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Abstract

The purpose of this study is to examine the effects of a measured aggregate productivity shock on asset returns. To achieve this, a simple equilibrium business cycle model is presented to show that an aggregate productivity shock can be identified as a factor affecting asset returns. The paper uses the Solow residual to measure productivity changes, but deviates from standard practice by incorporating variations in capital utilization rates. The paper first develops the theoretical link between productivity shocks and asset returns with no adjustment costs, and then tests that link with the two measures of productivity, the Solow residual with and without variation in capital utilization. Results based on U.S post-war data show significant differences in the dynamic impacts of these two measures of productivity. The VAR evidence suggests that technology changes, measured with variation in capital utilization, have a delayed impact on asset returns, a distinct finding. Finally, policy implications of the findings are discussed.

JEL Classification: O47; O30; G12

Key Words: Aggregate Productivity Shock; Asset Returns; Adjusted Solow Residual; Capital Utilization

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1 Introduction

This study aims to identify an aggregate productivity shock as a macroeconomic factor, measure it appropriately, and then, empirically evaluate its effects on asset returns using post World War II U.S. data. This study contributes to the literature by theoretically exploring the relationship between aggregate productivity shocks and asset returns and by empirically evaluating this relationship. In particular, this study attempts to explore the asset pricing implications of variable capital utilization adjustment. The study documents substantial differences between the *conventional* Solow residual and the *adjusted* Solow residual in terms of their dynamic effects on asset returns.[1] While the aggregate productivity shocks measured by the conventional Solow residual generate an impact effect on asset returns, those measured by the adjusted Solow residual generate a delayed effect.

It is a well-known fact that asset returns are affected not only by firm-specific risks but also by macroeconomic risks. Although firm-specific risks (diversifiable risks) can be avoided by building a good portfolio, macroeconomic risks (undiversifiable risks), owing to their nature, cannot be avoided. Since macroeconomic risks are unavoidable and yet have significant effects on the asset returns, many researchers have attempted to identify one or more variables as macroeconomic risks and have analyzed their impacts.

In the empirical finance literature, macroeconomic risks are considered as *factors*. The first significant study in this area was conducted by Chen, Roll, and Ross (1986), who explored a set of economic variables that systematically affected asset returns.[2] Although a wide range of variables have been chosen as factors in this line of research, little consensus has been obtained on why such variables should be the factors. In other words, the existing literature often fails to provide a theoretical justification for the chosen factors, particularly when they are selected by fitting returns, rather than by deriving from explicit theoretical frameworks.

This study adopts a different approach by establishing a link between theory and empirics. In particular, it identifies one of the factors based on a simple equilibrium model, and then empirically assesses its effects on asset returns. Based on an equilibrium business cycle model, this study shows that an aggregate productivity shock can be identified as one of the factors that affect asset returns. In other words, this study provides a theoretical justification for an identifiable factor before it proceeds to the empirics.

Over the years, consumption-based asset pricing models have provided theoretical foundations for analyzing asset returns. Under these models, the key relationship between asset returns and a stochastic discount factor is closely

[1] The *conventional* Solow residual refers to the standard Solow residual, and the *adjusted* Solow residual takes into account variable capital utilization.

[2] The variables include (1) the spread between long- and short-term interest rates, (2) expected inflation, (3) unexpected inflation, (4) industrial production, and (5) the spread between high- and low-grade bonds.

related to the first order condition of an investor's consumption and portfolio choice problem.[3] Another similar approach has considered at the production side. Cochrane (1996), Lamont (2000), Hall (2001), Jermann (1998), and Rouwenhorst (1995) studied asset pricing implications from the perspective of the production side of an economy. They derived the asset pricing relationship from the first order condition of the producer's problem. Indeed, this paper takes an approach similar to existing production-side asset pricing models, where asset returns equal capital returns.

This line of study is particularly useful for showing how equilibrium business cycle models can be used to study various issues in finance. In the standard one-sector business cycle model, an aggregate productivity shock is important because it is considered to be one of the major sources of fluctuations in most macroeconomic variables in the absence of other shocks, such as preference and monetary shocks. Clearly, a single source of uncertainty, the aggregate productivity shock in the model can be a natural candidate for a macroeconomic factor. The capital returns are exposed to the aggregate productivity shock in the equilibrium business cycle model, and the capital returns equal the asset returns in the production-side asset pricing model. Thus, if the two models are combined, the link between the aggregate productivity shock and the asset returns can be established.

To evaluate the quantitative aspects of the relationship between aggregate productivity shock and asset returns, this paper attempts to estimate the fundamental equations using the U.S. data, rather than to calibrate the model and run simulations to match the observed data. In this sense, the present study shares a spirit with Lettau and Ludvigson (2001), who first identified the consumption-wealth ratio as one of the factors based on an equilibrium model and then empirically evaluated its effects.

For empirical investigations, the paper uses the Solow residual as a proxy for the measured aggregate productivity shock, but deviates from the standard practice by incorporating variations in capital utilization rates. In particular, the conventional Solow residual is constructed based on the standard growth accounting framework. On the other hand, the adjusted Solow residual is obtained after controlling for variable capital utilization. The major differences between the two Solow residuals are their cyclical variations.

One of the well-known characteristics of the conventional Solow residual is its procyclicality.[4] A number of existing studies point out that one source of the procyclicality might arise from unaccounted variations in inputs. Since a rise in factor utilization leads to an increase in output, the former should be considered when the Solow residual is constructed or else the Solow residual would be spuriously procyclical. In other words, variable factor utilization provides one possible explanation for the observed cyclicality.

[3] For a good survey on the equity premium puzzle, see Kocherlakota (1996).

[4] Procyclicality, a comovement between the conventional Solow residual and output growth is one of the key features in the RBC models. This study finds that the correlation coefficient between the conventional Solow residual and output growth is 0.9 while that between the adjusted Solow residual and output growth is 0.2.

Starting from Greenwood, Hercowitz, and Huffman (1988), a growing number of authors have studied variable factor utilization and its implications for equilibrium business cycle models. In particular, Shapiro (1996) argued that capital stock needed to be adjusted for variable capital-utilization rates to properly measure the Solow residual. Further, he showed that the procyclicality of the Solow residual almost disappeared after the adjustment.[5] Prior to his work, Burnside, Eichenbaum, and Rebelo (1995) considered electricity use as a proxy for the flow of capital service and argued that the Solow residual was not very procyclical. Basu, Fernald, and Shapiro (2001) also considered variable factor utilization along with adjustment costs when they measured the Solow residual. In addition, Paquet and Robidoux (1997) used the adjusted Solow residual while testing its exogeneity based on U.S. and Canadian data. To accommodate these recent developments, this study follows the approach adopted by Paquet and Robidoux (1997) and Shapiro (1996) for constructing the adjusted Solow residual.

The adjustment is not trivial because it substantially changes some characteristics of the conventional Solow residual. First, the variability of the Solow residual reduces considerably after the adjustment. Second, the Solow residual becomes far less procyclical. Indeed, Basu, Fernald, and Kimball (2004) argued that the technology improvements were contractionary after the adjustments.

While the existing literature examines the business cycle implications of the adjustment, few studies investigate the asset pricing implications. This study examines the effects of the aggregate productivity shock on asset returns based on two alternative measures: the conventional Solow residual and the adjusted Solow residual.

Empirical investigations of the study comprise two parts. First, the linear approximation of the fundamental equations, derived from the equilibrium model, is estimated to evaluate the size of the effects of the adjusted Solow residual (and the conventional Solow residual) on asset returns. The results show that variable capital utilization adjustment does lead to large differences in the outcomes of the estimations. However, the Granger-causality test, which could empirically verify the direction of the causality implied by the model, suggests that the data do not support the implied casuality; the test results are, at best, ambiguous. Since the first step of the analysis uncovers interesting dynamic effects arising from the adjusted Solow residual, Vector Autoregressions (VARs) are employed as a second step for a better understanding of the dynamic effects of the measured productivity shock on asset returns. The VAR evidence suggests that technology changes, measured with variation in capital utilization, have a delayed impact on asset returns – a distinct finding.

The remainder of the paper is organized as follows. Sections 2 and 3 describe the model economy and the empirical specifications, respectively. Section 4 describes the data and the estimation results, after which it discusses the implications of the findings. Section 5 concludes the paper.

[5] He showed that the correlation between the adjusted Solow residual and output growth was close to zero.

2 Model

A simple version of the equilibrium business cycle model is presented to derive the key equations for empirical investigations. The economy is composed of a large number of homogeneous households whose utilities are determined by the consumption of goods and labor. On the production side of the economy, identical firms produce homogenous goods. The firms own the capital stock of the economy. There is only one source of uncertainty, an aggregate productivity shock. The markets are competitive and complete.

2.1 Households

A representative agent maximizes her expected lifetime utility subject to her budget constraint. The agent's utility function is assumed to be concave, strictly increasing, and twice continuously differentiable, while the type of the utility function is not assumed.[6] In particular, the household problem becomes

$$\max_{C,L} E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t), \tag{2.1}$$

$$0 < \beta < 1,$$

where β is the discount factor, C_t is the consumption in period t, and L_t is the labor in period t. E denotes the conditional expectation operator given the information set. The sequential budget constraint is:

$$C_t + P_t Z_t = w_t L_t + Z_{t-1}(P_t + D_t), \tag{2.2}$$

where P_t is the asset price measured in consumption goods at time t, Z_t denotes the number of shares owned by the consumer at the beginning of t, and D_t represents the dividend. Then, the consumption Euler equation associated with this problem becomes[7]

$$u_{c_t} = \beta E_t u_{c_{t+1}} R_t, \tag{2.3}$$

where $R_t = \frac{P_{t+1}+D_{t+1}}{P_t}$ represents the rate of return on the asset between time t and $t+1$.[8]

[6]This section does not intend to solve the model in detail by specifying all the functional forms, but it does aim to derive the key relationship between an aggregate productivity shock and asset returns.

[7]See Appendix A.1 for the details of the derivations.

[8]In Section 2.2, one period return R_t is tied to the capital return in period t, which is exposed to the current aggregate productivity shocks. To avoid confusion, this paper deviates from the conventional timing in the consumer problem, where R_{t+1} is used in the consumption Euler equation.

2.2 Firms

A representative firm has a constant-returns-to-scale production function with output augmenting (Hicks-neutral) technical progress. A_t is the aggregate productivity level (level of technology) in period t. The firm chooses labor, L_t, and utilized capital stock, $u_t K_t$, to maximize the expected discounted present value of the firm. The production function can be rewritten as

$$Y_t = A_t F(u_t K_t, L_t), \tag{2.4}$$

where Y_t is the output in period t, K_t is the capital stock at the beginning of the period, and u_t is the rate of capital utilization.

The capital stock is accumulated with the variable depreciation rate, δ_t. There are no adjustment costs for capital. As in Greenwood et al. (1988), the depreciation rate is the increasing function of the utilization rate. This study assumes that the depreciation rate of capital, δ_t, is given by

$$\delta_t = \delta u_t^{\phi}, \tag{2.5}$$

where $0 < \delta < 1$ and $\phi > 1$. Since $\phi > 1$, the depreciation rate increases with the utilization rate in a convex manner. In this specification, ϕ can be interpreted as the elasticity of marginal rate of depreciation. As ϕ increases, the shape of the depreciation rate curve becomes more convex, indicating that it is costlier to change the utilization rate.

The stock of capital is given by

$$K_{t+1} = (1 - \delta_t)K_t + I_t. \tag{2.6}$$

When output is produced, the payments to labor, w_t, and investment, I_t, are made and the remaining portion forms the dividend, D_{t+1}, which is paid out at the beginning of the next period,

$$D_{t+1} = Y_t - w_t L_t - I_t. \tag{2.7}$$

The firm maximizes its net present discounted value:[9]

$$\max_{K_t, L_t, u_t} E_0 \sum_{t=0}^{\infty} \beta^t \nu_{t+1}(D_{t+1}) \tag{2.8}$$
$$\text{subject to (2.4) \& (2.6)},$$

where ν_{t+1} represents the price of capital or the marginal rate of substitution of the firm owners between time 0 and $t+1$. Then, the first order conditions for capital, labor, and the utilization rate are

$$\beta E_t \nu_{t+1}(A_t F_{1,t} u_t + 1 - \delta_t) = \nu_t, \tag{2.9}$$

[9] In the case of the firm's problem, the choice variables are capital stock, K, labor, L, and the utilization rate, u.

$$A_t F_{2,t} = w_t, \tag{2.10}$$

$$A_t F_{1,t} K_t = \delta \phi u_t^{\phi - 1} K_t, \tag{2.11}$$

where $F_{1,t}$ is the derivative of F with respect to the first argument,$u_t K_t$, and $F_{2,t}$ is the derivative of F with respect to the second argument, L_t.

According to Equations (2.9) and (2.10), the price of capital, ν_t is equal to the expected marginal value product of the next period, and the wage rate is equal to the marginal product of labor. Equation (2.11) shows that the value of additional output from a higher utilization rate is equal to the replacement cost, which is the cost of replacing an additional unit of capital that is worn out due to a higher utilization rate. From Equation (2.11), the optimal utilization rate is obtained as

$$u_t^* = (\frac{A_t F_{1,t}}{\delta \phi})^{\frac{1}{\phi - 1}}. \tag{2.12}$$

Furthermore, once the optimal utilization rate is chosen, its associated depreciation rate is determined by using (2.5):

$$\delta_t^* = \delta (\frac{A_t F_{1,t}}{\delta \phi})^{\frac{\phi}{\phi - 1}}. \tag{2.13}$$

Here, it should be noted that $F_{1,t}$, the derivative of F with respect to the first argument, is a function of u_t, L_t, and K_t. Thus, Equations (2.12) and (2.13) say that the optimal utilization rate and the optimal depreciation rate are nonlinear functions of A_t and K_t, L_t, and u_t.

2.3 Equilibrium

An equilibrium is defined as a set of endogenous variables, where firms maximize their present discounted values given their production technology and households maximize their utilities subject to their budget constraints. The equilibrium is efficient since it satisfies all the efficient allocation conditions. In addition, all goods produced are either invested or consumed.

$$Y_t = C_t + I_t. \tag{2.14}$$

Labor markets and financial markets also clear so that neither excess demand nor supply exists.

2.4 Key Relationships

Under the assumption of constant-returns-to-scale production and competitive markets, the key relationship between asset returns and aggregate productivity shock can be derived.[10] The capital payment is the remainder of the value of

[10] As Benhabib, Meng, and Nishimura (2000) elaborated, deviations from these assumptions could cause theoretical problems, such as indeterminacy or multiple equilibria. However, a number of researchers have concluded that returns to scale of the aggregate U.S. economy appear to be roughly constant. See Burnside, Eichenbaum, and Rebelo (1995), Basu and

the output after the payment to labor because of the homogeneity of degree one. Then, the dividend equation can be written as:

$$D_{t+1} = Y_t - w_t L_t - I_t = Y_t - (Y_t - A_t F_{1,t} u_t K_t) - (K_{t+1} - K_t - \delta_t K_t). \quad (2.15)$$

By rearranging the terms, Equation(2.15) can be written as

$$\frac{D_{t+1} + K_{t+1}}{K_t} = A_t F_{1,t} u_t + 1 - \delta_t. \quad (2.16)$$

Using the first order condition for the capital from the firm's problem and the household Euler equation from the households problem, it is shown that $K_t = P_t$.[11] Then, the above equation becomes

$$\frac{D_{t+1} + K_{t+1}}{K_t} = \frac{D_{t+1} + P_{t+1}}{P_t} = R_t. \quad (2.17)$$

Equation(2.17) says that asset returns are determined by the dividend and the asset prices. Combined with Equation (2.16), Equation (2.17) becomes,

$$R_t = A_t F_{1,t} u_t + 1 - \delta_t. \quad (2.18)$$

Using the optimal utilization rate and its associated depreciation rate, R_t – the one-period return between t and $t+1$ – can be written as:

$$R_t = A_t F_{1,t} u_t + 1 - [\delta(\frac{A_t F_{1,t}}{\delta\phi})^{\frac{\phi}{\phi-1}}] \quad (2.19)$$

Equation (2.19) represents the relationship between asset returns and net marginal product of capital.[12]

Fernald (1997) for examples. While the assumptions of the study are based on these empirical studies, the results of the study are consistent with the ones by Basu, Fernald, and Kimball (2004), who allowed for non-constant returns and imperfect competition when they measured *purified technological progress*, which is similar to *adjusted Solow residual* in this study.

[11] For the detailed derivation, see Appendix A.2. Without adjustment costs, Tobin's q is equal to 1. Consequently, the price of the equity in the model is equal to the value of the capital stock. When the adjustment costs are introduced, the price of equity in the model could differ from the value of capital stock. In fact, using annual data from two-digit industries, Hall (2004) found relatively strong evidence against substantial adjustment costs. The result supported his earlier work (2001), where he measured intangible capital based on the value of the capital stock market, assuming a low rate of adjustment.

[12] Alternatively, the one-period return is directly related to the depreciation rate, $R_t = 1 + (\phi - 1)\delta_t$, if the optimality condition for the capacity utilization, $A_t F_{1,t} = \delta\phi u_t^{\phi-1}$, is plugged into Equation (2.18). This is, indeed, the basis of the empirical specification, which will be discussed in Equation (3.1) in Section 3. If the optimal utilization rate and its associated depreciation rate are used, the on-period return can be rewritten as, $R_t = 1 + (\phi - 1)[\delta(\frac{A_t F_{1,t}}{\delta\phi})^{\frac{\phi}{\phi-1}}]$. Because $F_{1,t}$ is a function of K_t, L_t, and u_t, the above equation says that R_t is nonlinearly related to A_t, K_t, L_t, and u_t. Thus, the empirical specification in (3.1) does not change.

2.5 Constant capacity utilization

When a 100% utilization rate is assumed, $u_t = u = 1.0$, $\delta_t = \delta$.[13] Thus, the depreciation rate becomes constant. The production function becomes,

$$Y_t = A_t F(u_t K_t, L_t) \underbrace{\Rightarrow}_{when\ u_t = 1.0} Y_t = A_t F(K_t, L_t). \tag{2.20}$$

The stock of capital is accumulated according to equation (2.21),

$$K_{t+1} = (1 - \delta)K_t + I_t. \tag{2.21}$$

The objective function for the firm has not been changed; it maximizes its net present discounted value.[14] However, there are only two first order conditions (capital and labor):

$$\beta E_t \nu_{t+1}(A_t F_{1,t} + 1 - \delta) = \nu_t, \tag{2.22}$$

$$A_t F_{2,t} = w_t. \tag{2.23}$$

Finally, the key relationship is derived in the same manner as it was done in the previous section, using the dividend relationship and the linear homogeneity assumption of the production function. In particular, the one-period return, R_t, is obtained as

$$R_t = A_t F_{1,t} + 1 - \delta. \tag{2.24}$$

In fact, Equation (2.24) is a special case of Equation (2.18) when $u_t = 1.0$, for all t.

3 Empirical Strategies

According to Equation (2.19), R_t depends on the level of the aggregate productivity, A_t, the capital stock, K_t, the labor input, L_t, and the utilization rate, u_t. At this stage, one of the difficult tasks is to choose a particular functional form for the aggregate production function and to derive the exact relationship. This study begins with a general function, H, which simply assumes that R_t is nonlinearly related to A_t, K_t, L_t, and u_t,

$$R_t = H(A_t, K_t, L_t, u_t). \tag{3.1}$$

[13]From the optimal utilization, $u_t = (\frac{A_t F_{1,t}}{\delta\phi})^{\frac{1}{\phi-1}}$, if $\phi \to \infty$, then $\frac{1}{\phi-1} \to 0$. Thus, $u_t \to 1.0$. In this case, it becomes too costly to vary the intensive margin, and the quantity of capital service does not respond to changes in the marginal production of these services. When $\phi = 0$ (although it is not allowed in the model due to a parameter restriction, $\phi > 1$), the level of utilization is not determined and the depreciation rate becomes constant, i.e., $\delta_t = \delta u_t^0 = \delta$ regardless of the level of utilization.

[14]In the model of the constant utilization rate, the choice variables are capital stock and labor only. The utilization rate is no longer a choice variable.

Next, the first-order Taylor approximation is applied,

$$R_t = H^* + H_1^*(A_t - A^*) + H_2^*(K_t - K^*) + H_3^*(L_t - L^*) \\ + H_4^*(u_t - u^*) + \text{Higher Order Terms}, \quad (3.2)$$

where H^* is evaluated at the stationary equilibrium levels of all its arguments and H_1^*, H_2^*, H_3^*, and H_4^* are the derivatives of the function H with respect to A_t, K_t, L_t, and u_t, respectively.

By rearranging the terms, the linear approximation of the asset return equation is obtained:

$$R_t = \beta_0 + \beta_1 A_t + \beta_2 K_t + \beta_3 L_t + \beta_4 u_t + \epsilon_t, \quad (3.3)$$

where ϵ_t includes an approximation error and factors other than A_t, K_t, L_t, and u_t that affect asset returns at time t. $\beta_0 = H^* - H_1^* A^* - H_2^* K^* - H_3^* L^* - H_4^* u^*$, $\beta_1 = H_1^*$, $\beta_2 = H_2^*$, $\beta_3 = H_3^*$, and $\beta_4 = H_4^*$.

Before the estimation, this study examines the time series properties of all the variables. To avoid possible spurious regression results in the presence of unit roots, a set of unit root tests is conducted. In particular, a recently developed test —Ng and Perron's unit root test— and a conventional unit root test —the Augmented Dickey–Fuller test— are performed to determine whether these variables are indeed I(0).[15]

The unit-root test results given in Tables 1 and 2 show that all the variables except for the utilization rate are nonstationary.[16] Thus, the first-differenced specification is considered,

$$\Delta R_t = \beta_1 \Delta A_t + \beta_2 \Delta K_t + \beta_3 \Delta L_t + \beta_4 \Delta u_t + \eta_t, \quad (3.4)$$

where $\{\eta_t\} \sim i.i.d.(0, \sigma_\eta^2)$.

Equation (3.4) is the final specification for empirical investigations under the variable utilization. In this specification, the parameters in equation (3.4) have elasticity interpretations because this study takes the natural log of all the level variables in Equation (3.3).

The final specification under the constant utilization can be obtained as a special case of Equation (3.4). Once the utilization rate is constant, $\Delta u_t = 0$, and Equation (3.4) becomes

$$\Delta R_t = \gamma_1 \Delta A_t + \gamma_2 \Delta K_t + \gamma_3 \Delta L_t + \nu_t, \quad (3.5)$$

[15] Ng and Perron (2001) argued that many unit root tests suffered from size distortion and that, in some cases, unit root tests tended to produce an overrejection of the unit root hypothesis. They developed a unit root test based on GLS detrended series, and provided modified information criteria to determine the number of lagged variables to be used in the unit root test.

[16] This study also conducts other available unit root tests such as the Phillips–Perron test; the Kwiatkowski, Phillips, Schmidt, and Shin test; and the Elliot, Rothenberg, and Stock test. This study finds that the results of the unit root tests are often dependent on the number of lagged variables and the detrending method. The results from various unit root tests are robust for all the variables except for real asset returns.

where $\{\nu_t\} \sim i.i.d.(0, \sigma_\nu^2)$.

4 Data and Estimation

4.1 Empirical Investigation : Benchmark Cases

The sample period of the study runs from 1949 to 2001. Asset returns are calculated based on Standard & Poor's 500 composite index. An one-year time horizon begins on January 1 and ends on December 31. Thus, an investor purchases one unit of asset at the beginning of the period (January 1) and then sells it at the end of the period (December 31). In the interim, a dividend payment for her share is made before selling the asset so that the dividend is included in one period's return. Real asset returns are computed after adjusting for inflation from nominal asset returns. To measure aggregate productivity shocks (both the conventional Solow residual and the adjusted one), real gross domestic product (GDP), the number of employees, average hours worked, nonresidential real capital stock, the capacity utilization rate, and the average labor share are used.[17] The real GDP and the labor share data are obtained from the Bureau of Economic Analysis (BEA). The average labor share is computed from the annual series. The real capital stock is taken from the BEA. The capital stock includes private and public capital stock, excluding residential capital stock. Consumer price index (CPI) data that is taken from the Bureau of Labor Statistics (BLS) is used as a deflator. The capacity utilization rate is from the Federal Reserve Board (FRB).[18] Finally, all the labor data (hours worked and the number of employees) are obtained from the BLS.

Table 3 summarizes the descriptive statistics. There is no significant difference between the conventional and the adjusted Solow residuals in terms of the average annual growth rates. The average annual aggregate productivity growth rates are 1.58% and 1.48% based on the conventional Solow residual and the adjusted Solow residual respectively. The volatility, measured in terms of the standard deviation, is 0.01 for the adjusted Solow residual and 0.02 for the conventional Solow residual.[19] Table 4 shows that after the adjustment,

[17] See Appendix A.3. for details on measuring aggregate productivity shocks.

[18] According to the FRB, the capacity utilization rate is equal to the output index divided by the capacity index, and FRB's capacity indexes capture the concepts of sustainable maximum output. In fact, Shapiro (1989) criticized the official utilization series produced by the FRB as an economically meaningful measure of utilization. It is true that the official series falls short of being a satisfactory measure of the capacity utilization rate. However, it is the changes in, not the levels of, the capital utilization rate that this study uses in the empirical analysis. In addition, the characteristics of the adjusted Solow residual are qualitatively rather similar to the ones found in the previous studies that are based on other measures of the capacity utilization rate — Burnside, Eichenbaum, and Rebelo (1995); Shapiro (1996); Basu, Fernald, and Kimball (2004). A similar justification was provided by Paquet and Robidoux (1997) when they used the official capacity utilization rate obtained from the FRB to adjust the Solow residual. This study compares the official capacity utilization rate series from the FRB with the one used in Burnside, Eichenbaum, and Rebelo (1995). As observed in Figure 14, these two series move closely together, and the correlation coefficient is over 0.9.

[19] The difference is quite substantial in terms of the coefficient of variation. While the

the correlation between the Solow residual and output growth reduces dramatically from 0.9 to 0.2. Interestingly, the correlation between the adjusted Solow residual and labor hour growth becomes negative.[20]

According to the model in this study, the aggregate productivity shock that hits the economy affects the asset returns. Thus, the causality runs from the aggregate productivity shock to the asset returns. However, Kiyotaki and Moore (1997) argued for a reverse causality.[21] To empirically verify the relationship, the Granger-causality tests are performed. Table 5 shows that the overall results based on the Granger-causality tests are, at best, inconclusive. The data indicate that the casuality could run in both directions.

Figure 1 shows the plots of the changes in the asset returns and the two Solow residuals, and Table 6 presents the estimation results.[22] As shown in Table 6, the conventional Solow residual has a significant effect on asset returns in both Case I and Case II. The estimated coefficient from Case I implies that a 1% increase in the aggregate productivity level raises the asset returns by 5.67%.[23] Approximately 25% of the variation in the changes in asset returns is explained by the regression line in Case I.

On the other hand, in Case I, the estimated coefficient on the adjusted Solow residual is 5.87. Although the sign is consistent with the model's prediction, the coefficient becomes insignificant. Instead, the utilization rate appears important for explaining the asset returns. Roughly, a 1% increase in the utilization rate raises the asset returns by 2.12%. It should be noted that the coefficient of the utilization rate captures two effects on the asset returns: the first order direct effect from the utilization rate and the second order indirect effect from the depreciation rate. Thus, it is not easy to distinguish the effects of the depreciation rate on the asset returns based on the regression results.

Among other things, Cases III and IV, shown in Table 6, reveal that the adjusted Solow residual generates dynamic effects on the asset returns. Perhaps, the delayed effect might arise from the changes in the depreciation rate. Once variable capital utilization is taken into account, the contemporaneous effect of the aggregate productivity shock on the asset returns becomes weaker and the

coefficient of variation for the conventional Solow residual is 129.6%, it is 84.6% for the adjusted Solow residual.

[20]It could imply that a positive aggregate technology shock might be contractionary. In fact, Galí (1999) and Basu, Fernald, and Kimball (2004) documented a negative contemporary correlation between the adjusted Solow residual and input growth. They argued that a technology shock could reduce input usages to accommodate an increase in productivity, particularly in the short run, with some price rigidity.

[21]A consumer can only borrow against her collateral, and the value of the collateral is linked to the asset price. When the asset market booms, investors can invest, which leads to productivity improvements.

[22]With regard to the first two plots in Figure 1, the effects of other variables on changes in the asset returns are already partialled out in the Solow residuals. Thus, these plots show the relationship between the conventional Solow residuals and the asset returns (the first figure) and that between the adjusted Solow residual and the asset returns (the second figure), after controlling for other variables.

[23]Since the first-differenced specification is used, there is no constant term. The estimated slope coefficient can be used to interpret the parameter in the level specification.

utilization rate becomes important. Moreover, the shock generates interesting dynamic effects. In order to gain a better understanding of the dynamic effects of the measured aggregate productivity shock on the asset returns, this study considers a VAR analysis.

Before proceeding to the VAR analysis, Ramsey's RESET test is conducted to test possible specification errors in the regression specifications.[24] The testing results are informative because this study derives the final specifications by the first order linear approximation, where the higher-order approximation terms are ignored. Thus, the final specifications may not describe the correct relationship between the asset returns and the measured aggregate productivity. The test results provided in Table 7 validate the empirical specifications of this study. The null hypotheses of no specification errors are not rejected at a 5% significance level for both cases.[25]

4.2 Vector Autoregressions: VARs

Based on the final specifications, the first-differenced form of the variables are considered in the VARs.[26] In particular, this study employs two VAR specifications [VAR(1) and VAR(6)] for the adjusted Solow residual:

$$\mathbf{X_a(t)} = \mathbf{B_1 X_a(t-1)} + \varepsilon_{\mathbf{a,t}}, \tag{4.1}$$

$$\mathbf{X_a(t)} = \mathbf{B_1 X_a(t-1)} + \cdots + \mathbf{B_6 X_a(t-6)} + \eta_{\mathbf{a,t}} \tag{4.2}$$

where $B_1 \cdots B_6$ are 5 x 5 matrices and $\mathbf{X_a(t)} = [\Delta A_{adj,t}, \Delta L_t, \Delta K_t, \Delta u_t, \Delta R_t]'$.[27] In addition, the following two VAR specifications [VAR(1) and VAR(6)] are considered for the conventional Solow residual:

$$\mathbf{X_c(t)} = \mathbf{C_1 X_c(t-1)} + \varepsilon_{\mathbf{c,t}}, \tag{4.3}$$

$$\mathbf{X_c(t)} = \mathbf{C_1 X_c(t-1)} + \cdots + \mathbf{C_6 X_c(t-6)} + \eta_{\mathbf{c,t}}, \tag{4.4}$$

where $C_1 \cdots C_6$ are 4 x 4 matrices and $\mathbf{X_c(t)} = [\Delta A_{conv,t}, \Delta L_t, \Delta K_t, \Delta R_t]'$.

[24] The Ramsey RESET test is often used for specification errors, which produce a nonzero mean for an error term. Thus, $H_o : u \sim N(0, \sigma^2), H_a : u \sim N(\mu, \sigma^2), \mu \neq 0$.

[25] In general, the rejection of the null hypothesis indicates one or all of the possible specification errors, including omitted variables, an incorrect functional form, measurement errors in regressors, and serially correlated disturbance.

[26] The first-differenced form of variables are used because of stationarity considerations. The impulse responses from level specifications show that the impacts do not fade even after 20 years, which does not provide meaningful interpretations of the dynamic effects of the shock. This study also considers level specifications using R_t instead of ΔR_t.

[27] This study is concerned about the implications for the sampling error of including too many lags and hence, an excessive number of parameters. Accordingly, this study uses Akaike Information Criterion (AIC) to determine the lag length, which results in the choice of one yearly lag, VAR(1). In addition, as in other business cycle models, in order to use VAR(6), this study implicitly assumes that the shocks could last up to 24 quarters (6 years).

4.2.1 Identification

Since the VARs are reduced–form models, the reduced-form errors are linear combinations of primitive shocks to the system.[28] This study follows Sim's (1980) method of orthogonalizing innovations. For identification purposes, this study introduces a lower–triangular matrix with 1 on the main diagonal.[29]

While the triangular identification scheme provides a set of residuals that are uncorrelated with the residuals associated with the equations ordered before them, it is a well-known fact that the order of the variables is rather important in this identification scheme.[30] Thus, with regard to the impulse responses and the variance decompositions, this paper chooses the orders consistent with the theoretical parts of the study.[31]

For benchmark cases, this study assumes that an aggregate productivity shock hits the economy at the beginning of the period, after which firms optimally choose capital (and utilization rate) and labor, and finally, asset returns are determined.[32] In addition, the Kiyotaki and Moore (1997) specifications are considered. In their model, a shock hits the asset market at the beginning of the period. Thus, the asset returns change first. Then, firms hire capital (and utilization rate) and labor. As a result, the aggregate productivity changes.

4.2.2 Impulse Response Functions and Variance Decompositions

Figures 2 and 3 show impulse response functions for the adjusted Solow residual based on VAR(1) and VAR(6). The impulse response functions reconfirm the

[28] Shocks to unorthogonalized innovations (reduced– form errors) generally do not provide useful interpretations, particularly when the shocks are correlated with each other. An alternative approach would be to use the structural VAR (SVAR). In fact, there ample discussions on the importance of the SVAR in the business cycle literature. In particular, Chari, Kehoe, and McGrattan (2005) provided a critique of the SVAR procedure using economic models. In addition, Fernandez-Villaverde, Rubio-Ramirez, and Sargent (2005) addressed issues related to the possible invertibility problems of mapping from VAR shocks to economic shocks. On the other hand, Erceg, Guerrieri, and Gust (2005) argued that the SVAR performed reasonably well in the business cycle models. Indeed, the recursive VAR approach used in this study is still subject to the small sample bias elaborated by Erceg, Guerrieri, and Gust (2005).

[29] The conventional and the adjusted Solow residuals are assumed to be more exogenous in the model. Thus, shocks to the conventional and adjusted Solow residuals transmit to other variables, but the converse is not true.

[30] One of the problems with this identification scheme is that the decomposition of the variance of the reduced–form error is not unique. Further, the lower triangularity of the transition matrix imposes a recursive structure on the system.

[31] To check the robustness of the VAR results, a sensitivity analysis is conducted by changing the order of the variables. The results (not reported in this paper) show that the impulse responses and variance decompositions do not change as long as the aggregate productivity comes first and the asset returns come last for the benchmark case. In addition, the results are robust for the Kiyotaki and Moore (1997) specifications as long as the asset returns comes first and the the aggregate productivity comes last. While the order of the variables in the recursive VAR generally matters, the results of the sensitivity analysis conducted in this study by changing the order of input choices (labor, capital, and utilization) suggest that the order does not matter much.

[32] When variable utilization is allowed, firms optimally choose capital, labor, and the utilization rate.

results of the previous section. In both cases, the shock does not affect the asset returns in the first period and generates dynamic effects on the asset returns. In addition, the shock reduces input usages as documented in Basu, Fernald, and Kimball (2004). Most importantly, the next–period asset returns respond to the current–period productivity shock. The impulse response functions for the level specifications are shown in Figures 4 and 5. The qualitative results do not change. A shock to technology generates a delayed effect on asset returns.

Figures 6 and 7 show the impulse response functions for the conventional Solow residual. In these cases, there are immediate effects on two inputs and the asset returns. Thus, these are consistent with the prediction of the standard equilibrium business cycle models. An improvement in technology has an impact effect on these inputs and asset returns. In addition, the level specifications shown in Figures 8 and 9 also confirm the impact effect of technology shock on asset returns.

This study reports additional impulse response functions for the Kiyotaki and Moore (1997) specifications. In cases that use the adjusted Solow residual, it appears difficult to explain the dynamic effects of the shock using their model. With a positive shock, there is no contemporaneous effect on the aggregate productivity. Furthermore, in the next period, the aggregate productivity declines as shown in Figures 10 and 11. In cases that use the conventional Solow residual, Figures 12 and 13 show that the shock to asset returns has an impact effect on two inputs and the aggregate productivity, as predicted by their model.

Finally, Table 8 presents the forecast error variance decompositions to summarize the impacts of the shock. For VAR(1) in Case 1, in the first period, approximately 0.3% of the variance of the changes in the asset returns is affected by the adjusted Solow residual. However, in the second period, this proportion jumps to 13% and it remains around that level after the third period. The main qualitative results do not change with the level specifications for VAR(6). The variance of the changes in the asset returns is substantially influenced by the adjusted Solow residual from the second period. On the other hand, when the conventional Solow residual is used, approximately 27% of the variance is affected in the first period for VAR(1) in Case 2. In the second period, the proportion rises slightly to 32%. Over the long forecast horizon, approximately 33% of the variance of the changes in the asset returns are attributed to the measured aggregate productivity. The patterns do not change with the VAR(6) specification. The results from the variance decompositions confirm the first-period impact.

The variance decompositions from Cases 1 and 2 reveal two aspects regarding the dynamic effects of the shock. First, the conventional Solow residual has larger impacts on the asset returns than the adjusted Solow residual. Second, while most of the effects of the conventional Solow residual occur in the first period, the effects of the adjusted Solow residual become significant from the second period; the effects of the first period differ substantially from the two.[33]

[33] Table 9 presents the variance decompositions for level specifications. Even with level specifications, there are sizable differences in the first period. In Case 2, which uses the

In the Kiyotaki and Moore (1997) specifications (Cases 3 and 4), where a shock hits the asset returns first, the measured aggregate productivity plays a minimal role. In particular, in Case 3, when the adjusted Solow residual is used, approximately 4% of the variance is affected in the second period and the proportion rises slightly to 7% in the third period.[34] When the conventional Solow residual is used, the variance of the changes in the asset returns is not affected even in the second period. In the third period, approximately 1% of the variance is attributed to the conventional Solow residual.

4.2.3 Discussions

In summary, the results from the impulse responses and the variance decompositions suggest that the measured aggregate productivity shock is important in understanding asset returns. In particular, the causality could run from aggregate productivity to asset returns. Thus, the model studied in this paper provides a framework to understand the direction of the relationship. Indeed, under the equilibrium business cycle model with constant utilization, the depreciation rate is constant, and the asset returns increase unambiguously on impact with a favorable aggregate productivity shock.

However, the dynamic effects of the aggregate productivity shock measured by the adjusted Solow residual are remarkably different from the ones based on the conventional Solow residual. It appears that the depreciation rate, which depends on the utilization rate, plays a nonnegligible role in explaining the observed dynamics in the model with variable capital utilization. Once variable utilization is controlled for, technology improvements become input saving. Thus, firms use less labor and decrease capital utilization. These changes somehow offset an impact effect of a favorable technology shock on asset returns and generate a delayed effect.

What is the intuition behind these results? When variable utilization is allowed, the asset returns become a function of the nonconstant depreciation rate, which is determined by the variable utilization rate. Since the optimal utilization rate is a nonlinear function of capital, labor, and the aggregate productivity, the degrees of substitutability and complementarity among them appear to matter. Recall the key equation presented in footnote 12. The one-period return equation can be written as,

$$R_t = 1 + (\phi - 1)[\delta(\frac{A_t F_{1,t}}{\delta\phi})^{\frac{\phi}{\phi-1}}]. \tag{4.5}$$

Equation (4.5) says that R_t depends on the level of the aggregate productivity, A_t, the capital stock, K_t, the labor input, L_t, and the utilization rate, u_t. It

conventional Solow residual, the first-period impacts are obvious in VAR(1) and VAR(6). On the other hand, in Case 1, which uses the adjusted Solow residual, the variance of the asset returns is mostly influenced by the technology shock from the second period in VAR(1) and VAR(6).

[34] The variance of the changes in the asset returns in the first period is not affected by the aggregate productivity in the Kiyotaki and Moore (1997) specifications.

should be noted that $\phi > 1$ and $F_{1,t}$ is a function of K_t, L_t, and u_t. Thus, when A_t increases, R_t should rise on impact, holding $F_{1,t}$ constant. Now, suppose an improvement in A_t causes L_t and u_t to decrease. Then, the impact effect of a technology improvement on R_t could be cancelled by a decrease in $F_{1,t}$. Therefore, the overall impact effect on R_t could be ambiguous. Consequently, the asset returns might not respond to technology improvements on impact. Furthermore, it could possibly generate a delayed effect.

While the above explanation is consistent with what Basu, Fernald, and Kimball (2004) argued, particularly with regard to the input saving technological progress, the observed results are somewhat difficult to be rationalized based on the frictionless model studied in this paper. In fact, it is believed that a technology shock is amplified in the model with variable utilization.[35] The idea is that when technology improves, it is relatively easier for a firm to change capital services by adjusting the level of capital utilization without changing the level of physical capital. Thus, in the short run, capital service supply becomes upward sloping rather than vertical. Consequently, the favorable technology shock would increase the utilized capital and create an amplification effect.

According to the prediction of the model, technology improvements should have a positive impact effect on utilization. Thus, on impact, due to the amplification, the aggregate productivity shocks are expected to affect asset returns to a greater extent in the model with variable utilization than in the model with constant utilization. However, contradictory results are obtained from the impulse responses. A favorable technology improvement actually reduces utilization. As a result, in order to rationalize the observed phenomenon, the model needs a mechanism for generating slow adjustments. As suggested by Basu, Fernald, and Kimball (2004), a prediction from the sticky–price model might be consistent with the findings of the study. Given the output level, technology improvements could be input saving.

The policy implications of the study are apparent. The monetary policy could help to resolve some puzzles on the observed pattern of the asset returns found in this study. Since the standard equilibrium business cycle model with constant utilization rate primarily relies on aggregate technology shocks and the models' predictions are consistent with the observed dynamics verified by the VARs, it would be difficult to discuss the role of demand shocks such as monetary policy shocks. However, this study highlights the inconsistency between the observed dynamics from the VARs and predictions suggested by the models with variable utilization. Furthermore, the results are somewhat difficult to be rationalized based on the model studied in this study. Thus, this study's results could validate the importance of other types of models such as sticky–price models, along with other types of shocks such as monetary shocks.

If the monetary policy authority reacts to the aggregate productivity shocks, the link between the aggregate productivity shocks and the asset returns established in this study could be altered. Consequently, in the presence of monetary

[35] The existing literature documents that allowing variable utilization in the equilibrium business cycle model generally amplifies rather than delays the effects of the shock. For examples, see King and Rebelo (1999) and Baxter and Farr (2001).

shocks, the asset returns could behave differently from the behavior predicted by the model without monetary shocks. Indeed, many researchers have already studied the link between monetary policy and stock markets.[36] Although the question of whether monetary policy should respond to the stock market is still open and it is somewhat difficult to measure the extent to which changes in monetary policy affect asset returns, the existing literature has documented that monetary policy has been affecting the stock market.[37] This paper leaves the unsolved issues for future research.

5 Conclusion

This paper examines and documents the effects of a measured aggregate productivity shock on asset returns. Using a simple equilibrium business cycle model, this study derives the relationship between the aggregate productivity shock and asset returns. Then, it uses Solow residual to measure productivity changes, but deviates from standard practice by incorporating variations in capital utilization rates.

This study reiterates the importance of variable capital utilization when the Solow residual is constructed as a proxy for the measured productivity shock. It reconfirms that variable capital utilization substantially reduces the cyclical variation in the Solow residual. More importantly, the VAR evidence suggests that once variable utilization is controlled for, technology improvements become input saving and generate a delayed effect on the asset returns.

This study presents a method for empirically assessing the implications of a measured aggregate productivity shock for asset returns based on an equilibrium business cycle theory. While the theory successfully identifies the aggregate productivity shock as a macroeconomic factor affecting the asset returns and helps to understand the direction of the causality, the model with variable utilization does not appear to rationalize the empirical findings presented in the study. Given these results, further research needs to be conducted in order to resolve the unanswered issues documented in this study; in the interim, the Solow residual should be used with caution for its relevance in the analysis of asset returns.

[36]See Rigobon and Sack (2004) for a notable study on these topics.

[37]Mishkin and White (2002) argued that it would be optimal for monetary policy makers to focus on financial stability rather than on the stock market. In addition, Bernanke and Kuttner (2005) argued that monetary shocks would affect stock prices by influencing the risk–premium.

A Appendix

A.1 Derivation of Equation (2.3)

This section derives the Euler equation for a consumer problem. Define the consumer's wealth at time t as W_t. Using the definition of one–period return, $R_t = \frac{P_{t+1}+D_{t+1}}{P_t}$, the budget constraint for the household can be rewritten as,

$$W_{t+1} = R_t(W_t - C_t) + w_t L_t. \tag{A.1}$$

In this problem, the household's wealth, W_t, and technology, A_t, can be defined as state variables and consumption, c_t, as a control variable along with L_t.

Now, the Bellman's functional equation for this problem is given as

$$V(W_t, A_t) = \max_{h_t, L_t} u(c_t, 1 - L_t) + \beta E_t V\left(R_t(W_t - C_t) + w_t L_t, A_{t+1}\right). \tag{A.2}$$

The first order condition for h_t is

$$u_{c_t} - \beta E_t V_1(W_{t+1}, A_{t+1}) R_t = 0. \tag{A.3}$$

By the Envelope theorem,

$$V_1(W_{t+1}, A_{t+1}) = u_{c_{t+1}}. \tag{A.4}$$

Using Equations (A.3) and (A.4), the consumption Euler equation can be obtained,

$$u_{c_t} = \beta E_t u_{c_{t+1}} R_t. \tag{A.5}$$

A.2 Derivation of Equation (2.17)

This section shows that $P_t = K_t$. Begin with the first order condition for capital,

$$\beta E_t \nu_{t+1}(A_t F_{1,t} u_t + 1 - \delta_t) = \nu_t,$$

where v_{t+1} is the price of capital or the marginal rate of substitution between time 0 and $t+1$. In other words, $v_{t+1} = \frac{u_{c_{t+1}}}{u_{c_0}}$. Using Equation (2.16), the first order condition given above can be rewritten as

$$\beta E_t v_{t+1}\left(\frac{D_{t+1} + K_{t+1}}{K_t}\right) = v_t. \tag{A.6}$$

By the definition of $v_{t+1} = \frac{u_{c_{t+1}}}{u_{c_0}}$, Equation (A.6) becomes

$$\beta E_t u_{c_{t+1}}\left(\frac{D_{t+1} + K_{t+1}}{K_t}\right) = u_{c_t}. \tag{A.7}$$

When $\frac{D_{t+1}+K_{t+1}}{K_t}$ is replaced by R_t, Equation (A.7) becomes the consumption Euler equation, as in Equation (A.5). Therefore, $P_t = K_t$.

A.3 Measuring Aggregate Productivity Shocks

The aggregate productivity shock is one of the key variables in the empirical investigations. To measure it, this study uses the growth accounting framework. Under the assumptions of competitive markets and constant returns to scale, the growth accounting framework decomposes output growth into two parts: portions attributed to growth in inputs and changes in aggregate productivity.[38]

From the production function introduced in Section 2, the conventional Solow residual can be constructed as follows:

$$\Delta A_{conv} = \Delta Y - \alpha \Delta K - (1-\alpha)\Delta L, \tag{A.8}$$

where ΔY is the growth rate of output, α is the factor share distributed to capital, ΔL is the rate of labor hour growth, ΔK is the rate of growth of physical capital, and ΔA_{conv} is the conventional Solow residual.[39]

Thus, the adjusted Solow residual can be constructed,

$$\begin{aligned} \Delta A_{adj} &= \Delta Y - \alpha \Delta K - (1-\alpha)\Delta L - \alpha \Delta u \\ &= \Delta A_{conv} - \alpha \Delta u, \end{aligned} \tag{A.9}$$

where ΔA_{adj} is the adjusted Solow residual, and Δu is the growth rate of the utilization rate.

Equation (A.9) shows that as long as the utilization rate is not constant, i.e., $\Delta u \neq 0$, the adjusted and the conventional Solow residual are not equivalent, i.e., $\Delta A_{adj} \neq \Delta A_{conv}$.

[38] Hall(1990) said that the following theorem would hold under Solow's assumptions: *the productivity residual is uncorrelated with any variable that is uncorrelated with the rate of growth of true productivity.* This is a restatement of Solow's basic results in which the residual measures the shift of the production function.

[39] This study constructs the conventional and the adjusted Solow residuals using both the actual and the average shares. The results of the study are not sensitive to the capital share parameter, α

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Table 1: Ng-Perron Unit Root Tests

This table reports the results of the Ng-Perron unit root tests. Significant coefficients at 5% are indicated with **. The sample period is from 1949 to 2001. All variables are in natural logarithm. REALR stands for gross real asset returns, K for capital stock, L for labor hours, and u for utilization rate, SRL for aggregate productivity level, and ADJSRL for adjusted aggregate productivity level.

Ng-Perron test statistics		MZa	MZt	MSB	MPT
REALR		-1.357	-0.682	0.503	14.549
K		-1.925	-0.700	0.363	9.854
L		-7.364	-1.709	0.232	4.056
u		-20.754**	-3.131**	0.151**	1.494**
SRL		-1.425	-0.549	0.385	11.23
ADJSRL		1.186	0.965	0.814	50.15
	1%	-13.8	-2.58	0.174	1.78
Asymptotic Critical Values	5%	-8.1	-1.98	0.233	3.17
	10%	-5.7	-1.62	0.275	4.45

Table 2: Augmented Dickey-Fuller Unit Root Tests

This table reports the results of the Augmented Dickey-Fuller unit root tests.

		Augmented Dickey-Fuller test statistics
REALR		-1.45
K		-2.39
L		0.28
u		-4.188**
SRL		-1.52
ADJSRL		-2.11
MacKinnon's critical values	1%	-3.56
	5%	-2.92
	10%	-2.60

Table 3: Descriptive Statistics

GREALR stands for gross real asset return growth, SR for conventional Solow residual, ADJSR for adjusted Solow residual, GY for real GDP growth, GK for capital stock growth, GL for labor-hour growth, and GCU for utilization growth. The sample period is from 1949 to 2001.

	GREALR	SR	ADJSR	GY	GL	GK	GCU
Mean	0.0271	0.0158	0.0148	0.0358	0.0139	0.0300	0.0026
Median	-0.0434	0.0174	0.0147	0.0375	0.0153	0.0308	0.0023
Max	0.9318	0.0879	0.0499	0.0875	0.0474	0.0473	0.1000
Min	-0.3317	-0.0220	-0.0106	-0.0203	-0.0227	0.0138	-0.1151
SD	0.2454	0.0205	0.0126	0.0243	0.0149	0.0071	0.0452

Table 4: Contemporaneous Correlations

	GREALR	SR	ADJSR	GY	GL	GK	GCU
GREALR	1.00						
SR	0.48	1.00					
ADJSR	0.19	0.53	1.00				
GY	0.45	0.91	0.23	1.00			
GL	0.14	0.17	-0.57	0.55	1.00		
GK	-0.09	0.02	0.02	0.22	0.24	1.00	
GCU	0.42	0.79	-0.09	0.90	0.61	0.01	1.00

Table 5: Pairwise Granger-Causality Tests

This table reports the results of the Granger-Causality tests. GREALR stands for gross real asset return growth, SR for conventional Solow residual, ADJSR for adjusted Solow residual. The sample period is from 1949 to 2001. The number of lagged variable is 1.

Null Hypothesis:	F-Statistic	Probability
SR does not Granger Cause GREALR	1.699	0.199
GREALR does not Granger Cause SR	1.801	0.186
ADJSR does not Granger Cause GREALR	11.328	0.002**
GREALR does not Granger Cause ADJSR	18.409	0.000**

Table 6: Linear Regression Results

The sample period is from 1950 to 2001. Significant coefficients at 5% and 10% are indicated with ** and *, respectively. The numbers in the parentheses are standard errors. GREALR stands for gross real asset return growth, SR for conventional Solow residual, GK for capital stock growth, GL for labor-hour growth, ADJSR for adjusted Solow residual, and GCU for utilization growth.

	Case I	Case II	Case III	Case IV
Conventional Solow Residual				
SR	5.67*	4.22*		
	(1.50)	(1.20)		
GK	-2.85			
	(1.52)			
GL	1.50			
	(2.16)			
SR(-1)			-1.99	-2.53**
			(1.31)	(1.41)
GK(-1)				5.56**
				(1.44)
GL(-1)				-8.55**
				(2.04)
R-squared	0.246	0.187	0.036	0.320
Adj. R-squared	0.215	0.187	0.036	0.292

Adjusted Solow Residual				
ADJSR	5.87	2.62		
	(3.24)	(1.78)		
GK	-3.04			
	(3.07)			
GL	1.71			
	(3.69)			
GCU	2.12*			
	(0.96)			
ADJSR(-1)			3.44**	4.76*
			(1.70)	(2.58)
GK(-1)				-1.05
				(2.42)
GL(-1)				-1.47
				(2.86)
GCU(-1)				-2.74**
				(0.73)
R-squared	0.245	0.031	0.067	0.443
Adj. R-squared	0.197	0.031	0.067	0.406

Table 7: Ramsey RESET Tests

The table reports results of the Ramsey RESET tests. The tests are based on the results from the regressions (Case I) in Table 6.

Adjusted Solow Residual			
F-statistic	1.996	Probability	0.164
Log likelihood ratio	2.214	Probability	0.136

Conventional Solow Residual			
F-statistic	1.348	Probability	0.252
Log likelihood ratio	1.442	Probability	0.230

Table 8: Variance Decompositions for Changes in Real Asset Returns due to Adjusted Solow Residual & Conventional Solow Residual

The table reports the results of the variance decompositions. FE stands for forecast error, GREALR for gross real asset return growth, SR for conventional Solow residual, ADJSR for adjusted Solow residual, GK for capital stock growth, GL for labor-hour growth, and GCU for utilization growth.

The orders: Case 1 (Benchmark with adjusted Solow residual): ADJSR, GL, GCU, GK, GREALR; Case 2 (Benchmark with conventional Solow residual): SR, GL, GK, GREALR; Case 3 (Kiyotaki & Moore with adjusted Solow residual): GREALR, GL, GCU, GK, ADJSR; Case 4 (Kiyotaki & Moore with conventional Solow residual): GREALR, GL, GK, SR.

VAR1	Case1		Case2		Case3		Case4	
Period	F.E.	(%)	F.E.	(%)	F.E.	(%)	F.E.	(%)
1	0.175	0.33	0.193	26.85	0.175	0.00	0.193	0.00
2	0.246	13.47	0.254	32.46	0.246	3.35	0.254	0.34
3	0.255	15.76	0.255	32.21	0.255	6.49	0.255	0.70
4	0.259	15.46	0.256	32.45	0.259	6.29	0.256	0.71
5	0.260	15.90	0.256	32.50	0.260	6.51	0.256	0.74
6	0.260	15.89	0.256	32.52	0.260	6.52	0.256	0.75
7	0.261	15.87	0.256	32.53	0.261	6.51	0.256	0.75
8	0.261	15.89	0.256	32.53	0.261	6.53	0.256	0.75
9	0.261	15.89	0.256	32.53	0.261	6.53	0.256	0.75
10	0.261	15.89	0.256	32.53	0.261	6.53	0.256	0.75
VAR6								
Period	F.E.	(%)	F.E.	(%)	F.E.	(%)	F.E.	(%)
1	0.205	16.42	0.220	33.27	0.205	0.00	0.220	0.00
2	0.324	38.75	0.316	32.58	0.324	6.28	0.316	0.23
3	0.351	44.18	0.318	32.42	0.351	7.66	0.318	0.64
4	0.371	43.08	0.329	30.48	0.371	10.54	0.329	3.28
5	0.375	42.37	0.334	30.05	0.375	10.48	0.334	4.08
6	0.386	42.43	0.339	29.40	0.386	11.82	0.339	4.79
7	0.392	41.18	0.340	29.79	0.392	11.48	0.340	5.27
8	0.398	42.04	0.346	28.99	0.398	11.89	0.346	6.38
9	0.401	41.49	0.348	28.65	0.401	11.84	0.348	7.02
10	0.407	42.51	0.350	28.75	0.407	11.96	0.350	7.23

Table 9: Variance Decompositions for Real Asset Returns due to Adjusted Solow Residual & Conventional Solow Residual - Level Specifications

The table reports the results of the variance decompositions. FE stands for forecast error, REALR for gross real asset return, SR for conventional Solow residual, ADJSR for adjusted Solow residual, GK for capital stock growth, GL for labor-hour growth, and GCU for utilization growth.

The orders: Case 1 (Benchmark with adjusted Solow residual): ADJSR, GL, GCU, GK, REALR; Case 2 (Benchmark with conventional Solow residual): SR, GL, GK, REALR.

VAR1	Case1		Case2	
Period	F.E.	%	F.E.	%
1	0.147	0.43	0.153	23.11
2	0.170	18.81	0.169	19.04
3	0.173	18.24	0.173	21.13
4	0.173	18.63	0.173	21.14
5	0.174	18.59	0.173	21.27
6	0.174	18.65	0.173	21.35
7	0.174	18.66	0.173	21.37
8	0.174	18.66	0.173	21.37
9	0.174	18.66	0.173	21.37
10	0.174	18.66	0.173	21.37
VAR6				
Period	F.E.	%	F.E.	%
1	0.141	9.82	0.161	35.33
2	0.192	26.47	0.174	37.14
3	0.196	25.81	0.175	36.74
4	0.206	27.12	0.185	33.16
5	0.219	29.03	0.193	30.40
6	0.225	28.08	0.196	29.62
7	0.229	27.58	0.198	30.08
8	0.243	30.40	0.208	32.66
9	0.246	30.44	0.211	33.62
10	0.247	30.67	0.212	33.55

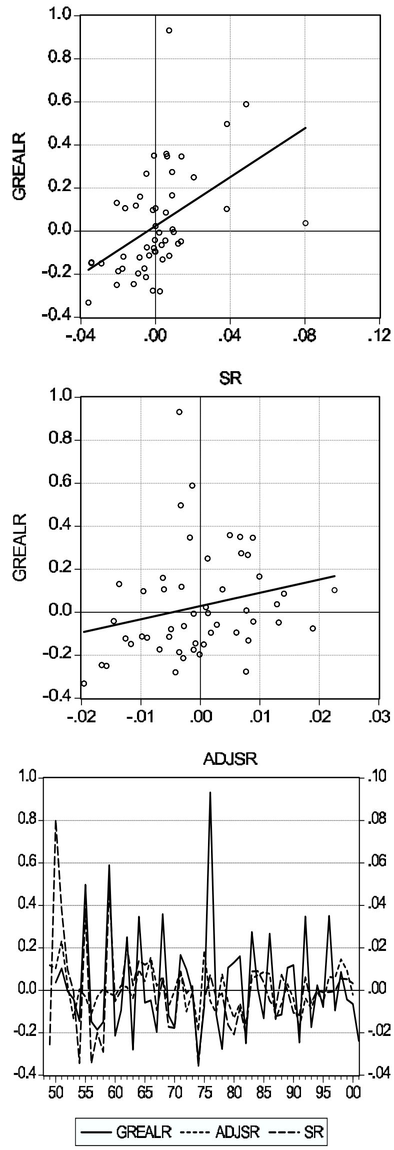

Figure 1: Changes in Asset Returns vs. the Adjusted Solow Residual & Changes in Asset Returns vs. the Conventional Solow Residual : In the first plot, the effects of other variables (GK, GL) are partialled out in SR. In the second plot, the effects of other variables (GK, GL, GCU) affecting changes in asset returns are partialled out in ADJSR.

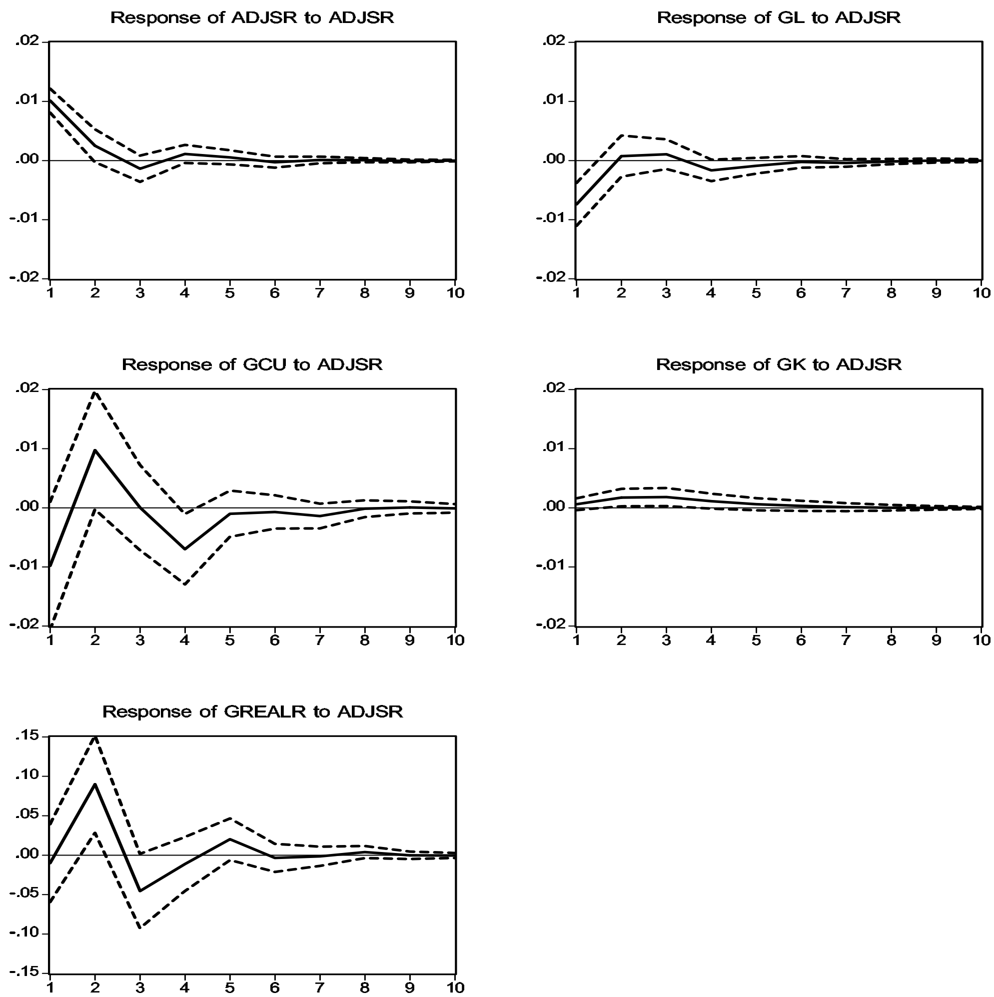

Figure 2: Impulse Response Functions, VAR(1): one standard deviation perturbation to adjusted Solow residual. GREALR stands for gross real asset return growth, ADJSR for adjusted Solow residual, GK for capital stock growth, GL for labor-hour growth, and GCU for utilization growth. The order - ADJSR, GL, GCU, GK, GREALR.

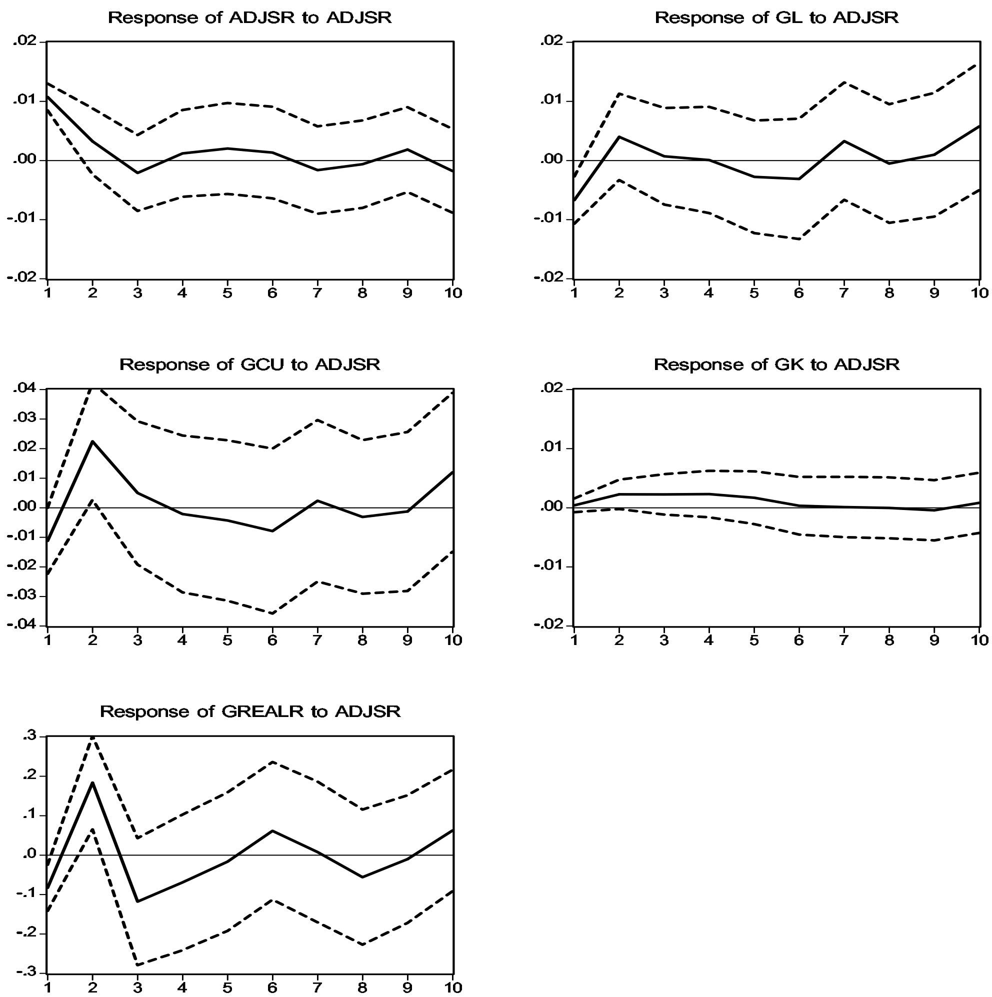

Figure 3: Impulse Response Functions, VAR(6): one standard deviation perturbation to adjusted Solow residual. The order - ADJSR, GL, GCU, GK, GREALR.

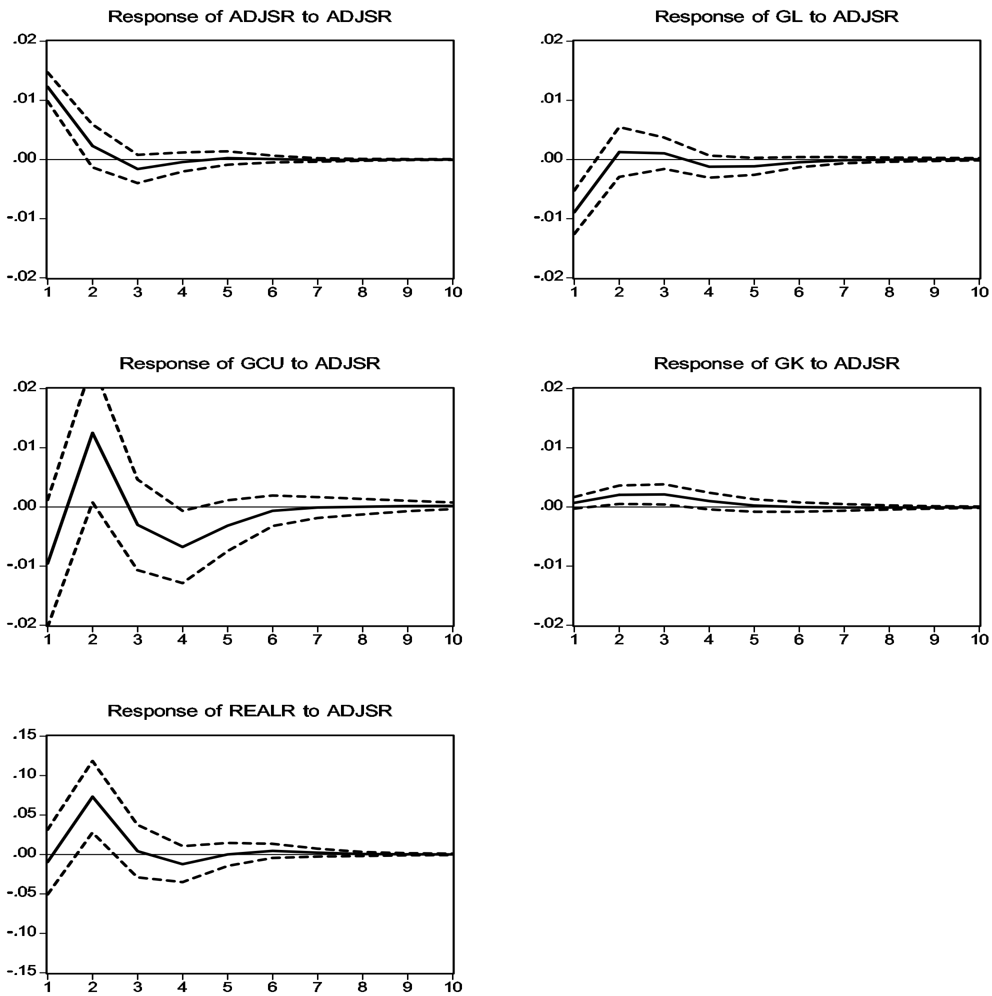

Figure 4: Impulse Response Functions, VAR(1) - Level: one standard deviation perturbation to adjusted Solow residual. REALR stands for gross real asset return. The order - ADJSR, GL, GCU, GK, REALR.

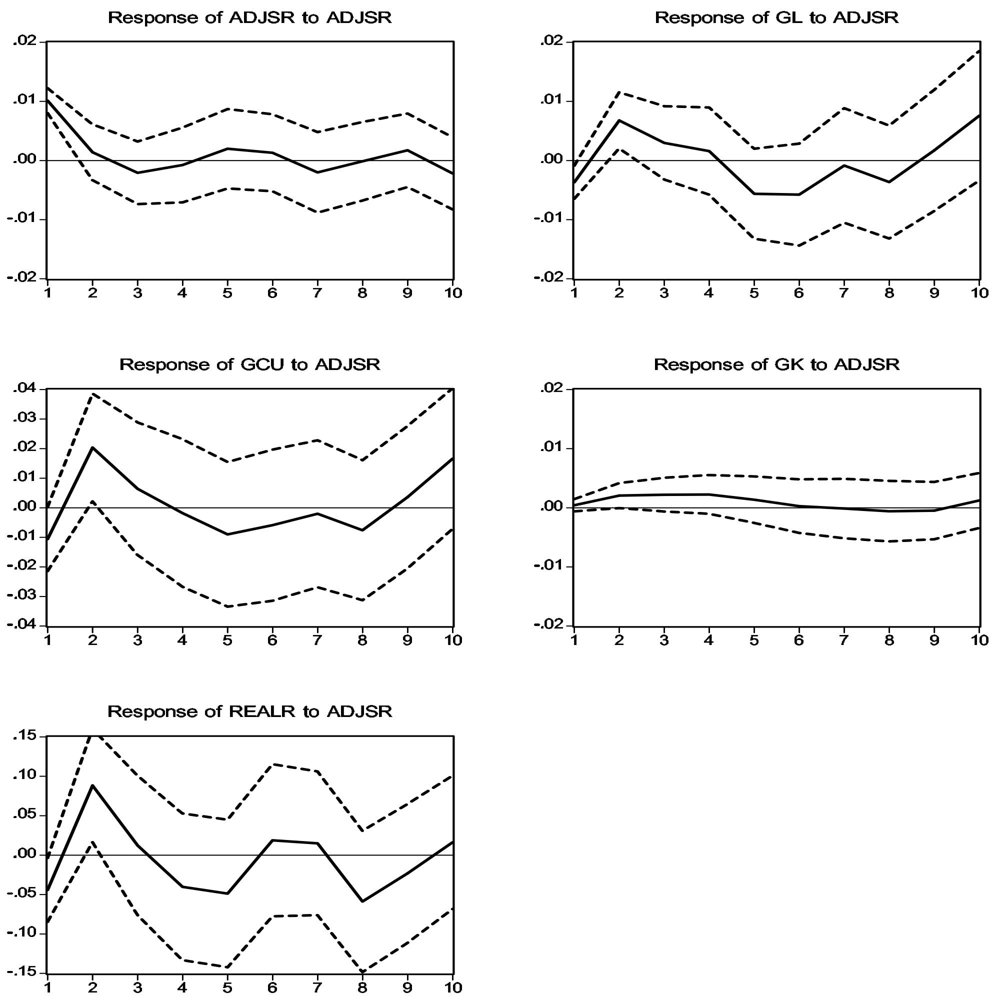

Figure 5: Impulse Response Functions, VAR(6) - Level: one standard deviation perturbation to adjusted Solow residual. The order - ADJSR, GL, GCU, GK, REALR.

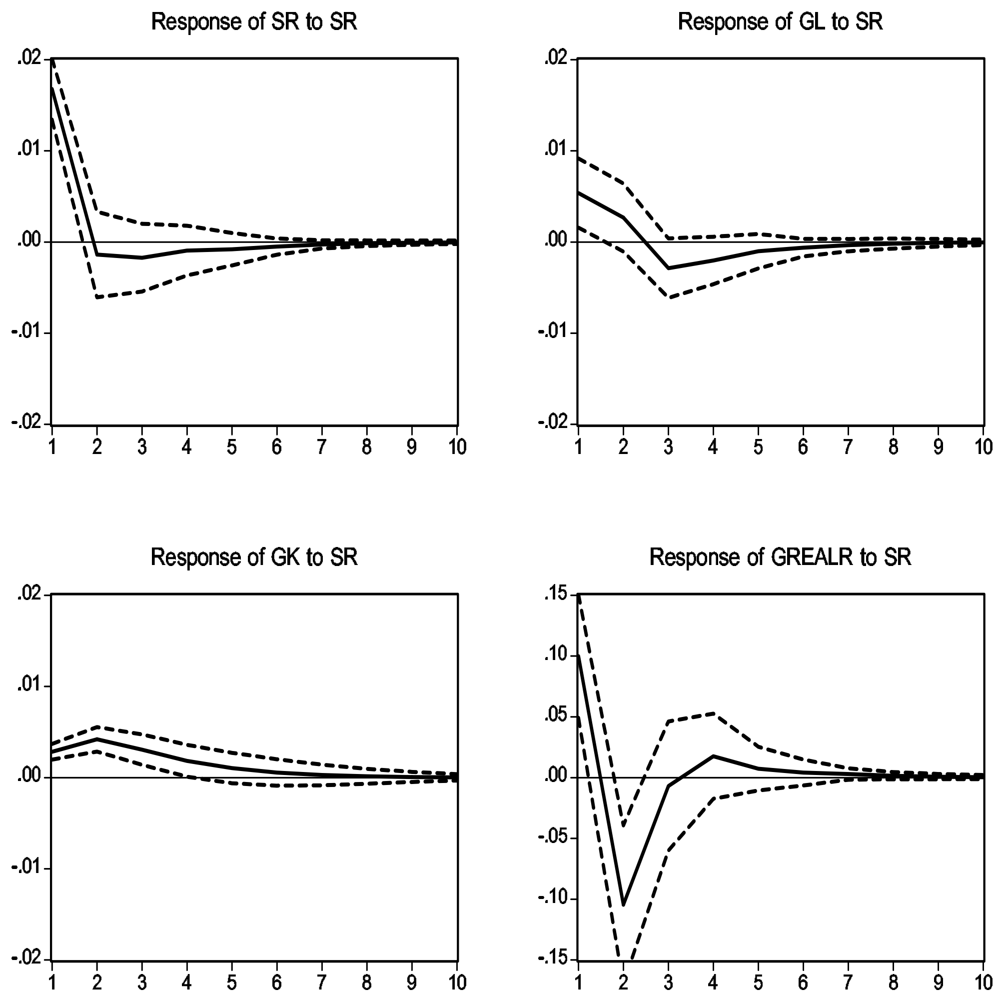

Figure 6: Impulse Response Functions, VAR(1): one standard deviation perturbation to conventional Solow residual. The order - SR, GL, GK, GREALR.

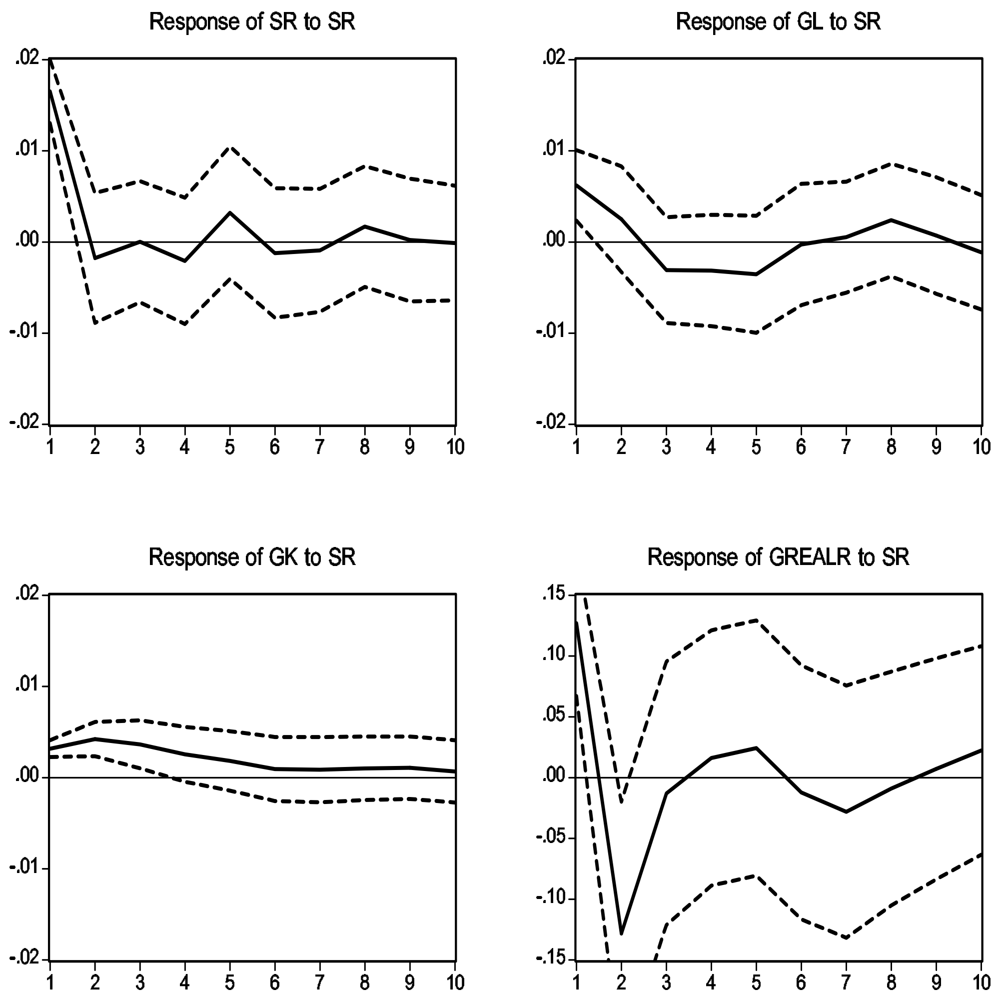

Figure 7: Impulse Response Functions, VAR(6): one standard deviation perturbation to conventional Solow residual. The order - SR, GL, GK, GREALR.

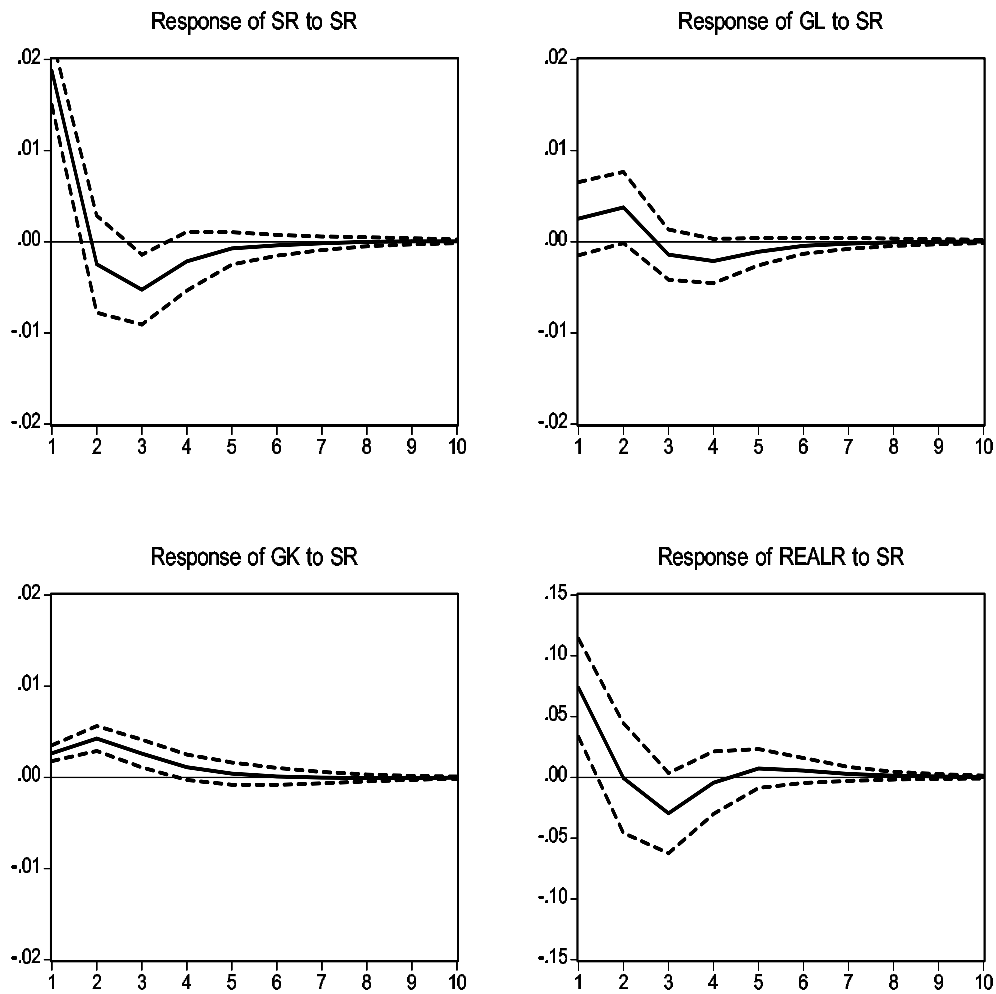

Figure 8: Impulse Response Functions, VAR(1) - Level: one standard deviation perturbation to conventional Solow residual. The order - SR, GL, GK, REALR.

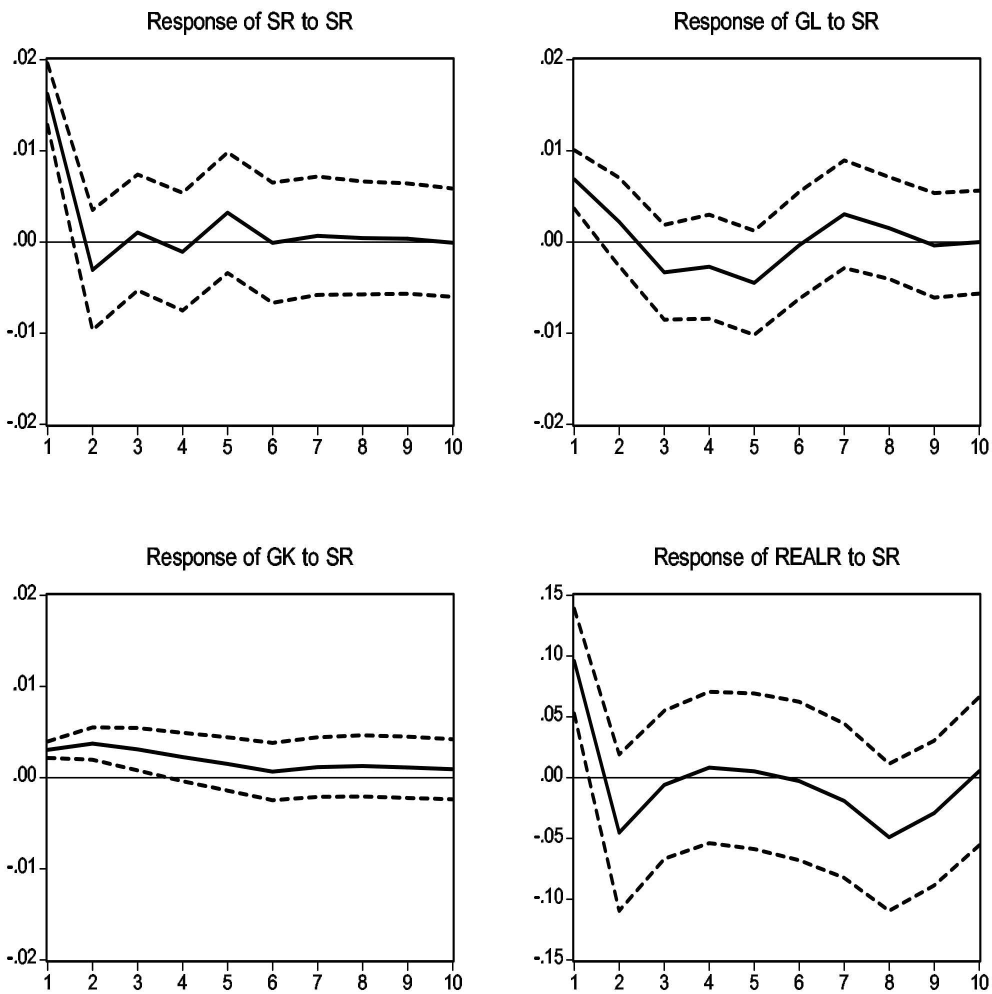

Figure 9: Impulse Response Functions, VAR(6) - Level: one standard deviation perturbation to conventional Solow residual. The order - SR, GL, GK, REALR.

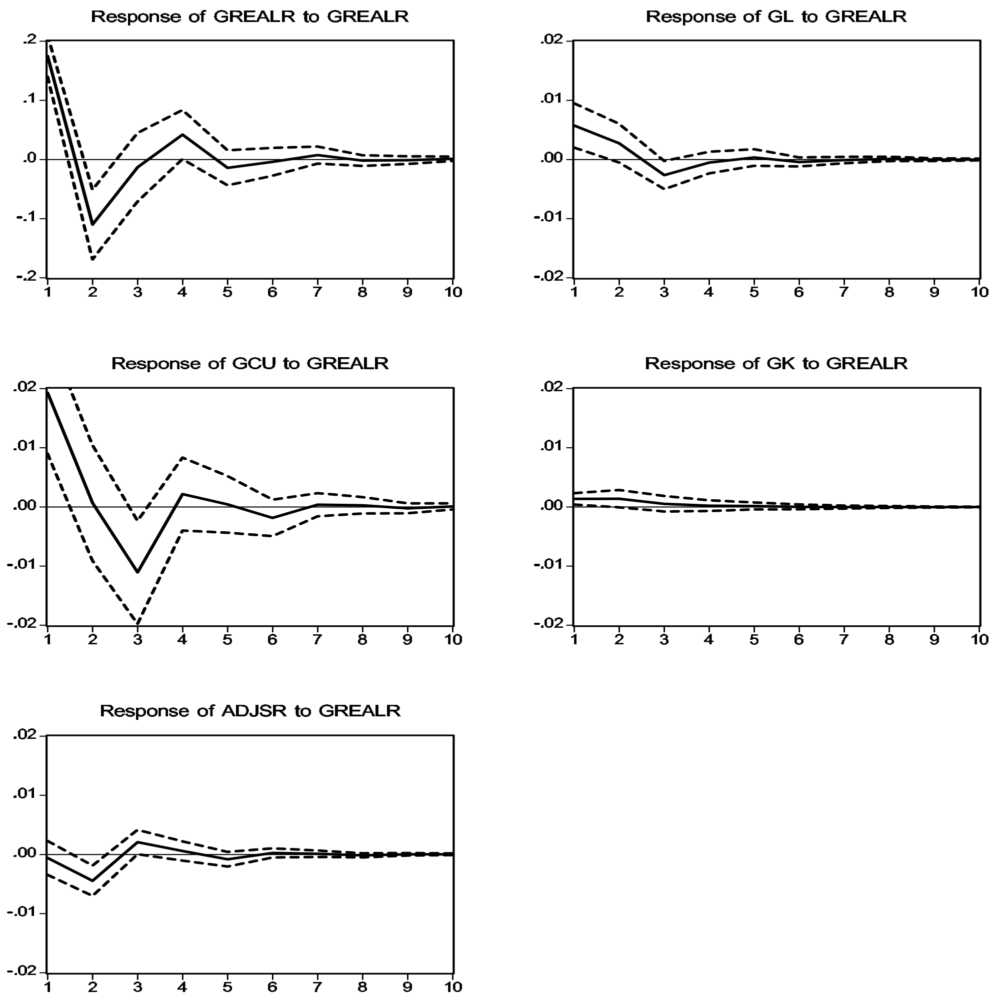

Figure 10: Impulse Response Functions, VAR(1): one standard deviation perturbation to changes in gross real asset returns. The order - GREALR, GL, GCU, GK, ADJSR.

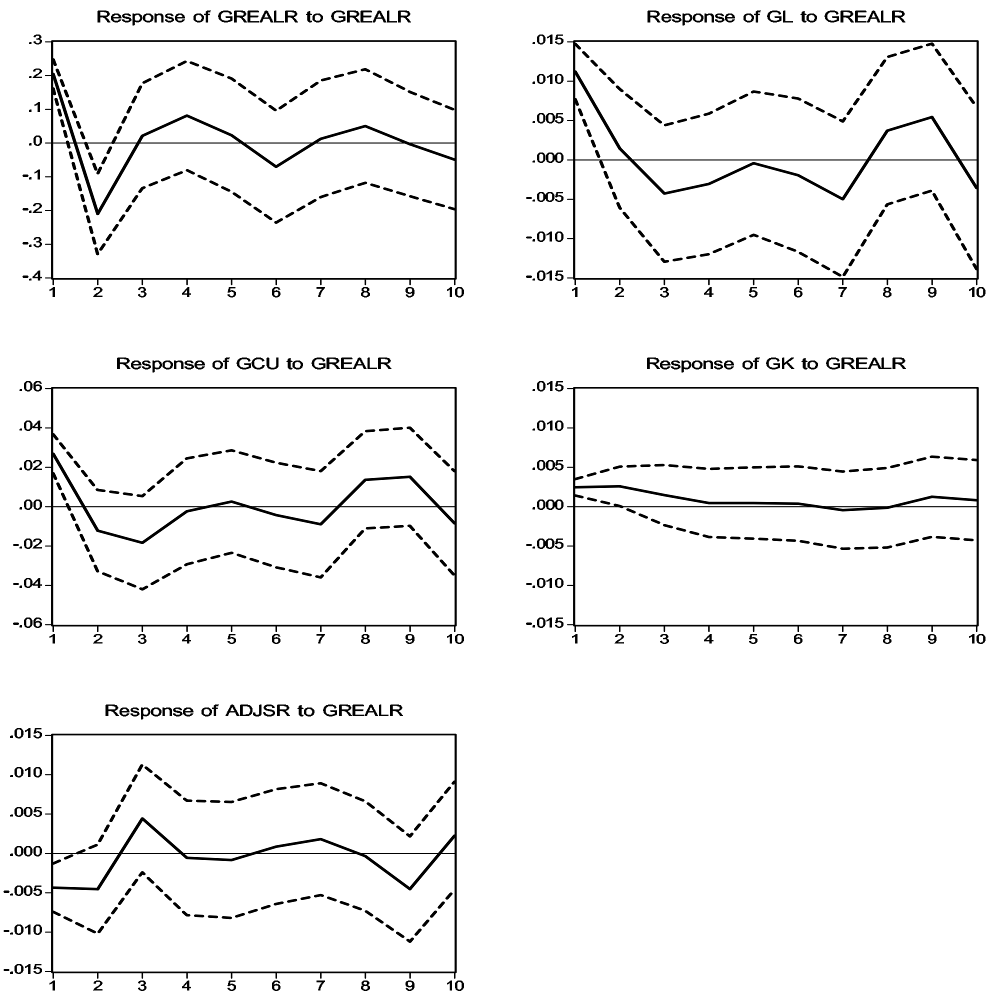

Figure 11: Impulse Response Functions, VAR(6): one standard deviation perturbation to changes in gross real asset returns. The order - GREALR, GL, GCU, GK, ADJSR.

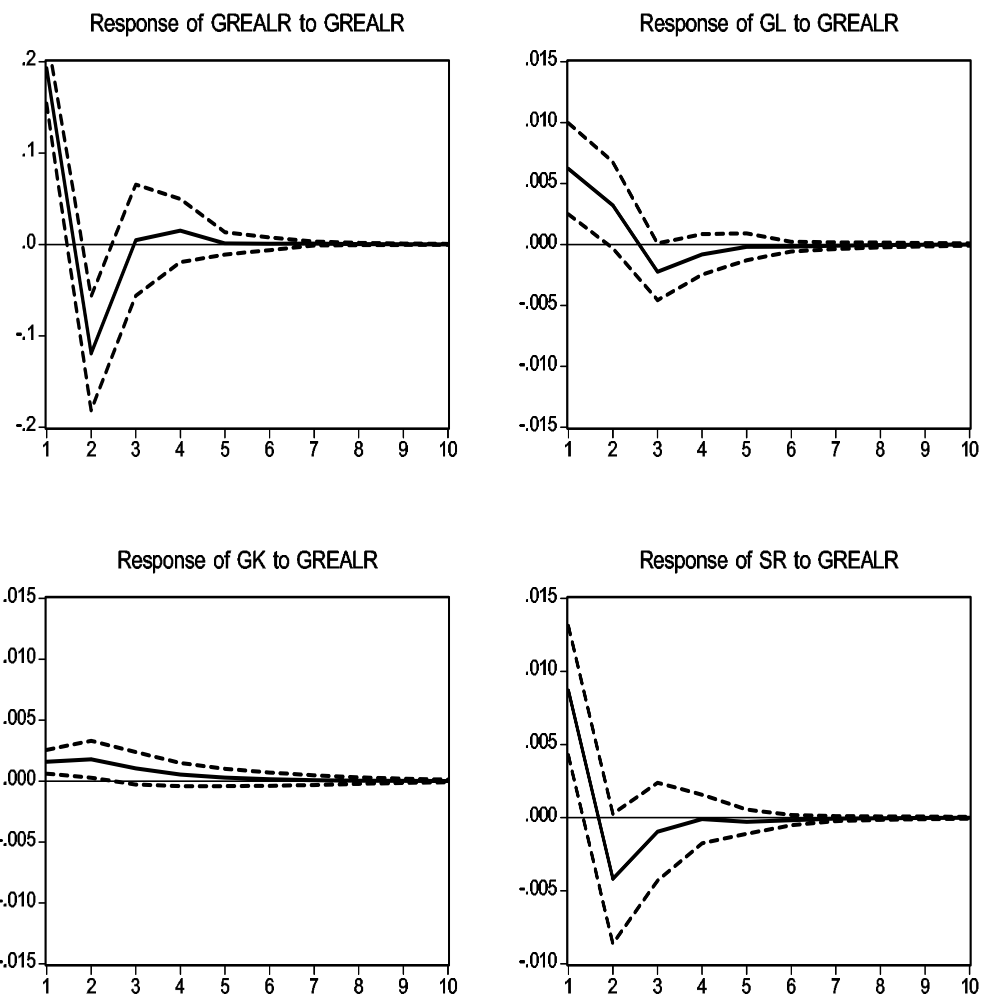

Figure 12: Impulse Response Functions, VAR(1): one standard deviation perturbation to changes in gross real asset returns. The order - GREALR, GL, GK, SR.

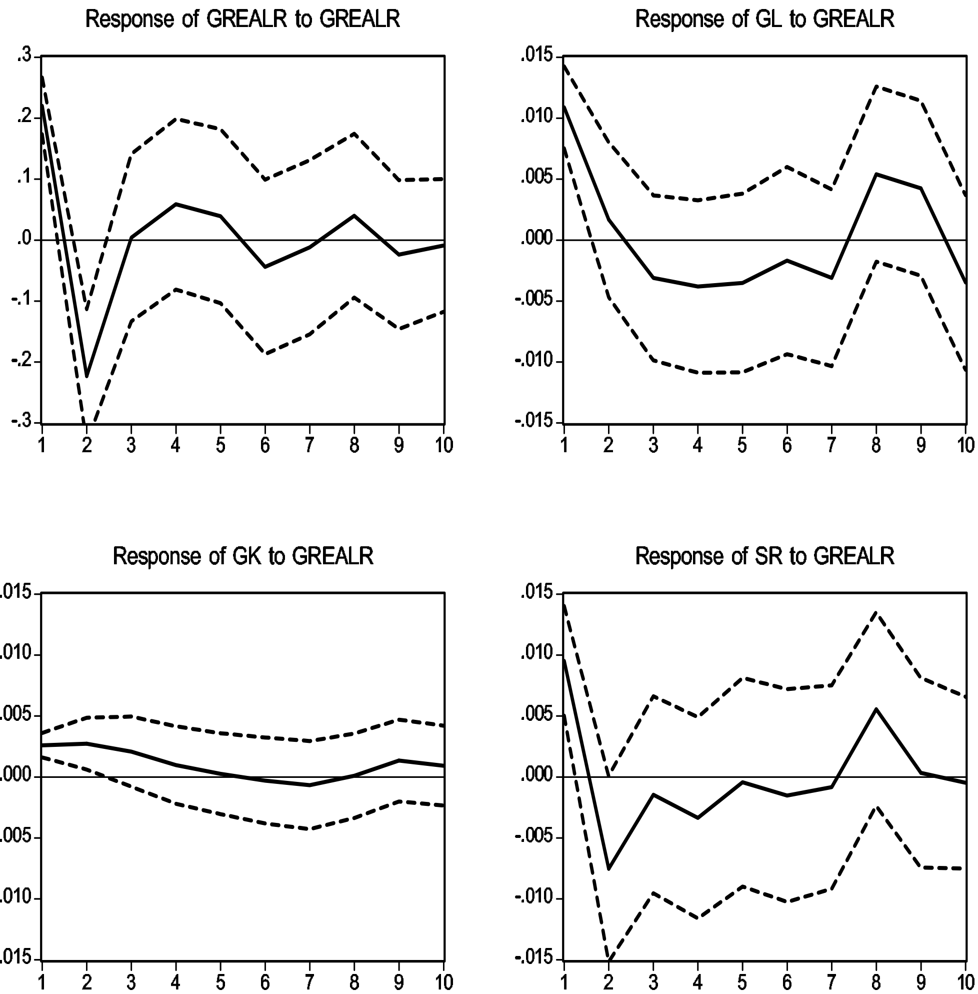

Figure 13: Impulse Response Functions, VAR(6): one standard deviation perturbation to changes in gross real asset returns. The order - GREALR, GL, GK, SR.

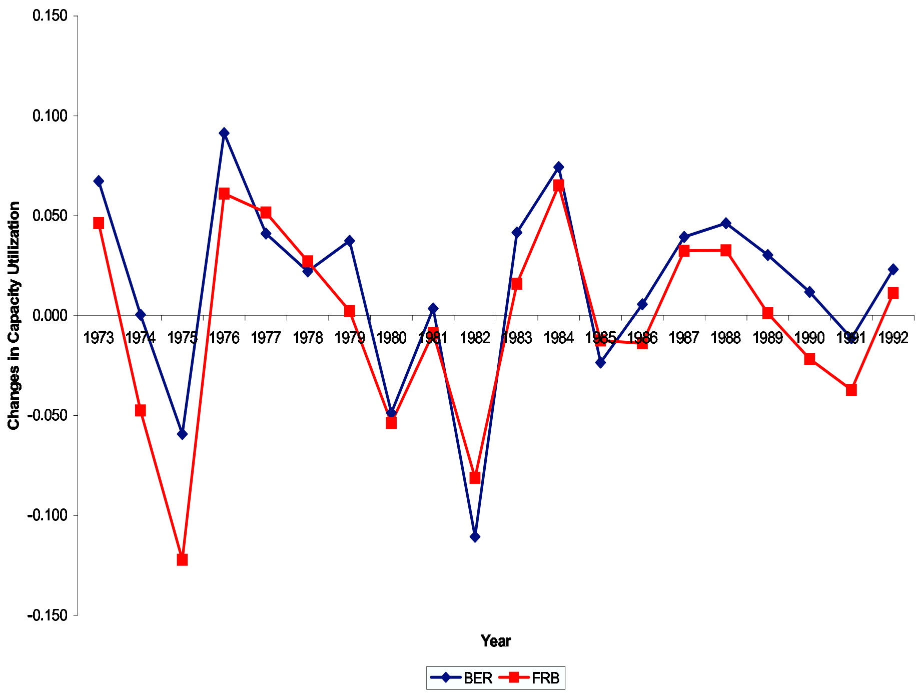

Figure 14: Capital Utilization Rates: A Comparison between FRB official series vs. capital utilization data used by Burnside, Eichenbaum, and Rebelo (1995): The sample period runs from 1972 to 1992.