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Abstract

This paper introduces a new concept of market mechanism design into general economic environments with finite but many traders, where multiple objects are traded and any combination of complements and substitutes is permitted. The auctioneer randomly divides traders into multiple groups. Within each group, trades occur at the market-clearing price vector of another group. With private values, any undominated strategy profile mimics price-taking behavior, enforcing perfect competition. With interdependent values, any twice iteratively undominated strategy profile mimics the rational expectations equilibrium, enforcing ex post efficiency. Our mechanisms are detail-free, i.e., they do not depend on the details of model specification.

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1. Introduction

The hypothesis of perfect competition assumes that traders are non-strategic and adopt price-taking behavior. In order to provide a strategic foundation to the hypothesis, several studies after Wilson (1977) investigated standard models with private values and a single object. These include double auctions where there exist finite but many traders who announce supplies and demands; the auctioneer calculates the price at which the announced supplies and demands are balanced, and trades occur at this market-clearing price. Clearly, in the case of a continuum of traders, every trader's dominant strategy is to behave as a price taker because her announcement does not influence her trading price. However, when the number of traders is finite, each trader may be able to manipulate the market-clearing price for her benefit. In fact, in this case, none of the traders adopts a dominant strategy of behaving as a price taker. Hence, several studies such as Rustichini, Satterthwaite, and Williams (1994), Fudenberg, Mobius, and Szeidl (2003), and Jackson and Swinkels (2004) replaced the dominant strategy with the Bayesian Nash equilibrium,[1] thereby clarifying that traders are involved in a complicated strategic interaction that relies heavily on a strong knowledge assumption of their rational behavior.

Instead of using the standard models, this paper introduces a new concept for designing a market mechanism with finite but many traders. The first part of this paper demonstrates that in the *private* value case, even if the number of traders is finite, *the dominant strategy mimics price-taking behavior, and the auctioneer almost certainly achieves the approximate competitive equilibrium allocation*. The basic idea of our mechanism design is as follows. In contrast to the standard models, the auctioneer makes stochastic decisions and sets multiple prices. The auctioneer randomly divides the traders into two groups and deals with each group as a *separate* group. The auctioneer calculates the price at which the demands and supplies announced by the traders in each group are balanced, i.e., the market-clearing price within each group. It should be noted that in order to enforce trades within a group, the auctioneer does not use the market-clearing price of the same group but that of *the other group*. Hence, a trader's announcement does not influence her trading price, which is the force that derives the price-taking behavior to be dominant. From the law of large numbers, it is clear that when the number of traders is sufficiently large, the market-clearing price in each group almost certainly mimics the unified market-clearing price, at which all the demands and supplies by all the traders are balanced; in other words, the market-clearing price in each group mimics the competitive equilibrium price.

The basic concept of *random grouping* proposed here has very high potential to achieve approximate efficiency in general economic environments. For instance, it enables

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[1] Rustichini, Satterthwaite, and Williams (1994) investigated the symmetric Bayesian Nash equilibrium in the case of independent private signals with single-unit supplies and demands. Fudenberg, Mobius, and Szeidl (2003) investigated the symmetric Bayesian Nash equilibrium in the case of correlated private signals with single-unit supplies and demands. Jackson and Swinkels (2004) investigated the mixed strategy Bayesian Nash equilibrium in a variety of multi-unit double auctions with private values.

us to consider the trading of multiple commodities, where each trader demands multiple units for each commodity. Through a careful design of rationing rules, we can also allow each trader any combination of substitutes and complements.

The second part of this paper deals with the *interdependent* value case, where a trader's payoff depends not only on her private signal but also on the private signals of other traders. The standard analysis of perfectly competitive markets assumes that each trader utilizes not only her private signal but also any other available information, such as the market price, and examines the rational expectation equilibrium.[2] Each trader updates her belief based on the market price and a forecast function that maps private signal profiles to trading prices, and then maximizes the updated expected utility as a non-strategic price taker. The rational expectation equilibrium depends critically on the assumption that traders have knowledge of this forecast function, which is supported as a rule through the assumption that trading behavior is common knowledge among traders. This makes it impossible to regard price-taking behavior as a dominant strategy. In fact, irrespective of whether or not a continuum of traders exists, price-taking behavior is no more than a Bayesian Nash equilibrium where traders are involved in a more complicated strategic interaction with interdependent values rather than private values.[3]

Instead of using the forecast function as a tool for information transmission, the second part of this paper will propose the design of a new trading procedure. In this procedure, each trader announces her demand or supply *three times*; she first observes a part of the other traders' first announcements and updates her belief before finally announcing her second and third announcements. The concept of random grouping also plays a very significant role in the interdependent value case, where the auctioneer randomly divides traders into multiple groups. In this case, it is assumed that not only the number of the traders belonging to each group but also the number of groups is sufficiently large. The auctioneer calculates the market-clearing price of each group, at which the supplies and demands of the traders in the group based on their *second* announcements are balanced. In order to enforce trades within each group, the auctioneer uses the *third* announcements of the traders in the group together with the market-clearing price of *the precedent group*.

The solution concept we use is a *twice iteratively undominated strategy*, which is defined by eliminating the dominated strategies for each trader only *twice*. In this case, we require only a very weak knowledge assumption regarding the rationale that each trader expects other traders not to employ strategies that are dominated, i.e., are deleted just by the *first* round of iterative removals. The traders' rationale is not required to be common knowledge. This paper demonstrates that the designed procedure succeeds in information transmission. *Any twice iteratively undominated strategy profile mimics price-taking behavior and almost certainly enforces approximate rational expectation equilibrium*

[2] See Allen (1986), and Mas-Colell, Whinston, and Green (1992, Chapter 19.H)).

[3] In order to provide a strategic foundation for the rational expectation equilibrium with finite traders, Reny and Perry (2003) investigated Bayesian Nash equilibria in standard double auction models with single-object and single-unit, where traders' private signals are assumed to be strictly affiliated.

allocations, i.e., enforces approximate ex post efficiency. This permissive result holds in a wide class of economic environments that requires only minor restrictions on the private signal structure, where trading in multiple objects is permitted, and every trader is allowed any combination of complements and substitutes.

It is of particular importance that our designed mechanisms are *detail-free,* i.e., they do not depend on the details of model specification such as the utility functions of traders, their private signal spaces, and the prior distribution. This is in contrast with previous studies on mechanism design such as d'Aspremont and Gererd-Varet (1979), Myerson (1981), Myerson and Satterthwaite (1983), and Cremer and McLean (1988). These studies on mechanism design assumed that the auctioneer or the central planner possessed complete knowledge of the fine details of model specification upon which the designed mechanisms crucially depended. As a criticism of the study of mechanism design, Hurwicz (1972), Wilson (1985, 1987), and Dasgupta and Maskin (2000) have indicated that this assumption makes the mechanisms difficult to put into practice. Hence, the first step toward a practically useful theory is to formulate a method to design mechanisms that are detail-free but well-behaved.

From the experimental viewpoint, it is well-known that the concept of twice iterative dominance, which is utilized in the latter part of this paper, is significantly much effective in predicting real human behavior as compared to any concept based on three or more rounds of the iterative removal of dominated strategies. For instance, refer to the experimental research by Cost-Comes, Crawford, and Broseta (2001).

This paper is organized as follows. Section 2 describes the basic model. Section 3 investigates the private value case, and Section 4 investigates the interdependent value case.

2. The Model

There exist infinitely many agents $i \in \{1,2,...\}$ and k distinct commodities to be traded. Each agent $i \in \{1,2,...\}$ has an initial endowment vector denoted by

$$e_i = (e_i(h))_{h=1}^{k} \in \{0,...,l-1\}^k,$$

where $l \geq 2$, and for each $h \in \{1,...,k\}$, agent i possesses an amount $e_i(h)$ of commodity h. The first nr agents participate in the trading procedure as *traders*, where $n \geq 2$ and $r \geq 2$ are positive integers, and nr is sufficiently large so that it satisfies

$$nr \geq l.$$

We assume that $e = (e_i)_{i=1}^{nr}$ is known to the auctioneer and is verifiable to the court. We also assume that

(1) $$\sum_{i=1}^{nr} e_i(h) \geq l \quad \text{for all} \quad h \in \{1,...,k\}.$$

Let $P = \{0, \frac{1}{T}, ..., 1\}^k$ denote the finite set of price vectors, where T is a positive integer. Here, the existence of positive price grid $\frac{1}{T} > 0$ is irrelevant to our argument. In fact, we can choose T as large as possible. An *allocation* is defined as the combination $a = (x, q) \in \{0,...,l\}^{knr} \times P^{nr}$, where

$$x = (x_i)_{i=1}^{nr}, \quad x_i = (x_i(h))_{h=1}^{k}, \quad x_i(h) \in \{0,...,l\},$$

$$q = (q_i)_{i=1}^{nr}, \quad q_i = (q_i(h))_{h=1}^{k} \in P, \quad q_i(h) \in \{0, \frac{1}{T}, ..., 1\},$$

(2) $$\sum_{i=1}^{nr} \{e_i(h) - x_i(h)\} = 0 \quad \text{for all} \quad h \in \{1,...,k\},$$

and

(3) $$\sum_{i=1}^{nr} \sum_{h=1}^{k} q_i(h)\{x_i(h) - e_i(h)\} = 0.$$

Trader i sells and buys each commodity $h \in \{1,...,k\}$ at a price $q_i(h)$ and receives an amount $x_i(h)$ of each commodity h. The total monetary amount received is given by

$$\sum_{h=1}^{k} q_i(h)\{e_i(h) - x_i(h)\}.$$

The inequalities (2) imply *market-clearing* in the sense that for each commodity, the total amount allocated to traders and the total supply are equalized.[4] Inequality (3) implies *budget-balancing*. Each trader possesses an initial endowment of each commodity that is less than or equal to $l-1$, whereas she can consume up to amount l, i.e.,

$$e_i(h) \leq l-1 \quad \text{and} \quad x_i(h) \leq l.$$

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[4] Hence, $e_i - x_i$ represents the *excess* demand vector for trader i.

Here, the existence of the upper bounds described by l is irrelevant to our argument. In fact, we can choose l as large as possible. Let $A \subset \{0,...,l\}^{knr} \times \{0, \frac{1}{T},...,1\}^{knr}$ denote the set of allocations satisfying inequalities (2) and (3), and let $\Delta(A)$ denote the set of lotteries over allocations.

A *mechanism* is defined by $G = (M, g)$, where $M = \prod_{i=1}^{nr} M_i$, $g : M \to \Delta(A)$, and M_i is the set of messages for trader $i \in \{1,...,nr\}$. Let D denote the set of *demand functions* $d = (d(h))_{h=1}^{k} : P \to \{0,...,l\}^k$. Let Φ denote the set of permutations over traders $\phi : \{1,...,nr\} \to \{1,...,nr\}$. $\varepsilon \in (0, \frac{1}{3})$ is set arbitrarily to a value that is positive but close to zero. Set a function $\hat{p} : D^n \times \{0,...,nl\}^k \to P$, where $y = (y(h))_{h=1}^{k} \in \{0,...,nl\}^k$ and

$$\hat{p}((d_i)_{i=1}^{n}, y) \in \arg\min_{p \in P} \sum_{h=1}^{k} \left| \sum_{i=1}^{n} d_i(h)(p) - y(h) \right|,$$

which approximates the *market-clearing price vector* among n traders when their demand functions and the total supply vector are given by $(d_i)_{i=1}^{n}$ and y, respectively.

3. Private Values

This section investigates the *private* value case, where none of the traders possesses any private information about the other traders' preferences. Trader $i's$ utility function is given by $u_i : A \to R$. Let $u_i(\alpha) = \sum_{a \in A} u_i(a)\alpha(a)$ for all $\alpha \in \Delta(A)$, where the expected utility hypothesis is assumed. This section assumes that the first $2n$ agents participate in the trading procedure as traders, i.e.,

$$r = 2.$$

The auctioneer determines an allocation according to the following trading procedure, which consists of three stages.

Stage 0: The auctioneer *randomly* selects a combination of a permutation $\phi \in \Phi$ and a price vector $p \in P$ with probability $\frac{1}{(2n)!(T+1)^k}$. The auctioneer divides the traders into *two groups*, namely group 1 and group 2, where n traders $\phi(1)$, …, $\phi(n)$ belong to group 1, and the remaining n traders $\phi(n+1)$, …, $\phi(2n)$ belong to group 2. No trader observes (ϕ, p).

Stage 1: Each trader i announces a demand function $d_i \in D$.

Stage 2: For each $\beta \in \{1,2\}$ and $p' \in P$, let $\bar{n} = \bar{n}(\beta, p')$ denote the maximal integer $n' \in \{0,...,n\}$ such that

$$\sum_{j=(\beta-1)n+1}^{\beta n} e_{\phi(j)}(h) - \sum_{j=(\beta-1)n+1}^{(\beta-1)n+n'-1} d_{\phi(j)}(h)(p') \geq l \quad \text{for all} \quad h \in \{1,...,k\}.$$ [5]

Let $\underline{n} = \underline{n}(\beta, p')$ denote the minimal integer $n' \in \{1,...,n\}$ such that

$$\sum_{j=(\beta-1)n+1}^{\beta n} e_{\phi(j)}(h) - \sum_{j=(\beta-1)n+1}^{(\beta-1)n+n'-1} d_{\phi(j)}(h)(p') > (n-n')l \quad \text{for some} \quad h \in \{1,...,k\}.$$

Let

(4) $$\tilde{n} = \tilde{n}(\beta, p') \equiv \min[\bar{n}(\beta, p'), \underline{n}(\beta, p') - 1].$$

After the first $\tilde{n}$ traders in group β receive the same amounts as their announced demands, it happens for some commodity that the remaining part of the total supply is less than l or more than $(n - \tilde{n} - 1)l$. It should be noted that if the remaining part of the total supply is more than $(n - \tilde{n} - 1)l$ and the demand announced by the $(\tilde{n}+1)-th$ trader,

[5] We denote $\beta = 2$ as $\beta + 1 = 1$ and $\beta = 1$ as $\beta - 1 = 2$.

$d_{\phi((\beta-1)n+\tilde{n}+1)}(p')$, is replaced with zero, then the remaining part of the total supply cannot be allocated to the remaining $n-\tilde{n}-1$ traders with no remainder.

With probability $1-\varepsilon$, the auctioneer determines $a=(x,q)$ according to Rationing Rule 1 described below.

> **Rationing Rule 1:** $\beta \in \{1,2\}$ is set arbitrarily. The auctioneer buys all initial endowments possessed by the traders in group β at the price vector $\hat{p}(d_{group\beta-1}, y_{group\beta-1})$, where $d_{group\beta-1}$ is the profile of demand functions announced in group β, and $y_{group\beta-1}$ is the total supply vector in group β, i.e.,
>
> $$d_{group\beta-1} \equiv (d_{\phi(i)})_{i=(\beta-2)n+1}^{(\beta-1)n} \in D^n \text{ and } y_{group\beta-1} = \sum_{i=(\beta-2)n+1}^{(\beta-1)n} e_{\phi(i)} \in \{0,...,nl\}^k .$$
>
> Denote $\tilde{n} = \tilde{n}(\beta, \hat{p}(d_{group\beta-1}, y_{group\beta-1}))$. The auctioneer sells
>
> $$x_i = d_i(\hat{p}(d_{group\beta-1}, y_{group\beta-1}))$$
>
> to *the first* $\tilde{n}$ *traders* $i=\phi(j)$ *in group* β, where
>
> $$(\beta-1)n+1 \le j \le (\beta-1)n+\tilde{n} .$$
>
> Recursively, for every $j \in \{(\beta-1)n+\tilde{n}+1,...,\beta n\}$, the auctioneer sells amount
>
> $$x_i(h) = \min[l, \sum_{j'=(\beta-1)n+1}^{\beta n} e_{\phi(j')}(h) - \sum_{j'=(\beta-1)n+1}^{j-1} x_{\phi(j')}(h)]$$
>
> of each commodity h to trader $i=\phi(j)$. Here, the auctioneer sets the price vector q_i equal to $\hat{p}(d_{group\beta-1}, y_{group\beta-1})$ for each trader i in group β. The allocation determined according to Rationing Rule 1 is denoted by
>
> $$a = a(\phi, p, (d_i)_{i=1}^{2n}, 1) .$$

With probability ε, the auctioneer determines $a=(x,q)$ according to Rationing Rule 2 described below. The auctioneer sets the trading price vector equal to p in common with all traders. Only the first among the traders receives the same amounts as her announced demands. The rest of the total supply is allocated to the remaining $2n-1$ traders with no remainder.

> **Rationing Rule 2**: The auctioneer buys all the initial endowments possessed by the traders at the price vector p. The auctioneer sells
>
> $$x_{\phi(1)} = d_{\phi(1)}(p)$$
>
> to *the first among the traders*, i.e., trader $\phi(1)$. Recursively, for every $j \in \{2,..,2n\}$, the auctioneer sells amount
>
> $$x_i(h) = \min[l, \sum_{j'=(\beta-1)n+1}^{\beta n} e_{\phi(j')}(h) - \sum_{j'=(\beta-1)n+1}^{j-1} x_{\phi(j')}(h)]$$
>
> of each commodity h to trader $i=\phi(j)$. In this case, the auctioneer sets the

price vector q_i equal to p for all the traders. The allocation determined according to Rationing Rule 2 is denoted by

$$a = a(\phi, p, (d_i)_{i=1}^{2n}, 2).$$

From inequality (1), $2n \geq l$, $e_i(h) \leq l-1$, and $x_i(h) \leq l$, it is clear that $a(\phi, p, (d_i)_{i=1}^{2n}, 2)$ is well-defined.

Example: Let $n = 4$, $k = 1$, and $l = 3$. Consider group 1, which includes four traders, i.e., traders $\phi(1),...,\phi(4)$. Suppose that the initial endowments of the traders in group 1 are given by

$$e_{\phi(1)} = e_{\phi(2)} = e_{\phi(3)} = e_{\phi(4)} = 2.$$

Then, the total supply in group 1 is 8. Suppose

$$d_{\phi(1)}(\hat{p}(d_{group2}, e_{group2})) = 1,$$
$$d_{\phi(2)}(\hat{p}(d_{group2}, e_{group2})) = d_{\phi(3)}(\hat{p}(d_{group2}, e_{group2})) = 2$$
$$d_{\phi(4)}(\hat{p}(d_{group2}, e_{group2})) = 3$$

and therefore, the total demand in group 1 is 8, which equals the total supply. Hence, we have $\overline{n} = 4$. However, in accordance with Rationing Rule 1, only traders $\phi(1)$ and $\phi(2)$ receive the same amounts as their announced demands, i.e., $\tilde{n} = 2$. If the demand announced by trader $\phi(3)$, which is 2, is replaced with zero, then the remaining part of the total supply $8-1-2=5$ is greater than $l = 3$, i.e., the upper bound of trader $\phi(4)'s$ demand. This implies $\underline{n} = 3$, and therefore, we have $\tilde{n} = 2$. Hence, according to Rationing Rule 1, the auctioneer decides

$$x_{\phi(1)} = 1,\ x_{\phi(2)} = 2,\ x_{\phi(3)} = 3, \text{ and } x_{\phi(4)} = 2.$$

The role of the requirement that the remaining part of the total supply be allocated to the remaining traders with no remainder, i.e., $\tilde{n} \leq \underline{n} - 1$, is important for inducing traders to play price-taking behavior as being dominant. This example shows that this requirement suffers efficiency losses. Fortunately, when the number of traders is sufficiently large and the trading price approximately equalizes the total demand and supply, this requirement hardly makes a severe distortion. In fact, it is almost certain that both the proportions $\frac{\overline{n}}{n}$ and $\frac{\underline{n}}{n}$ are close to unity, i.e., most traders can receive the same amount as their announced demands.

Based on the above procedure, we specify a mechanism $G^* = (M, g)$ by

$$M_i = D \text{ for all } i \in \{1,...,2n\},$$

and

$$g(m)(a)=\rho_1\frac{1-\varepsilon}{(2n)!(T+1)^k}+\rho_2\frac{\varepsilon}{(2n)!(T+1)^k} \quad \text{for all } m\in M \text{ and } a\in A,$$

where, for each $\gamma=\{1,2\}$, ρ_γ is the number of (ϕ,p) such that

$$a=a(\phi,p,m,\gamma).$$

Since $\varepsilon>0$ is close to zero, it can be stated that the auctioneer almost certainly selects an allocation according to Rationing Rule 1; in other words, the auctioneer almost certainly selects $a(\phi,p,m,1)$.

It is particularly noteworthy that the designed mechanism G^* is *detail-free* in the sense that it does not depend on the details of model specification such as the utility functions of the traders (However, it depends on the profile of the initial endowment vectors of traders, i.e., $(e_i)_{i=1}^{2n}$.).

From the specification of G^*, it is clear that the following properties hold.

(i) The message m_i of every trader i does not influence the determination of her trading price q_i.

(ii) Each trader i receives either the same amount vector as her announced demand vector, i.e., $m_i(q_i)$, or an amount vector that is determined independent of her message m_i.

(iii) The probability of each trader i receiving $m_i(q_i)$ is independent of her message m_i. These properties are the driving force for price-taking behaviors being the dominant strategy.

We define $M_i^*\subset M_i$ as the set of *undominated* strategies m_i for trader i where there exists no $m_i'\in M_i$ such that

$$u_i(g(m))<u_i(g(m_i',m_{-i})) \quad \text{for all } m_{-i}\in M_{-i}.$$

We assume that there exists a function $v_i:\{0,...,l\}^k\times R\to R$ such that

$$u_i(a)=v_i(x_i,\sum_{h=1}^{k}q_i(h)\{e_i(h)-x_i(h)\}),$$

which implies that all commodities are *private goods*. Here, we do not need any restriction on the shape of the function v_i. For instance, we can allow any mixture of substitutes and complements for each trader. We define $D_i^*\subset D$ as the set of demand functions d such that for every $p\in P$,

$$d(p)\in\arg\max_{x_i} v_i(x_i,\sum_{h=1}^{k}p(h)\{e_i(h)-x_i(h)\}),$$

which implies that trader i behaves as a price taker. The following theorem shows that any undominated strategy mimics price-taking behavior, i.e., price-taking behavior is regarded as being dominant.

Theorem 1: *For every* $i \in N$,

$$M_i^* = D_i^* .$$

Proof: From properties (i), (ii), and (iii), it follows that irrespective of the messages announced by the other traders, trader $i's$ utility maximization implies the maximization of

$$v_i(m_i(q_i), \sum_{h=1}^{k} q_i(h)\{e_i(h) - m_i(h)(q_i)\})$$

with respect to $m_i(q_i)$ for all $q_i \in P$. Hence, we have proved that $M_i^* = D_i^*$.

Q.E.D.

We now demonstrate that when the number of traders, $2n$, is sufficiently large, any undominated strategy profile almost certainly induces an approximate competitive equilibrium allocation. We denote $T = T^{(n)}$, $\varepsilon = \varepsilon^{(n)}$, $\hat{p} = \hat{p}^{(n)}$, $G^* = G^{*(n)}$, and so on. We assume that

$$\lim_{n\to\infty} (T^{(n)}, \varepsilon^{(n)}) = (\infty, 0) ,$$

which implies that the price grid and the probability of Rationing Rule 2 being applicable converge at zero in the limit as n increases. We assume that there exists $\hat{p} \in [0,1]^k$ such that for every infinite sequence of demand function profiles $(d_i^{(2)})_{i=1}^{4}, (d_i^{(3)})_{i=1}^{6}, \dots$, if

$$d_i^{(n)} \in D_i^* \text{ for all } n = 2,3,\dots \text{ and all } i \in \{1,\dots,2n\} ,$$

then

$$\lim_{n\to\infty} \hat{p}^{(2n)}((d_i^{(n)}, e_i)_{i=1}^{2n}) = \hat{p} .$$

This implies that $\hat{p}$ approximates the *competitive equilibrium* price vector when the number of traders is sufficiently large and the traders behave as price takers. An infinite sequence of undominated strategy profiles, $m^{(2)}, m^{(3)}, \dots$, is set arbitrarily, where Theorem 1 implies

$$m_i^{(n)} \in D_i^* \text{ for all } n = 2,3,\dots \text{ and all } i \in \{1,\dots,2n\} .$$

It is clear from the law of large numbers that for every sufficiently large n, the auctioneer almost certainly selects ϕ such that both $\hat{p}^{(n)}(d_{group1}, e_{group1})$ and $\hat{p}^{(n)}(d_{group2}, e_{group2})$ are approximated by the unified market-clearing price vector $\hat{p}^{(2n)}((d_i^{(n)}, e_i)_{i=1}^{2n})$ and therefore approximated by $\hat{p}$, where $d_i = m_i^{(n)}$. The specification of Rationing Rule 1 implies that whenever n is sufficiently large, $\dfrac{\tilde{n}(\beta, \hat{p}^{(n)}(d_{group\beta-1}, e_{group\beta-1}))}{n}$ is approximated by 1; therefore, each trader almost certainly receives the same amount vector as her announced demand vector. Hence, we conclude that when the number of traders is sufficiently large, any undominated strategy profile almost certainly induces an approximate competitive equilibrium allocation. It is clear from this property that for every sufficiently large n, *individual rationality* typically holds in that

$$u_i(g(m)) > v_i(e_i, 0) \quad \text{for all } i \text{ and all } m \in M^*,$$

where $v_i(e_i, 0)$ denotes trader $i's$ outside opportunity.

Remark 1: As shown in this section, *stochastic* decisions play a significant role, particularly in *economic* environments with *no* public goods. In contrast to our permissive result, Barberà and Jackson (1995) have shown that in economic environments with no public goods, strategy-proof social choice functions are never efficient, even in the limit as the number of agents increases. Their negative result relies crucially on the restriction that stochastic decisions were excluded. Gibbard (1977) and Benoit (2002) investigated *general* social choice environments, where there are *no* strategy-proof non-trivial stochastic social choice functions.

Remark 2: In general environments with quasi-linear preferences, there is a celebrating work by Groves (1973) that designed the so-called the Groves mechanisms, where truth-telling is a dominant strategy and achieves efficiency. Groves' work suffers from the drawback that the Groves mechanisms do *not* satisfy budget-balancing;[6] in contrast, our mechanism does.

McAfee (1992) proposed an alternative concept of double auction design, in which budgetary deficit does not occur. McAfee's analysis relies crucially on the assumption that each buyer (seller) has only *single-unit* demand (supply). In contrast, our concept of random grouping can be applied to extremely general cases with multiple objects and multi-unit demands and supplies, where each trader is allowed any combination of complements and substitutes.

Remark 3: In contrast to the uniform price auction, our mechanism G^* fails to conform to *the law of one price*, i.e., the auctioneer may assign vastly different price vectors to different groups. As Milgrom (2004, Chapter 7) explains, from a practical viepoint, non-conformance to the law of one price is generally regarded as a disadvantage of mechanism design.

However, this problem can be solved by modifying the specification of G^* as

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[6] See also Vickery (1961) and Clarke (1971). Vickery (1961) studied dominant strategy auctions, where only buyers are players, and it is assumed that the seller sells a commodity irrespective of the trading price determined. This assumption contradicts individual rationality.

follows. Instead of two groups, the auctioneer divides the traders into a sufficiently large number of groups. Trades occur within each group at a price vector that practically equalizes the total demands and supplies announced by the traders outside the group. It should be noted that this trading price vector always approximates the unified market-clearing price vector because the total demands and supplies within any single group are trivial when compared to the total demands and supplies of the traders outside the group. This implies that it is *certain* that the trading price vector is practically identical across all traders.

4. Interdependent Values

This section investigates the *interdependent* value case, where each trader observes a *private signal* ω_i and her utility depends not only on her private signal but also on the private signals of the other traders. Let Ω_i denote the finite set of trader $i's$ possible private signals. We assume that

$$\Omega_i = \Omega_1 \text{ for all } i \in \{2,...,nr\}.$$

The probability of the occurrence of a private signal profile $\omega = (\omega_i)_{i=1}^{nr}$ is given by $f(\omega) > 0$. There exists a *macro shock* ω_0 that is *unobservable* to all traders and is randomly determined according to the probability function $f_0 : \Omega_0 \to (0,1]$, where Ω_0 is the finite set of macro shocks. The traders' private signals are correlated through the macro shock, i.e., there exist $f(\cdot \mid \omega_0) : \Omega_1 \to (0,1]$ for all $\omega_0 \in \Omega_0$ such that

$$f(\omega) = \sum_{\omega_0} f_0(\omega_0) \prod_{i=1}^{nr} f(\omega_i \mid \omega_0).$$

Each trader $i's$ utility function is given by $u_i : A \times \Omega_0 \times \Omega_i \to R$, where her utility depends on the macro shock as well as her private signal. Since the private signals of traders are correlated through this macro shock, $u_i(a, \omega) \equiv E[u_i(a, \omega_0, \omega_i) \mid \omega]$ depends on $\omega_{-i} = (\omega_1,..., \omega_{i-1}, \omega_{i+1},..., \omega_{nr})$, which implies interdependent values.

This section assumes that the number of groups is three or more, i.e.,

$$r \geq 3.$$

The auctioneer will determine an allocation according to the following procedure, which consists of five stages.

Stage 0: The auctioneer randomly selects (ϕ, p) with probability $\frac{1}{(nr)!(T+1)^k}$. She divides the traders into r groups, $1,...,r$, where for each $\beta \in \{1,...,r\}$, n traders, $\phi((\beta-1)n+1)$, …, $\phi(\beta n)$, belong to group β. No trader observes (ϕ, p).

Stage 1: Each trader i announces a demand function $d_i^1 \in D$ as her *first* announcement. For each $\beta \in \{1,...,r\}$, every trader in group β observes the first announcements by the traders in the *precedent* group $\beta - 1$, i.e., she observes

$$d^1_{group\beta-1} \equiv (d^1_{\phi(i)})_{i=(\beta-2)n+1}^{(\beta-1)n} \in D^n.$$ [7]

[7] We denote $\beta + 1 = 1$ for $\beta = r$, and $\beta - 1 = r$ for $\beta = 1$.

Stage 2: Each trader i announces a demand function $d_i^2 \in D$ as her *second* announcement. After the second announcements are made, every trader in each group $\beta \in \{1,...,r\}$ observes the *first* announcements by the traders who do not belong to group β, i.e., she observes

$$d^1_{-group\beta} \equiv (d^1_{group1},...,d^1_{group\beta-1},d^1_{group\beta+1},...,d^1_{groupr}) \in D^{(r-1)n}.$$ [8]

Stage 3: Each trader i announces a demand function $d_i^3 \in D$ as her *third* announcement.

Stage 4: For each $\beta \in \{1,...,r\}$ and $p' \in P$, let $\overline{n} = \overline{n}(\beta, p')$ denote the maximal integer $n' \in \{0,...,n\}$ such that

$$l \leq \sum_{j=(\beta-1)n+1}^{\beta n} e_{\phi(j)}(h) - \sum_{j=(\beta-1)n+1}^{(\beta-1)n+n'-1} d^3_{\phi(j)}(h)(p') \geq l \quad \text{for all} \quad h \in \{1,...,k\}.$$

Let $\underline{n} = \underline{n}(\beta, p')$ denote the minimal integer $n' \in \{1,...,n\}$ such that

$$\sum_{j=(\beta-1)n+1}^{\beta n} e_{\phi(j)}(h) - \sum_{j=(\beta-1)n+1}^{(\beta-1)n+n'-1} d^3_{\phi(j)}(h)(p') > (n-n')l \quad \text{for some} \quad h \in \{1,...,k\}.$$

Let

$$\tilde{n} = \tilde{n}(\beta, p') \equiv \min[\overline{n}(\beta, p'), \underline{n}(\beta, p') - 1].$$

This definition corresponds to (4) in the private value case, where we replace $d_{\phi(j)}$ with $d^3_{\phi(j)}$.

With probability $1-3\varepsilon$, the auctioneer determines $a = (x, q)$ according to Rationing Rule 3 given below, which corresponds to Rationing Rule 1 in the private value case. To calculate the market-clearing price vector in each group, the auctioneer uses the *second* announcements. To determine the allocation within each group β, the auctioneer uses the *third* announcements in each group together with the calculated market-clearing price vector in the *precedent* group, $\beta - 1$.

> **Rationing Rule 3:** $\beta \in \{1,...,r\}$ is set arbitrarily. The auctioneer buys all initial endowments possessed by the traders in group β at the price vector $\hat{p}(d^2_{group\beta-1}, y_{group\beta-1})$. We denote $\tilde{n} = \tilde{n}(\beta, \hat{p}(d^2_{group\beta-1}, y_{group\beta-1}))$. The auctioneer sells
>
> $$x_i = d_i^3(\hat{p}(d^2_{group\beta-1}, y_{group\beta-1}))$$
>
> to the first $\tilde{n}$ traders $i = \phi(j)$ in group β, where

---

[8] It should be noted that any trader can not observe the announcements by the traders in the same group. This is the force that simplifies the strategic relationship among the traders.

$(\beta-1)n+1\leq j\leq(\beta-1)n+\tilde{n}$.

Recursively, for every $j\in\{(\beta-1)n+\tilde{n}+1,...,\beta n\}$, the auctioneer sells amount

$$x_i(h)=\min[l,\sum_{j'=(\beta-1)n+1}^{\beta n}e_{\phi(j')}(h)-\sum_{j'=(\beta-1)n+1}^{j-1}x_{\phi(j')}(h)]$$

of each commodity h to trader $i=\phi(j)$. In this case, the auctioneer sets the price vector q_i equal to $\hat{p}(d^2_{group\beta-1},y_{group\beta-1})$ for each trader i in group β. The allocation determined according to Rationing Rule 3 is denoted by

$$a=a(\phi,p,(d_i^1,d_i^2,d_i^3)_{i=1}^{nr},3).$$

With probability ε, the auctioneer determines $a=(x,q)$ according to Rationing Rule 4 given below, which corresponds to Rationing Rule 2 in the private value case, where the auctioneer uses the *first* announcements to determine the allocation.

Rationing Rule 4: The auctioneer buys all initial endowments possessed by the traders at the price vector p. The auctioneer sells

$$x_{\phi(1)}=d^1_{\phi(1)}(p)$$

to trader $\phi(1)$. Recursively, for every $j\in\{2,..,nr\}$, the auctioneer sells amount

$$x_i(h)=\min[l,\sum_{j'=(\beta-1)n+1}^{\beta n}e_{\phi(j')}(h)-\sum_{j'=(\beta-1)n+1}^{j-1}x_{\phi(j')}(h)]$$

of each commodity h to trader $i=\phi(j)$. In this case, the auctioneer sets the price vector q_i equal to p for all traders. The allocation determined according to Rationing Rule 4 is denoted by

$$a=a(\phi,p,(d_i^1,d_i^2,d_i^3)_{i=1}^{nr},4).$$

From inequality (1), $nr\geq l$, $e_i(h)\leq l-1$, and $x_i(h)\leq l$, it is clear that $a(\phi,p,(d_i^1,d_i^2,d_i^3)_{i=1}^{nr},4)$ is well-defined.

With probability ε, the auctioneer determines $a=(x,q)$ according to Rationing Rule 5 given below, which corresponds to Rationing Rule 2 in the private value case, where the auctioneer uses the *second* announcements to determine the allocation.

Rationing Rule 5: The auctioneer buys all initial endowments possessed by the traders at the price vector p. The auctioneer sells

$$x_{\phi(1)}=d^2_{\phi(1)}(p)$$

to trader $\phi(1)$. Recursively, for every $j\in\{2,..,nr\}$, the auctioneer sells amount

$$x_i(h)=\min[l,\sum_{j'=(\beta-1)n+1}^{\beta n}e_{\phi(j')}(h)-\sum_{j'=(\beta-1)n+1}^{j-1}x_{\phi(j')}(h)]$$

of each commodity h to trader $i=\phi(j)$. In this case, the auctioneer sets the price vector q_i equal to p for all traders. The allocation determined according to Rationing Rule 5 is denoted by

$$a = a(\phi, p, (d_i^1, d_i^2, d_i^3)_{i=1}^{nr}, 5).$$

From inequality (1), $nr \geq l$, $e_i(h) \leq l-1$, and $x_i(h) \leq l$, it is clear that $a(\phi, p, (d_i^1, d_i^2, d_i^3)_{i=1}^{nr}, 5)$ is well-defined.

With probability ε, the auctioneer determines $a = (x, q)$ according to Rationing Rule 6 given below, which corresponds to Rationing Rule 2 in the private value case, where the auctioneer uses the *third* announcements to determine the allocation.

Rationing Rule 6: The auctioneer buys all initial endowments possessed by the traders at the price vector p. The auctioneer sells

$$x_{\phi(1)} = d_{\phi(1)}^3(p)$$

to trader $\phi(1)$. Recursively, for every $j \in \{2,..,nr\}$, the auctioneer sells amount

$$x_i(h) = \min[e_i(h), \sum_{j'=(\beta-1)n+1}^{\beta n} e_{\phi(j')}(h) - \sum_{j'=(\beta-1)n+1}^{j-1} x_{\phi(j')}(h)]$$

of each commodity h to trader $i = \phi(j)$. In this case, the auctioneer sets the price vector q_i equal to p for all traders. The allocation determined according to Rationing Rule 6 is denoted by

$$a = a(\phi, p, (d_i^1, d_i^2, d_i^3)_{i=1}^{nr}, 6).$$

From inequality (1), $nr \geq l$, $e_i(h) \leq l-1$, and $x_i(h) \leq l$, it is clear that $a(\phi, p, (d_i^1, d_i^2, d_i^3)_{i=1}^{nr}, 6)$ is well-defined.

Based on the above procedure, a mechanism $G^{**} = (M, g)$ is specified by

$$M_i = M_i^1 \times M_i^2 \times M_i^3,$$

$$M_i^1 = D,$$

M_i^2 is the set of functions $m_i^2 : D^n \to D$,

and

M_i^3 is the set of functions $m_i^3 : D^{(r-1)n} \to D$.

For each $m \in M$ and $a \in A$, we specify

$$g(m)(a) = \rho_3 \frac{1-3\varepsilon}{(nr)!(T+1)^k} + (\rho_4 + \rho_5 + \rho_6) \frac{\varepsilon}{(nr)!(T+1)^k}.$$

Here, for each $\gamma \in \{3,4,5,6\}$, ρ_γ is the number of (ϕ, p) such that

$$a = a(\phi, p, (d_i^1, d_i^2, d_i^3)_{i=1}^{nr}, \gamma),$$

where for every $\beta \in \{1,...,r\}$ and $j \in \{(\beta-1)n+1,...,\beta n\}$,

$$d^1_{\phi(j)} = m^1_{\phi(j)},\ \ d^2_{\phi(j)} = m^2_{\phi(j)}(d^1_{group\beta-1}) \text{, and } \ d^3_{\phi(j)} = m^3_i(d^1_{-group\beta}) .$$

Here, m^1_i , $m^2_j(d^1_{group\beta-1})$, and $m^3_i(d^1_{-group\beta})$ denote the first, second, and third announcements of a trader i belonging to group β. The first announcement plays the role of *information transmission* and the second announcement, the role of *trading price vector calculations*. It should be noted that the value of $\varepsilon > 0$ is close to zero; therefore, the auctioneer generally applies Rationing Rule 3. Hence, it follows that the third announcement of a trader plays the role of her demand vector that the auctioneer generally *accepts for trading*.

A *strategy* for trader i is defined by $s_i : \Omega_i \to M_i$. Let S_i denote the set of strategies for trader i and $s = (s_i) \in S = \prod_{i=1}^{nr} S_i$. Trader $i's$ interim expected utility is defined as $u_i(s, \omega_i) = E[u_i(g(s(\omega)), \omega_0, \omega_i) \mid \omega_i]$. We define $S^1_i \subset S_i$ as the set of *undominated* strategies for trader i , i.e., the set of strategies $s_i \in S_i$ satisfying the condition that for each $\omega_i \in \Omega_i$, there exists no $s'_i \in S_i$ such that

$$u_i(s, \omega_i) < u_i(s'_i, s_{-i}, \omega_i) \ \text{ for all } \ s_{-i} \in S_{-i} .$$

We define $S^2_i \subset S_i$ as the set of *twice iteratively undominated strategies* for trader i , i.e., the set of undominated strategies $s_i \in S^1_i$ satisfying the condition that for each $\omega_i \in \Omega_i$, there exists no $s'_i \in S^1_i$ such that

$$u_i(s, \omega_i) < u_i(s'_i, s_{-i}, \omega_i) \ \text{ for all } \ s_{-i} \in S^1_{-i} .$$

In particular, the mechanism G^{**} is *detail-free* in the sense that it does not depend on the details of model specification such as the utility functions of traders, the probability structure of private signals, and macro shocks (However, it depends on the profile of traders' initial endowment vectors.).

Since the number of groups r is three or more, the following properties hold.

(iv) Irrespective of the strategies employed by the other traders, each trader $i's$ message m_i does not influence the determination of her trading price q_i .

(v) When the auctioneer conforms to Rationing Rules 4 (Rationing Rule 5), each trader i in each group β receives either her first- (second-) announced demand vector $m^1_i(q_i)$ $(m^2_i(d^1_{group\beta-1})(q_i))$ or an amount vector that is determined independent of her message m_i . The probability of each trader i receiving $m^1_i(q_i)$ ($m^2_i(d^1_{group\beta-1})(q_i)$) is independent of her message m_i , and the determination of q_i does not include any information about the private signals possessed by other traders (other traders who do not belong to group $\beta - 1$).

(vi) Similar to property (v), when the auctioneer conforms to either Rationing Rules 3 or 6, each trader i in each group β receives either her third-announced demand vector $m_i^3(d_{-group\beta}^1)(q_i)$ or an amount vector that is determined independent of her message m_i. The probability of each trader i receiving $m_i^3(d_{-group\beta}^1)(q_i)$ is independent of her message m_i, and the determination of q_i does not include any information about the private signals possessed by other traders in group β.

(vii) When the auctioneer conforms to Rationing Rules 4 (Rationing Rules 5 or 6), the probability of each trader i receiving $m_i^1(q_i)$ ($m_i^2(d_{group\beta-1}^1)(q_i)$) is independent of the strategies of other traders.

(viii) In contrast to property (vii), when the auctioneer conforms to Rationing Rule 3, the probability of each trader i receiving $m_i^3(d_{-group\beta}^1)(q_i)$ may depend on the strategies employed by the other traders in group β.

Properties (iv), (v), and (vii) are the driving force for price-taking behaviors since the first announcement represents undominated and price-taking behaviors and the second represents a twice iteratively undominated strategy. Property (viii) is slightly problematic because a trader's best third announcement may depend on her expectation of the behavior of other traders. However, we demonstrate that the twice iteratively undominated strategies of the traders mimic price-taking behavior in their third announcements. We assume *symmetry* and *quasi-linearity* in that there exists a function $v:\{0,...,l\}^k \times \Omega_0 \times \Omega_1 \to R$ such that for every $i \in \{1,...,nr\}$,

$$u_i(a,\omega_0,\omega_i) = v(x_i,\omega_0,\omega_i) + \sum_{h=1}^{k} q_i(h)\{e_i(h) - x_i(h)\}.$$

This assumption implies that the sets of undominated strategies and twice iteratively undominated strategies are identical across traders, i.e.,

(5) $$S_i^1 = S_1^1 \text{ and } S_i^2 = S_1^2 \text{ for all } i \in \{1,...,nr\}.$$

We define $D^1(\omega_1) \subset D$ as the set of demand functions $d \in D$ such that

$$d(p) \in \arg\max_{x_1} \{E[v(x_1,\omega_1,\omega_0) \mid \omega_1] - \sum_{h=1}^{k} p(h)x_1(h)\} \text{ for all } p \in P,$$

which implies that trader 1 behaves as a price taker when she has knowledge of only her private signal. The following assumption implies that the price-taking behaviors mentioned above are different for distinct private signals.

Assumption 1: For every $\omega_1 \in \Omega_1$ and $\omega_1' \in \Omega_1 \setminus \{\omega_1\}$,

$$D^1(\omega_1) \cap D^1(\omega_1') = \phi.$$

It follows from Assumption 1 that whenever traders behave as price takers in terms of their first announcements, each trader's first announcement *fully* reveals her private signal.

We define $D^2(\omega_1,...,\omega_{n+1}) \subset D$ as the set of demand functions $d \in D$ such that

$$d(p) \in \arg\max_{x_1} \{E[v(x_1,\omega_1,\omega_0) \mid \omega_1,...,\omega_{n+1}] - \sum_{h=1}^{k} p(h)x_1(h)\} \text{ for all } p \in P,$$

which implies that trader 1 behaves as a price taker when she has knowledge of not only her private signal but also the private signals of n other traders. Moreover, $D^3(\omega_1,...,\omega_{(r-1)n+1}) \subset D$ is defined as the set of demand functions $d \in D$ satisfying the condition that for every $p \in P$ and $x_1 \in \{0,...,l\}^k$, there exists $(\omega_{(r-1)n+2},...,\omega_{nr})$ such that

$$E[v(d(p),\omega_1,\omega_0) \mid \omega] - \sum_{h=1}^{k} p(h)d(p)(h)$$
$$\geq E[v(x_1,\omega_1,\omega_0) \mid \omega] - \sum_{h=1}^{k} p(h)x_1(h),$$

which implies that whenever trader 1 has knowledge of not only her private signal but also the private signals of $(r-1)n$ other traders, she does not announce any demand function that is dominated, *irrespective of the private signals the remaining* $n-1$ *traders receive*. It should be noted that when the number of groups r is sufficiently large, the information included in the private signals $(\omega_{(r-1)n+2},...,\omega_{nr})$ of the $n-1$ traders $(r-1)n+2,\ldots,nr$ is trivial in comparison to the private signals $(\omega_1,...,\omega_{(r-1)n+1})$ of other $(r-1)n+1$ traders. This implies that $D^3(\omega_1,...,\omega_{(r-1)n+1})$ is practically the same as the set of demand functions $d \in D$ such that

$$d(p) \in \arg\max_{x_1} \{E[v(x_1,\omega_1,\omega_0) \mid \omega_1,...,\omega_{nr}] - \sum_{h=1}^{k} p(h)x_1(h)\} \text{ for all } p \in P,$$

which implies that trader 1 behaves as a price taker when she has knowledge of the private signals of all traders. The following theorem, together with equalities (5), shows that for every trader, any undominated strategy mimics price-taking behavior in terms of the first announcement and any twice iteratively undominated strategy mimics price-taking behavior in terms of the second and third announcements.

Theorem 2: *For every* $s_1 \in S_1^1$ *and* $\omega_1 \in \Omega_1$,

$$s_1^1(\omega_1) \in D^1(\omega_1).$$

For every $s_1 \in S_1^2$, $(\omega_1,...,\omega_{n+1}) \in \Omega_1^n$, *and* $(d_2,...,d_{n+1}) \in \prod_{i=2}^{n+1} D^1(\omega_i)$,

$$s_1^2(\omega_1)(d_2,...,d_{n+1}) \in D^2(\omega_1,...,\omega_{n+1}).$$

Moreover, for every $s_1 \in S_1^2$, $(\omega_1,...,\omega_{(r-1)n+1}) \in \Omega_1^{(r-1)n}$, *and* $(d_2,...,d_{(r-1)n+1}) \in \prod_{i=2}^{(r-1)n+1} D^1(\omega_i)$,

$$s_1^3(\omega_1)(d_2,...,d_{(r-1)n+1}) \in D^3(\omega_1,...,\omega_{(r-1)n+1}).$$

Proof: From properties (iv), (v), and (vii), it follows that, if ω_1 is given, then irrespective of the strategies employed by the other traders, trader 1's utility maximization with respect to the first announcement implies the maximization of

$$E[v(x_1,\omega_1,\omega_0) \mid \omega_1] - \sum_{h=1}^{k} q_1(h)x_1(h)$$

with respect to x_1 for all $q_1 \in P$. Hence, we have proved that for every $s_1 \in S_1^1$ and $\omega_1 \in \Omega_1$,

$$s_1^1(\omega_1) \in D^1(\omega_1).$$

From equalities (5), the above holds for every trader. Henceforth, we assume that every trader $i \in \{1,...,nr\}$ employs a strategy s_i such that for every $\omega_i \in \Omega_i$,

$$s_i^1(\omega_i) \in D^1(\omega_i).$$

Assumption 1 implies that prior to making her second announcement, trader 1 has knowledge of all the private signals possessed by the traders belonging to the precedent group $\beta - 1$, under the assumption that trader 1 belongs to group β. From properties (iv), (v), and (vii), it follows that, if ω_1 is given, trader 1's utility maximization with respect to the second announcement implies the maximization of

$$E[v(x_1,\omega_1,\omega_0) \mid \omega_1, \omega_{group\beta-1}] - \sum_{h=1}^{k} q_1(h)x_1(h)$$

with respect to x_1 for all $q_1 \in P$ and $\omega_{group\beta-1} \in \Omega_1^n$. Hence, we have proved that for every $s_1 \in S_1^2$, $(\omega_1,...,\omega_{n+1}) \in \Omega_1^n$, and $(d_2,...,d_{n+1}) \in \prod_{i=2}^{n+1} D^1(\omega_i)$,

$$s_1^2(\omega_1)(d_2,...,d_{n+1}) \in D^2(\omega_1,...,\omega_{n+1}).$$

Assumption 1 also implies that prior to making her third announcement, trader 1 has knowledge of all the private signals possessed by the traders outside group β. From properties (iv), (vi), and (viii), it follows that a necessary condition for trader 1's utility maximization with respect to the third announcement is that, given $(\omega_1, \omega_{-group\beta})$,[9] she does not request x_1 that is dominated by some an x_1'; i.e., for every $(\omega_1,...,\omega_{nr}) \in \Omega_1^{nr}$ that is consistent with $(\omega_1, \omega_{-group\beta})$,

$$E[v(x_1',\omega_1,\omega_0) \mid \omega_1,...,\omega_{nr}] - \sum_{h=1}^{k} q_1(h)x_1'(h)$$

[9] Here, we denote $\omega_{-group\beta} = (\omega_{\phi(1)},...,\omega_{\phi((\beta-1)n)}, \omega_{\phi(\beta n)},...,\omega_{\phi(nr)})$.

$$> E[v(x_1,\omega_1,\omega_0)\,|\,\omega_1,...,\omega_{nr}] - \sum_{h=1}^{k} q_1(h)x_1(h)\,.$$

Hence, we have proved that for every $s_1 \in S_1^2$, $(\omega_1,...,\omega_{(r-1)n+1}) \in \Omega_1^{(r-1)n}$, and $(d_2,...,d_{(r-1)n+1}) \in \prod_{i=2}^{(r-1)n+1} D^1(\omega_i)$,

$$s_1^3(\omega_1)(d_2,...,d_{(r-1)n+1}) \in D^3(\omega_1,...,\omega_{(r-1)n+1})\,.$$

Q.E.D.

We will now demonstrate that when the number of traders in each group n and the number of groups r are sufficiently large, any twice iteratively undominated strategy profile almost certainly induces an approximate *rational expectations equilibrium* allocation and therefore induces approximate *ex post* efficiency. We denote $T = T^{(n)}$, $\varepsilon = \varepsilon^{(n)}$, $r = r^{(n)}$, $\hat{p} = \hat{p}^{(n)}$, $G^{**} = G^{**(n)}$, and so on. It is assumed that

$$\lim_{n\to\infty} (T^{(n)}, \varepsilon^{(n)}, r^{(n)}) = (\infty, 0, \infty)\,,$$

which implies that as n increases in the limit, the price grid converges at zero, the probability of Rationing Rule 3 being applicable converges at unity, and the number of groups diverges into infinity. It is assumed that the probability distribution over private signals is different across macro shocks, i.e., the following assumption is made.

Assumption 2: For every $\omega_0 \in \Omega_0$ and $\omega_0' \in \Omega_0 \setminus \{\omega_0\}$,

$$f(\cdot\,|\,\omega_0) \neq f(\cdot\,|\,\omega_0')\,.$$

Suppose that n is sufficiently large and ω_0 and ω_1 are the realizations of the macro shock and the private signal for trader 1 respectively, where it is assumed that trader 1 belong to a group β. In this case, it is almost certain that for each $\theta \in \Omega_1$, the proportion of the traders i in group β possessing $\omega_i = \theta$ is approximated by $f(\theta\,|\,\omega_0)$. Together with Assumption 2 and Theorem 2, this implies that from the observation of the first announcements in group $\beta - 1$, trader 1 can almost certainly infer with reasonable accuracy the macro shock that actually occurred. This holds true for every trader due to the symmetry across traders. Hence, the price vector determined according to Rationing Rule 3 almost certainly approximates the market-clearing price vector that corresponds to the case where the realization of the macro shock is known to all traders. Since the value of $\varepsilon = \varepsilon^{(n)} > 0$ is close to zero, the auctioneer almost certainly conforms to Rationing Rule 3.

Since the number of groups $r = r^{(n)}$ is sufficiently large, it follows that the information that the private signals of any $n-1$ traders include is *trivial* in comparison to the private signals for the remaining $(r-1)n+1$ traders. This implies that any third announcement included in $D^3(\omega_1,...,\omega_{(r-1)n+1})$ mimics the price-taking behavior associated with full knowledge about the macro shock that actually occurred. Based on the above observations, and along with the same argument as that presented subsequent to the proof of Theorem 1 in Section 3, we conclude that when the number of traders is sufficiently large, the mechanism $G^{**} = G^{**(n)}$ will succeed in information transmission and any twice iteratively undominated strategy profile induces the rational expectations equilibrium allocation, i.e., induces ex post efficiency. From this property, it is clear that for every sufficiently large n, *interim individual rationality* typically holds in that for every $i \in \{1,...,nr\}$ and every $\omega_i \in \Omega_i$,

$$u_i(s,\omega_i) > E[v(e_i,\omega_0,\omega_i) \mid \omega_i] \text{ for all } s \in \prod_{j=1}^{nr} S_j^2 ,$$

where $E[v(e_i,\omega_0,\omega_i) \mid \omega_i]$ represents trader $i's$ outside opportunity when she possesses ω_i.

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