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Hitoshi Matsushima
University of Tokyo

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Hitoshi Matsushima[*]

Faculty of Economics, University of Tokyo

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Abstract

In this paper, we investigate the collective decision problem with incomplete information and side payments. We show that a direct mechanism associated with the social choice function that satisfies budget balancing, incentive compatibility, and interim individual rationality exists for generic prior distributions. We consider the possibility that a risk-averse principal extracts full surplus in agency problems with adverse selection. Additionally, with regard to generic prior distributions, we show that there exists a modified direct mechanism associated with the virtual social choice function, which satisfies budget balancing and interim individual rationality, such that truth telling is the unique three times iteratively undominated message rule profile.

Journal of Economic Literature Classification Numbers: C70, D44, D60, D71, D78, D82

Key Words: Incentive Compatibility, Budget Balancing, Interim Individual Rationality, Iterative Dominance, Full Surplus Extraction, Risk-Averse Principal.

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[*] Faculty of Economics, University of Tokyo, Hongo, Bunkyo-Ku, Tokyo 113, Japan. E-mail: hitoshi@e.u-tokyo.ac.jp

1. Introduction

This paper investigates the collective decision problem with incomplete information where players' utilities are quasi-linear. Each player receives a private signal and makes a public announcement about this signal. Based on these announcements and the mechanism constructed in advance, the players collectively choose an alternative and make side payments with budget balancing. Each player's utility may depend not only on her private signal but also on those of the other players, i.e., the players may have interdependent values. Their private signals may contain information about payoff-irrelevant factors such as their interim outside values.

Since the players' private signals are not contractible, each player may misrepresent her private signal without being punished for such misrepresentation. Moreover, after receiving a private signal that implies a higher interim outside value than the interim expected utility that she can obtain from a collective decision, each player has an incentive not to participate in the collective decision. Hence, it would be important for the players to construct a mechanism with budget balancing that satisfies incentive compatibility such that truth telling is considered to be a Bayesian Nash equilibrium, and also satisfies interim individual rationality such that each player can always obtain an interim expected utility larger than or equivalent to the interim outside value by participating in the collective decision after receiving her private signal.[1] Ideally, truth telling should be described by the unique equilibrium behavior; from a practical viewpoint, we should achieve this uniqueness by operating only a small number of iterative removals of undominated strategies.[2]

The first purpose of this paper is to present a sufficient condition for prior distribution, i.e., Conditions 1 and 2 in Section 3, under which there exists a direct mechanism, irrespective of the social choice function, that satisfies budget balancing, incentive compatibility, and interim individual rationality (Theorem 2). Conditions 1 and 2 are certainly more restrictive than Condition C proposed by D'Aspremont, Crèmer, and Gerard-Varet (1990), which is sufficient for the existence of budget balancing mechanisms with incentive compatibility but without interim individual rationality. However, with minor restrictions on the sizes of private signal spaces, Conditions 1 and 2 may be extremely weak. In particular, we will show that with three or more players, if their private signals are correlated and the size of each player's private signal space is approximately fixed, then such a direct mechanism does exist for generic prior distributions.

This positive result should be regarded as a distinct contribution to the mechanism

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[1] There exist many studies on mechanism design with side payments, showing their respective existence theorems of budget balancing mechanisms with incentive compatibility that do not necessarily satisfy interim individual rationality. See D'Aspremont and Gerard-Varet (1979), Arrow (1979), D'Aspremont, Crèmer, and Gerard-Varet (1990, 2004), Crèmer and Riordan (1985), Rob (1989), Aoyagi (1998), Chung (1999), and others.

[2] There exists a huge volume of studies on unique implementation with incomplete information. See the survey by Palfrey (1992). Most of the studies assumed budget balancing mechanisms to either allow no side payments or only small side payments; therefore, these mechanisms were assumed to require a social choice function to be incentive compatible with no side payments for its implementation. This paper allows large side payments and does not require incentive compatibility of the social choice function without side payments.

design literature because previous studies have failed to provide good explanations for the compatibility of interim individual rationality with incentive compatibility. For instance, according to the impossibility result shown by Myerson and Satterthwaite (1983), in a model of bilateral trading with independent private signals, private values, and zero interim outside values, there exist no such mechanisms. In contrast, this paper shows a possibility result in the case that the number of players is three or more and their private signals are correlated.[3] According to Fudenberg, Levine, and Maskin (1995), with three or more players and correlated signals, there exists a mechanism that satisfies incentive compatibility, interim individual rationality, and a weaker version of budget balancing such that the expected value of the sum of side payments is non-positive. In contrast, this paper requires complete budget balancing so that the sum of side payments is always zero.

As a corollary of the above result, we show that a player can extract the full surplus despite the constraints of incentive compatibility, interim individual rationality, and budget balancing. Crèmer and McLean (1985, 1988) showed that a principal can extract the full surplus in a multiagent problem, where three or more agents receive their private signals, but the principal does not receive any. In contrast, this paper shows that it is possible to extract the full surplus even when all players including the principal receive their private signals. According to McAfee and Reny (1992), in a model of bilateral trading similar to Myerson and Satterthwaite's model, with continuums of private and correlated signals, there exists a mechanism with budget balancing and incentive compatibility that allows the seller to virtually extract the full surplus. In their model, the maximal value of side payments diverges to infinity as the agents' total rent approaches the value of zero. In contrast, this paper shows that it is possible to precisely extract the full surplus even with bounded transfers.

As an example, we consider agency problems wherein a risk-averse uninformed principal hires multiple risk-neutral agents with private information. We show that it might be impossible for the principal to extract the full surplus either with two agents or with independent private signals. However, with three or more agents and correlated signals, it is generically possible to extract the full surplus.

Although most previous studies on agency problems with adverse selection concentrated on the case of risk-neutral principals, the study of risk-averse principals might have a high potential ability for explaining real economic phenomena. As an example of this high potential ability, we consider auctions with a risk-averse seller and multiple risk-neutral buyers, showing that it is possible for the seller to extract the full surplus without harming the incentive compatibility and interim individual rationality of the buyers.

The first half of this paper does not require truth telling to be considered as the unique Bayesian Nash equilibrium. However, the purpose of the second half of this paper is to show several sufficient conditions under which there exists a mechanism with budget balancing, incentive compatibility, and interim individual rationality that allows truth telling to be considered as the unique iteratively undominated strategy profile. In this case, we require only two or three rounds of iterative removal of

[3] McAfee (1991) reinvestigated Myerson and Satterthwaite's model and pointed out that their negative result was based on the assumption that zero trade was efficient if the seller's cost was the highest or the buyer's valuation was the lowest.

undominated strategies. In particular, in the case of private values and four or more players, even if we restrict our attention to direct mechanisms, unique implementation in terms of twice iterative dominance might be generically possible. Moreover, even in the absence of any substantial restriction on utility functions such as private values, we show that unique and virtual implementation in terms of three times iterative dominance may be generically possible by slightly modified direct mechanisms. In other words, unique and virtual implementation is generically possible when the interim preferences of players are not common knowledge.

Abreu and Matsushima (1992) showed the possibility of uniquely and virtually implementing social choice functions in terms of iterative dominance. They used only small side payments, constructed mechanisms that are more complicated than direct mechanisms, and required many rounds of iterative removal in order to achieve the unique strategy profile. However, several experimental reports indicate that real individuals are boundedly rational and that they possibly refrain from calculating the unique profile after practicing only two or three rounds of iterative removal.[4] Hence, in practice, the use of only a small number of iterative removals would be an important restriction. Based on the above viewpoint of bounded rationality, the present paper constructed only simple mechanisms and used only two or three rounds of iterative removal in order to achieve the unique profile.

The key finding is that under a slightly stronger version of Conditions 1 and 2, the constraint of interim individual rationality is trivial in that whenever there exists a budget balancing side payment function that uniquely implements a social choice function, there exists another budget balancing side payment function uniquely implementing the same social choice function that satisfies interim individual rationality (Theorem 8). Based on this, we only have to check the compatibility between incentive compatibility and budget balancing, without explicitly taking interim individual rationality into account.

The organization of this paper is as follows. Section 2 defines the model. Section 3 investigates the direct mechanisms that satisfy budget balancing, incentive compatibility, and interim individual rationality. Sections 4 and 5 investigate the possibility of the full surplus extraction. Section 4 considers the agency problem with a risk-averse principal and multiple risk neutral agents. Section 5 considers the partnership problem with risk-neutral players. Section 6 shows a sufficient condition on the prior distribution under which the existence of mechanisms with interim individual rationality is trivial. Section 7 investigates the possibility of uniquely and exactly implementing the social choice function via direct mechanisms in terms of an iterative dominance. Sections 8 and 9 investigate the possibility of uniquely and virtually implementing the social choice function in terms of twice or three times iterative dominance. Section 10 concludes the paper.

[4] For example, see Costa-Gomes, Crawford, and Broseta (2001).

2. The Model

Let $N \equiv \{1,2,...,n\}$ denote the finite set of *players*. We will assume that $n \geq 3$, except in Proposition 1 of this section, Proposition 3 of Section 3, and the first part of Proposition 4 of Section 4. Each player $i \in N$ receives her *private signal* ω_i, and Ω_i denotes the *finite* set of private signals for player i. Let $\Omega \equiv \prod_{i \in N} \Omega_i$, $\Omega_{-i} \equiv \prod_{j \in N/\{i\}} \Omega_j$, and $\Omega_{-i-j} \equiv \prod_{h \in N/\{i,j\}} \Omega_h$. A private signal profile $\omega \equiv (\omega_i)_{i \in N} \in \Omega$ is randomly drawn in accordance with a *common* prior distribution $p : \Omega \to [0,1]$. We assume that p has *full* support, i.e., $p(\omega) > 0$ for all $\omega \in \Omega$. The conditional probabilities are denoted by

$$p_i(\omega_i) \equiv \sum_{\omega_{-i} \in \Omega_{-i}} p(\omega),$$

$$p_i(\omega_{-i} \mid \omega_i) \equiv \frac{p(\omega)}{p_i(\omega_i)},$$

$$p_i^j(\omega_j \mid \omega_i) \equiv \sum_{\omega_{-i-j} \in \Omega_{-i-j}} p_i(\omega_{-i} \mid \omega_i),$$

$$p_{ij}(\omega_{-i-j} \mid \omega_i) \equiv \sum_{\omega_j \in \Omega_j} p_i(\omega_{-i} \mid \omega_i),$$

and

$$p_{ij}(\omega_{-i-j} \mid \omega_i, \omega_j) \equiv \frac{p_i(\omega_{-i} \mid \omega_i)}{p_i^j(\omega_j \mid \omega_i)},$$

where $\omega_{-i} \equiv (\omega_j)_{j \in N/\{i\}} \in \Omega_{-i}$ and $\omega_{-i-j} \equiv (\omega_h)_{h \in N/\{i,j\}} \in \Omega_{-i-j}$. For every subset $D_j \subset \Omega_j$, let

$$p_i^j(D_j \mid \omega_i) \equiv \sum_{\omega_j \in D_j} p_i^j(\omega_j \mid \omega_i).$$

Let A denote the set of *alternatives* and Δ the set of *simple lotteries* on A. We assume that each player $i's$ utility is *quasi-linear* in that when the private signal profile and the alternative are $\omega \in \Omega$ and $a \in A$, respectively, and player i receives the side payment $t_i \in R$, her utility is given by

$$u_i(a, \omega) + t_i.$$

We assume the *expected utility* hypothesis. For every simple lottery $\alpha \in \Delta$, let $u_i(\alpha, \omega) = \sum_{a \in \Gamma} u_i(a, \omega)\alpha(a)$, where Γ is the support of the lottery α, which is countable. A *social choice function* is defined by $f : \Omega \to A$, where $f(\omega)$ is regarded as the desirable alternative when the private signal profile is $\omega \in \Omega$.

For every $i \in N$, the set of messages for player i is denoted by M_i. Let $M \equiv \prod_{i \in N} M_i$ and $m = (m_i)_{i \in N} \in M$. A *message rule for player* i is defined as a function $\phi_i : \Omega_i \to M_i$. Let Φ_i denote the set of message rules for player i. Let $\Phi \equiv \prod_{i \in N} \Phi_i$ and $\phi \equiv (\phi_i)_{i \in N} \in \Phi$.

Fix the set of message profiles M arbitrarily. A *mechanism* is defined by (g, x), where $g: M \to \Delta$ is an outcome function, $x_i : M \to R$ is a side payment function for player i, and $x = (x_i)_{i \in N}$ is a side payment function with *budget balancing* in that

$$\sum_{i \in N} x_i(m) = 0 \quad \text{for all} \quad m \in M .$$

When all players announce a message profile $m \in M$, the resultant lottery and side payment for each player i are $g(m)$ and $x_i(m)$, respectively. When $g(m)$ is a degenerate lottery, it will be regarded as a pure alternative.

Fix a positive integer k arbitrarily. Let $\Phi_i^0 = \Phi_i$. For every positive integer h, we define $\Phi_i^h \subset \Phi_i$ as the set of message rules for player i, $\phi_i \in \Phi_i^{h-1}$, satisfying that there exists no $\phi_i' \in \Phi_i^{h-1} / \{\phi_i\}$ such that for every $\phi_{-i} \in \Phi_{-i}^{h-1}$,

$$\sum_{\omega \in \Omega} \{u_i(g(\phi_{-i}(\omega_{-i}), \phi_i'(\omega_i)), \omega) + x_i(\phi_{-i}(\omega_{-i}), \phi_i'(\omega_i))\} p(\omega)$$

$$> \sum_{\omega \in \Omega} \{u_i(g(\phi(\omega)), \omega) + x_i(\phi(\omega))\} p(\omega) .$$

A message rule profile $\phi \in \Phi$ is said to be *k times iteratively undominated* in (p, g, x) if $\phi \in \Phi^k$. We will use the concept of iterative dominance only in the latter part of this paper, i.e., in Sections 6, 7, 8, and 9, where we will consider only the case of $k \in \{2,3\}$. In all sections except Sections 6, 8, and 9, we consider only *direct mechanisms* where

$$M_i = \Omega_i \quad \text{for all} \quad i \in N$$

and

$$g = f .$$

Hence, a direct mechanism is denoted by (f, x), where

$$x_i : \Omega \to R \quad \text{for all} \quad i \in N .$$

The *honest message rule profile* in a direct mechanism is denoted by $\phi^* \in \Phi$ where

$$\phi^*(\omega) = \omega \quad \text{for all} \quad \omega \in \Omega .$$

For every $i \in N$ and every $\omega_i \in \Omega_i$, the *interim outside value* that player i can obtain when she observes her private signal ω_i and decides not to participate in the collective decision is denoted by $U_i^*(\omega_i) \in R$. We introduce the following requirement on x.

Interim Individual Rationality (IIR): For every $i \in N$ and every $\omega_i \in \Omega_i$,

$$\sum_{\omega_{-i} \in \Omega_{-i}} \{u_i(f(\omega)), \omega) + x_i(\omega)\} p_i(\omega_{-i} \mid \omega_i) \geq U_i^*(\omega_i) .$$

In the direct mechanism (f, x), when all players announce their private signals honestly, IIR requires the resultant interim expected utility for each player to be larger than or equivalent to her interim outside value $U_i^*(\omega_i)$; therefore, each player has the incentive to participate in the collective decision, irrespective of her private signal. The following proposition shows a necessary and sufficient condition for the existence of a budget balancing side payment function that satisfies IIR. We denote $\mu_i : \Omega_i \to R_+ \cup \{0\}$, $\mu = (\mu_i)_{i \in N}$, and $\lambda : \Omega \to R$.

Proposition 1: *Suppose* $n \geq 2$. *Then, there exists a budget balancing side payment function* x *that satisfies IIR if and only if*

(1) $$\sum_{i \in N} \sum_{\omega_i \in \Omega_i} \{ \sum_{\omega_{-i} \in \Omega_{-i}} u_i(f(\omega), \omega) p_i(\omega_{-i} \mid \omega_i) - U_i^*(\omega_i) \} p_i(\omega_i) \geq 0 .$$

Proof: By using Theorem 1 proposed by Fan (1956) in the same manner as used in D'Aspremont and Gèrard-Varet (1979, Theorem 7), we can show that a budget balancing side payment function x that satisfies IIR exists if and only if for every (μ, λ), whenever

(2) $$\lambda(\omega) = p_i(\omega_{-i} \mid \omega_i) \mu_i(\omega_i) \text{ for all } i \in N \text{ and all } \omega \in \Omega ,$$

then

(3) $$\sum_{i \in N} \sum_{\omega_i \in \Omega_i} \{ \sum_{\omega_{-i} \in \Omega_{-i}} u_i(f(\omega), \omega) p_i(\omega_{-i} \mid \omega_i) - U_i^*(\omega_i) \} \mu_i(\omega_i) \geq 0 .$$

In this case, we used $\mu_i(\omega_i) \in R_+ \bigcup \{0\}$ and $\lambda(\omega) \in R$ as multipliers for the inequality in the definition of IIR and equality in the definition of budget balancing, i.e., $\sum_{i \in N} x_i(\omega) = 0$, respectively.[5]

Suppose that (μ, λ) satisfies the equalities (2) and that there exist $i \in N$, $j \in N / \{i\}$, $\omega_i \in \Omega_i$, and $k \geq 0$ such that $\sum_{\omega_j \in \Omega_j} \mu_j(\omega_j) = k$ and $\mu_i(\omega_i) > k p_i(\omega_i)$. Then, for every $\omega_{-i} \in \Omega_{-i}$,

$$p_i(\omega_{-i} \mid \omega_i) \{ \mu_i(\omega_i) - k p_i(\omega_i) \} = p_j(\omega_{-j} \mid \omega_j) \{ \mu_j(\omega_j) - k p_j(\omega_j) \} > 0 .$$

This implies that $\mu_j(\omega_j) > k p_j(\omega_j)$ for all $\omega_j \in \Omega_j$; therefore, $\sum_{\omega_j \in \Omega_j} \mu_j(\omega_j) > k$, which is a contradiction. Hence, whenever (μ, λ) satisfies the equalities (2), then there exists $k \geq 0$ such that $\mu_i(\omega_i) = k p_i(\omega_i)$ for all $i \in N$ and $\omega_i \in \Omega_i$. From the inequality (3), it follows that

$$\sum_{i \in N} \sum_{\omega_i \in \Omega_i} \{ \sum_{\omega_{-i} \in \Omega_{-i}} u_i(f(\omega), \omega) p_i(\omega_{-i} \mid \omega_i) - U_i^*(\omega_i) \} \mu_i(\omega_i)$$
$$= k \sum_{i \in N} \sum_{\omega_i \in \Omega_i} \{ \sum_{\omega_{-i} \in \Omega_{-i}} u_i(f(\omega), \omega) p_i(\omega_{-i} \mid \omega_i) - U_i^*(\omega_i) \} p_i(\omega_i) \geq 0 ,$$

which implies the inequality (1).

Q.E.D.

The inequality (1) implies that the sum of players' ex ante expected utilities is greater than or equal to the sum of players' ex ante expected outside values. It should be noted that the inequality (1) corresponds to the *ex ante social rationality* suggested by Fudenberg, Levine, and Maskin (1994) when $U_i^*(\omega_i) = 0$ for all $i \in N$ and $\omega_i \in \Omega_i$. *Throughout this paper, we will assume that the inequality (1) holds.*

Similar to the manner observed in the proof of Proposition 1, even when it is

[5] The same technique used in the application of Fan's theorem can be found in the proofs of Theorem 2 and Propositions 3 and 7.

possible to choose any set of message profiles M, we can check that the inequality (1) is still necessary and sufficient for the existence of an indirect mechanism (g, x) with budget balancing and a message rule profile $\phi \in \Phi$ satisfying that

$$g(\phi(\omega)) = f(\omega) \quad \text{for all} \quad \omega \in \Omega$$

and that when players announce their messages according to the message rule profile ϕ, the resultant interim expected utility for each player $i \in N$ is larger than or equal to her interim outside value, i.e.,

$$\sum_{\omega_{-i} \in \Omega_{-i}} \{u_i(g(\phi(\omega))), \omega) + x_i(\phi(\omega))\} p_i(\omega_{-i} \mid \omega_i) \geq U_i^*(\omega_i) \quad \text{for all} \quad \omega_i \in \Omega_i .$$

We introduce the following requirement on x.

Incentive Compatibility (IC): For every $i \in N$, every $\omega_i \in \Omega_i$, and every $m_i \in \Omega_i$,

$$\sum_{\omega_{-i} \in \Omega_{-i}} \{u_i(f(\omega), \omega) + x_i(\omega)\} p_i(\omega_{-i} \mid \omega_i)$$
$$\geq \sum_{\omega_{-i} \in \Omega_{-i}} \{u_i(f(\omega_{-i}, m_i), \omega) + x_i(\omega_{-i}, m_i)\} p_i(\omega_{-i} \mid \omega_i) .$$

IC implies that the honest message rule profile ϕ^* is a Bayesian Nash equilibrium in the direct mechanism (f, x).

3. Interim Individual Rationality and Incentive Compatibility

We introduce the following two conditions on p.

Condition 1: For every $(\omega_1,\omega_2)\in\Omega_1\times\Omega_2$, the collection of probability distributions on Ω_{-1-2} given by

$$P_{12}(\omega_1,\omega_2)\equiv\{p_{12}(\cdot\,|\,\omega_1',\omega_2')\,|\,(\omega_1',\omega_2')\in\Omega_1\times\Omega_2,\ \textit{either}\ \omega_1'=\omega_1\ \textit{or}\ \omega_2'=\omega_2\}$$

is linearly independent; in other words, for every $(w(\omega_1'))_{\omega_1'\in\Omega_1}\in R^{|\Omega_1|}$ and every $(w(\omega_2'))_{\omega_2'\in\Omega_2/\{\omega_2\}}\in R^{|\Omega_2|-1}$, whenever

(4) $$\sum_{\omega_1'\in\Omega_1}w(\omega_1')p_{12}(\omega_{-1-2}\,|\,\omega_1',\omega_2)+\sum_{\omega_2'\in\Omega_2/\{\omega_2\}}w(\omega_2')p_{12}(\omega_{-1-2}\,|\,\omega_1,\omega_2')=0$$

for all $\omega_{-1-2}\in\Omega_{-1-2}$,

then

(5) $$w(\omega_1')=0\ \text{ for all }\ \omega_1'\in\Omega_1\ \text{ and }\ w(\omega_2')=0\ \text{ for all }\ \omega_2'\in\Omega_2/\{\omega_2\}.$$

Since $p_{12}(\cdot\,|\,\omega_1',\omega_2')$ is a $|\Omega_{-1-2}|$-dimensional vector, it follows that if $|\Omega_{-1-2}|\geq|\Omega_1|+|\Omega_2|-1$, then the $|\Omega_1|+|\Omega_2|-1$ vectors in $P_{12}(\omega_1,\omega_2)$ are linearly independent, i.e., Condition 1 holds, for generic prior distributions.

Condition 2: For every $i\in N/\{1,2\}$, the collection of probability distributions on Ω_{-i} given by

$$P_i(\omega_i)\equiv\{p_i(\cdot\,|\,\omega_i)\,|\,\omega_i\in\Omega_i\}$$

is linearly independent; in other words, for every $(w(\omega_i))_{\omega_i\in\Omega_i}\in R^{|\Omega_i|}$, whenever

(6) $$\sum_{\omega_i\in\Omega_i}w(\omega_i)p_i(\omega_{-i}\,|\,\omega_i)=0\ \text{ for all }\ \omega_{-i}\in\Omega_{-i},$$

then

(7) $$w(\omega_i)=0\ \text{ for all }\ \omega_i\in\Omega_i.$$

Since $p_i(\cdot\,|\,\omega_i)$ is a $|\Omega_{-i}|$-dimensional vector, it follows that if $|\Omega_{-i}|\geq|\Omega_i|$ for all $i\in N/\{1,2\}$, then the $|\Omega_i|$ vectors in $P_i(\omega_i)$ are linearly independent for all $i\in N/\{1,2\}$, i.e., Condition 2 holds, for generic prior distributions. Conditions 1 and 2 are not more restrictive than pairwise identifiability by Fudenberg, Levine, and Maskin (1994). In fact, pairwise identifiability requires a version of linear independence similar to Condition 1 not only for the pair of players 1 and 2 but also for all other pairs. Moreover, Condition 2 is weaker than pairwise identifiability. The following theorem states that if $|\Omega_{-1-2}|\geq|\Omega_1|+|\Omega_2|-1$ and $|\Omega_{-i}|\geq|\Omega_i|$ for all $i\in N/\{1,2\}$, there exists a budget balancing side payment function x for generic prior distributions that satisfies IC and IIR, irrespective of the social choice function f.

Theorem 2: *Suppose that* p *satisfies Conditions 1 and 2. Then, there exists a budget* *balancing side payment function* x *that satisfies IC and IIR.*

Proof: We denote $\alpha_i : \Omega_i^2 \to R_+ \cup \{0\}$ and $\alpha = (\alpha_i)_{i\in N}$. By using Theorem 1 proposed by Fan (1956) in the same manner as used in Proposition 1, we can show that a budget balancing side payment function x that satisfies IC and IIR exists if and only if for every (α, μ, λ), whenever

(8) $$\lambda(\omega) = p_i(\omega_{-i} \mid \omega_i) \sum_{\omega_i' \in \Omega_i} \alpha_i(\omega_i, \omega_i') - \sum_{\omega_i' \in \Omega_i} p_i(\omega_{-i} \mid \omega_i') \alpha_i(\omega_i', \omega_i)$$
$$+ p_i(\omega_{-i} \mid \omega_i) \mu_i(\omega_i) \text{ for all } i \in N \text{ and all } \omega \in \Omega,$$

then

(9) $$\sum_{i\in N} \sum_{\omega\in\Omega} \sum_{\omega_i' \in \Omega_i} \{u_i(f(\omega), \omega) - u_i(f(\omega_{-i}, \omega_i'), \omega)\} p_i(\omega_{-i} \mid \omega_i) \alpha_i(\omega_i, \omega_i')$$
$$+ \sum_{i\in N} \sum_{\omega_i \in \Omega_i} \{ \sum_{\omega_{-i} \in \Omega_{-i}} u_i(f(\omega), \omega) p_i(\omega_{-i} \mid \omega_i) - U_i^*(\omega_i)\} \mu_i(\omega_i) \geq 0.$$

In this case, we used $\alpha_i(\omega_i, \omega_i') \in R_+ \cup \{0\}$ as the multiplier for the inequality in the definition of IC. Fix (α, μ, λ) arbitrarily. Suppose that the equalities (8) hold. Fix $(\omega_1, \omega_2) \in \Omega_1 \times \Omega_2$ arbitrarily. Let

$$w(\omega_1') = p_1^2(\omega_2 \mid \omega_1) \alpha_1(\omega_1', \omega_1) \text{ for all } \omega_1' \in \Omega_1 / \{\omega_1\},$$
$$w(\omega_2') = -p_2^1(\omega_1 \mid \omega_2) \alpha_2(\omega_2', \omega_2) \text{ for all } \omega_2' \in \Omega_2 / \{\omega_2\},$$

and

$$w(\omega_1) = p_1^2(\omega_2 \mid \omega_1) \{\mu_1(\omega_1) + \sum_{\omega_1' \in \Omega_1 / \{\omega_1\}} \alpha_1(\omega_1, \omega_1')\}$$
$$- p_2^1(\omega_1 \mid \omega_2) \{\mu_2(\omega_2) + \sum_{\omega_2' \in \Omega_2 / \{\omega_2\}} \alpha_2(\omega_2, \omega_2')\}.$$

It should be noted that the equalities (4) hold. Condition 1 implies that the equalities (5) hold; therefore,

$$\alpha_1(\omega_1', \omega_1) = 0 \text{ for all } \omega_1' \in \Omega_1 / \{\omega_1\},$$
$$\alpha_2(\omega_2', \omega_2) = 0 \text{ for all } \omega_2' \in \Omega_2 / \{\omega_2\},$$

and

$$p_1^2(\omega_2 \mid \omega_1) \{\mu_1(\omega_1) + \sum_{\omega_1' \in \Omega_1 / \{\omega_1\}} \alpha_1(\omega_1, \omega_1')\}$$
$$- p_2^1(\omega_1 \mid \omega_2) \{\mu_2(\omega_2) + \sum_{\omega_2' \in \Omega_2 / \{\omega_2\}} \alpha_2(\omega_2, \omega_2')\} = 0.$$

Since the above equalities hold for all $(\omega_1, \omega_2) \in \Omega_1 \times \Omega_2$, it follows that

$$p_1^2(\omega_2 \mid \omega_1) \mu_1(\omega_1) = p_2^1(\omega_1 \mid \omega_2) \mu_2(\omega_2) \text{ for all } (\omega_1, \omega_2) \in \Omega_1 \times \Omega_2,$$

which implies that there exists $k \geq 0$ such that

$$\mu_1(\omega_1) = k p_1(\omega_1) \text{ for all } \omega_1 \in \Omega_1.$$

Hence, from the equalities (8), it follows that

$$\lambda(\omega) = k p(\omega) \text{ for all } \omega \in \Omega.$$

Fix $i \in N / \{1,2\}$ and $\omega_i \in \Omega_i$ arbitrarily. Let

$$z_i(\omega_i) = k p_i(\omega_i) - \sum_{\omega_i' \in \Omega_i / \{\omega_i\}} \alpha_i(\omega_i, \omega_i') - \mu_i(\omega_i)$$

and

$$z_i(\omega_i') = \alpha_i(\omega_i', \omega_i) \quad \text{for all} \quad \omega_i' \in \Omega_i / \{\omega_i\}.$$

It should be noted that the equalities (6) hold. Condition 2 implies that the equalities (7) hold; therefore, for every $i \in N / \{1,2\}$ and every $\omega_i \in \Omega_i$,

$$\alpha_i(\omega_i', \omega_i) = 0 \quad \text{for all} \quad \omega_i' \in \Omega_i / \{\omega_i\}$$

and

$$\mu_i(\omega_i) = kp_i(\omega_i).$$

Based on the above arguments and the assumption given by the inequality (1), we have proved that

$$\sum_{i \in N} \sum_{\omega \in \Omega} \sum_{\omega_i' \in \Omega_i} \{u_i(f(\omega), \omega) - u_i(f(\omega_{-i}, \omega_i'), \omega)\} p_i(\omega_{-i} \mid \omega_i) \alpha_i(\omega_i, \omega_i')$$
$$+ \sum_{i \in N} \sum_{\omega_i \in \Omega_i} \{ \sum_{\omega_{-i} \in \Omega_{-i}} u_i(f(\omega), \omega) p_i(\omega_{-i} \mid \omega_i) - U_i^*(\omega_i)\} \mu_i(\omega_i)$$
$$= k \sum_{i \in N} \sum_{\omega_i \in \Omega_i} \{ \sum_{\omega_{-i} \in \Omega_{-i}} u_i(f(\omega), \omega) p_i(\omega_{-i} \mid \omega_i) - U_i^*(\omega_i)\} p_i(\omega_i) \geq 0,$$

which implies the inequality (9).

Q.E.D.

Myerson and Satterthwaite (1983) investigated a model of bilateral trading and showed the nonexistence of a budget balancing side payment function that satisfies IC and IIR. Their negative result was based on the assumption that there existed only two players, i.e., $n = 2$, and that their private signals were independent, i.e., for every $i \in N$, $p_i(\cdot \mid \omega_i)$ was independent of $\omega_i \in \Omega_i$. In contrast, Theorem 2 of this paper assumes that $n \geq 3$ and that the players' private signals are *correlated*. The following proposition states that when $n = 2$, there may not exist a budget balancing side payment function x that satisfies IC and IIR, irrespective of whether the players' private signals are correlated.

Proposition 3: *Suppose $n = 2$, that the inequality (1) holds with equality, i.e.,*

(10) $$\sum_{i \in N} \sum_{\omega_i \in \Omega_i} \{ \sum_{\omega_{-i} \in \Omega_{-i}} u_i(f(\omega), \omega) p_i(\omega_{-i} \mid \omega_i) - U_i^*(\omega_i)\} p_i(\omega_i) = 0,$$

and that there exists $\hat{\omega} \in \Omega$ such that

(11) $$\sum_{\omega \in \Omega} \{u_1(f(\omega), \hat{\omega}_1, \omega_2) + u_2(f(\omega), \omega_1, \hat{\omega}_2)\} p_1(\omega_2 \mid \hat{\omega}_1) p_2(\omega_1 \mid \hat{\omega}_2)$$
$$> U_1^*(\hat{\omega}_1) + U_2^*(\hat{\omega}_2).$$

Then, there exists no budget balancing side payment function x that satisfies IIR and IC.

Proof: From IIR and the equality (10), for every $i \in N$ and every $\omega_i \in \Omega_i$, it must hold that

$$\sum_{\omega_j \in \Omega_j} \{u_i(f(\omega), \omega) + x_i(\omega)\} p_i(\omega_j \mid \omega_i) = U_i^*(\omega_i).$$

We denote $\eta_i : \Omega_i \to R$ and $\eta = (\eta_i)_{i \in N}$. We will use $\eta_i(\omega_i) \in R$ as the multiplier for this equality. By using Theorem 1 proposed by Fan (1956) in the same manner as used in Proposition 1, we can show that a budget balancing side payment function x that satisfies IIR and IC exists if and only if for every (α, η, λ), whenever

(12) $$\lambda(\omega) = p_i(\omega_j \mid \omega_i) \sum_{\omega_i' \in \Omega_i} \alpha_i(\omega_i, \omega_i') - \sum_{\omega_i' \in \Omega_i} p_i(\omega_j \mid \omega_i') \alpha_i(\omega_i', \omega_i)$$

$$+ p_i(\omega_j \mid \omega_i) \eta_i(\omega_i) \quad \text{for all } \ i \in N \ \text{ and all } \ \omega \in \Omega,$$

then

(13) $$\sum_{i \in N} \sum_{\omega \in \Omega} \sum_{\omega_i' \in \Omega_i} \{u_i(f(\omega), \omega) - u_i(f(\omega_i', \omega_j), \omega)\} p_i(\omega_{-i} \mid \omega_i) \alpha_i(\omega_i, \omega_i')$$

$$+ \sum_{i \in N} \sum_{\omega_i \in \Omega_i} \{ \sum_{\omega_j \in \Omega_j} u_i(f(\omega), \omega) p_i(\omega_j \mid \omega_i) - U_i^*(\omega_i) \} \eta_i(\omega_i) \geq 0 .$$

We specify

$$\alpha_i(\hat{\omega}_i, \omega_i) = p_j(\omega_i \mid \hat{\omega}_j) \quad \text{for all } \ \omega_i \in \Omega_i,$$

$$\alpha_i(\omega_i, \omega_i') = 0 \quad \text{for all } \ \omega_i \in \Omega_i / \{\hat{\omega}_i\} \ \text{ and } \ \omega_i' \in \Omega_i,$$

$$\eta_i(\hat{\omega}_i) = -1,$$

and

$$\eta_i(\omega_i) = 0 \quad \text{for all } \ \omega_i \in \Omega_i / \{\hat{\omega}_i\}.$$

From $n = 2$, it should be noted that for every $i \in N$ and every $\omega \in \Omega$,

$$p_i(\omega_j \mid \omega_i) \sum_{\omega_i' \in \Omega_i} \alpha_i(\omega_i, \omega_i') - \sum_{\omega_i' \in \Omega_i} p_i(\omega_j \mid \omega_i') \alpha_i(\omega_i', \omega_i) + p_i(\omega_j \mid \omega_i) \eta_i(\omega_i)$$

$$= -p_1(\omega_2 \mid \hat{\omega}_1) p_2(\omega_1 \mid \hat{\omega}_2).$$

Let

$$\lambda(\omega) = -p_1(\omega_2 \mid \hat{\omega}_1) p_2(\omega_1 \mid \hat{\omega}_2) \quad \text{for all } \ \omega \in \Omega .$$

It should be noted that (α, η, λ) satisfies the equalities (12). Based on the inequality (11), it follows that

$$\sum_{i \in N} \sum_{\omega \in \Omega} \sum_{\omega_i' \in \Omega_i} \{u_i(f(\omega), \omega) - u_i(f(\omega_i', \omega_j), \omega)\} p_i(\omega_{-i} \mid \omega_i) \alpha_i(\omega_i, \omega_i')$$

$$+ \sum_{i \in N} \sum_{\omega_i \in \Omega_i} \{ \sum_{\omega_j \in \Omega_j} u_i(f(\omega), \omega) p_i(\omega_j \mid \omega_i) - U_i^*(\omega_i) \} \eta_i(\omega_i)$$

$$= \sum_{i \in N} \sum_{\omega_j \in \Omega_j} \sum_{\omega_i' \in \Omega_i} \{u_i(f(\hat{\omega}_i, \omega_j), \hat{\omega}_i, \omega_j) - u_i(f(\omega_i', \omega_j), \hat{\omega}_i, \omega_j)\} p_i(\omega_j \mid \hat{\omega}_i) p_j(\omega_i' \mid \hat{\omega}_j)$$

$$- \sum_{i \in N} \{ \sum_{\omega_j \in \Omega_j} u_i(f(\hat{\omega}_i, \omega_j), \hat{\omega}_i, \omega_j) p_i(\omega_j \mid \hat{\omega}_i) - U_i^*(\hat{\omega}_i) \}$$

$$= -\sum_{\omega \in \Omega} \{u_1(f(\omega), \hat{\omega}_1, \omega_2) + u_2(f(\omega), \omega_1, \hat{\omega}_2)\} p_1(\omega_2 \mid \hat{\omega}_1) p_1(\omega_2 \mid \hat{\omega}_1)$$

$$+ U_1^*(\hat{\omega}_1) + U_2^*(\hat{\omega}_2) < 0 ,$$

which contradicts the inequality (13).

Q.E.D.

It must be noted that the inequality (11) holds if $U_1^*(\hat{\omega}_1)$ and $U_2^*(\hat{\omega}_2)$ are extremely small. Hence, Proposition 3 implies the nonexistence of a budget balancing side payment function that satisfies IC and IIR in the two-player case if for every player there exists a private signal such that the associated interim outside value is extremely small. Further implications will be shown in the next two sections.

4. Full Surplus Extraction by a Risk-Averse Principal

Consider the following situation in which a *risk-averse* principal hires n risk-neutral agents. Each agent $i \in N$ receives a private signal $\omega_i \in \Omega_i$, whereas the principal does not receive any. Each agent $i \in N$ announces her message $m_i \in M_i = \Omega_i$ in a direct mechanism (f, x) where the side payment function x may not correspond with budget balancing.[6] Based on their message profile $m \in M$, the agents collectively choose the alternative $f(m) \in A$. Each agent i receives her profit $u_i(f(m), \omega)$ and pays the money $-x_i(m) \in R$ to the principal. The principal's utility is given by $v(-\sum_{i \in N} x_i(m))$, where $v : R \to R$ is an increasing and concave function. Each agent $i's$ utility is given by $u_i(f(m), \omega) + x_i(m)$. We assume that the interim outside value for each player is set at a value equal to zero, i.e.,

$$U_i^*(\omega_i) = 0 \text{ for all } i \in N \text{ and all } \omega_i \in \Omega_i .$$

Similar to the manner observed in the proof of Proposition 1, we can check that there exists a side payment function x such that

(14) $$-\sum_{i \in N} x_i(\omega) = \sum_{i \in N, \omega' \in \Omega} u_i(f(\omega'), \omega') p(\omega') \text{ for all } \omega \in \Omega ,$$

and for every $i \in N$, player $i's$ interim expected utility is always equal to zero, i.e.,

$$\sum_{\omega_{-i} \in \Omega_{-i}} \{u_i(f(\omega), \omega) - x_i(\omega)\} p_i(\omega_{-i} \mid \omega_i) = 0 \text{ for all } \omega_i \in \Omega_i .$$

In this case, when all the agents announce honestly in accordance with ϕ^*, the principal receives a fixed amount of money $\sum_{i \in N, \omega' \in \Omega} u_i(f(\omega'), \omega') p(\omega')$, irrespective of what the true private signal profile is; therefore, the principal's expected utility is given by

$$\sum_{\omega \in \Omega} v(-\sum_{i \in N} x_i(\omega)) p(\omega) = v(\sum_{i \in N, \omega' \in \Omega} u_i(f(\omega'), \omega') p(\omega')) .$$

Since v is increasing and concave, this value is equivalent to the maximal expected utility for the principal with the constraints of IIR for all agents. *The principal can be said to extract the full surplus* if there exists x that satisfies IC, IIR, and the equalities (14). Similar to the manner observed in the proofs of Theorem 2 and Proposition 3, we can prove the following proposition.

Proposition 4: *If* $n = 2$ *and there exists* $\hat{\omega} \in \Omega$ *such that*

(15) $$\sum_{\omega \in \Omega} \{u_1(f(\omega), \hat{\omega}_1, \omega_2) + u_2(f(\omega), \omega_1, \hat{\omega}_2)\} p_1(\omega_2 \mid \hat{\omega}_1) p_2(\omega_1 \mid \hat{\omega}_2)$$
$$> \sum_{\omega \in \Omega} \{u_1(f(\omega), \omega) + u_2(f(\omega), \omega)\} p(\omega),$$

then the principal cannot extract the full surplus. If $n \geq 3$ *and* p *satisfies Conditions 1 and 2, then the principal can extract the full surplus.*

It should be noted that the inequality (15) holds in the two-player case if each player

[6] The side payments between the principal and the agents are always in accordance with budget balancing.

$i \in \{1,2\}$ possesses high utility when she receives the private signal $\hat{\omega}_i$, irrespective of the other player's private signal and the collective decision; in other words, for every $\omega \in \Omega$,

$$u_1(f(\omega), \hat{\omega}_1, \omega_2) > \sum_{\omega' \in \Omega} u_1(f(\omega'), \omega') p(\omega')$$

and

$$u_2(f(\omega), \omega_1, \hat{\omega}_2) > \sum_{\omega' \in \Omega} u_2(f(\omega'), \omega') p(\omega').$$

Hence, the first part of Proposition 4 implies that the principal cannot extract the full surplus in the two-agent case if for each player there exists a private signal that provides her with an extremely high utility. On the other hand, the latter part of Proposition 4 implies that in the case of three or more agents, the principal can extract the full surplus for generic prior distributions if $|\Omega_{-1-2}| \geq |\Omega_1| + |\Omega_2| - 1$ and $|\Omega_{-i}| \geq |\Omega_i|$ for all $i \in N / \{1,2\}$.

As an example, we consider *auctions* with a single risk-averse seller and multiple risk-neutral buyers and with private values. The seller has one unit of commodity for sale. Each buyer $i's$ private signal ω_i implies her own valuation, where Ω_i is given by a finite set of positive real numbers. The set of alternatives is given by $A = N$, where the alternative $i \in A$ implies that the commodity is transferred to buyer i. Hence, for every buyer $i \in N$,

$$u_i(a, \omega) = \omega_i \text{ if } a = i,$$

and

$$u_i(a, \omega) = 0 \text{ if } a \neq i.$$

The social choice function f is efficient, i.e., for every $\omega \in \Omega$ and every $i \in N$,

$$\omega_i \geq \omega_j \text{ for all } j \in N \text{ whenever } f(\omega) = i.$$ [7]

The expected full surplus is given by

$$\sum_{i \in N} \sum_{\omega \in \Omega} u_i(f(\omega), \omega) p(\omega) = \sum_{\omega \in \Omega} (\max_{i \in N} \omega_i) p(\omega).$$

The latter part of Proposition 4 implies that with three or more buyers, if $|\Omega_{-1-2}| \geq |\Omega_1| + |\Omega_2| - 1$ and $|\Omega_{-i}| \geq |\Omega_i|$ for all $i \in N / \{1,2\}$, then the full surplus extraction by the seller is generically possible. However, with only two buyers, the risk-averse seller may not be able to extract the full surplus. For every $i \in N$, buyer $i's$ highest possible valuation is denoted by $\overline{\omega}_i \in \Omega_i$, where

$$\overline{\omega}_i > \omega_i \text{ for all } \omega_i \in \Omega_i / \{\overline{\omega}_i\}.$$

Assume that each buyer's highest possible valuation is greater than the expected full surplus, i.e.,

$$\overline{\omega}_i > \sum_{\omega \in \Omega} (\max_{j \in N} \omega_j) p(\omega) \text{ for all } i \in N.$$

This is a trivial assumption when each buyer has the same highest possible valuation. It must be noted that when $n = 2$, this assumption implies the inequality (15). Hence, from the first part of Proposition 4, it follows that the risk-averse seller cannot extract

[7] Without any substantial change of our arguments, we can allow $f(\omega)$ to be a lottery over multiple agents whose evaluations are the highest.

the full surplus when there are only two buyers.

Proposition 1 implies that if incentive compatibility is not required, the seller can always extract the full surplus even in the two-buyer case. Moreover, we can check that if we require ex ante individual rationality, instead of interim individual rationality, that

$$\sum_{\omega\in\Omega}\{u_i(f(\omega),\omega)+x_i(\omega)\}p(\omega)=0 \quad \text{for all} \quad i\in N,$$

then the seller can extract the full surplus in a wide class of environments with two buyers. Suppose that (p,f) satisfies the "sorting" condition that for every $i\in\{1,2\}$, every $\omega_i\in\Omega_i/\{\overline{\omega}_i\}$, and every $\widetilde{\omega}_i\in\Omega_i/\{\omega_i\}$,

$$\sum_{\omega_j\in\Omega_j}\{u_i(f(\omega_i',\omega_j),\omega)-u_i(f(\omega),\omega)\}p_i(\omega_j\mid\omega_i)$$
$$\geq \sum_{\omega_j\in\Omega_j}\{u_i(f(\omega_i',\omega_j),\widetilde{\omega}_i,\omega_j)-u_i(f(\omega),\widetilde{\omega}_i,\omega_j)\}p_i(\omega_j\mid\widetilde{\omega}_i) \quad \text{if} \quad \widetilde{\omega}_i<\omega_i,$$

and

$$\sum_{\omega_j\in\Omega_j}\{u_i(f(\omega_i',\omega_j),\omega)-u_i(f(\omega),\omega)\}p_i(\omega_j\mid\omega_i)$$
$$\leq \sum_{\omega_j\in\Omega_j}\{u_i(f(\omega_i',\omega_j),\widetilde{\omega}_i,\omega_j)-u_i(f(\omega),\widetilde{\omega}_i,\omega_j)\}p_i(\omega_j\mid\widetilde{\omega}_i) \quad \text{if} \quad \widetilde{\omega}_i>\omega_i,$$

where $\omega_i'\in\Omega_i/\{\omega_i\}$ is the smallest element of Ω_i that is larger than ω_i. Then, according to standard analysis, for every $i\in\{1,2\}$, it follows that there exists $s_i:\Omega_i\to R$ such that

$$\sum_{\omega_j\in\Omega_j}\{u_i(f(\omega),\omega)+s_i(\omega_i)\}p_i(\omega_j\mid\omega_i)$$
$$\geq \sum_{\omega_j\in\Omega_j}\{u_i(f(\widetilde{\omega}_i,\omega_j),\omega)-s_i(\widetilde{\omega}_i)\}p_i(\omega_j\mid\omega_i) \quad \text{for all} \quad \widetilde{\omega}_i\in\Omega_i.$$

For every $i\in\{1,2\}$, let

$$x_i(\omega)=s_i(\omega_i)-s_j(\omega_j)-\sum_{\widetilde{\omega}\in\Omega}\{u_i(f(\widetilde{\omega}),\widetilde{\omega})+s_i(\widetilde{\omega}_i)-s_j(\widetilde{\omega}_j)\}p(\widetilde{\omega}),$$

which satisfies the incentive compatibility and ex ante individual rationality with equality. Hence, without interim individual rationality, the seller can extract the full surplus even in the two-buyer case.

5. Full Surplus Extraction by a Risk-Neutral Player

Fix any player $i^* \in N$ arbitrarily. We will say that *player* i^* *can extract the full surplus* if there exists a budget balancing side payment function x that satisfies IC and IIR where the properties of IIR hold with equality for every player except player i^*, i.e., for every $i \in N/\{i^*\}$ and every $\omega_i \in \Omega_i$,

$$\sum_{\omega_{-i} \in \Omega_{-i}} \{u_i(f(\omega)), \omega) + x_i(\omega)\} p_i(\omega_{-i} \mid \omega_i) = U_i^*(\omega_i). \tag{16}$$

One of the interpretations is that before receiving a private signal, player i^* will design a mechanism in order to maximize her ex ante expected payoff. The following proposition shows a sufficient condition under which player i^* can extract the full surplus.

Proposition 5: *Suppose that* p *satisfies Conditions 1 and 2. Then, player* i^* *can extract the full surplus.*

Proof: The inequality (1) implies that we can choose $U_{i^*}^{**}(\omega_{i^*})$ for each $\omega_{i^*} \in \Omega_{i^*}$ such that

$$U_{i^*}^{**}(\omega_{i^*}) \geq U_{i^*}^{*}(\omega_{i^*}) \text{ for all } \omega_{i^*} \in \Omega_{i^*},$$

and

$$\{ \sum_{\omega_{-i^*} \in \Omega_{-i^*}} u_{i^*}(f(\omega), \omega) p_{i^*}(\omega_{-i^*} \mid \omega_{*_i}) - U_{i^*}^{**}(\omega_{*_i}) \} p_{i^*}(\omega_{*_i})$$
$$+ \sum_{i \in M/\{i^*\}} \sum_{\omega_i \in \Omega_i} \{ \sum_{\omega_{-i} \in \Omega_{-i}} u_i(f(\omega), \omega) p_i(\omega_{-i} \mid \omega_i) - U_i^*(\omega_i) \} p_i(\omega_i) = 0. \tag{17}$$

Based on Theorem 2 and the equality (17), if Conditions 1 and 2 hold, it follows that there exists a budget balancing side payment function x that satisfies IC such that the equality (16) holds for all $i \in N/\{i^*\}$ and all $\omega_i \in \Omega_i$ and

$$\sum_{\omega_{-i^*} \in \Omega_{-i^*}} u_{i^*}(f(\omega), \omega) p_{i^*}(\omega_{-i^*} \mid \omega_{*_i}) = U_{i^*}^{**} \text{ for all } \omega_{i^*} \in \Omega_{i^*}.$$

This implies that player i^* can extract the full surplus.

Q.E.D.

Crèmer and McLean (1985, 1988) investigated the auctions of a single seller and three or more bidders and showed that the seller can extract the full surplus if the bidders' private signals are correlated. They assumed that the seller does not receive any private signals. In contrast, the present paper permits the seller to receive her private signal and considers the constraints of IC and IIR for this seller as well as the bidders. The following proposition states that if there exists a player whose private signal is independent, player i^* may not be able to extract the full surplus.

Proposition 6: *Fix* $i \in N/\{i^*\}$ *arbitrarily. Suppose that there exist* $\hat{\omega}_i \in \Omega_i$ *and* $\tilde{\omega}_i \in \Omega_i/\{\hat{\omega}_i\}$ *such that*

(18) $$\sum_{\tilde{\omega}_{-i}\in\Omega_{-i}}\{u_i(f(\tilde{\omega}),\tilde{\omega})p_i(\tilde{\omega}_{-i}\mid\tilde{\omega}_i)-u_i(f(\tilde{\omega}),\tilde{\omega}_{-i},\hat{\omega}_i)p_i(\tilde{\omega}_{-i}\mid\hat{\omega}_i)\}$$
$$<U_i^*(\tilde{\omega}_i)-U_i^*(\hat{\omega}_i).$$

Suppose that $p_i(\cdot\mid\omega_i)$ *is independent of* $\omega_i\in\Omega_i$. *Then, there exists no side payment function* x_i *for player* i *such that for every* $\omega_i\in\Omega_i$,

(19) $$\sum_{\omega_{-i}\in\Omega_{-i}}\{u_i(f(\omega)),\omega)+x_i(\omega)\}p_i(\omega_{-i}\mid\omega_i)$$
$$\geq\sum_{\omega_{-i}\in\Omega_{-i}}\{u_i(f(\omega_{-i},\omega_i')),\omega)+x_i(\omega_{-i},\omega_i')\}p_i(\omega_{-i}\mid\omega_i)\ \ \textit{for all}\ \ \omega_i'\in\Omega_i,$$

and

(20) $$\sum_{\omega_{-i}\in\Omega_{-i}}\{u_i(f(\omega)),\omega)+x_i(\omega)\}p_i(\omega_{-i}\mid\omega_i)=U_i^*(\omega_i).$$

Proof: Suppose that x_i satisfies the inequalities (19) and the equalities (20). Based on the inequalities (20), the fact that $p_i(\cdot\mid\omega_i)$ is independent of $\omega_i\in\Omega_i$, and the inequality (18), it follows that

$$\sum_{\tilde{\omega}_{-i}\in\Omega_{-i}}\{u_i(f(\tilde{\omega})),\tilde{\omega}_{-i},\hat{\omega}_i)+x_i(\tilde{\omega})\}p_i(\tilde{\omega}_{-i}\mid\hat{\omega}_i)$$
$$-\sum_{\tilde{\omega}_{-i}\in\Omega_{-i}}\{u_i(f(\tilde{\omega}_{-i},\hat{\omega}_i)),\tilde{\omega}_{-i},\hat{\omega}_i)+x_i(\tilde{\omega}_{-i},\hat{\omega}_i)\}p_i(\tilde{\omega}_{-i}\mid\hat{\omega}_i)$$
$$=\sum_{\tilde{\omega}_{-i}\in\Omega_{-i}}\{u_i(f(\tilde{\omega})),\tilde{\omega}_{-i},\hat{\omega}_i)+x_i(\tilde{\omega})\}p_i(\tilde{\omega}_{-i}\mid\hat{\omega}_i)-U_i^*(\hat{\omega}_i)$$
$$=\sum_{\tilde{\omega}_{-i}\in\Omega_{-i}}\{u_i(f(\tilde{\omega}),\tilde{\omega}_{-i},\hat{\omega}_i)p_i(\tilde{\omega}_{-i}\mid\hat{\omega}_i)-u_i(f(\tilde{\omega}),\tilde{\omega})p_i(\tilde{\omega}_{-i}\mid\tilde{\omega}_i)\}+U_i^*(\tilde{\omega}_i)$$
$$-U_i^*(\hat{\omega}_i)>0.$$

This contradicts the inequalities (19).

Q.E.D.

Proposition 6 implies that if there exists a player $i\in N/\{i^*\}$ such that $p_i(\cdot\mid\omega_i)$ is independent of $\omega_i\in\Omega_i$ and the inequality (18) holds for some $\hat{\omega}_i\in\Omega_i$ and $\tilde{\omega}_i\in\Omega_i/\{\hat{\omega}_i\}$, then player i^* cannot extract the full surplus. It should be noted that the inequality (18) holds when the difference in the interim outside values between the private signals $\hat{\omega}_i$ and $\tilde{\omega}_i$ is large. It should also be noted that even if the interim outside values $U_i^*(\hat{\omega}_i)$ and $U_i^*(\tilde{\omega}_i)$ are equivalent, the inequality (18) holds when player $i's$ payoff in the case of $\hat{\omega}_i$ is better than that in the case of $\tilde{\omega}_i$, i.e.,

$$u_i(f(\tilde{\omega}),\tilde{\omega}_{-i},\hat{\omega}_i)>u_i(f(\tilde{\omega}),\tilde{\omega})\ \ \text{for all}\ \ \tilde{\omega}_{-i}\in\Omega_{-i}.$$

6. Uniqueness and Interim Individual Rationality

We introduce the following two conditions on p.

Condition 3: The collection of probability functions on Ω_{-1-2} given by

$$\overline{P}_{12}(\omega_1,\omega_2)\equiv\{p_{12}(\cdot\,|\,\omega_1,\omega_2)\,|\,(\omega_1,\omega_2)\in\Omega_1\times\Omega_2\}$$

is linearly independent; in other words, for every $(w(\omega_1,\omega_2))_{(\omega_1,\omega_2)\in\Omega_1\times\Omega_2}\in R^{|\Omega_1|\times|\Omega_2|}$, whenever

$$\sum_{(\omega_1,\omega_2)\in\Omega_1\times\Omega_2} w(\omega_1,\omega_2)p_{12}(\omega_{-1-2}\,|\,\omega_1,\omega_2)=0 \text{ for all } \omega_{-1-2}\in\Omega_{-1-2},$$

then

$$w(\omega_1,\omega_2)=0 \text{ for all } (\omega_1,\omega_2)\in\Omega_1\times\Omega_2.$$

Since $p_{12}(\cdot\,|\,\omega_1',\omega_2')$ is a $|\Omega_{-1-2}|$-dimensional vector, if $|\Omega_{-1-2}|\geq|\Omega_1|+|\Omega_2|$, then it follows that the $|\Omega_1|+|\Omega_2|-1$ vectors in $\overline{P}_{12}(\omega_1,\omega_2)$ are linearly independent, i.e., Condition 3 holds, for generic prior distributions. However, it must be noted that Condition 3 is more restrictive than Condition 1.[8]

Condition 4: For every $i\in N/\{2\}$, the collection of probability functions on Ω_{-2-i} given by

$$\overline{P}_{2i}(\omega_i)\equiv\{p_{2i}(\cdot\,|\,\omega_i)\,|\,\omega_i\in\Omega_i\}$$

is linearly independent; in other words, for every $(w(\omega_i))_{\omega_i\in\Omega_i}\in R^{|\Omega_i|}$, whenever

$$\sum_{\omega_i\in\Omega_i} w(\omega_i)p_{2i}(\omega_{-2-i}\,|\,\omega_i)=0 \text{ for all } \omega_{-2-i}\in\Omega_{-2-i},$$

then

$$w(\omega_i)=0 \text{ for all } \omega_i\in\Omega_i.$$

Since $p_{2i}(\cdot\,|\,\omega_i)$ is a $|\Omega_{-2-i}|$-dimensional vector, if $|\Omega_{-2-i}|\geq|\Omega_i|$ for all $i\in N/\{1,2\}$, then it follows that the $|\Omega_i|$ vectors in $\overline{P}_{2i}(\omega_i)$ are linearly independent for all $i\in N/\{1,2\}$, i.e., Condition 4 holds, for generic prior distributions. However, it must be noted that Condition 4 is more restrictive than Condition 2. The following proposition will be useful for constructing a side budget balancing payment function that satisfies IIR without harming other properties such as incentive compatibility and uniqueness.

Proposition 7: *Consider an arbitrary collection of functions (v_i) where $v_i:\Omega_i\to R$ for all $i\in N$, and*

$$\sum_{i\in N,\omega_i\in\Omega_i} v_i(\omega_i)p_i(\omega_i)=0.$$

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[8] It must be noted that if $|\Omega_{-2-i}|\geq|\Omega_i|$ for all $i\in N/\{1\}$ and $|\Omega_{-1-2}|\geq|\Omega_1|\times|\Omega_2|$, then it must hold that $n\geq 4$.

Suppose that p *satisfies Conditions 3 and 4. Then there exists a collection of functions* (y_i) *such that* $y_i : \Omega_{-i} \to R$ *for all* $i \in N$*,*

$$\sum_{i \in N} y_i(\omega_{-i}) = 0 \ \textit{for all} \ \ \omega \in \Omega$$

and

$$\sum_{\omega_{-i} \in \Omega_{-i}} y_i(\omega_{-i}) p_i(\omega_{-i} \mid \omega_i) = v_i(\omega_i) \ \textit{for all} \ \ i \in N \ \ \textit{and all} \ \ \omega_i \in \Omega_i .$$

Proof: Condition 4 implies that for every $i \in N / \{1,2\}$, there exists t_i such that

$$\sum_{\omega_{-2-i} \in \Omega_{-2-i}} t_i(\omega_{-2-i}) p_{i2}(\omega_{-2-i} \mid \omega_i) = v_i(\omega_i) \ \ \text{for all} \ \ \omega_i \in \Omega_i .$$

Let

$$w_2(\omega_2) = v_2(\omega_2) + \sum_{\omega_{-2} \in \Omega_{-2}} \{ \sum_{i \in N/\{1,2\}} t_i(\omega_{-2-i}) \} p_2(\omega_{-2} \mid \omega_2) .$$

It should be noted that

$$(21) \qquad \sum_{\omega_1 \in \Omega_1} v_1(\omega_1) p_1(\omega_1) + \sum_{\omega_2 \in \Omega_2} w_2(\omega_2) p_2(\omega_2) = 0 .$$

By using Theorem 1 proposed by Fan (1956) in the same manner as used in Proposition 1, it follows that there exists $z : \Omega_1 \times \Omega_2 \to R$ such that

$$\sum_{\omega_2 \in \Omega_2} z(\omega_1, \omega_2) p_1(\omega_2 \mid \omega_1) = v_1(\omega_1) \ \ \text{and}$$

$$\sum_{\omega_1 \in \Omega_1} z(\omega_1, \omega_2) p_2(\omega_1 \mid \omega_2) = -w_2(\omega_2)$$

if and only if for every (η_1, η_2), whenever

$$(22) \qquad \eta_1(\omega_1) p_1(\omega_2 \mid \omega_1) = \eta_2(\omega_2) p_2(\omega_1 \mid \omega_2) \ \ \text{for all} \ \ (\omega_1, \omega_2) \in \Omega_1 \times \Omega_2 ,$$

then

$$(23) \qquad \sum_{\omega_1 \in \Omega_1} \eta_1(\omega_1) v_1(\omega_1) + \sum_{\omega_2 \in \Omega_2} \eta_2(\omega_2) w_2(\omega_2) = 0 .$$

Similar to the manner observed in the proof of Theorem 2, it follows that whenever (η_1, η_2) satisfies the equalities (22), then there exists $k \geq 0$ such that

$$\eta_i(\omega_i) = k p_i(\omega_i) \ \ \text{for all} \ \ i \in \{1,2\} \ \ \text{and all} \ \ \omega_i \in \Omega_i .$$

Hence, based on the equality (21), it follows that the equality (23) holds; therefore, we have shown that such a function $z : \Omega_1 \times \Omega_2 \to R$ exists. Condition 3 implies that there exists $t_1 : \Omega_{-1-2} \to R$ such that

$$\sum_{\omega_{-1-2} \in \Omega_{-1-2}} t_1(\omega_{-1-2}) p_{12}(\omega_{-1-2} \mid \omega_1, \omega_2) = z(\omega_1, \omega_2) .$$

This implies that

$$\sum_{\omega_{-1-2} \in \Omega_{-1-2}} t_1(\omega_{-1-2}) p_{12}(\omega_{-1-2} \mid \omega_1) = v_1(\omega_1) \ \ \text{for all} \ \ \omega_1 \in \Omega_1$$

and

$$-\sum_{\omega_{-1-2} \in \Omega_{-1-2}} t_1(\omega_{-1-2}) p_{12}(\omega_{-1-2} \mid \omega_2) = w_2(\omega_2) \ \ \text{for all} \ \ \omega_2 \in \Omega_2 .$$

We specify (y_i) as follows. For every $\omega \in \Omega$,

$$y_i(\omega_{-i}) = t_i(\omega_{-2-i}) \ \ \text{for all} \ \ i \in N / \{2\}$$

and

$$y_2(\omega_{-2}) = -\sum_{i\in N/\{2\}} t_i(\omega_{-2-i}).$$

Based on the above arguments, it follows that

$$\sum_{i\in N} y_i(\omega_{-i}) = 0 \quad \text{for all} \quad \omega\in\Omega$$

and

$$\sum_{\omega_{-i}\in\Omega_{-i}} y_i(\omega_{-i}) p_i(\omega_{-i} \mid \omega_i) = v_i(\omega_i) \quad \text{for all} \quad i\in N \quad \text{and all} \quad \omega_i\in\Omega_i.$$

Q.E.D.

The following theorem states that under Conditions 3 and 4, the existence of a mechanism with budget balancing that implements the social choice function implies the existence of a mechanism with interim individual rationality as well as budget balancing that implements the social choice function; therefore, interim individual rationality is a trivial requirement.

Theorem 8: *Consider an indirect mechanism* (g,x) *with budget balancing. Suppose that* ϕ *is the unique k times iteratively undominated message rule profile in* (p,g,x). *Suppose that* p *satisfies Conditions 3 and 4, and that*

(24) $$\sum_{i\in N}\sum_{\omega_i\in\Omega_i}\{\sum_{\omega_{-i}\in\Omega_{-i}} u_i(g(\phi(\omega)),\omega) p_i(\omega_{-i}\mid\omega_i) - U_i^*(\omega_i)\} p_i(\omega_i) \geq 0.$$ [9]

Then, there exists a budget balancing side payment function $\hat{x}$ *such that* ϕ *is the unique k times iteratively undominated message rule profile in* $(p,g,\hat{x})$, *and it satisfies interim individual rationality in that for every* $i\in N$ *and every* $\omega_i\in\Omega_i$,

$$\sum_{\omega_{-i}\in\Omega_{-i}}\{u_i(g(\phi(\omega)),\omega) + x_i(\phi(\omega))\} p_i(\omega_{-i}\mid\omega_i) \geq U_i^*(\omega_i).$$

Proof: Based on the inequality (24), it follows that there exists (v_i) such that for every $i\in N$,

$$\sum_{i\in N}\sum_{\omega_i\in\Omega_i} v_i(\omega_i) p_i(\omega_i) = 0,$$

and for every $\omega_i\in\Omega_i$,

$$v_i(\omega_i) \geq U_i^*(\omega_i) - \sum_{\omega_{-i}\in\Omega_{-i}}\{u_i(g(\phi(\omega)),\omega) + x_i(\phi(\omega))\} p_i(\omega_{-i}\mid\omega_i).$$

From Proposition 7, it follows that there exists (y_i) that satisfies the properties in Proposition 7. We specify $\hat{x}$ by

$$\hat{x}_i(m) = x_i(m) + y_i(m_{-i}^2) \quad \text{for all} \quad i\in N \quad \text{and all} \quad m\in\Omega.$$

It should be noted that $(p,f,\hat{x})$ satisfies interim individual rationality in the above sense. Since y_i is independent of m_i, it follows that ϕ is the unique k-times iteratively undominated message rule profile in $(p,g,\hat{x})$.

Q.E.D.

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[9] The inequality (24) corresponds to the inequality (1) in direct mechanisms, which is regarded as a necessary condition for satisfying IIR as well as budget balancing in indirect mechanisms.

Hence, from next section onward, we will only show the existence of a budget balancing mechanism that uniquely implements the social choice function without explicitly requiring interim individual rationality.

7. Twice Iterative Dominance

We introduce the following condition on p.

Condition 5: For every $i \in N/\{1\}$, every $\omega_i \in \Omega_i$, and every $\omega_i' \in \Omega_i$,
$$p_i^1(\cdot \mid \omega_i) \neq p_i^1(\cdot \mid \omega_i').$$

It must be noted that if $|\Omega_1| \geq 2$, then Condition 5 holds for generic prior distributions. We introduce the following conditions on f.

Condition 6: There exists $d : \Omega \to R$ such that for every $\omega \in \Omega$ and every $\omega' \in \Omega$ that satisfies $\omega_1 \neq \omega_1'$,
$$u_1(f(\omega_{-1}', \omega_1), \omega) + d(\omega_{-1}', \omega_1) > u_1(f(\omega'), \omega) + d(\omega').$$

Suppose that player 1's utility $u_1(a, \omega)$ is independent of ω_{-1} and that player $i's$ utility $u_i(a, \omega)$ is independent of ω_1 for every $i \in N/\{1\}$. These suppositions are regarded as weaker versions of the private value assumption that every player's utility is independent of the private signals of the other players. Suppose that a social choice function f is *strictly efficient*; in other words, for every $\omega \in \Omega$,
$$\sum_{i \in N} u_i(f(\omega), \omega) > \sum_{i \in N} u_i(f(a), \omega) \quad \text{for all } a \in A/\{f(\omega)\}.$$
We specify that
$$d(\omega) = \sum_{i \in N/\{1\}} u_i(f(\omega), \omega).$$
It must be noted that Condition 6 holds in this case.[10] The following proposition states that under Condition 6, if $|\Omega_1| \geq 2$, then there exists a direct mechanism with budget balancing for generic prior distributions, in which truth telling is considered as the unique twice iteratively undominated message rule profile.

Proposition 9: *Suppose that Conditions 5 and 6 hold. Then, there exists a budget balancing side payment function x such that ϕ^* is the unique twice iteratively undominated message rule profile in (p, f, x).*

Proof: For every $i \in N/\{1\}$, we define $s_i : \Omega_1 \times \Omega_i \to R$ in a manner such that for every $(\omega_1, \omega_i) \in \Omega_1 \times \Omega_i$,
$$s_i(\omega_1, \omega_i) = -\{1 - p_i^1(\omega_1 \mid \omega_i)\}^2 - \sum_{\omega_1' \in \Omega_1/\{\omega_1\}} p_i^1(\omega_1' \mid \omega_i)^2.$$
Condition 5 implies that for every $i \in N/\{1\}$, every $\omega_i \in \Omega_i$, and every $\omega_i' \in \Omega_i/\{\omega_i\}$,

[10] This was first suggested by Groves (1973), D'Aspremont and Gerard-Varet (1979), and others. See also Matsushima (1990a) for dominant strategies.

(25) $\sum_{\omega_1 \in \Omega_1} s_i(\omega_1, \omega_i) p_i^1(\omega_1 \mid \omega_i) > \sum_{\omega_1 \in \Omega_1} s_i(\omega_1, \omega_i') p_i^1(\omega_1 \mid \omega_i)$.[11]

The following is the proof for the inequalities (25). Consider the maximization problem given by

$$\max_{(q(\omega_1)) \in R^{|\Omega_1|}} \sum_{\omega_1 \in \Omega_1} [-\{1 - q(\omega_1)\}^2 - \sum_{\omega_1' \neq \omega_1} q(\omega_1')^2\} p_i^1(\omega_1 \mid \omega_i)$$

subject to $\sum_{\omega_1 \in \Omega_1} q(\omega_1) = 1$.

The first order condition is

$$2 p_i^1(\omega_1 \mid \omega_i) - 2q(\omega_1) = \rho \quad \text{for all} \quad \omega_1 \in \Omega_1,$$

where ρ is a real number. Together with $\sum_{\omega_1 \in \Omega_1} q(\omega_1) = 1$, this implies that $q(\omega_1) = p_i^1(\omega_1 \mid \omega_i)$; therefore, this implies the inequalities (25).

Fix a positive real number $k > 0$ arbitrarily and specify x as follows. For every $m \in M$,

$$x_1(m) = d(m),$$

$$x_2(m) = -d(m) + k\{s_2(m_1, m_2) - \frac{1}{n-2} \sum_{j \in N/\{1,2\}} s_j(m_1, m_j)\},$$

and for every $i \in N/\{1,2\}$,

$$x_i(m) = k\{s_i(m_1, m_i) - \frac{1}{n-2} \sum_{j \in N/\{1,i\}} s_j(m_1, m_j)\}.$$

It should be noted that x satisfies budget balancing. Based on the inequalities (25), it follows that for every sufficiently large k, every $\phi \in \Phi$ and every $i \in N/\{1\}$, if $\phi_1 = \phi_1^*$ and $\phi_i \neq \phi_i^*$, then

$$\sum_{\omega \in \Omega} \{u_i(f(\phi(\omega)/\phi_i^*(\omega_i)), \omega) + x_i(\phi(\omega)/\phi_i^*(\omega_i))\} p(\omega)$$
$$> \sum_{\omega \in \Omega} \{u_i(f(\phi(\omega)), \omega) + x_i(\phi(\omega))\} p(\omega).$$

This implies that whenever player 1 announces honestly, then all other players have a strict incentive to do the same. Condition 6 implies that for every $\phi \in \Phi$, if $\phi_1 \neq \phi_1^*$, then

$$\sum_{\omega \in \Omega} \{u_1(f(\phi(\omega)), \omega) + x_1(\phi(\omega))\} p(\omega)$$
$$= \sum_{\omega \in \Omega} \{u_1(f(\phi(\omega)), \omega_1) + d(\phi(\omega))\} p(\omega)$$
$$< \sum_{\omega \in \Omega} \{u_1(f(\omega_1, \phi_{-1}(\omega_{-1})), \omega_1) + d(\omega_1, \phi_{-1}(\omega_{-1}))\} p(\omega)$$
$$= \sum_{\omega \in \Omega} \{u_1(f(\phi_1^*(\omega_1), \phi_{-1}(\omega_{-1})), \omega) + x_1(\phi_1^*(\omega_1), \phi_{-1}(\omega_{-1}))\} p(\omega).$$

This implies that player 1 has the strict incentive to announce honestly, irrespective of what the other players' message rules are. Hence, we have proved that ϕ^* is the unique

[11] This is based on the idea of proper scoring rules. For its application to mechanism design, see Johnson, Pratt, and Zeckhauser (1990), Matsushima (1990b, 1993), Aoyagi (1998), and others.

twice iteratively undominated message rule profile in (p, f, x).

Q.E.D.

Matsushima (1990a) showed a sufficient condition on the common prior distribution under which, with private values, there exists a budget balancing side payment function that considers truth telling to be the unique twice iteratively undominated message rule profile when the social choice function is strictly efficient. This sufficient condition requires linear independence of the conditional distributions, which is more restrictive than Condition 5. Hence, Proposition 9 is regarded as the generalization of Matsushima (1990a).[12]

[12] Arya, Glover, and Young (1995) also showed the possibility of uniquely and virtually implementing social choice functions in terms of twice iterative dominance on the private value assumption.

8. Virtual Implementation

It should be noted that Condition 6 excludes a wide class of environments with interdependent values. By using a modified version of the direct mechanism, this section aims to show that a social choice function is uniquely, and not exactly but virtually, implementable in terms of twice iterative dominance, even in a wide class of environments with interdependent values.[13] We consider indirect mechanisms (g, x) where the set of messages for each player $i \in N$ is specified as

$$M_i = \Omega_i^2 .$$

At one time, each player makes two announcements about her private signals. We specify that $m_i \equiv (m_i^1, m_i^2) \in M_i$ and $\phi_i \equiv (\phi_i^1, \phi_i^2) \in \Phi$, where

$$\phi_i^h : \Omega_i \to \Omega_i \quad \text{for each} \quad h \in \{1,2\} .$$

The honest message profile is denoted by $\phi^{**} \in \Phi$, where

$$\phi_i^{**h}(\omega_i) = \omega_i \quad \text{for each} \quad h \in \{1,2\} .$$

For every $\varepsilon > 0$, a mechanism (g, x) is said to satisfy the $\varepsilon -$*closeness to* f if for every $\omega \in \Omega$,

$$g(\phi^{**}(\omega))(f(\omega)) \geq 1 - \varepsilon .$$

We introduce the following conditions on (p, f).

Condition 7: For every $i \in N$, every $\omega_i \in \Omega_i$, and every $\omega_i' \in \Omega_i / \{\omega_i\}$, there does not exist $\beta \in R$ such that for every $a \in A$,

$$\sum_{\omega_{-i} \in \Omega_{-i}} u_i(a, \omega) p_i(\omega_{-i} \mid \omega_i) = \sum_{\omega_{-i} \in \Omega_{-i}} u_i(a, \omega_i', \omega_{-i}) p_i(\omega_{-i} \mid \omega_i') + \beta .$$

Condition 7 implies that each player has different preferences with regard to the pure alternatives across her private signals. Since Condition 7 is weaker than Condition 6, we can say that it holds in a wide class of environments with interdependent values. We introduce the following condition on p.

Condition 8: For every $i \in N$, there exists $\iota(i) \in N / \{i\}$ such that for every $\omega_i \in \Omega_i$ and every $\omega_i' \in \Omega_i / \{\omega_i\}$,

$$p_{i\iota(i)}(\cdot \mid \omega_i) \neq p_{i\iota(i)}(\cdot \mid \omega_i') .$$

It should be noted that whenever $|\Omega_i| \geq 2$ for at least two players $i \in N$, then Condition 8 holds for generic prior distributions. The following proposition states that under Condition 7, for generic prior distributions, there exist mechanisms with budget balancing that uniquely and virtually implement the social choice function in terms of twice iterative dominance.

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[13] There exist many papers on unique and virtual implementation with incomplete information, such as Abreu and Matsushima (1992), Matsushima (1993), Duggan (1997), Serrano and Vohra (2000, 2001), and others.

Proposition 10: *Suppose that Conditions 7 and 8 hold. Then, for every* $\varepsilon > 0$*, there exists a mechanism with budget balancing* (g,x) *that satisfies the* $\varepsilon-$*closeness to* f *such that* ϕ^{**} *is the unique twice iteratively undominated message rule profile in* (p,g,x).

Proof: Based on Condition 7, for every $i \in N$, it follows that there exist $l_i : \Omega_i \to \Delta$ and $e_i : \Omega_i \to R$ such that for every $\omega_i \in \Omega_i$,

(26) $$\sum_{\omega_{-i}\in\Omega_{-i}} u_i(l_i(\omega_i),\omega) p_i(\omega_{-i} \mid \omega_i) + e_i(\omega_i) > \sum_{\omega_{-i}\in\Omega_{-i}} u_i(l_i(\omega_i'),\omega) p_i(\omega_{-i} \mid \omega_i) + e_i(\omega_i')$$

for all $\omega_i' \in \Omega_i \{\omega_i\}$.

We will occasionally regard $f(\omega)$ as the degenerate lottery that assigns probability 1 to the alternative $f(\omega)$. We specify g by

$$g(m) = (1-\varepsilon) f(m^2) + \frac{\varepsilon}{n} \sum_{i\in N} l_i(m_i^1).$$

For every $i \in N/\{1\}$, we define $s_i : \Omega_{-\iota(i)} \to R$ in a manner such that for every $\omega_{-\iota(i)} \in \Omega_{-\iota(i)}$,

$$s_i(\omega_{-\iota(i)}) = -\{1 - p_{-i-\iota(i)}(\omega_{-i-\iota(i)} \mid \omega_i)\}^2 - \sum_{\omega_i'\in\Omega_1/\{\omega_1\}} p_{-i-\iota(i)}(\omega_{-i-\iota(i)} \mid \omega_i')^2 .$$

Similar to the manner observed in the proof of Proposition 9 (the inequalities (25)), from Condition 8, it follows that for every $i \in N/\{1\}$ and $\omega_i \in \Omega_i$,

(27) $$\sum_{\omega_{-i-\iota(i)}\in\Omega_{-i-\iota(i)}} s_i(\omega_{-\iota(i)}) p_{i\iota(i)}(\omega_{-i-\iota(i)} \mid \omega_i) > \sum_{\omega_{-i-\iota(i)}\in\Omega_{-i-\iota(i)}} s_i(\omega_i', \omega_{-i-\iota(i)}) p_{i\iota(i)}(\omega_{-i-\iota(i)} \mid \omega_i)$$

for all $\omega_i' \in \Omega_i/\{\omega_i\}$.

Fix a positive real number $k > 0$ arbitrarily. We specify x in a manner such that for every $i \in N$ and every $m \in M$,

$$x_i(m) = \varepsilon e_i(m_i^1) - \frac{\varepsilon}{n-1} \sum_{j\in N/\{i\}} e_j(m_j^1) + k\{s_i(m_{-i-\iota(i)}^1, m_i^2) - \sum_{j\in N/\{i\}:i=\iota(j)} s_i(m_{-i-j}^1, m_j^2)\}.$$

It should be noted that (g,x) is in accordance with budget balancing and satisfies the $\varepsilon-$closeness to f. Based on the inequalities (26), it follows that for every $i \in N$ and every $\phi \in \Phi$, if $\phi_i^1 \neq \phi_i^{**1}$, then

$$\sum_{\omega\in\Omega} \{u_i(g(\phi(\omega)),\omega) + x_i(\phi(\omega))\} p(\omega)$$
$$- \sum_{\omega\in\Omega} \{u_i(g(\phi_{-i}(\omega_{-i}), \phi_i'(\omega_i)),\omega) + x_i(\phi_{-i}(\omega_{-i}), \phi_i'(\omega_i)))\} p(\omega)$$
$$= \varepsilon \sum_{\omega\in\Omega} \{u_i(l_i(\phi_i^1(\omega_i)),\omega) + e_i(\phi_i^1(\omega_i)) - u_i(l_i(\omega_i),\omega) - e_i(\omega_i)\} p(\omega) < 0,$$

where $\phi_i' = (\phi_i^{**1}, \phi_i^2)$. This implies that each player has a strict incentive to make an honest announcement about her first message, irrespective of the other players' announcements about their first message. Based on the inequalities (27), it follows that for every sufficiently large k, every $\phi \in \Phi$, and every $i \in N$, if $\phi_j^1 = \phi_j^{**1}$ for all $j \in N$, and $\phi_i^2 \neq \phi_i^{**2}$, then

$$\sum_{\omega\in\Omega} \{u_i(g(\phi(\omega)),\omega) + x_i(\phi(\omega))\} p(\omega)$$

$$- \sum_{\omega\in\Omega}\{u_i(g(\phi_{-i}(\omega_{-i}),\phi_i^{**}(\omega_i)),\omega) + x_i(\phi_{-i}(\omega_{-i}),\phi_i^{**}(\omega_i)))\}p(\omega)$$
$$= \sum_{\omega\in\Omega}\{(1-\varepsilon)u_i(f(\phi^2(\omega)),\omega) + ks_i(\omega_{-i-\iota(i)},\phi_i^2(\omega_i)))$$
$$-(1-\varepsilon)u_i(f(\phi_{-i}^2(\omega),\omega_i),\omega) - ks_i(\omega_{-\iota(i)})\}p(\omega) < 0\,.$$

This implies that whenever all players make honest announcements about their first message, each player has a strict incentive to make an honest announcement about her second message, irrespective of the other players' announcements about their second message. Hence, we have proved that ϕ^{**} is the unique twice iteratively undominated message rule profile in (p,g,x).

Q.E.D.

9. Three Times Iterative Dominance

It should be noted that Condition 7 excludes an important class of environments in which each player's private signal may have information only about payoff-irrelevant factors such as the interim outside values. As shown by Matsushima (2004), no inconstant social choice function is uniquely implementable when the private signals of players do not have any payoff-relevant information; therefore, players' *ex post* preference profile is common knowledge in the sense that for every $i \in N$, every $\omega \in \Omega$, and every $\omega' \in \Omega / \{\omega\}$, there exists $\beta \in R$ such that

$$u_i(\cdot, \omega') = u_i(\cdot, \omega) + \beta .$$

Serrano and Vohra (2001) also showed this result by providing an example with interim individual rationality. By using another modification of the direct mechanism, this sections aims to show that any inconstant social choice function is virtually implementable in terms of three times iterative dominance when the players' interim preferences are not common knowledge. We introduce the following conditions on (p, f).

Condition 9: There exist a nonempty proper subset $D_1 \subset \Omega_1$, two lotteries $\hat{\alpha} \in \Delta$, $\tilde{\alpha} \in \Delta$, and two real numbers $\hat{t} \in R$ and $\tilde{t} \in R$ that satisfy the following properties.

(i) For every $\omega_1 \in D_1$,

$$\sum_{\omega_{-1} \in \Omega_{-1}} u_1(\hat{\alpha}, \omega) p_1(\omega_{-1} \mid \omega_1) + \hat{t} > \sum_{\omega_{-1} \in \Omega_{-1}} u_1(\tilde{\alpha}, \omega) p_1(\omega_{-1} \mid \omega_1) + \tilde{t} ,$$

and for every $\omega_1 \in \Omega_1 / D_1$,

$$\sum_{\omega_{-1} \in \Omega_{-1}} u_1(\hat{\alpha}, \omega) p_1(\omega_{-1} \mid \omega_1) + \hat{t} < \sum_{\omega_{-1} \in \Omega_{-1}} u_1(\tilde{\alpha}, \omega) p_1(\omega_{-1} \mid \omega_1) + \tilde{t} .$$

(ii) For every $i \in N / \{1\}$, every $\omega_i \in \Omega_i$, and every $\omega_i' \in \Omega_i / \{\omega_i\}$,

$$p_i^1(D_1 \mid \omega_i) \neq p_i^1(D_1 \mid \omega_i') .$$

The first property of Condition 9 implies that each player's *interim* preference is not common knowledge. It should be noted that with private values, players' *ex post* preferences are not common knowledge only if the first property holds. It should be noted that the second property holds for generic prior distributions. We introduce the following condition on p.

Condition 10: There exists $\iota(1) \in N / \{1\}$ such that for every $\omega_1 \in \Omega_1$ and every $\omega_1' \in \Omega_1 / \{\omega_1\}$,

$$p_{1\iota(1)}(\cdot \mid \omega_1) \neq p_{1\iota(1)}(\cdot \mid \omega_1') .$$

It should be noted that Condition 10 holds for generic prior distributions. Serrano and Vohra (2000) showed a condition on the prior distribution under which every incentive compatible social choice function is virtually implementable in Bayesian Nash equilibrium. Conditions 9 and 10 are weaker than this condition.

We consider indirect mechanisms (g, x) where the set of messages for each player

$i \in N/\{1\}$ is specified as

$$M_i = \Omega_i,$$

and the set of messages for player 1 is specified as

$$M_1 = M_1^1 \times M_1^2 = \{0,1\} \times \Omega_1.$$

Similar to the direct mechanism, each player makes a single announcement about her private signal, except player 1. The latter does not only make a single announcement about her private signal but also announces either 0 or 1. We specify that $m_1 = (m_1^1, m_1^2) \in M_1$ and $\phi_1 = (\phi_1^1, \phi_1^2) \in \Phi$, where

$$\phi_1^1 : \Omega_1 \to \{0,1\} \text{ and } \phi_1^2 : \Omega_1 \to \Omega_1.$$

The honest message profile is denoted by $\hat{\phi} \in \Phi$, where $\hat{\phi}_i = \phi_i^*$ for all $i \in N/\{1\}$,

$$\hat{\phi}_1^2(\omega_1) = \omega_1 \text{ for all } \omega_1 \in \Omega_1,$$

$$\hat{\phi}_1^1(\omega_1) = 0 \text{ for all } \omega_1 \in D_1,$$

and

$$\hat{\phi}_1^1(\omega_1) = 1 \text{ for all } \omega_1 \in \Omega_1 / D_1.$$

The following proposition states that under Conditions 9 and 10, for generic prior distributions, there exist indirect mechanisms with budget balancing that virtually implement the social choice function f in terms of three times iterative dominance.

Proposition 11: *Suppose that Conditions 9 and 10 hold. Then, for every $\varepsilon > 0$, there exists an indirect mechanism with budget balancing (g, x) that satisfies the ε-closeness to f such that $\hat{\phi}$ is the unique three times iteratively undominated message rule profile in (p, g, x).*

Proof: Let

$$l(0) \equiv \hat{\alpha} \text{ and } l(1) \equiv \tilde{\alpha}.$$

We specify g as

$$g(m) = (1-\varepsilon) f(m_1^2, m_{-1}) + \varepsilon l(m_1^1),$$

where we regard $f(\omega)$ as the degenerate lottery that assigns probability 1 to $f(\omega)$. For every $i \in N/\{1\}$, we define $s_i : \{0,1\} \times \Omega_i \to R$ in a manner such that for every $\omega_i \in \Omega_i$,

$$s_i(0, \omega_i) = -\{1 - p_i^1(D_1 \mid \omega_i)\}^2 - \sum_{\omega_i' \in \Omega_i / \{\omega_i\}} p_i^1(D_1 \mid \omega_i')^2$$

and

$$s_i(1, \omega_i) = -\{1 - p_i^1(\Omega_1 / D_1 \mid \omega_i)\}^2 - \sum_{\omega_i' \in \Omega_i / \{\omega_i\}} p_i^1(\Omega_1 / D_1 \mid \omega_i')^2.$$

We define $s_1 : \Omega_{-\iota(1)} \to R$ in a manner such that for every $\omega_{-\iota(1)} \in \Omega_{-\iota(1)}$,

$$s_1(\omega_{-\iota(1)}) = -\{1 - p_{1\iota(1)}(\omega_{-1-\iota(1)} \mid \omega_1)\}^2 - \sum_{\omega_1' \in \Omega_1 / \{\omega_1\}} p_{1\iota(1)}(\omega_{-1-\iota(1)} \mid \omega_1')^2.$$

The second property of Condition (9) implies that for every $i \in N/\{1\}$ and every $\omega_i \in \Omega_i$,

(28) $$s_i(0, \omega_i) p_i^1(D_1 \mid \omega_i) + s_i(1, \omega_i) p_i^1(\Omega_1 / D_1 \mid \omega_i)$$

$$> s_i(0,\omega_i)p_i^1(D_1 \mid \omega_i') + s_i(1,\omega_i)p_i^1(\Omega_1 / D_1 \mid \omega_i') \text{ for all } \omega_i' \in \Omega_i / \{\omega_i\}.$$

Condition (10) implies that for every $\omega_1 \in \Omega_1$,

(29) $$\sum_{\omega_{-1-\iota(1)} \in \Omega_{-1-\iota(1)}} s_1(\omega_{-\iota(1)}) p_{1\iota(1)}(\omega_{-1-\iota(1)} \mid \omega_1) > \sum_{\omega_{-1-\iota(1)} \in \Omega_{-1-\iota(1)}} s_1(\omega_1', \omega_{1-\iota(1)}) p_{1\iota(1)}(\omega_{-1-\iota(1)} \mid \omega_1)$$

for all $\omega_1' \in \Omega_1 / \{\omega_1\}$.

Let

$$e(0) \equiv \hat{t} \text{ and } e(1) \equiv \tilde{t}.$$

Fix a positive real number $k > 0$ arbitrarily and specify x in a manner such that for every $i \in N / \{1, \iota(1)\}$ and every $m \in M$,

$$x_i(m) = -\frac{\varepsilon e(m_1^1)}{n-2} + k\{s_i(m_1^1, m_i) - \frac{\sum_{j \in N/\{i,1\}} s_j(m_1^1, m_j)}{n-2}\},$$

$$x_1(m) = \varepsilon e(m_1^1) + k s_1(m_1^2, m_{-1-\iota(1)}),$$

and

$$x_{\iota(1)}(m) = k\{s_{\iota(1)}(m_1^1, m_{\iota(1)}) - s_1(m_1^2, m_{-1-\iota(1)}) - \frac{\sum_{j \in N/\{1,\iota(1)\}} s_j(m_1^1, m_j)}{n-2}\}.$$

It should be noted that (g, x) is in accordance with budget balancing and satisfies the ε – closeness to f. Based on the first property of Condition 9, it follows that for every $\phi \in \Phi$, if $\phi_1^1 \neq \hat{\phi}_1^1$, then

$$\sum_{\omega \in \Omega} \{u_1(g(\phi(\omega)), \omega) + x_1(\phi(\omega))\} p(\omega)$$
$$- \sum_{\omega \in \Omega} \{u_1(g(\phi_{-1}(\omega_{-1}), \phi_1'(\omega_1)), \omega) + x_1(\phi_{-1}(\omega_{-1}), \phi_1'(\omega_1)))\} p(\omega)$$
$$= \varepsilon \sum_{\omega \in \Omega} \{u_1(l(\phi_1^1(\omega_i)), \omega) + e(\phi_1^1(\omega_1)) - u_1(l_1(\omega_1), \omega) - e(\omega_1)\} p(\omega) < 0,$$

where $\phi_1' = (\hat{\phi}_1^1, \phi_i^2)$. This implies that player 1 has a strict incentive to make an honest announcement about her first message. Based on the inequalities (28), it follows that for every sufficiently large k, every $\phi \in \Phi$, and every $i \in N / \{1\}$, if $\phi_1^1 = \hat{\phi}_1^1$ and $\phi_i \neq \hat{\phi}_i$, then

$$\sum_{\omega \in \Omega} \{u_i(g(\phi(\omega)), \omega) + x_i(\phi(\omega))\} p(\omega)$$
$$- \sum_{\omega \in \Omega} \{u_i(g(\phi_{-i}(\omega_{-i}), \hat{\phi}_i(\omega_i)), \omega) + x_i(\phi_{-i}(\omega_{-i}), \hat{\phi}_i(\omega_i))\} p(\omega)$$
$$= \sum_{\omega \in \Omega} \{u_i(g(\phi(\omega)), \omega) + k s_i(\hat{\phi}_1^1(\omega_1), \phi_i(\omega_i))$$
$$- u_i(g(\phi_{-i}(\omega_{-i}), \hat{\phi}_i(\omega_i)), \omega) - k s_i(\hat{\phi}_1^1(\omega_1), \hat{\phi}_i(\omega_i))\} p(\omega) < 0.$$

This implies that whenever player 1 makes an honest announcement about her first message, each player has a strict incentive to make an honest announcement, except player 1. Based on the inequalities (29), it follows that for every sufficiently large k and every $\phi \in \Phi$, if $\phi_1^1 = \hat{\phi}_1^1$ and $\phi_i = \hat{\phi}_i$ for all $i \in N / \{1\}$, then

$$\sum_{\omega \in \Omega} \{u_1(g(\phi(\omega)), \omega) + x_1(\phi(\omega))\} p(\omega)$$

$$
\begin{aligned}
&-\sum_{\omega\in\Omega}\{u_1(g(\phi_{-1}(\omega_{-1}),\phi_1'(\omega_1)),\omega)+x_1(\phi_{-1}(\omega_{-1}),\phi_1'(\omega_1)))\}\,p(\omega)\\
&=\sum_{\omega\in\Omega}\{u_1(g(\phi(\omega)),\omega)+ks_1(\phi_1^2(\omega_1),\hat{\phi}_{-1-t(1)}(\omega_{-1-t(1)}))\\
&-u_1(g(\hat{\phi}(\omega)),\omega)-ks_1(\hat{\phi}(\omega))\}\,p(\omega)<0\,.
\end{aligned}
$$

Hence, we have proved that $\hat{\phi}$ is the unique three times iteratively undominated message rule profile in (p,g,x).

Q.E.D.

10. Conclusion

This paper investigated mechanism design with incomplete information and quasi-linearity. We showed that with three or more players and a restriction on the size of the private signal space for each player, there exists a side payment function that satisfies budget balancing, incentive compatibility, and interim individual rationality. With regard to agency problems with adverse selection, we showed that the risk-averse principal could extract the full surplus without harming the incentive compatibility and interim individual rationality of the agents. These possibility results depended on the assumptions that there exist three or more players and that their private signals are correlated. We showed that the full surplus extraction might be impossible either when there exists only two players or when the players' private signals are independent. We also investigated the possibility of uniquely implementing social choice functions by practicing only a small number of iterative removals of undominated strategies. We showed that whenever players' interim preferences are not common knowledge, then, for generic prior distributions, every social choice function is uniquely and virtually implementable in terms of three times iterative dominance via a simple modification of the direct mechanism.

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