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Loss of Skill and Retraining in a Matching Model

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Abstract

This is a matching model with two types of skills, low and high, where the high skill is acquired through training in schools, the introduction of which is the main innovation of this paper. The search externalities make the competitive equilibrium differ from the social planner's steady state and in order to make the decentralized economy efficient, some public policies are studied along with their viability.

Keywords: Unemployment, Search and Matching, Loss of Skill, Retraining.
JEL Classification: J31, J41.

1 Introduction

Long term unemployment is a big problem in many countries, specially the ones with high unemployment rates. One of the consequences of this long term unemployment is the fact that workers, by not using their skills, tend to lose them and it becomes more difficult for them to find jobs in firms that require that type of skills.

The average unemployment rate of the Euro Zone countries is 10 percent, and nearly half of them (47.4 percent) have been unemployed for more than 12 months. Hence, if unemployment is to be reduced in these countries, it is essential to reduce the percentage of long-term unemployed which are at high risk of loosing their skills and who will be forced to take jobs which are under their original abilities. One way of doing so is by retraining the unemployed in the skills which are demanded by the firms.

There exist some previous literature which addresses economies with workers with heterogenous skills. Using the search and matching framework in the style of Pissarides (1990), Albrecht and Vroman (2002) develop a model with two skills (low and high), where there is also two types of firms which demand workers with either type of skill. In their model the individuals are born with an exogenously determined skill and cannot change it. Within this same environment, this is a problem addressed by Coles and Masters (1998), where workers are born with identical skill and it depreciates over time if they do not find a job. This model however, considers that all the firms are the same and that it is the firm which can train the worker to acquire a higher skill. Similarly, Larsen (2001) also develops a model where workers lose their skill by being unemployed, but they recover it just by working in the less technological advanced sector. Finally, but in a different environment, Acemoglu (1995), studies the effects of certain types of policies to retrain the unemployed workers. Among these policies, he works with subsidies and public training.

This paper tries to model an economy in the style of Pissarides (1990) where all workers are born with a low skill and can acquire the high skill through training in schools, which they meet randomly. The focus is then on the effects of these schools on the composition of low skill and high skill firms and unemployed workers in the economy. Through numerical methods, it is proved, that given the externalities in the model, the competitive equilibrium is not efficient, since it differs from the Social Planners equilibrium. Some policies are then studied to try to make efficient the decentralized economy.

The paper is structured as follows. Section 2 explains the model and states the equations which determine the competitive equilibrium steady state, whereas the Social Planner's derivations are left for Appendix A. Section 3 reports the results of the calibration. Section 4 studies how to make the competitive equilibrium efficient through four types of policies: subsidies to the training of low skill workers, subsidies to the opening of schools, subsidies to the opening of firms and changes in the unemployment benefit. Finally, Section 5 states the conclusions of the paper.

2 The Model

2.1 Environment

This is a continuous-time model in which workers live forever.

All individuals are born with low skill, but they can become high skill through training at a cost z. The training is instantaneous, but it only takes place if the low skill unemployed worker meets with a school.

The high skill individual can loose the skill if it does not use it, and the probability of loosing it follows a poison process with arrival rate λ.

There are two types of firms and one type of school.

High skill firms: They can only hire high skill workers and they produce y_h.

Low skill firms: They can hire both low and high skill workers and they produce $y_l < y_h$.

Schools: They train low skill unemployed workers, for which they receive z and have to pay a cost k to be opened.

Unemployed workers and vacancies meet randomly according to a matching function $m(u, v)$, where $u = u_l + u_h$, u is the total unemployment rate, u_l is the low skill unemployment rate, u_h is the high skill unemployment rate and v is the measure of vacancies. The matching function is assumed to be constant returns to scale, which implies

$$m(u, v) = m\left(1, \frac{v}{u}\right) u = m(\theta) u \text{ where } \theta = \frac{v}{u} \text{ is the market tightness of vacancies.}$$

Hence the arrival rate for firms is $\frac{m(u,v)}{v} = \frac{m(\theta)}{\theta}$. Since high firms can only hire high skill workers, their effective arrival rate is $\frac{m(\theta)}{\theta}(1-\gamma)$, where γ is the proportion of unemployed workers who are low skill. On the other hand low skill firms can hire both high skill and low skill workers, so their arrival rate is $\frac{m(\theta)}{\theta}$.

The arrival rate for workers is $\frac{m(u,v)}{u} = m(\theta)$. Again we must distinguish the arrival rate for low and for high skill workers, since the later can work at either a low or a high skill firm, whereas the former can only work at a low skill firm. Hence the arrival rate of vacancies is $m(\theta)\phi$ for low skill workers, and $m(\theta)$ for high skill workers, where ϕ is the proportion of low skill vacancies.

Low skill unemployed workers also meet with schools randomly and according to the matching function $s(u_l, N)$, where N is the number of schools in the economy. Again, this matching function is assumed to be constant returns to scale and therefore

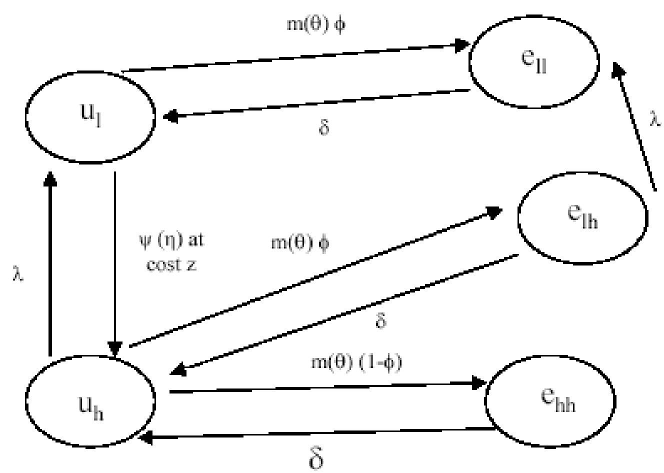
$$s(u_l, N) = s\left(1, \frac{N}{u_l}\right) u_l = \psi(\eta) u_l \text{ where } \eta = \frac{N}{u_l} \text{ is the market tightness of schools.}$$

Hence, the arrival rate for schools is $\frac{s(u_l,N)}{N} = \frac{\psi(\eta)}{\eta}$ and for workers $\frac{s(u_l,N)}{u_l} = \psi(\eta)$.

Finally. matches are destroyed according to a poison process with arrival rate δ, and there is no on-the-job search.

Figure 1 shows the different states in which the worker can be and the probabilities of changing it.

Figure 1: States and flows of the economy.



2.2 Value Functions (Competitive Equilibrium[1])

2.2.1 Employment

Low skill worker in a low skill firm (E_{ll})

A low skill worker who is hired by a low skill firm will obtain a wage ω_{ll}, but will loose the job according to a poison process with arrival rate δ. Therefore the value for a low skill individual of working for a low skill firm is:

$$rE_{ll} = \omega_{ll} - \delta\left(E_{ll} - U_l\right)$$

$$E_{ll} = \frac{1}{(r+\delta)}\left(\omega_{ll} + \delta U_l\right).$$

High skill worker in a low skill firm (E_{lh})

Unlike the low skill worker, the high skill individual can be hired by both the low and high skill firms. If the match occurs with the former, the value for the worker will be the wage received ω_{lh}, but again the match will be dissolved according to a poison process with arrival rate δ. Furthermore, since the worker is not using the high skill, which was previously acquired through training, there is a probability that he will be hit by a negative shock and loose it for not using it (the high skill is not required in this type of firm). Since the skill is perfectly observable, he becomes a low skill

[1]In order to study the efficiency of the competitive equilibrium, the derivation of the social planner's economy is done in Appendix A.

worker and the match will be of the type low skill - low skill. Hence the value for the worker of this type of match is:

$$rE_{lh} = \omega_{lh} - \delta\left(E_{lh} - U_h\right) - \lambda\left(E_{lh} - E_{ll}\right)$$

$$E_{ll} = \frac{1}{r+\delta+\lambda}\left(\omega_{lh} + \delta U_h + \lambda E_{ll}\right).$$

High skill worker in a high skill firm(E_{hh})

Similarly to the low skill - low skill match, the value for a high skill worker to be matched with a high skill firm is:

$$rE_{hh} = \omega_{hh} - \delta\left(E_{hh} - U_h\right)$$

$$E_{hh} = \frac{1}{(r+\delta)}\left(\omega_{hh} + \delta U_h\right).$$

2.2.2 Filled Job

Low skill Firm hiring a low skill worker (J_{ll})

Low skill firms produce y_l, for which they pay ω_{ll} if they hire a low skill worker. The match is dissolved with certain probability, and therefore the value for a low skill firm of hiring a low skill worker is:

$$rJ_{ll} = y_l - \omega_{ll} - \delta\left(J_{ll} - V_l\right)$$

$$J_{ll} = \frac{1}{(r+\delta)}\left(y_l - \omega_{ll} + \delta V_l\right).$$

Low skill firm hiring a high skill worker (J_{lh})

If the low skill firm matches with a high skill worker, it will produce y_l, pay a wage ω_{lh}, dissolve the match as before and in this case, if the worker looses the skill, the match will become of the low skill - low skill type. Hence the value for the low skill firm to match with a high skill worker is:

$$rJ_{lh} = y_l - \omega_{lh} - \delta\left(J_{lh} - V_h\right) - \lambda\left(J_{lh} - J_{ll}\right)$$

$$J_{lh} = \frac{1}{r+\delta+\lambda}\left(y_l - \omega_{lh} + \delta V_h + \lambda J_{ll}\right).$$

High skill firm hiring a high skill worker (J_{hh})

Similarly to the low skill - low skill match, the value for a high skill firm to be matched with a high skill worker is:

$$rJ_{hh} = y_h - \omega_{hh} - \delta\left(J_{hh} - V_h\right)$$

$$J_{hh} = \frac{1}{(r+\delta)}\left(y_h - \omega_{hh} + \delta V_h\right).$$

2.2.3 Unemployment

Low skill unemployment (U_l)

A low skill worker without job receives an unemployment benefit of b . He will be matched with a low skill firm with arrival rate $m(\theta)\phi$, and will be matched with a school to acquire the high skill with arrival rate $\psi(\eta)$. Hence the value of being unemployed for a low skill worker is:

$$rU_l = b + m(\theta)\phi(E_{ll} - U_l) + \psi(\eta)(U_h - U_l - z).$$

High skill unemployment (U_h)

A high skill worker can look for jobs in high skill and low skill firms, but only while he keeps the skill. The arrival rate of low skill firms is $m(\theta)\phi$, of high skill firms is $m(\theta)(1-\phi)$, and of the loss of skill shock λ. Therefore, the value of unemployment for a high skill worker is:

$$rU_h = b + m(\theta)\phi(E_{lh} - U_h) + m(\theta)(1-\phi)(E_{hh} - U_h) - \lambda(U_h - U_l).$$

2.2.4 Vacancy

Low skill vacancy (V_l)

Vacancies are opened by firms, and they can only hire one worker at a time. There is a cost, c, associated with having the vacancy posted. Since low skill firms can hire both low and high skill workers, the value of a low skill vacancy is:

$$rV_l = -c + \frac{m(\theta)}{\theta}\left[\gamma(J_{ll} - V_l) + (1-\gamma)(J_{lh} - V_l)\right].$$

High skill vacancy (V_l)

The value of a high skill vacancy, opened to hire only high skill workers, is:

$$rV_h = -c + \frac{m(\theta)}{\theta}(1-\gamma)(J_{hh} - V_h).$$

It will be assumed free entry of firms in the market and hence the value of a vacancy in equilibrium will be zero.

2.2.5 School

The value of having a school opened is:

$$rS = -k + \frac{\psi(\eta)}{\eta}z.$$

Again, the assumption of free entry of schools is made, which implies that the value of a school in equilibrium will be zero.

2.2.6 Wages

Wages are determined according to a Nash bargaining process in which the worker obtains a proportion β of the aggregate surplus of the match. There are three different wages according to three types of matches.

Low skill worker in a low skill Firm (ω_{ll})

$$E_{ll} - U_l = \beta[E_{ll} + J_{ll} - U_l - \underbrace{V_l]}_{=0 \text{ in eq.}}$$

$$\omega_{ll} = \beta y_l + (1-\beta)\, rU_l.$$

High skill worker in a low skill firm (ω_{lh})

$$E_{lh} - U_h = \beta[E_{lh} + J_{lh} - U_h - \underbrace{V_l]}_{=0 \text{ in eq.}}$$

$$\omega_{lh} = \beta y_l + (1-\beta)\, rU_h + (1-\beta)\, \lambda\, (U_h - U_l)\,.$$

High skill worker in a high skill firm (ω_{hh})

$$E_{hh} - U_h = \beta[E_{hh} + J_{hh} - U_h - \underbrace{V_h]}_{=0 \text{ in eq.}}$$

$$\omega_{hh} = \beta y_h + (1-\beta)\, rU_h$$

One thing to note is that, in this framework, where the workers can loose the skill for not using it, the wages obtained in equilibrium may have the unappealing property that high skill individuals working in a low skill firm will be paid more than if they worked in the high skill firm. Given the wage equations, this will happen if $(1-\beta)\,\lambda\,(U_h - U_l) > \beta\,(y_h - y_l)$, which could be the case when λ is very big, that is when workers loose the skill very quick, or when the difference between the high and low skill firms in not very big. Hence, when calibrating the model, this problem will be taken into account.

2.2.7 Cost of Training

The cost of training is also obtained through Nash bargaining, where β_s is the bargaining power of the worker.

$$(1-\beta_s)\,(U_h - U_l - z) = \beta_s z$$

$$z = (1-\beta_s)\,(U_h - U_l)\,.$$

2.2.8 Flows

In order to be able to determine the equilibrium in this economy the only thing left are the flows in and out of every one of the states.

Low skill unemployment

$$\dot{u}_l = \delta e_{ll} + \lambda u_h - [m(\theta)\phi + \psi(\eta)] u_l$$

High skill unemployment

$$\dot{u}_h = \delta(e_{hh} + e_{lh}) + \psi(\eta) u_l - [\lambda + m(\theta)] u_h$$

Low skill job filled by low skill worker

$$\dot{e}_{ll} = m(\theta)\phi u_l + \lambda e_{lh} - \delta e_{ll}$$

Low skill job filled by high skill worker

$$\dot{e}_{lh} = m(\theta)\phi u_h - (\lambda + \delta) e_{lh}$$

High skill job filled by high skill worker

$$\dot{e}_{hh} = m(\theta)(1 - \phi) u_h - \delta e_{hh}$$

The labor force is normalize to one for simplicity, therefore $u_l + u_h + e_{ll} + e_{lh} + e_{hh} = 1$.

2.2.9 Steady state equilibrium[2]

The steady state equilibrium of the competitive economy is a set of
$\{\theta, \eta, \phi, E_{ll}, E_{lh}, E_{hh}, J_{ll}, J_{lh}, J_{hh}, U_l, U_h, \omega_{ll}, \omega_{lh}, \omega_{hh}, z, u_l, u_h, e_{ll}, e_{lh}, e_{hh}\}$ which satisfy the following equations [3]:

$$\begin{aligned}
rE_{ll} &= \omega_{ll} - \delta\left(E_{ll} - U_l\right) \\
rE_{lh} &= \omega_{lh} - \delta\left(E_{lh} - U_h\right) - \lambda\left(E_{lh} - E_{ll}\right) \\
rE_{hh} &= \omega_{hh} - \delta\left(E_{hh} - U_h\right) \\
rJ_{ll} &= y_l - \omega_{ll} - \delta J_{ll} \\
rJ_{lh} &= y_l - \omega_{lh} - \delta J_{lh} - \lambda\left(J_{lh} - J_{ll}\right) \\
rJ_{hh} &= \left(y_h - \omega_{hh}\right) - \delta J_{hh} \\
rU_l &= b + m(\theta)\,\phi\left(E_{ll} - U_l\right) + \psi(\eta)\left(U_h - U_l - z\right) \\
rU_h &= b + m(\theta)\,\phi\left(E_{lh} - U_h\right) + m(\theta)(1-\phi)\left(E_{hh} - U_h\right) - \lambda\left(U_h - U_l\right) \\
0 &= -c + \frac{m(\theta)}{\theta}\left[\gamma J_{ll} + (1-\gamma)\,J_{lh}\right] \\
0 &= -c + \frac{m(\theta)}{\theta}(1-\gamma)\,J_{hh} \\
0 &= -k + \frac{\psi(\eta)}{\eta} z \\
\omega_{ll} &= \beta y_l + (1-\beta)\,rU_l \\
\omega_{lh} &= \beta y_l + (1-\beta)\,rU_h + (1-\beta)\,\lambda\left(U_h - U_l\right) \\
\omega_{hh} &= \beta y_h + (1-\beta)\,rU_h \\
z &= \left(1-\beta_s\right)\left(U_h - U_l\right) \\
\delta &= \left(2\theta^{\frac{1}{2}} + 2\eta^{\frac{1}{2}} + \delta\right) u_l + (\delta - \lambda)\,u_h + \delta e_2 + \delta e_{hh} \\
0 &= 2\eta^{\frac{1}{2}} u_l - \left(\lambda + 2\theta^{\frac{1}{2}}\right) u_h + \delta e_{lh} + \delta e_{hh} \\
0 &= 2\theta^{\frac{1}{2}} \phi u_h - (\delta + \lambda)\,e_{lh} \\
0 &= m(\theta)(1-\phi)\,u_h - \delta e_{hh} \\
e_{ll} &= 1 - u_l - u_h - e_{lh} - e_{hh}
\end{aligned}$$

[2] It has been assumed all along that it is worth for firms and workers to form a match of the type low skill firm - high skill worker. In order for this happen, one condition has to be satisfied: $E_{lh} + J_{lh} \geq U_h + V_l$, which implies that the parameters should be such that $(r+\delta+\lambda)\,y_l \geq (r+\lambda)(r+\delta)\,U_h - \lambda\delta U_l$. This condition will hold under the parametrization used in the calibration part of this paper, so in terms of Albrecht and Vroman (2002), a cross matching equilibrium will hold in this economy.

[3] Remember that in the equilibrium $V_l = V_h = S = 0$, due to the free entry of firms and schools. Also in the ss: $\dot{e}_{ll} = \dot{e}_{lh} = \dot{e}_{hh} = \dot{u}_l = \dot{u}_h = 0$. Finally $\gamma = \frac{u_l}{u_l + u_h}$.

3 Parameterization

Once we know the equations which determine the steady state of this economy, in order to solve for it, we have make some assumptions about the parameters and form of some of the functions in the model. Albrecht and Vroman (2002) is taken as a benchmark for this calibration.

Only the cross skill matching scenario is studied here, mainly since it represents a more realistic representation of reality, given that any high skill worker can do a job which does not require any skill, or a low skill that they have.

The baseline parameters values are chosen so that, within reasonable bounds, they will deliver values of the endogenous variables which are also reasonable.

The matching functions is assumed to be Cobb-Douglas with elasticity equal to $\frac{1}{2}$:

$$
\begin{aligned}
m(u,v) &= 2u^{\frac{1}{2}}v^{\frac{1}{2}} \Rightarrow m(\theta) = 2\theta^{\frac{1}{2}} \\
s(u_l,N) &= 2u_l^{\frac{1}{2}}N^{\frac{1}{2}} \Rightarrow \psi(\eta) = 2\eta^{\frac{1}{2}}
\end{aligned}
$$

The arrival rate of the shock which destroys the skill of the high skill worker is assumed to be $\lambda = 2/3$, which implies that on average the worker will take one year and a half to loose the high skill. The interest rate is set to be 5The arrival rate for the dissolution of the match is assumed to be $\delta = 0.2$, which implies an average for the live of a match of five years. Given the difficulties to estimate the bargaining power of workers in the determination of the wages and the cost of schooling and following Albrecht and Vroman (2002) and Pissarides (1992), $\beta = \beta_s = 0.5$. Also following Albrecht and Vroman (2002), the productivity of the low and high skill firms are set to $y_l = 1$, $y_h = 1.2$. The unemployment benefit is $b = 0.1$. Finally, the cost of having a vacancy and a school opened are respectively $c = 0.4$, $k = 0.4$.

The results of the simulation for the competitive equilibrium and for the social planner are shown in Table 1. We see how the unemployment rate implied is close to seven percent, with an average duration of unemployment of a little more than four months $\left(12 \cdot \left(1/\left(2\sqrt{1.78}\right)\right)\right)$, and vacancies being empty for an average of 8 months $\left(12 \cdot \left(1.78/\left(2\sqrt{1.78}\right)\right)\right)$ We also note that most of the workers are in the low skill sector (80%) and only a 11% of the labor force is working in the high skill sector. The proportion of low skill vacancies in the economy is around 70%. The average time a low skill worker will wait to match with a school is a little more than a year, but it is worth remembering that he will keep the skill for an average of 18 months.

One thing to note is that the results from the competitive equilibrium differ slightly from the ones for the social planner. Given that the bargaining power of the workers in negotiations of both wages and cost of training, are set to equal the elasticity of matching with respect to vacancies and schools respectively, this implies that the prove of Hosios (1990) of how to make the competitive equilibrium efficient, does not hold in this framework. In this model there are three two matching functions and four bargainings in place. In contrast, the framework used in Hosios' proof has a unique bargaining and matching function. The increased number of relations and flows introduced by the dual labor market must be the reason why the Hosios rule does not hold here. However, as will be seen the next section, it is possible, through public policy, to attain the social planner levels in a decentralized economy.

Table 1: Baseline simulation for the competitive and social planner economies.
$(b_l = 0.1, r = 0.05, \delta = 0.2, \lambda = 2/3, y_h = 1.2, y_l = 1, c = 0.4, k = 0.4, \beta = 0.5, \beta_s = 0.5)$

	Social Planner	**Competitive Equilibrium**
θ - Mkt tightness for firms$(\frac{v}{u})$.	1.7556	1.7800
η - Mkt tightness for schools $(\frac{N}{u_l})$.	0.5036	0.4493
$\phi - Proportion$ of LS vacancies.	0.7631	0.6893
u_l - Ls unemployment rate.	0.0428	0.0424
u_h - Hs unemployment rate.	0.0274	0.0273
e_{ll} - LL employment rate.	0.7801	0.7591
e_{lh} - LH employment rate.	0.0638	0.0579
e_{hh}-HH employment rate.	0.0859	0.1132

4 Policy Experiments

Section 4 has proven numerically that the competitive equilibrium is not efficient, even when the bargaining power of the workers is the same as the elasticity of substitution with respect to matching, as would be the case in the basic Pissarides (1990) model.

Given the assumption about the parameters of the model, we observe that the competitive equilibrium has too a low level of unemployment in both sectors and too high employment in the high skill sector.

In this section four type of public policies are studied to bring the decentralized economy back to efficient levels:

Subsidy to the training of low skill workers.

Subsidies to the opening of schools.

Subsidies to the opening of firms.

Changes in the unemployment benefit (including different benefits for the different skill levels).

We can see in Table 2, how, although there is always a level of these policies that will make the economy efficient, maybe not all of them would be put in place, in particular the taxation of unemployed workers.

The first thing we observe is that a very small tax or subsidy on the opening of low and high skill vacancies respectively, will bring us to the efficient levels, without the need for any other type of policy. We would obtain the same result by subsidizing either the training of low skill unemployed workers a 62,7 or the opening of new schools a 30 (0.12 out of the 0.4). In order for the government to achieve the efficient economy by changing the unemployment benefit, it would have to tax the unemployed workers, decreasing the unemployment benefit from 0.1 to -0.79. If the government wanted to discriminate by giving different levels of subsidies according to the skill of the worker, it could either tax the low skill unemployed workers a 170 (from 0.1 to -0.07), or subsidize the high skill workers a 120 (from 0.1 to 0.32).

Table 2: Policies that would make the make the competitive eq. efficient.
(Baseline parameters remain unchanged, and only the policy parameter is altered)

Policy	**Value**
Subsidy on the cost of opening a LS firm (f_l)	-5.7637×10^{-6}
Subsidy on the cost of opening a HS firm (f_h)	2.2466×10^{-6}
Percentage subsidy on the cost of training for the LS worker (s_z)	0.627
Subsidy on the cost of opening a school (s_k)	0.1274
Change in the unemployment benefit (b)	-0.8921
Change in the unemployment benefit for LS (b_l, ($b_h = 0.1$))	-0.1778
Change in the unemployment benefit for HS (b_h, ($b_l = 0.1$))	0.2221

5 Conclusions

This paper presents a search and matching model, in the style of Pissarides (1990), with a dual labor market with low and high skill workers and firms. Workers are low skill until they acquire the high skill. The innovation in this paper comes in the way the training of workers is modeled. A new market, for schools which train low skill workers, is introduced. This market behaves in the same manner as the labor market. Workers and schools meet randomly, there is a cost of having a school opened and the worker has to pay a price to acquire the training. Price which is bargained by the school and the worker.

The results of the model show that the competitive equilibrium is inefficient, as it occurs in simplified versions of this type of models if the bargaining power of the worker differs from the elasticity of matching with respect to unemployed workers (Hosios rule), but in this case the inefficiency goes further. This paper proves, numerically, that even in the case where the Hosios rule is applied, the steady state of the competitive economy still differs from the social planner's equilibrium. However it is possible to make the solutions coincide through public policy. Mainly through subsidies to the cost of opening schools, to the cost to the worker of getting the high skill, to the cost of vacancies for firms or by changing the unemployment subsidy, although the latter would most probably be never put in place, since it implies taxing the unemployed.

In future developments of this framework, it will interesting to study the effects of the business cycle on unemployment when the loss of skill of the long-term unemployed workers is taken into account.

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Appendix A

A.1 The problem

The Social Planner in this economy will choose σ, ϕ, θ to

$$max \sum_{t=0}^{\infty} e^{-rt} \left\{ y_l \left(e_{ll} + e_{lh} \right) + y_h \left(e_{hh} \right) + b \left(ul + u_h \right) - c\theta \left(u_l + u_h \right) - k\eta u_l \right\}$$

subject to

$$\begin{aligned}
\dot{u}_l &= \delta e_{ll} + \lambda u_h - \left[m(\theta) \phi + \psi(\eta) \right] u_l \\
\dot{u}_h &= \delta \left(e_{hh} + e_{lh} \right) + \psi(\eta) u_l - \left[\lambda + m(\theta) \right] u_h \\
\dot{e}_{ll} &= m(\theta) \phi u_l + \lambda e_{lh} - \delta e_{ll} \\
\dot{e}_{lh} &= m(\theta) \phi u_h - (\lambda + \delta) e_{lh} \\
\dot{e}_{hh} &= m(\theta)(1 - \phi) u_h - \delta e_{hh} \\
1 &= u_l + u_h + e_{ll} + e_{lh} + e_{hh}.
\end{aligned}$$

We can substitute the last constraint into the problem as $e_{ll} = 1 - u_l - u_h - e_{lh} - e_{hh}$, and write the corresponding Hamilitonian:

$$\begin{aligned}
\text{choose } \sigma, \phi, \theta \text{ to } \max H &= -\left(y_l - b + c\theta + k\eta \right) u_l - \left(y_l - b + c\theta \right) u_h + \left(y_h - y_l \right) e_{hh} + y_l + \\
&+ \mu_l \left[-\left(m(\theta) \phi + \psi(\eta) + \delta \right) u_l + (\lambda - \delta) u_h - \delta e_{ll} - \delta e_{ll} + \delta \right] + \\
&+ \mu_h \left[\psi(\eta) u_l - (\lambda + m(\theta)) u_h + \delta e_{lh} + \delta e_{hh} \right] + \\
&+ \mu_{lh} \left[m(\theta) \phi u_h - (\lambda + \delta) e_{lh} \right] + \\
&+ \mu_{hh} \left[m(\theta)(1 - \phi) u_h - \delta e_{hh} \right]
\end{aligned}$$

First order conditions:

Market tightness of vacancies (θ)

$$\begin{aligned}
\frac{\partial H}{\partial \theta} &= -c\left(u_l + u_h \right) - \mu_l \left(m'(\theta) \phi u_l \right) - \mu_h \left(m'(\theta) u_h \right) + \\
&+ \mu_{lh} \left(m'(\theta) \phi u_h \right) + \mu_{hh} \left(m'(\theta)(1 - \phi) u_h \right) = 0
\end{aligned}$$

$$c\left(u_l + u_h \right) = m'(\theta) \left[-\phi u_l \mu_l - u_h \mu_h + \phi u_h \mu_{lh} + (1 - \phi) u_h \mu_{hh} \right]$$

Market tightness of schools (η)

$$\frac{\partial H}{\partial \eta} = -k u_l + \mu_l \left(-\psi'(\eta) u_l \right) + \mu_h \left(\psi'(\eta) u_l \right) = 0$$

$$k = \psi'(\eta) \left(\mu_h - \mu_l \right)$$

Proportion of low skill vacancies (ϕ)

$$\begin{aligned}\frac{\partial H}{\partial \phi} &= \mu_l\left(-m(\theta)\,u_l\right) + \mu_{lh}\left(m(\theta)\,u_h\right) + \mu_{hh}\left(-m(\theta)\,u_h\right) = \\ &= m(\theta)\left[\left(\mu_{ll} - \mu_l\right)u_l - \left(\mu_{hh} - \mu_{lh}\right)u_h\right]\end{aligned}$$

$$\phi = \begin{cases} 1 & \text{if} \quad -\mu_l u_l - \left(\mu_{hh} - \mu_{lh}\right)u_h > 0 \\ [0,1] & \text{if} \quad -\mu_l u_l - \left(\mu_{hh} - \mu_{lh}\right)u_h = 0 \\ 0 & \text{if} \quad -\mu_l u_l - \left(\mu_{hh} - \mu_{lh}\right)u_h < 0 \end{cases}$$

The rest of the derivations assume that the solution for ϕ is interior, since otherwise the problem becomes trivial.

Unemployment rate of low skill workers (u_l)

$$b - y_l - c\theta - k\eta - \mu_l\left(m(\theta)\,\phi + \psi(\eta) + \delta\right) + \mu_h\left(\psi(\eta)\right) = r\mu_l - \dot{\mu}_l$$

In the steady state $\dot{\mu}_l = 0$, and therefore

$$r\mu_l = \left(b - y_l - c\theta - k\eta\right) - \left(m(\theta)\,\phi + \psi(\eta) + \delta\right)\mu_l + \psi(\eta)\,\mu_h$$

Unemployment rate of high skill workers (u_h)

$$b - c\theta + \mu_l\left(\lambda - \delta\right) - \mu_h\left(\lambda + m(\theta)\right) + \mu_{lh}\left(m(\theta)\,\phi\right) + \mu_{hh}\left(m(\theta)\,(1-\phi)\right) = r\mu_h - \dot{\mu}_h$$

$$r\mu_h = b - c\theta + \left(\lambda - \delta\right)\mu_l - \left(\lambda + m(\theta)\right)\mu_h + m(\theta)\,\phi\mu_{lh} + m(\theta)\,(1-\phi)\,\mu_{hh}$$

Employment rate of low skill workers in high skill firms (e_{lh})

$$-\mu_l(\delta) + \mu_h(\delta) - \mu_{lh}\left(\lambda + \delta\right) = r\mu_{lh} - \dot{\mu}_{lh}$$

$$r\mu_{lh} = -\delta\mu_l + \delta\mu_h - \left(\lambda + \delta\right)\mu_{lh}$$

Employment rate of high skill workers in high skill firms (e_{hh})

$$\left(y_h - y_l\right) - \mu_l(\delta) + \mu_h(\delta) - \mu_{hh}(\delta) = r\mu_{hh} - \dot{\mu}_{hh}$$

$$r\mu_{hh} = \left(y_h - y_l\right) - \delta\mu_l + \delta\mu_h - \delta\mu_{hh}$$

A.2 Steady state equilibrium

The steady state equilibrium of the social planner will be a set of $\{\theta, \eta, \phi, \mu_h, \mu_l, \mu_{lh}, \mu_{hh}, e_{ll}, e_{lh}, e_{hh,} u_l, u_h\}$ which satisfies the following equations:

$$
\begin{aligned}
c\left(u_l+u_h\right) &= m'\left(\theta\right)\left[-\phi u_l \mu_l - u_h \mu_h + \phi u_h \mu_{lh} + \left(1-\phi\right) u_h \mu_{hh}\right] \\
k &= \psi'\left(\eta\right)\left(\mu_h - \mu_l\right) \\
0 &= \mu_l u_l + \left(\mu_{hh} - \mu_{lh}\right) u_h \\
r\mu_l &= \left(b - y_l - c\theta - k\eta\right) - \left(m\left(\theta\right)\phi + \psi\left(\eta\right) + \delta\right)\mu_l + \psi\left(\eta\right)\mu_h \\
r\mu_h &= b - c\theta + \left(\lambda - \delta\right)\mu_l - \left(\lambda + m\left(\theta\right)\right)\mu_h + m\left(\theta\right)\phi\mu_{lh} + m\left(\theta\right)\left(1-\phi\right)\mu_{hh} \\
r\mu_{lh} &= -\delta\mu_l + \delta\mu_h - \left(\lambda + \delta\right)\mu_{lh} \\
r\mu_{hh} &= y_l - \delta\left(\mu_{hh} - \mu_h\right) \\
\delta &= \left(2\theta^{\frac{1}{2}} + 2\eta^{\frac{1}{2}} + \delta\right) u_l + \left(\delta - \lambda\right) u_h + \delta e_2 + \delta e_{hh} \\
0 &= 2\eta^{\frac{1}{2}} u_l - \left(\lambda + 2\theta^{\frac{1}{2}}\right) u_h + \delta e_{lh} + \delta e_{hh} \\
0 &= m\left(\theta\right)\phi u_h - \left(\lambda + \delta\right) e_{lh} \\
0 &= m\left(\theta\right)\left(1-\phi\right) u_h - \delta e_{hh}
\end{aligned}
$$