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Why some Distressed Firms Have Low Expected Returns

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Abstract

In recent years, empirical researchers show that firms with higher credit risk have much smaller average stock returns. This finding is opposite to the risk-reward principle and is often attributed to mispricing and market anomalies. We investigate how credit risk and expected stock return are determined in a model with production, capital structure and aggregate uncertainty. We show that, contrary to the conventional wisdom, a firm with higher credit risk can have less risky stock than the one with lower credit risk.

1 Introduction

Most of the analyses on credit risk have focused on default probability or the pricing of defaultable securities such as bonds or credit derivatives. The relationship between credit risk and stock return is rarely thoroughly investigated.

In recent years, however, researchers have investigated how the credit risk is reflected in the cross-sectional stock return, and most of the papers show results opposite to our intuition: firms with higher credit risk have much smaller average stock returns (e.g. Dichev(1998), Griffin and Lemmon(2002) and Campbell, Hilscher, and Szilagyi(2005)). Furthermore, this phenomenon appears even after the market, value and size effect are adjusted.

We investigate how credit risk and expected stock return are determined by economic primitives, such as tastes and technology, in the neoclassical framework with rational expectations. We show that, contrary to the conventional wisdom, a firm with higher credit risk can have less risky stock than the one with lower credit risk.

In the model we consider a firm with a zero-coupon bond. When the firm faces high default risk, the value of the cash flow after the maturity for stockholders is small. However, the firm pays dividend before the maturity to the stockholders, which is the main determinant of the stock value. Since the value of the cash flow before the maturity is more stable than the value of the cash flow after the maturity, the stock has low risk and low return, which explains the anomaly of the relationship between credit risk and stock return.

This paper is related to two strands of the financial economics literature. On one hand it shares with a series of recent papers such as Berk, Green and Naik (1999), Gomes, Kogan and Zhang (2003), Carlson, Fisher and Giammarino (2004), Zhang (2005) and Obreja (2006), with the objective of explaining both the predictable variations in equity returns through time and the cross-sectional relation between equity returns and characteristics. However, these models focus on all-equity firms only except for Obreja (2006), while ours focuses on financially leveraged firms.

Our theoretical model also shares with the credit risk literature on default probability and pricing of bonds. Important contribution to this literature include: Merton (1974), Black and Cox (1976), Geske (1977), Leland (1994), Leland and Toft(1996). From this perspective, the novelty in our model stems from the fact that the default probability is related to time-varying price of risk.

This paper is organized as follows. In section 2, we present the empirical research on the relationship between credit risk and stock return. In section 3, we present the model, outlining the technology, the ownership structure, the objectives, and the decisions of the firms. Then we discuss the valuation of the corporate bond and stocks. In section 4, we present the results of the paper, and finally we conclude in section 5. The proofs of the propositions are in Appendix.

2. Empirical research

Empirical research on the relationship between credit risk and stock return can be classified into three groups according to the method of performing the measurement of the likelihood of bankruptcy; (1) using accounting information, such as O score and Z score[1], (2) statistical computation using hazard models, and (3) using structural models such as Merton (1974).

Dichev (1998) shows that there is a negative correlation between stock return and the likelihood of bankruptcy with method of (1), using Z-score. This is confirmed using the same method by Griffin and Lemmon (2002), using O-score. Griffin and Lemmon (2002) also claim that the phenomenon is caused by that the bad performance of firms with low Book-to-Market ratios and high distress risks. They claim that such stocks are often mispriced in the market.

Campbell, Hilscher, and Szilagyi (2005) analyze the same issues using (2). In that paper, a firm with high likelihood of bankruptcy has also a low stock average return and they assert that the stock market is mispricing the distress risk.

Vassalou and Xing (2004) examine the problem with method (3). In that paper, contrary to the other research, it is reported that a firm with high likelihood of bankruptcy has a high average stock return. And in such firms, those with lower capital size and higher book-to-market ratio have even higher average returns.

Da and Gao (2006) dispute the conclusion of Vassalou and Xing (2004). They propose that the short-term return reversal and the liquidity risk can explain most of the results in Vassalou and Xing (2004), and conclude that there is little evidence that the

[1] See Altman (1968) for Z score and Ohlson (1980) for O score.

likelihood of bankruptcy and the subsequent average stock return have positive correlation.

Garlappi, Shu, and Yan (2006) also analyze the relationship between the likelihood of bankruptcy and the stock expected return, using EDF (Expected Default Frequency). They show that when likelihood of bankruptcy is high, the average stock return does not necessarily become high. In the paper, they suggest that when a firm has fallen into serious financial crisis there will be a negotiation between a stockholder and a bondholder. And they claim that likelihood of bankruptcy and a stock expected return are decided depending on bargaining ability using the model of Fan and Sundaresan (2000).

3. The model

We construct a neoclassical production model (e.g., Lucas (1967), Lucas and Prescott (1971)) augmented with aggregate uncertainty and capital structure. Section 3.A describes the economic environment. Section 3.B puts in the capital structure and default. Then section 3.C discusses valuation of stocks and debts. Appendix contains the proof.

A. Environment

A.1. Technology

Firms own technologies that use one input, capital, and exhibit decreasing-returns-to-scale. The output in dollar unit, y, is uncertain, depending on the realization of both an aggregate shock, x, and an idiosyncratic shock z. In this model, different firms have different idiosyncratic shocks, leading to heterogeneity in the production economy.

Let k_t denote the stock of capital at time t and the production function be given by

$$y_t = f(k_t; x_t, z_t) = k_t^{\alpha} \exp(x_t + z_t), \tag{3.A.1}$$

where $0 < \alpha < 1$, which denotes the capital share.

We assume that the productivity shocks have stationary and monotone Markov transition functions. Specifically, x and z follow stationary autonomous dynamics of the following form:

$$x_{t+1} = x_t + (1-\rho_x)(\overline{x} - x_t) + \sigma_x \varepsilon_{t+1}^{x} \tag{3.A.2}$$

$$z_{t+1} = z_t + (1-\rho_z)(-z_t) + \sigma_z \varepsilon^z_{t+1}, \tag{3.A.3}$$

where ε^x_{t+1} and ε^z_{t+1} are I.I.D. standard normal shocks. The central tendency of the aggregate shock is captured by $\overline{x}$, while the speed of the reversion and the conditional volatility are captured by ρ_x and σ_x, respectively. The parameters ρ_z and σ_z for idiosyncratic process, carry similar interpretations.

A.2. Stochastic Discount Factor

We parameterize directly the pricing kernel without explicitly modeling the consumers' problem. We assume the process of the pricing kernel M_t follows the specifications in Zhang (2005), that is

$$\log M_{t+1} = \log\beta - \gamma_t (x_{t+1} - x_t) \tag{3.A.3}$$

$$\gamma_t = \gamma_0 + \gamma_1 (x_t - \overline{x}), \tag{3.A.4}$$

where β, $\gamma_0 > 0$ and $\gamma_1 < 0$ are constant parameters. γ_t is time-varying and decreasing with the demeaned aggregate productivity $x_t - \overline{x}$. γ_t can be interpreted as the market price of aggregate risk, while the constraint on $\alpha < 0$ ensures that this price of risk is countercyclical. The central tendency of the market price of aggregate risk is captured by γ_0.

A.3. Capital Accumulation, Adjustment Costs, and Tax

The law of motion for capital is driven by depreciation, expressed as a fraction, δ, of the level of capital, and the rate of investment, as a fraction of capital, i_t. The rate of capital growth is given by the following equation:

$$k_{t+1} = k_t + (i_t - \delta) k_t. \tag{3.A.5}$$

We assume that investment is reversible, but to adjust the level of capital, firms incur adjustment costs. Following the literature on investment with adjustment costs, we assume that the cost function is given by the quadratic:

$$h(i_t, k_t) = \frac{\theta}{2} i_t^2 k_t. \tag{3.A.6}$$

Notice that firms are subject to adjustment costs not only when they invest but also when they disinvest.

Further, A firm pays taxes on realized profits, $\tau(y_t - \delta k_t)$. Here, τ denotes the flat tax rate on corporate profits.

B. Capital Structure and Default

The entire corporate debt of a firm is modeled as a single zero-coupon bond of face value F maturing at time T. If the after-tax firm value at time T does not exceed the actual principal, then default occurs and the shareholders give the ownership to the bondholders. Otherwise we assume that the firm redeems the bond and continues with no liability.

The bankruptcy process is assumed to be instantaneous but costly. We assume that the bankruptcy costs are expressed as a fraction, ξ, of the asset value[2].

C. Valuation

This section deals with the valuation of the outstanding stocks and debt. Upon observing the shocks at the beginning of period t, a firm makes optimal investment decisions to maximize expected present cash flows for stockholders.

In our model, since time-homogeneity holds only after the redemption of a zero-coupon bond, stock value before the maturity cannot be represented as the solution of a Bellman's equation. In order to solve the problem, first we need to obtain the firm value at the maturity by solving a Bellman's equation, and then solve the dynamic program from T recursively.

C.1 Stock

We start with the valuation of the outstanding stocks.

The value of the firm at time t after a firm redeems the bond is priced as the solution to the following dynamic program:

$$W(k_t, x_t, z_t) = (1-\tau)(y_t - \delta k_t) + \delta k_t + \max_{i_t} \left\{ -(i_t k_t + h(i_t, k_t)) + E_t(M_{t+1} W(k_{t+1}, x_{t+1}, z_{t+1})) \right\}, \tag{3.C.1}$$

subject to $k_{t+1} = k_t + (i_t - \delta) k_t$.

In this case, we obtain the following results:

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2 There is no incentive for a firm to decide to default before maturity because we assume that the production function is always positive. Therefore, default can occur only at maturity.

Proposition1

There exists a unique value function $W(k_t, x_t, z_t)$ that satisfies (3.C.1) and is continuous, increasing and differentiable in k_t, x_t and z_t, and concave in k_t.□

Proof:

See appendix.□

At maturity, the value of the stocks is,

$$V_T(k_T, x_T, z_T; F) = \max(W(k_T, x_T, z_T) - (1-\tau)F, 0), \tag{3.C.2}$$

Then, we obtain stock value at $t < T$ recursively:

$$\begin{aligned} V_t(k_t, x_t, z_t; F) = (1-\tau)(y_t - \delta k_t) + \delta k_t \\ + \max_{i_t} \{-(i_t k_t + h(i_t, k_t)) + E_t(M_{t+1} V_{t+1}(k_{t+1}, x_{t+1}, z_{t+1}; F))\}, \end{aligned} \tag{3.C.3}$$

subject to $k_{t+1} = k_t + (i_t - \delta) k_t$.

Proposition2

There exists i_t^* that maximizes $(-(i_t k_t + h(i_t, k_t)) + E_t(M_{t+1} V_{t+1}(k_{t+1}, x_{t+1}, z_{t+1}; F)))$, and $V_t(k_t, x_t, z_t; F)$ at $t < T$ is continuous, increasing and differentiable in k_t, x_t and z_t, and concave in k_t.□

Proof:

It is almost obvious from Proposition 1.□

Notice that, V depends on time t since the model is not time-homogeneity because of the existence of a zero-coupon bond,

C.2 Debt

We price a zero-coupon bond. At maturity, the bond price is as follows:

$$B_T(k_T, x_T, z_T; F) = \begin{cases} F & \text{if } W(k_T, x_T, z_T) > (1-\tau)F \\ \xi W(k_T, x_T, z_T) & \text{else} \end{cases}, \tag{3.C.4}$$

We obtain the bond value at $t < T$ recursively:

$$B_t(k_t, x_t, z_t; F) = E_t[M_{t+1} B_{t+1}(k_{t+1}, x_{t+1}, z_{t+1}; F)]. \tag{3.C.5}$$

4. Results

This chapter presents the main results of our model. First, we describe the calibrated values of the parameters used in solving the model. Second, we analyze the relationship between the credit risk and expected stock returns and optimal investment rates.

4.1 Calibration

The parameters of the model and their calibrated values are summarized in Table 1.

Parameter	Value
Productivity Shocks	
Monthly persistence of aggregate productivity	0.983
Monthly conditional volatility of aggregate productivity	0.00233
Long-run average of the aggregate productivity	−3.47
Monthly persistence of idiosyncratic productivity	0.97
Monthly conditional volatility of idiosyncratic productivity	0.1
Capital Dynamics	
Capital share	0.3
Monthly depreciation rate	0.01
Adjustment cost of investment	15
Pricing Kernel	
β	0.994
γ_0	50
γ_1	−1000
Tax	
Tax rate on corporate income	0.35
Bankruptcy Process	
Bankruptcy costs	0.03

Table 1: Calibration values for the parameters of the model

The monthly persistence, ρ_x , and conditional volatility, σ_x , of the aggregate productivity shock, are consistent with the quarterly estimates of Cooley and Prescott(1995). The long run average, is calibrated so that the rate of return on a unit of invested capital, for an average firm, is, about 30 percent per year, consistent with the choice of Berk, Green, and Naik (1999). The monthly persistence, ρ_z , and volatility, σ_z , of the idiosyncratic productivity shock correspond to those in Zhang (2005), and they are consistent with the empirical evidence of Pastor and Veronesi (2003).

The parameters driving the capital flow are calibrated as follows: the capital share α is similar to the choice in Kydland and Prescott (1982), the monthly depreciation rate, δ, is consistent with the estimates in Cooper and Haltiwanger (2000), and the adjustment cost parameter, θ, corresponds to the choice in Zhang (2005) and is consistent with the empirical estimates of Whited (1992).

The parameters driving the dynamics of pricing kernel, γ_0, γ_1 and β are similar to those in Zhang (2005), and they are calibrated to match the average Sharp ratio, the, average real interest rate and the volatility of interest rates.

The parameters governing the tax system, τ, is consistent with the annual calibrated values employed by Moyen (2004).

Finally, the bankruptcy parameter, ξ, in line with the empirical estimates of Altman (1991).

4.2 Analysis

We analyze the relationship between expected stock return and default probability[3]. Expected stock return is defined as follows:

$$E_t[R_{t+1}] = \frac{E_t[V_{t+1}]}{V_t - d_t}, \tag{4.2.1}$$

where d_t denotes dividend at time t:

$$\begin{aligned} d_t &= d_t(k_t, x_t, z_t) \\ &= (1-\tau)(y_t - \delta k_t) + \delta k_t - (i_t^* k_t + h(i_t^*, k_t)) \end{aligned} \tag{4.2.2}$$

$V_t - d_t$ is ex-dividend market value of equity.

Default probability at t is defined as:

$$DP_t = E_t(1_{V_T > 0}) \tag{4.2.3}$$

A. After the maturity

The plots of Figure 1(a) capture expected stock returns after the maturity T in "good"

[3] In following study, we set $k = 0.5$, $F = 2$ and $T = 20$, and following expected return and default probability are all conditional.

and "bad" economy[4]. The expected returns are higher in bad economy. This pattern is driven by the countercyclical price of risk: In bad economy, since investors become more risk averse, the price of risk is higher and the future cash flow is priced lower. However, with respect to idiosyncratic shocks, the expected return is almost constant in both good and bad economy. This implies that in this model, with no liability, investors price aggregate shocks, while investors hardly care about idiosyncratic shocks[5].

The plots of Figure 1(b) capture optimal investment rate after the maturity T in good and bad economy With respect to optimal investment rate, firms with lower idiosyncratic or aggregate shocks choose lower investment rate. We can interpret this as follows. When the idiosyncratic shock is lower, because of its persistence, the future cash flow is also lower, which means that the value of the capital is smaller. What is more, when the aggregate shock is lower, since not only the productivity is lower, but also the price of risk is higher, the value of the future cash flow, or capital, becomes even lower. Then, in such case, since marginal value of investment is also lower, a firm choose lower investment rate. If the productivity is extremely low, a firm may even disinvest capitals if it is not restricted by the bond covenants.

B. Before the maturity

B.1 Relationship between expected stock return and credit risk

The plots of Figure 2 capture default probability to idiosyncratic shocks before the maturity T in bad economy at different times. As Figure2 shows, with these parameters, the default probability of each firm is very high, especially when the time to maturity is short. In the following study, we investigate the relationship between expected stock return and default probability.

The plots of Figure 3 capture expected stock returns to idiosyncratic shocks before the maturity T in bad economy at different times. Figure 3 shows an interesting result. First when the time to maturity is long ($t = 0$), as idiosyncratic shock is higher, the expected stock return becomes lower. However, when the time to maturity is shorter ($t = 10,15,18$), as idiosyncratic shock is higher the expected return is not always lower:

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[4] In following study, we set $x_t = -3.467$ in "good" economy, and $x_t = -3.493$ in "bad" economy. Notice that, since a firm loads no debts after the maturity, expected stock returns and optimal investment rates of Figure1 does not depend on the time.

[5] Zhang (2005) shows that introducing asymmetric capital adjustment cost, the expected returns is significantly high when the idiosyncratic shock is low, which generates value premium in cross-section.

Rather, in lower idiosyncratic shocks, expected return is increasing. From Figure 2 and 3, when a firm has very high default probability, the expected stock return of the firm can be lower than the one of the firm with lower default probability.

We analyze this puzzling result as follows. We can decompose the ex-dividend market value of equity, $V_t - d_t$, into the cash flows before the maturity and the ones after the maturity:

$$\begin{aligned} V_t - d_t &= E_t(M_{t+1}V_{t+1}) \\ &= E_t(M_{t+1}(d_{t+1} + M_{t+2}V_{t+2})) \\ &= \cdots \\ &= \sum_{i=t+1}^{T-1} E_t(\mathbf{M}_{t+1,i} \cdot d_i) + E_t(\mathbf{M}_{t+1,T} \cdot V_T) \\ &= \sum_{i=t+1}^{T-1} E_t(\mathbf{M}_{t+1,i} \cdot d_i) + E_t(\mathbf{M}_{t+1,T} \cdot \max(W_T - (1-\tau)F, 0)) \\ &= SW_t + CO_t, \end{aligned} \tag{4.2.4}$$

where

$$\mathbf{M}_{t,s} \equiv \prod_{j=t}^{s} M_j \tag{4.2.5}$$

$$SW_t \equiv \sum_{i=t+1}^{T-1} E_t(\mathbf{M}_{t+1,i} \cdot d_i) \tag{4.2.6}$$

$$CO_t \equiv E_t(\mathbf{M}_{t+1,T} \cdot \max(W_T - (1-\tau)F, 0)) \tag{4.2.7}$$

In this decomposition, we can interpret the first term as the value of a swap receiving floating interest, SW_t, and the second term as the value of a call option CO_t on the firm value. We also decompose the stock return into returns of swap and call option:

$$\begin{aligned} R_{t+1} &= \frac{V_{t+1}}{V_t - d_t} \\ &= \frac{d_{t+1} + SW_{t+1} + CO_{t+1}}{SW_t + CO_t} \\ &= w_t^{SW} R_{t+1}^{SW} + w_t^{CO} R_{t+1}^{CO}, \end{aligned} \tag{4.2.8}$$

where

$$w_t^{SW} \equiv \frac{SW_t}{SW_t + CO_t} \tag{4.2.9}$$

$$w_t^{CO} \equiv \frac{CO_t}{SW_t + CO_t} \tag{4.2.10}$$

$$R_{t+1}^{SW} \equiv \frac{d_{t+1} + SW_{t+1}}{SW_t} \tag{4.2.11}$$

$$R_{t+1}^{CO} \equiv \frac{CO_{t+1}}{CO_t}. \tag{4.2.12}$$

In this way, we can interpret that the stock is a portfolio of the swap and the call option, and that the expected return and variation of the stock are dependent upon the proportions of the swap and the call option as contributions to the total stock value.

To understand the relationship of expected stock returns and default probability, this decomposition is useful.

The point is as follows. When the default probability of a firm is high, the value of the call option is small, while that of the swap is not as small as that of the call option. The reason is that, even if there is little possibility of survival at the maturity, dividends are still paid to the stockholders before the maturity. This cash flow is less risky than the cash flow after the maturity. Therefore, when the default probability is high, the expected return of the swap is also much lower than that of call option.

This explains the puzzling result of Figure 3. When the default probability is low, as the plots $t = 0$ in Figure3, we can interpret that the portfolio that represents the total stock value has high weight on the call option. If idiosyncratic shock is higher, the weight on the call option of the portfolio becomes higher and, at the same time, the risk of the call option becomes lower. As a result, the risk of portfolio becomes lower and the expected return of it becomes lower. This is the reason why the plot of $t = 0$ in Figure 3 is decreasing.

On the other hand, take a look at the case when the default probability is high as the plots $t = 10,15,18$ in Figure3. When the default probability is high, the portfolio has high weight on the swap. If idiosyncratic shock is higher, since the value of swap does not goes up as much as that of call option, the weight on the call option becomes higher. As idiosyncratic shock is higher, the call option becomes less risky. Still, when the default probability is very high, the risk of call option is much higher than that of swap. Therefore, by raising the weight on the riskier call option, the portfolio becomes riskier and its expected return becomes higher as idiosyncratic shock is higher. This is why the plots of $t = 15,18$ in Figure3 are increasing. In case idiosyncratic shock is much higher, since the expected return of call option is not much higher compared to that of the swap, the portfolio becomes less risky and its expected return becomes lower. This is the reason that the plot of $t = 10$ starts to decrease if the idiosyncratic shock is high.

B.2 Optimal investment rate

The plots of Figure 4 capture optimal investment rate to idiosyncratic shocks before the maturity T in bad economy at different times. This figure implies that the higher default probability is, the lower the optimal investment rate is. By investment, a firm exchanges today's cash for future cash flow. If it expects good future cash flow by investment, a firm invests a lot. However, if it doesn't, a firm invests little, or even disinvests and increases dividend if allowed by the bond covenants. In case a firm has liability and default probability is high, with low expected cash flow after the maturity, a firm will make little investment.

How does investment change the expected stock return, compared with the case in which a firm does not invest at any time? If a firm has no debts, expected stock return will be low because by investment stockholders can increase the stock value and decrease its risk. Then consider a firm with liability. In such case, investment will also increase stock value. However, if the firm has high default probability, since the weight of call option will increase, the portfolio will be riskier. Therefore, in a high default probability, investment can increase the value of call option more than, the one of swap, and make the expected stock return higher.

4.3 Discussion

The result that the cross-sectional expected stock return is lower when the cross-sectional credit risk is higher is consistent with the empirical research of Dichev(1998), Griffin and Lemmon(2002) and Campbell, Hilscher, and Szilagyi (2005).

However, some of the results from the model need further discussion. First, the results implies that if default probability is higher, since the weight of swap value is higher, dividend yield, d_t / V_t, becomes higher. Though this seems at odds, there is no empirical research on the relationship between credit risk and dividend yield, further research is needed.

Second, in the model, even if a firm defaults at the maturity, the firm still has always positive profit. One of the solutions is the introduction of nonnegative fixed cost f to profit function (3.A.1),

$$y_t = k_t^{\alpha} \exp(x_t + z_t) - f. \tag{4.3.1}$$

With this introduction, there will be the possibility of default with negative profit and therefore there is possibility of default before the maturity of the debt because of the fixed cost. Due to the potential negative profit, there will be the case of a negative stock value before the maturity if a firm continues. Even if we introduce the fixed cost, since the weight of swap value is larger than that of call option in higher default probability, the lower expected stock return still holds. In this paper for simplicity, the profit function has no fixed cost.

Finally, the relationship between the investment and credit risk has not been confirmed in empirical research. It can provide fresh directions for future research.

5. Conclusion

In this paper, we propose a neoclassical production augmented with capital structure and aggregate uncertainty. In our model, we show that, although the credit risk is high, under investors' expectation for stable dividend payments before the maturity, expected stock return can be lower than when credit risk is low. The puzzling phenomenon, interpreted by Griffin and Lemmon (2002) and Campbell, Hilscher, and Szilagyi (2005) as mispricing, is therefore consistent with rational expectations. Still, to be persuasive, the model needs future research in this area.

Appendix

Proof of Proposition 1

It is a special case of Zhang (2005). We prove the former half of the proposition, the uniqueness of the function of the asset value only.

Let T be the functional operator associated with the dynamic program in (3.5.1). That is:

$$T(W)(k_t, x_t, z_t) = y_t + \max_{i_t} \left\{ -(i_t k_t + h(i_t, k_t)) + E_t(M_{t+1} W(k_{t+1}, x_{t+1}, z_{t+1})) \right\} \tag{A.1}$$

To see whether the operator T has a unique solution, it is sufficient to check the Blackwell's sufficient conditions for a contraction mapping. The monotonicity condition follows immediately from the linearity of the T and V. The discounting condition follows from the fact that

$$\begin{aligned} T(W+a)(k_t, x_t, z_t) &= y_t + \max_{i_t} \left\{ -(i_t k_t + h(i_t, k_t)) + E_t(M_{t+1}(W(k_{t+1}, x_{t+1}, z_{t+1}) + a) \right\} \\ &= y_t + \max_{i_t} \left\{ -(i_t k_t + h(i_t, k_t)) + E_t(M_{t+1} W(k_{t+1}, x_{t+1}, z_{t+1})) \right\} + E_t(M_{t+1}) a \end{aligned} \tag{A.2}$$

for any real $a > 0$. Therefore, as long as $E_t(M_{t+1}) < 1$, the discounting property is satisfied and T is a contraction mapping. This ensures that the operator T has a unique fixed point in the space of continuous and bounded function.□

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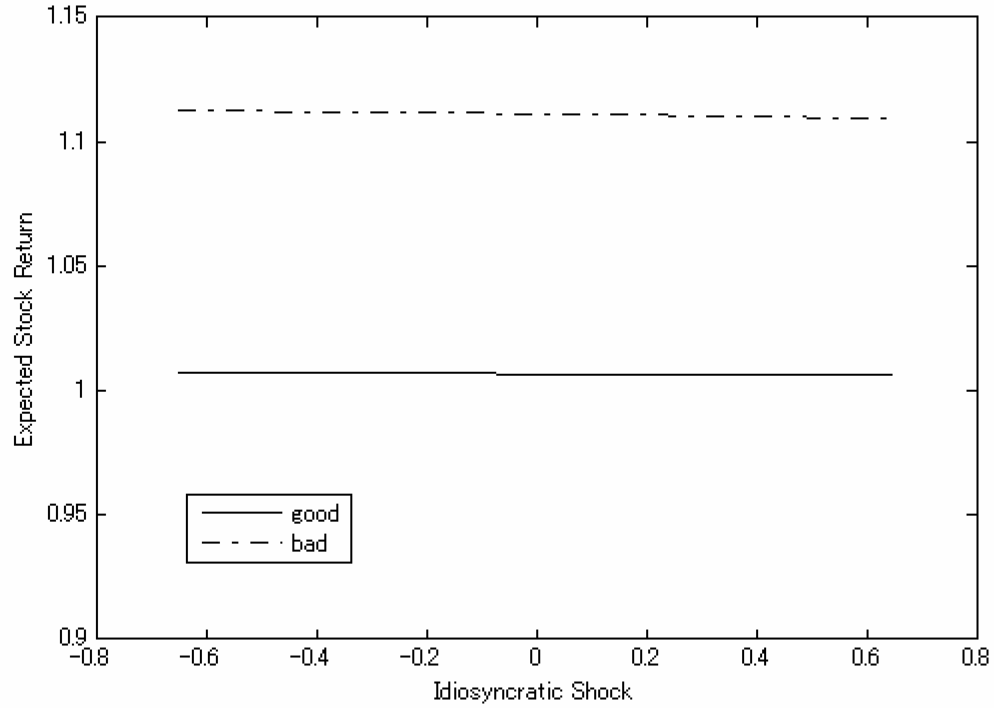

Figure1 (a): Conditional Expected stock return to idiosyncratic shocks after the maturity

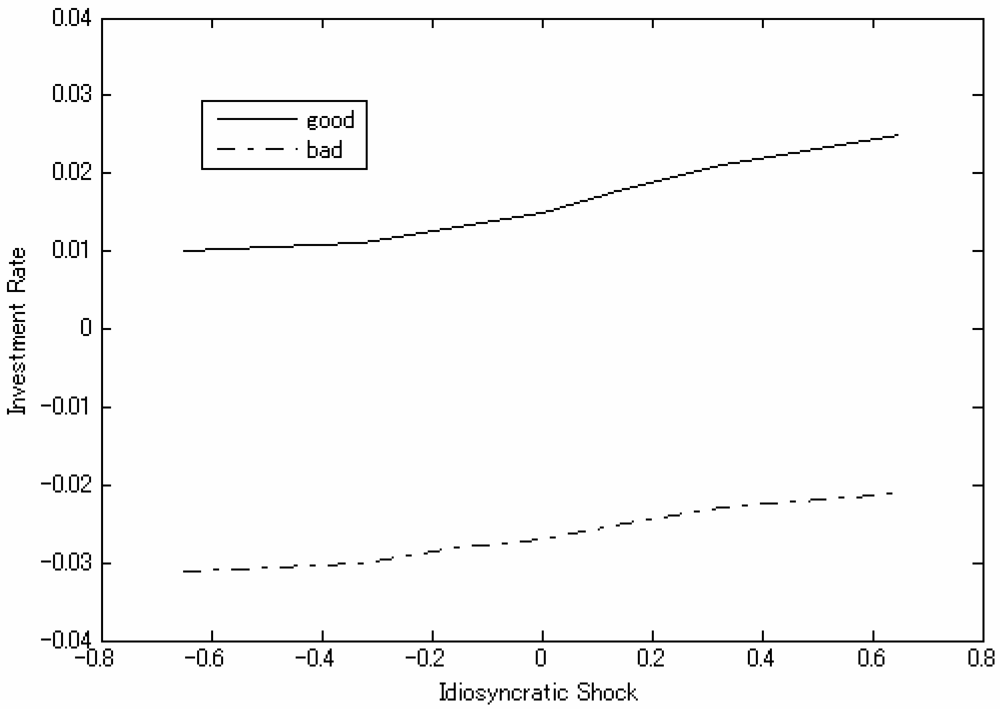

Figure1 (b): Optimal investment rate to idiosyncratic shocks after the maturity

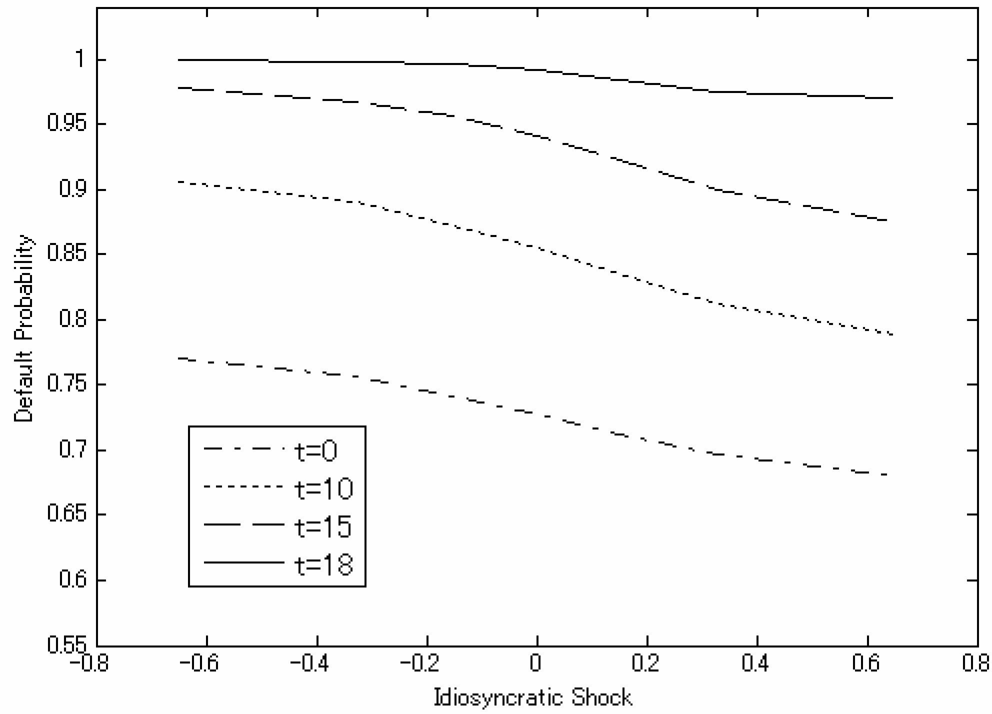

Figure 2: Conditional default probability to idiosyncratic shocks in bad economy

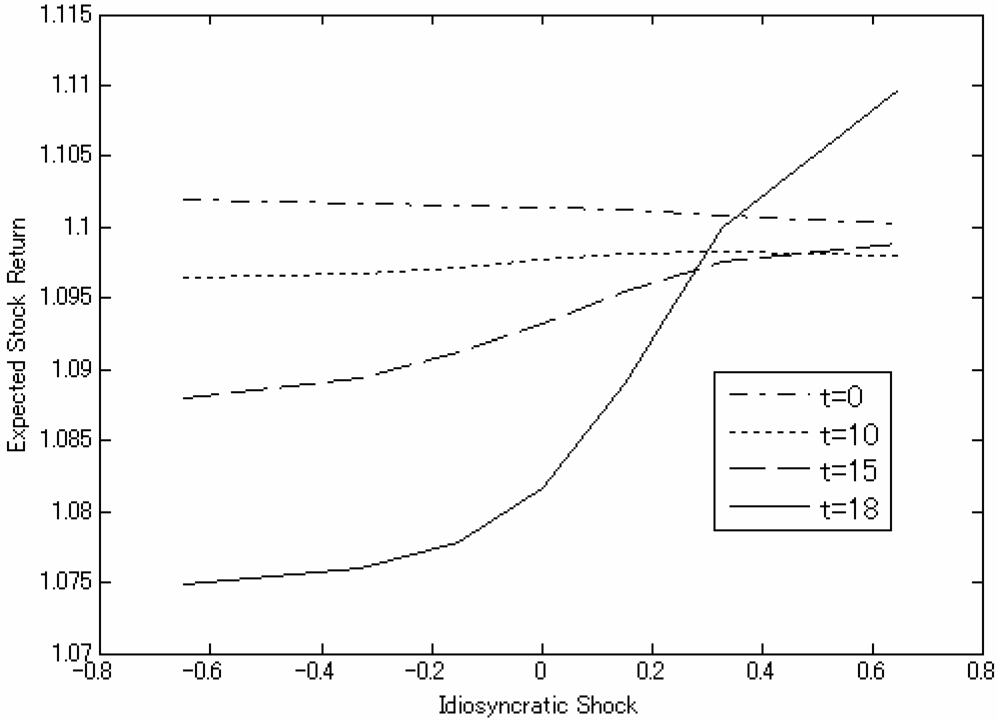

Figure 3: Conditional expected stock returns to idiosyncratic shocks in bad economy

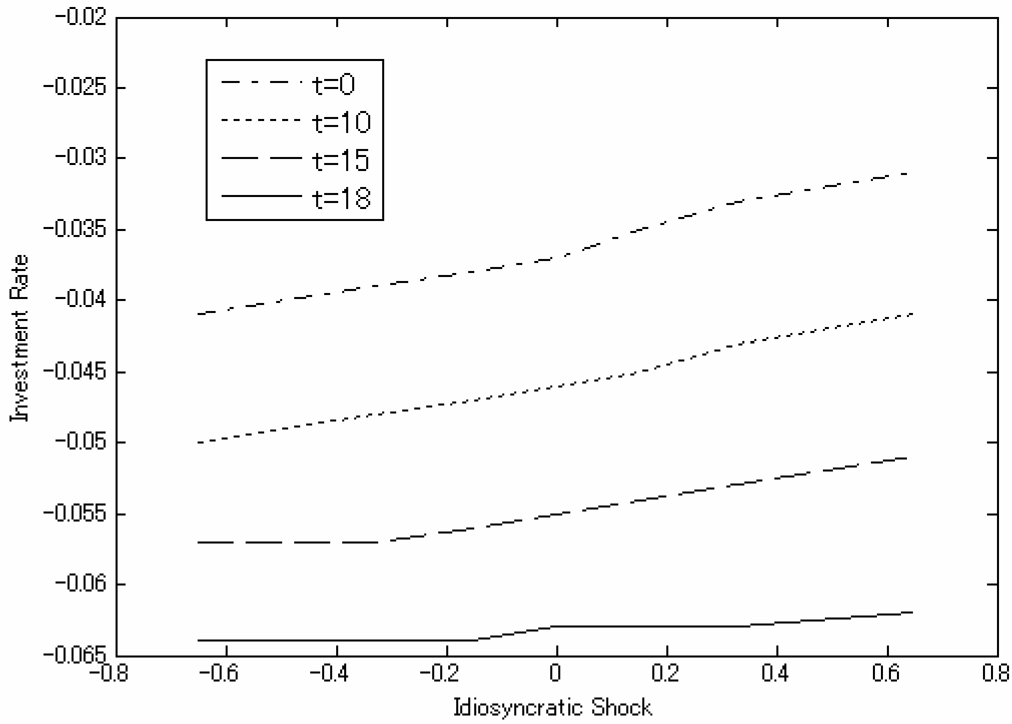

Figure 4: Optimal investment rate to idiosyncratic shocks in bad economy