hours decline in response to improvements in technology using U.S. data.

zero nominal interest rate bound hampers the ability of the monetary authority to ease interest rates and output, investment and consumption now fall. The effects of the zero bound are also quite large. Output falls by over 9 percent now as compared to a 3 percent increase when the zero bound constraint is not imposed. The reason for this response is the large increase in the markup, which is acting like a tax on investment and labor input. With a higher markup investment now falls by 29 percent as compared to a 4 percent increase when the constraint is not imposed and hours now decline by 25 percent as compared to 5 percent before.

To summarize, our results indicate that the dynamic response of the economy to either shock is quite different when the zero nominal interest rate bound is modeled *and* a shock arrives that leads the constraint to bind. In this sense, a binding zero nominal interest rate constraint ties the hands of the monetary authority. The effects of this constraint are most pronounced for technology shocks but a binding zero constraint also amplifies the responses of output and hours to negative government purchases shocks. We now turn to investigate the quantitative performance of our model during the 1990s in Japan.

### 4.2 Accounting for the facts from the lost decade

Our objective is to provide a quantitative assessment of alternative monetary policies during the lost decade. In order to do this we need a baseline specification that can account for the main facts from this period. In our view the most important real facts are that per capita output grew at 0.68 percent per annum, per capita investment fell at 1.4 percent per annum and hours worked declined by 1.18 percent per annum. The key nominal facts from the 1990s are that the nominal interest rate fell from 7.4 percent to 0.1 percent and the inflation rate, as measured by the growth rate of the GDP deflator, fell from 2.3 to -1.8 percent.

Our model has two exogenous sources of variation: government purchases and technology. Since the model assumes perfect foresight there is only one surprise to households and it occurs in 1990 when households see two infinite sequences of TFP and government purchases realizations. To compute a solution we need to specify the entire sequence $(\epsilon_0, \epsilon_1, \epsilon_2, ...)$ in equation (A23) and the initial condition $\hat{k}_0$. This is done in the following way. The initial capital stock is set to match its 1990 value in Japanese data.[9] For the 1990s we condition on the actual time path of TFP and government purchases. For the period beyond 2000 we first assumed that TFP growth was 0.3 percent per annum, and that the share of government purchases in output was 15 percent in each period. These are the same assumptions made by Hayashi and Prescott (2002). Monetary policy follows the baseline Taylor rule with an output coefficient of 0.4 and inflation coefficient of 1.7 and set the long-run nominal interest rate was initially set to 2.3 percent so that it implied a zero long-run inflation rate. However, these conditioning assumptions did not reproduce the real and nominal facts for the 1990s. In particular, the nominal interest rate did not fall to zero. We then experimented with a variety of other conditioning assumptions. We found that if the long-run government share of output is set to 20 percent and the long-run nominal interest rate is set to 0.3 percent then the model does a reasonable job of explaining the real and nominal facts from the lost decade.[10]

[9] We use Hayashi and Prescott's (2002) measure of the capital stock. The presence of adjustment costs on capital in our model means that their measure of the capital stock should be adjusted to account for these costs. However, we have made no such adjustment.

[10] We also assume a 15 year transition from the year 2000 for TFP and government purchases

More generally, our experiments suggest that it is difficult to produce a binding zero nominal interest rate and the other facts from the 1990s with higher long-run nominal interest rate targets and lower long-run government purchase shares of output.

To understand why these particular conditioning assumptions work it is helpful to refer back to the impulse response analysis in Section 4.1. Note that a positive impulse to TFP and/or a negative impulse to government purchases is needed to produce a decline in the nominal interest rate and inflation. Low TFP growth in the long-run means that TFP in the 1990s is seen to be temporarily high by households. As a result, both the nominal interest rate and inflation rate fall.[11] Higher government purchases in the long-run also act to drive down the nominal interest rate and inflation rate in the 1990s, since households perceive government purchases in 1990s to be temporarily low.

Figure 2 reports model simulations and Japanese data for our baseline specification. From this figure one can see that the model does a surprisingly good job of accounting for both the real and nominal facts from Japan in the 1990s. The patterns in output, inflation, investment and the nominal interest rate fit the data well. The model's success in reproducing the pattern in hours is partially due to our assumption about $\nu$, the preference parameter weight on leisure. As noted above in Section 3.1, $\nu$ is calibrated so that the steady-state value of hours is 31.6 hours per week. The biggest gap between our theory and Japanese data is the 1990 value of the nominal interest rate. The model predicts a nominal interest rate of 5 percent whereas the value in the data is 7.4 percent. We are

to adjust to their new steady-state values.

[11] The initial capital output ratio also matters. We are implicitly assuming that the initial capital output ratio is low relative to its ultimate steadystate value. This turns out to be the case when we simulate the model using Japanese data.

not particularly concerned by this gap between our theory and the data since at this time interest rates were set at a high level by the monetary authority in an effort to slow the asset price bubble. The Taylor rule we are using does not incorporate this aspect of monetary policy. The model also slightly understates investment in 1990 and the capital output ratio in 2000. Based on these results we conclude that our model successfully accounts for the main real and nominal facts from Japan's lost decade.

## 4.3 Monetary policy during the lost decade

During this period monetary policy has come under considerable criticism. It has been argued that an alternative monetary policy might have stimulated economic activity during the 1990's and/or avoided the zero nominal interest rate bound. For example, Jinushi et al. (2000) and McCallum (2003) suggest that monetary policy was tight during the 1990s and Krugman (1998), Bernanke (2000), Svensson (2001) and Auerbach and Obstfeld (2003) have all proposed alternative policies that in their views would produce better outcomes. (See Ito and Mishkin (2004) for an excellent review of the large literature on this topic).

We next use our model as a laboratory to investigate how alternative monetary policies affect economic outcomes during the 1990s. We consider three general types of variations in the monetary policy rule: the magnitude of the reaction coefficients on output and inflation, the value of the long-run nominal interest rate and the length of the interval over which the interest rate is kept at zero.

**Variations in the Taylor rule reaction coefficients**

Table 2 reports simulation results for four settings of the Taylor rule reaction coefficients on output and inflation. In this table the baseline specification is referred to as Taylor rule 3. Fujiwara et al. (2004) find that Taylor rules 2 though 4 are all optimal rules for particular settings of the weight on the output gap in a quadratic loss function in the output gap and inflation gap for the central bank. Taylor rule 1 is reported for completeness. It provides information on the properties of a Taylor rule that assigns most weight to the output gap. All of the results in Table 2 condition on a long-run nominal interest rate of 0.3 percent. A comparison of these four rules indicates that none of them avoid the zero nominal interest rate bound. This result is quite striking since these rules span a set of orthodox policies that range from output stabilization targeting (Taylor rule 1) to a pure inflation stabilization target (Taylor rule 4). However, the four monetary policies do differ in other respects. Output contracts during the 1990s under Taylor rule 1, whereas Taylor rule 4 shows the strongest output growth during the 1990s and also the most deflation in 2000. Taylor rule 4 also looks attractive in other respects. It turns in the highest consumption and investment growth during the 1990s. Another distinction is in the length of the interval of time when the nominal interest rate is zero. Under Taylor Rule 1 the nominal interest rate is zero for 27 years while under Taylor rule 4 the constraint ceases to bind in 2009.

Given these differences among the four policies it is interesting to evaluate them on the basis of economic welfare. The last row of Table 2 reports welfare for each policy. Welfare is reported from the perspective of Taylor rule 3. We follow the methodology of Lucas (2003) and calculate the constant supplement to each period's consumption under Taylor rule 3 that renders present value utility under Taylor rule 3 equal to present value utility under each other Taylor rule.[12] Thus a value of 0.19 for Taylor Rule 1 implies that the welfare gain of moving from Taylor rule 3 to Taylor rule 1 is equivalent to a constant 0.20 percent of consumption.

A striking property of these welfare calculations is that Taylor rule 1 produces the highest welfare. To understand this result note that Taylor rule 1 has a zero nominal interest rate for the longest interval of time. Next note that whenever we change the long run interest rate target $R$ the long-run inflation target $\pi$ also changes one for one. Adjustment costs are relative to this long-run inflation target. Thus, when the long-run nominal interest rate target is 0.3 percent the Friedman rule is, to a first approximation, the optimal monetary policy in this economy. As pointed out in Aiyagari and Braun (1998) there are two factors that determine optimal monetary policy in this economy: the inflation tax effects emphasized by Friedman(1966) and Cooley and Hansen (1989) and the adjustment costs on prices. The former costs are minimized when the nominal interest rate is zero and the latter costs are minimized when inflation is at its target level. Under a long-run nominal interest rate target of 0.3 percent the adjustment costs for prices are centered at a value that is very close to the Friedman rule rate of deflation. Taylor rule 1 follows the Friedman Rule for the longest period of time and it is thus not surprising that it produces the highest welfare.

[12] The welfare measure we use is calculated as follows. First, we compute an equilibrium allocation for each rule. Let $\{c_t(i), h_t(i)\}$ be a sequence of consumption and hours in the equilibrium under Taylor rule $i$. And second, we compute for each Taylor rule $i$ a welfare measure $d(i)$ defined as a real number which satisfies

$$\begin{aligned}
&\textstyle\sum_{t=0}^{\infty} \beta^t \{\log c_t(i) + \nu \log(1 - h_t(i))\} \\
&\quad = \textstyle\sum_{t=0}^{\infty} \beta^t \{\log(1 + d(i))c_t(3) + \nu \log(1 - h_t(3))\} \\
&\quad = \log(1 + d(i))/(1 - \beta) + \textstyle\sum_{t=0}^{\infty} \beta^t \{\log c_t(3) + \nu \log(1 - h_t(3))\}.
\end{aligned}$$

We approximate the infinite sum of period utility by a finite sum from $t = 0$ to $T$. In Table 2, the $d(i)$'s are reported in terms of percentages.

**Other variations in monetary policy**

Next we consider three other monetary policies that have been proposed in the literature. Goodfriend (2000) suggests that a carry tax on money is an effective way to undo the zero nominal interest rate bound. Eggertsson and Woodford (2003) find that policy duration is part of an optimal monetary policy in a model with no capital accumulation. Finally, a higher setting of the long-run nominal interest rate avoids the zero nominal interest rate bound. Table 3 reports simulation results for these three cases and the baseline Taylor rule 3 specification. In all cases it is assumed that the output and inflation feedback coefficients are respectively 0.4 and 1.7. "Unconstrained" corresponds to the policy proposed by Goodfriend (2000).[13] "Policy duration" assumes that the nominal interest rate is kept at zero for five years beyond the point where the zero constraint ceases to bind. "Price stability" refers to a scenario where the long-run nominal interest rate target is 2.3 percent. This value is consistent with a stable long-run price level.

Observe that the policies labeled unconstrained and price stability have very similar properties. Growth rates of both real and nominal variables are nearly the same in all sub-samples. Both policies produce stronger consumption and output growth in the second half of the 1990s as compared to Taylor rule 3. However, between 2000 and 2009 the picture is reversed and the Taylor rule 3 specification exhibits stronger output and consumption growth. Welfare is also

[13] Goodfriend (2000) points out that a carry tax on money will undo the zero nominal interest rate constraint and suggests that taxing reserves is one way that a central bank can implement such a policy. In our model the easiest way to model this is to tax beginning of period holdings of money. Suppose that the tax on money is $\tau_{m,t}$, then it can be shown that setting $\tau_{m,t+1} = -R_t$ when the nominal interest rate is negative undoes the zero bound constraint. To see this note that with a carry tax (A3) is replaced with $c_t \Upsilon'(M_{t+1}/P_t) = \frac{R_t + \tau_{m,t+1}}{1+R_t}$. This change does not affect any other equilibrium conditions under a Taylor rule so the equilibrium values of all other variables are not affected.

nearly identical for the unconstrained and price stability policies. The overall magnitude of the welfare gains relative to Taylor rule 3 is about the same as the welfare gains for stabilizing business cycle fluctuations that Lucas (2003) has estimated using U.S. data. He estimates that the benefits of stabilizing business cycle fluctuations are about 0.05 percent in the U.S.

Policy duration produces higher output and consumption growth during the 1990s than Taylor rule 3. Moreover, policy duration also produces the highest welfare of the four policies. Only Taylor rule 1 in Table 2 produces higher welfare. The mechanisms underlying this result are neoclassical. Policy duration is better than the other monetary policies because it is a closer approximation to the Friedman rule. This result is not unique to Taylor rule 3. When policy policy duration experiments are performed for the other Taylor rules extending the period of time that the nominal interest rate is zero also enhances welfare.

Before concluding we wish to say a few words about the robustness of the welfare results to our modeling assumptions. Some of the welfare rankings reported in Tables 2 and 3 hinge crucially on the assumption of perfect foresight. From the impulse response analysis we know that a binding zero nominal interest rate has significant and negative impacts on the response of the economy to positive innovations in technology and negative shocks to government purchases. The welfare effects of these shocks can be large. For instance, if we compare welfare for the two impulse responses to technology reported in Figure 1B using the same method described above, imposing the zero bound constraint produces a welfare loss of 0.34 percent. This welfare loss is larger than any of the results reported in Table 2.[14] Although it is beyond the scope of this analysis to solve

[14]The welfare loss associated with imposing the zero interest rate constraint for the govern-

and compute equilibria for a stochastic version of our model, it is still interesting to conjecture what such an analysis might find. Our impulse response analysis shows that the arrival of positive TFP shocks in real time can have strong and negative implications for economic activity and welfare when the nominal interest rate is zero. This fact suggests that monetary policies such as Taylor rule 1 and/or policy duration might be very costly in a stochastic environment. Both of these specifications imply that the nominal interest rate is zero for many periods.

One solution to this problem is to undo the zero constraint via a carry tax on money as proposed by Goodfriend (2002). However, such a policy may be costly to implement and doesn't differ much in terms of welfare from the alternative of simply setting the long-run nominal interest rate at a higher level. Results from our model suggest that a long-run nominal interest rate target of 2.3 percent is probably sufficient to avoid the bound. This setting of the nominal interest rate is consistent with price stability and has the property that even extremely large shocks to government purchases fail to produce a binding zero constraint. In the case of technology shocks only very large positive shocks with a magnitude of 16 percent or larger would induce a binding constraint. In this sense setting the long-run nominal interest rate at a target level of 2.3 percent or higher is a good robust policy.

---

ment purchases shock in Figure 1A is much smaller: 0.016 percent.

# 5 Conclusion

In this paper we have considered the effects of monetary policy in the neighborhood of the zero nominal interest rate in a model with costly price adjustment and capital accumulation. We found a specification that accounts for both the nominal and real facts from the 1990s in Japan. Two key ingredients are needed to reproduce the measured decline in the nominal interest rate to zero. Households must expect higher government purchases in future years and the long-run nominal interest rate target of the monetary authority must be very low. We also performed some counterfactual experiments to ascertain the extent to which alternative monetary policies might have improved economic activity during the 1990s and/or avoided the zero nominal interest rate bound.

We find that the setting of the long-run inflation target is much more important for avoiding the zero bound than the short-run reaction of monetary policy to a particular sequence of exogenous shocks. We consider various settings of the Taylor rule reaction coefficients on output and inflation, and none of them avoid the zero nominal interest rate bound. We also consider other policies that have been suggested in the literature. Policies that keep the nominal interest rate low for long periods of time are good policies under the assumption of perfect foresight. Setting the long-run interest rate target in a way to maintain price stability is also a good policy. We conjecture that this latter finding would also apply in a stochastic environment.

In future work we plan to investigate the role of other shocks including shocks to the financial sector and develop computational methods that allow us to analyze the zero bound in an environment with uncertainty.

**Acknowledgments**

We wish to thank an anonymous referee, our discussant Franck Portier and participants at the 2004 CIRJE-TCER macro conference and CERGE-EI for their helpful comments. We owe a special thanks to Fumio Hayashi for his many constructive comments.The research of the first author was funded by grant numbers 12124202 and 15530282 from the Japanese Ministry of Education, Culture, Sports, Science and Technology.

# Appendix: solution method

In this appendix we describe our solution method more precisely. As in the literature, we first derive the equilibrium conditions and detrend variables so that all variables in the system are stationary. The equilibrium conditions are

$$
\begin{aligned}
1 &= \beta(c_t/c_{t+1})(1+R_t)/(1+\pi_t), && \text{(A1)}\\
w_t &= \nu c_t/(1-h_t), && \text{(A2)}\\
0 &= R_t/(1+R_t) - c_t \Upsilon'\Big(M_{t+1}/P_t\Big), && \text{(A3)}\\
0 &= \beta(c_t/c_{t+1})\Big\{(1-\tau)r_{t+1} + \tau\delta - \Phi\Big(k_{t+2}/k_{t+1}\Big) && \\
&\quad +\Phi'\Big(k_{t+2}/k_{t+1}\Big)(k_{t+2}/k_{t+1})\Big\} - \Phi'\Big(k_{t+1}/k_t\Big), && \text{(A4)}\\
0 &= 1-\theta+\theta\chi_t - \Gamma'\Big(\Pi_t/\Pi\Big)(\Pi_t/\Pi) && \\
&\quad -\beta(c_t/c_{t+1})(y_{t+1}/y_t)\Gamma'\Big(\Pi_{t+1}/\Pi\Big)(\Pi_{t+1}/\Pi), && \text{(A5)}\\
r_t &= \alpha\chi_t k_t^{\alpha-1}\big(A_t h_t\big)^{1-\alpha}, && \text{(A6)}\\
w_t &= (1-\alpha)\chi_t A_t^{1-\alpha} k_t^{\alpha} h_t^{-\alpha}, && \text{(A7)}\\
\chi_t &= r_t^{\alpha} w_t^{1-\alpha}/\big\{\alpha^{\alpha}(1-\alpha)^{1-\alpha} A_t^{1-\alpha}\big\}, && \text{(A8)}\\
T_t + g_t &= \tau(r_t-\delta)k_t + (M_{t+1}-M_t)/P_t && \\
&\quad +(B_{t+1}-(1+R_{t-1})B_t)/P_t, && \text{(A9)}\\
0 &= c_t + \Phi\Big(k_{t+1}/k_t\Big)k_t + g_t - \Big\{1-\Gamma\big(\Pi_t/\Pi\big)\Big\}y_t, && \text{(A10)}\\
y_t &= k_t^{\alpha}\big(A_t h_t\big)^{1-\alpha}, && \text{(A11)}
\end{aligned}
$$

and monetary policy. Note that we use symmetricity among intermediate firms to derive the above expressions.

As stated in the body, in our analysis we limit attention to equilibria in which the zero nominal interest rate constraint binds once for a finite number of periods so that $I_B = \{S, S+1, S+2, ..., T-2, T-1\}$. Because we have already described in Section 3.1 how to compute $S$, $T$ and $\hat{k}_T$, suppose in the following we know them.

**Log-linearization**

We linearize the system around the steady-state. All variables are log-linearized except $r$, $R$ and $\pi$ which are linearized without taking logarithms.

$$
\begin{aligned}
0 &= \hat{c}_t - \hat{c}_{t+1} + (1/(1+R))\hat{R}_t - (1/(1+\pi))\hat{\pi}_{t+1} && \text{(A12)}\\
\hat{w}_t &= \hat{c}_t + (h/(1-h))\hat{h}_t && \text{(A13)}\\
\hat{r}_t/r &= \hat{h}_t + \hat{w}_t - \hat{k}_t && \text{(A14)}\\
\hat{y}_t &= \alpha\hat{k}_t + (1-\alpha)\hat{h}_t + (1-\alpha)\hat{A}_t && \text{(A15)}\\
\hat{r}_t/r &= \hat{\chi}_t - (1-\alpha)\hat{k}_t + (1-\alpha)\hat{h}_t + (1-\alpha)\hat{A}_t && \text{(A16)}\\
\tilde{y}\hat{y}_t &= \tilde{c}\hat{c}_t + \tilde{g}\hat{g}_t + \tilde{x}\hat{x}_t && \text{(A17)}\\
(e^{\mu} - 1 + \delta)\hat{x}_t &= e^{\mu}\hat{k}_{t+1} - (1-\delta)\hat{k}_t && \text{(A18)}\\
0 &= \theta\chi\hat{\chi}_t - (\gamma/(1+\pi))\hat{\pi}_t + (\beta\gamma/(1+\pi))\hat{\pi}_{t+1} && \text{(A19)}\\
\hat{c}_t &= \hat{c}_{t+1} - \beta\phi e^{\mu}\hat{k}_{t+2} + (1+\beta)\phi e^{\mu}\hat{k}_{t+1} \\
&\quad -\phi e^{\mu}\hat{k}_t - \beta e^{-\mu}(1-\tau)\hat{r}_{t+1} && \text{(A20)}
\end{aligned}
$$

By assumption, we can log-linearize the monetary policy rule as

$$
\hat{R}_t = \begin{cases} -R & \text{if } S \le t \le T-1 \\ \rho_y \hat{y}_t + \rho_\pi \hat{\pi}_t & \text{if } t \le S-1 \text{ or } T \le t. \end{cases} \qquad \text{(A21)}
$$

**State Space Representation**

We can summarize the equations (A12)-(A20) by three equations eliminating endogenous variables other than $R$, $\pi$, $y$ and $k$. These three equations and the identity $\hat{k}_{t+1} = \hat{k}_{t+1}$ can be written using matrices and vectors as:

$$\mathbf{D}_1 \mathbf{z}_{t+1} = \mathbf{D}_2 \mathbf{z}_t + \mathbf{d}_3 \hat{R}_t + \mathbf{D}_4 \epsilon_t \tag{A22}$$

where $\mathbf{z_t} = (\hat{\pi}_t, \hat{y}_t, \hat{k}_{t+1}, \hat{k}_t)'$, $\epsilon_t = (\hat{g}_{t+1}, \hat{g}_t, \hat{A}_{t+1}, \hat{A}_t)'$, $\mathbf{D}_i$'s are appropriately defined $4 \times 4$ coefficients matrices and $\mathbf{d}_3$ is $4 \times 1$ coefficients vector.

We can rewrite $\rho_y \hat{y}_t + \rho_\pi \hat{\pi}_t$ as $\rho' \mathbf{z_t}$ where $\rho = (\rho_\pi, \rho_y, 0, 0)'$. Then we get

$$\mathbf{D}_1 \mathbf{z}_{t+1} = \begin{cases} \Big(\mathbf{D}_2 + \mathbf{d}_3 \rho'\Big)\mathbf{z}_t + \mathbf{D}_4 \epsilon_t & \text{if } t \leq S-1 \text{ or } T \leq t \\ \mathbf{D}_2 \mathbf{z}_t - \mathbf{d}_3 R + \mathbf{D}_4 \epsilon_t & \text{if } S \leq t \leq T-1 \end{cases} \tag{A23}$$

**Case 1:** $t \geq T$

For all $t \geq T+1$,

$$\begin{aligned} \mathbf{z}_{t+1} &= \mathbf{D}_1^{-1}\Big(\mathbf{D}_2 + \mathbf{d}_3 \rho'\Big)\mathbf{z}_t + \mathbf{D}_1^{-1}\mathbf{D}_4 \epsilon_t \\ &= \mathbf{F_1} \mathbf{z}_t + \mathbf{G} \epsilon_t \end{aligned} \tag{A24}$$

Under our parameterization we can diagonalize $\mathbf{F_1}$ as $\mathbf{V_1 \Lambda_1 V_1^{-1}}$. Thereby the equation (A24) is rewritten as

$$\mathbf{q}_{t+1} = \mathbf{\Lambda_1} \mathbf{q}_t + \mathbf{V_1^{-1}} \mathbf{G} \epsilon_t \tag{A25}$$

where $\mathbf{q}_t = \mathbf{V_1^{-1}} \mathbf{z}_t$. Since $\mathbf{\Lambda_1}$ is diagonal, we have

$$q_{t+1}(i) = \lambda_{1,i} q_t(i) + (\mathbf{V_1^{-1}G})_i \epsilon_t \tag{A26}$$

for $i = 1, 2, 3, 4$ where $q_t(i)$ is the $i$-th element of $\mathbf{q}_t$, $\lambda_{1,i}$ is the $i$-th diagonal element of $\mathbf{\Lambda_1}$ and $(\mathbf{V_1^{-1}G})_i$ is the $i$-th row of $\mathbf{V_1^{-1}G}$. The stability condition of this system is that $\mathbf{\Lambda_1}$ has only one stable root. If this condition is satisfied, we can solve the system of $q_t(i)$'s for $t \geq T$ in the following manner. Let $\lambda_{1,4}$ be the stable root, then for $i = 1, 2, 3$,

$$\begin{aligned} q_t(i) &= (1/\lambda_{1,i})(q_{t+1}(i) - (\mathbf{V_1^{-1}G})_i \epsilon_t) \\ &= -\textstyle\sum_{j=0}^{\infty}(1/\lambda_{1,i})^{j+1}(\mathbf{V_1^{-1}G})_i \epsilon_{t+j}, \end{aligned} \tag{A27}$$

and for $i = 4$,

$$q_{t+1}(4) = \lambda_{1,4} q_t(4) + (\mathbf{V_1^{-1}G})_4 \epsilon_t. \tag{A28}$$

Since our model is perfect foresight and we specify the entire path of $\epsilon_t$, we can compute $q_t(i)$ for $i = 1, 2, 3$ and $t \geq T$ from (A27). Because we know $\hat{k}_T$, we can compute $q_T(4)$ and $\mathbf{z}_T$ from the relationship $\mathbf{q}_T = \mathbf{V_1^{-1}}\mathbf{z}_T$ since here we have four equations and four unknowns $(\hat{\pi}_T, \hat{y}_T, \hat{k}_{T+1}, q_T(4))$. Finally, by iterating forward, we can obtain the whole sequence of $q_t(4)$ and $\mathbf{z}_t$ for $t \geq T$.

**Case 2:** $S \leq t \leq T-1$

For all $t$ such that $S \leq t \leq T-1$,

$$\begin{aligned} \mathbf{z}_{t+1} &= \mathbf{D}_1^{-1}\mathbf{D}_2\mathbf{z}_t - \mathbf{D}_1^{-1}\mathbf{d}_3 R + \mathbf{D}_1^{-1}\mathbf{D}_4\epsilon_t \\ &= \mathbf{F_2}\mathbf{z}_t - \mathbf{D}_1^{-1}\mathbf{d}_3 R + \mathbf{G}\epsilon_t \end{aligned}$$

Under our parameterization $\mathbf{F_2}$ is invertible. Thus from $\mathbf{z}_T$, we can obtain the entire sequence of $\mathbf{z}_t$ for all $S \leq t \leq T-1$ by sequential backward substitution such as;

$$
\begin{aligned}
\mathbf{z}_{T-1} &= \mathbf{F_2^{-1}}\mathbf{z}_T + \mathbf{F_2^{-1}}\Big(\mathbf{D}_1^{-1}\mathbf{d}_3 R - \mathbf{G}\epsilon_{T-1}\Big), \\
\mathbf{z}_{T-2} &= \mathbf{F_2^{-1}}\mathbf{z}_{T-1} + \mathbf{F_2^{-1}}\Big(\mathbf{D}_1^{-1}\mathbf{d}_3 R - \mathbf{G}\epsilon_{T-2}\Big), \\
&\vdots \\
\mathbf{z}_S &= \mathbf{F_2^{-1}}\mathbf{z}_{S+1} + \mathbf{F_2^{-1}}\Big(\mathbf{D}_1^{-1}\mathbf{d}_3 R - \mathbf{G}\epsilon_S\Big).
\end{aligned}
$$

**Case 3:** $t \leq S$

For all $t$ such that $t \leq S$, we again have

$$
\mathbf{z}_{t+1} = \mathbf{F_1}\mathbf{z}_t + \mathbf{G}\epsilon_t.
$$

Under our parameterization $\mathbf{F_1}$ is invertible. Therefore, we can calculate the sequence of $\mathbf{z}_t$ for all $t \leq S-1$ from $\mathbf{z}_S$ using the sequential backward substitution again.

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Table 1: Model Parameterization

| | |
|---|---|
| Capital share, $\alpha$ | 0.362 |
| Preference discount factor, $\beta$ | 0.98 |
| Depreciation rate on capital, $\delta$ | 0.089 |
| Tax rate on capital income, $\tau$ | 0.48 |
| CES parameter, $\theta$ | 1+1/0.15 |
| Leisure weight in preference, $\nu$ | 2.4202 |
| Adjustment cost on capital, $\phi$ | 2 |
| Adjustment cost on prices, $\gamma$ | 101.4 |

**TABLE 2***

**Simulation results for alternative Taylor rules and Japanese Data**

| | Output growth | Consumption Growth | Investment Growth | Hours Growth | Nominal Interest rate | | Inflation rate | | Interval when Nominal interest rate is zero | Welfare |
|---|---|---|---|---|---|---|---|---|---|---|
| Year | 1990-2000 | 1990-2000 | 1990-2000 | 1990-2000 | 1990 | 2000 | 1990 | 2000 | | |
| Taylor Rule 1 ($\rho_y$=2, $\rho_\pi$=0.3) | -0.19 | 0.28 | -2.60 | -2.22 | 6.11 | 0.00 | 4.96 | -2.09 | 1997-2023 | 0.20 |
| Taylor Rule 2 ($\rho_y$=0.7, $\rho_\pi$=1.6) | 0.38 | 0.61 | -1.20 | -1.41 | 5.46 | 0.00 | 3.13 | -2.13 | 1997-2013 | 0.08 |
| Taylor Rule 3 (Baseline) ($\rho_y$=0.4, $\rho_\pi$=1.7) | 0.58 | 0.72 | -0.64 | -1.09 | 4.97 | 0.00 | 2.31 | -2.17 | 1997-2010 | 0.00 |
| Taylor Rule 4 ($\rho_y$=0.0, $\rho_\pi$=1.5) | 1.04 | 0.94 | 0.79 | -0.32 | 3.43 | 0.00 | 0.08 | -2.27 | 1997-2008 | -0.24 |
| **Data** | 0.68 | 1.49 | -1.39 | -1.18 | 7.40 | 0.11 | 2.30 | -1.83 | -------------- | ------------ |

*Consumption, investment and output are average annual percentage growth rates for the period 1990-2000 and the nominal interest rate and inflation are annual rates expressed as percentages. In all simulations the long-run nominal interest rate is set to 0.3 percent.

**Table 3***

**A comparison of alternative monetary policies in the presence of a zero bound constraint on the nominal interest rate**

| | Output growth | Consumption Growth | Investment growth | Hours growth | Nominal interest rate | Inflation |
|---|---|---|---|---|---|---|
| **1990-1994** | | | | | | |
| Taylor Rule 3 | 1.19 | 1.17 | -0.77 | -0.59 | 3.38 | 0.85 |
| Unconstrained | 1.23 | 1.14 | -0.53 | -0.44 | 4.07 | 1.46 |
| Policy Duration | 1.50 | 1.31 | 0.26 | -0.00 | 4.79 | 1.95 |
| Price Stability | 1.24 | 1.15 | -0.49 | -0.42 | 6.19 | 3.51 |
| **1995-1999** | | | | | | |
| Taylor Rule 3 | -0.06 | 0.23 | -0.55 | -1.72 | 0.22 | -1.80 |
| Unconstrained | 0.52 | 0.70 | 0.87 | -0.98 | 1.50 | -0.92 |
| Policy Duration | 0.17 | 0.54 | -0.33 | -1.68 | 1.40 | -0.72 |
| Price Stability | 0.52 | 0.70 | 0.87 | -0.98 | 3.57 | 1.10 |
| **2000-2004** | | | | | | |
| Taylor Rule 3 | 0.85 | 0.10 | 0.21 | 0.57 | 0.00 | -2.12 |
| Unconstrained | 0.50 | -0.06 | -0.92 | -0.10 | -0.25 | -2.05 |
| Policy Duration | 0.69 | -0.01 | -0.21 | 0.34 | 0.00 | -1.97 |
| Price Stability | 0.51 | -0.06 | -0.92 | -0.10 | 1.79 | -0.05 |
| **2005-2009** | | | | | | |
| Taylor Rule 3 | 0.65 | 0.05 | -0.57 | 0.52 | 0.00 | -2.05 |
| Unconstrained | 0.48 | -0.12 | -0.90 | 0.37 | -0.42 | -2.33 |
| Policy Duration | 0.65 | 0.03 | -0.52 | 0.57 | 0.00 | -2.02 |
| Price Stability | 0.48 | -0.11 | -0.90 | 0.37 | 1.62 | -0.33 |
| **1990-2009** | | | | | | |
| Taylor Rule 3 | 0.66 | 0.39 | -0.42 | -0.31 | 0.90 | -1.28 |
| Unconstrained | 0.68 | 0.42 | -0.37 | -0.29 | 1.23 | -0.96 |
| Policy Duration | 0.75 | 0.47 | -0.20 | -0.19 | 1.55 | -0.69 |
| Price Stability | 0.69 | 0.42 | -0.36 | -0.28 | 3.29 | 1.06 |

* Output, consumption, investment, and hours are expressed as average annual percentages and nominal interest rate and inflation are average levels over the indicated interval. Welfare is measured as the constant amount of consumption that renders welfare in the constrained case equal to welfare in the unconstrained case. Welfare for the unconstrained, policy duration and price stability simulations is respectively, 0.066%, 0.11% and 0.063%.

Figure 1: Impulse Responses in the Baseline Model*

A) Negative shock to government purchases

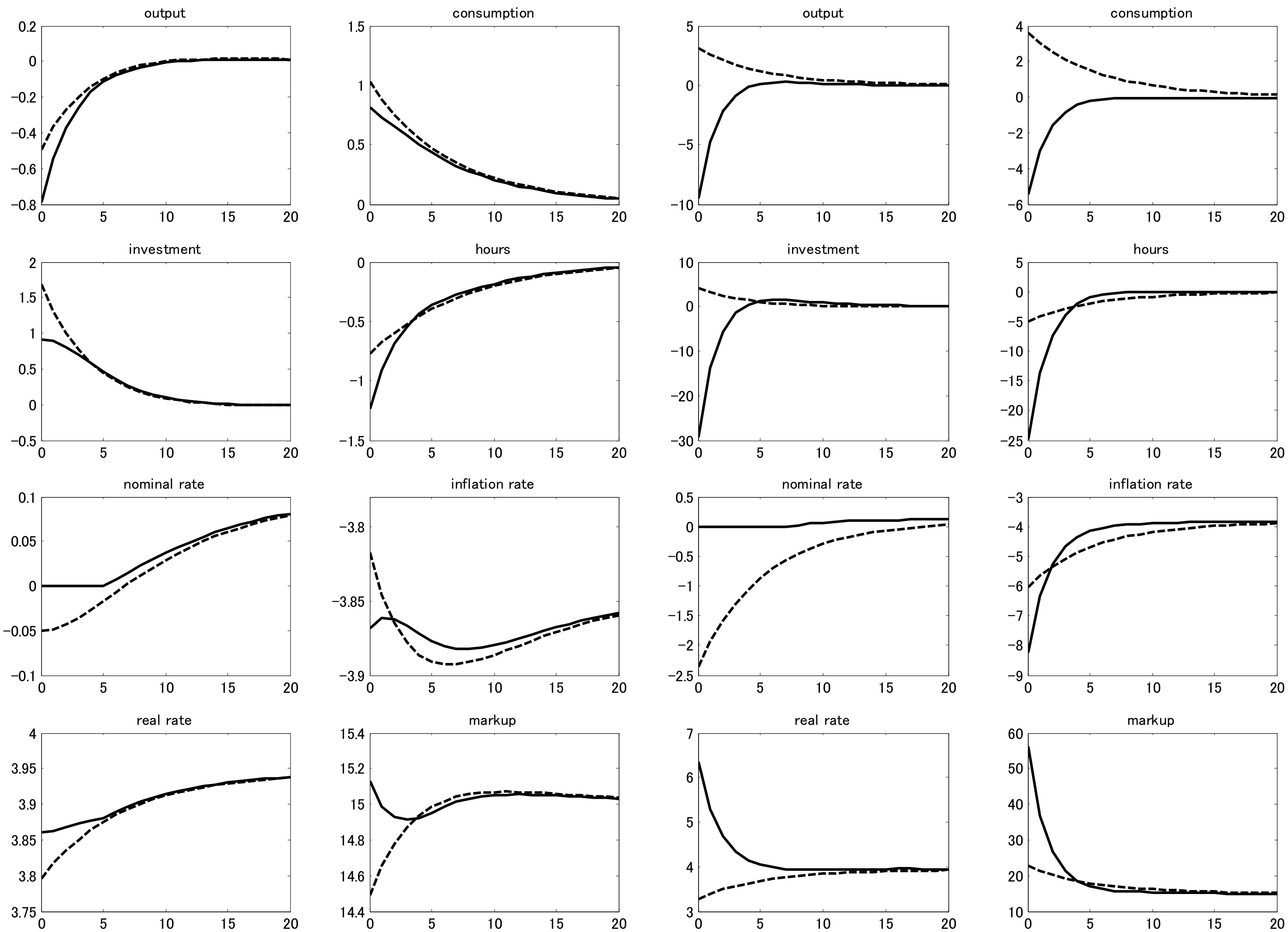
B) Positive shock to technology

[Insert Figure 1 here.]

* The dashed line shows the case where the zero bound is not imposed and the solid line shows the baseline case.

** Output, investment, consumption, hours worked, government purchases and TFP are expressed by percent deviation from steady state.

*** Other variables are expressed by percent levels.



output
consumption
output
consumption
investment
hours
investment
hours
nominal rate
inflation rate
nominal rate
inflation rate
real rate
markup
real rate
markup

Figure 2: Data and Simulation Results under Monetary Policy 3*

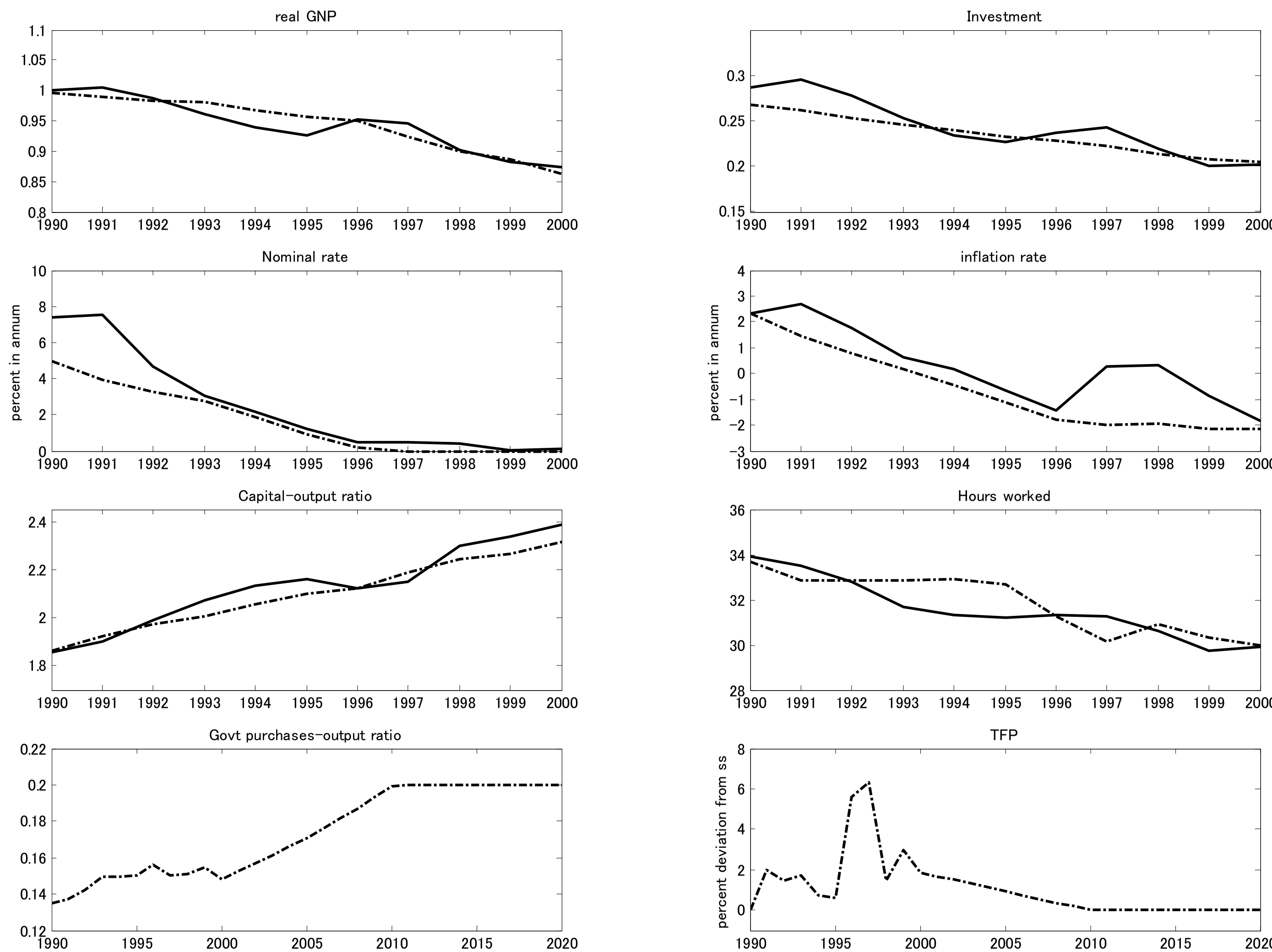
[Insert Figure 2 here.]

* The solid line shows Japanese data and the dashed line shows the simulation results except for the last two figures. The last two figures show the path of time-varying exogenous variables.

** Real GNP, investment, hours worked and government purchases are expressed by per capita terms.

*** Real GNP and investment are detrended by the 2% trend growth and are normalized by 1990 Japanese real GNP.

**** Data Source: Hayashi and Prescott (2002) and Bank of Japan web page.



real GNP
Investment
Nominal rate
percent in annum
inflation rate
percent in annum
Capital−output ratio
Hours worked
Govt purchases−output ratio
TFP
percent deviation from ss