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Realized Volatility, Covariance and Hedging Coefficient of the Nikkei-225 Futures with Micro-Market Noise

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Abstract

For the estimation problem of the realized volatility, covariance and hedging coefficient by using high frequency data with possibly micro-market noises, we use the Separating Information Maximum Likelihood (SIML) method, which was recently developed by Kunitomo and Sato (2008). By analyzing the Nikkei 225 futures and spot index markets, we have found that the estimates of realized volatility, covariance and hedging coefficient have significant bias by the traditional method which should be corrected. Our method can handle the estimation bias and the tick-size effects of Nikkei 225 futures by removing the possible micro-market noise in multivariate high frequency data.

Key Words

Realized Volatility, Realized Covariance, Realized Hedging Coefficient, Micro-Market Noise, High-Frequency Data, Separating Information Maximum Likelihood (SIML), Nikkei 225 Futures, Tick Size Effects.

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1. Introduction

The Nikkei-225 futures at the Osaka Securities Exchange (OSE) are the futures contracts for the Nikkei-225 Index and they are the most important futures contracts in the Japanese financial markets over the past 20 years. Because of their important role in financial markets, there have been basic questions to be answered on their performance and function as a hedging tool on the Nikkei-225 spot index as futures contracts. As the high frequency data of Nikkei-225 futures have become available, it may be natural to examine these problems because the majority of the past analyses are based on daily or monthly data. We may think that the finer data we use we have more accurate information on the performance of the futures contracts as some continuous financial models have suggested. We shall demonstrate in their paper, however, that the estimates obtained by the traditional realized variance, covariance and the hedging ratio are often not reliable and they should be corrected while the estimates we have obtained by another method give stable and reliable results on these key quantities. Then it is important to incorporate the micro-market noise when we estimate the realized volatility, covariance and the hedging ratio for practical purposes. We shall show that the new estimation method called the Separating Information Maximum Likelihood (SIML) approach recently proposed by Kunitomo and Sato (2008) gives an easy way to handle this problem and construct reliable estimates for the realized variance, covariance, correlation and the hedging ratio.

A considerable interest has been recently paid on the estimation problem of the realized volatility by using high-frequency data in financial econometrics. It may be partly because it is possible now to use a large number of high-frequency data in financial markets including the foreign exchange rates markets and stock markets. However, the earlier studies often had ignored the presence of micro-market noises in financial markets when they tried to estimate the underlying stochastic processes. Then several new statistical estimation methods have been developed. See Zhou (1998), Anderson, T.G., Bollerslev, T. Diebold,F.K. and Labys, P. (2000), Ait-

Sahalia, Y., P. Mykland and L. Zhang (2005), Hayashi and Yoshida (2005), Zhang, L., P. Mykland and Ait-Sahalia (2005), Barndorff-Nielsen, O., P. Hansen, A. Lunde and N. Shepard (2006), for further discussions on the related topics. In addition to these recent studies on the statistical methods on high frequency data, Kunitomo and Sato (2008) recently have developed the Separating Information Maximum Likelihood (SIML) estimation method for estimating the realized volatility and the realized covariance with possible micro-market noise by using high frequency data. The main merit of the SIML estimation is its simplicity and then it can be practically used for the multivariate (high frequency) financial time series with micro-market noise.

The main purpose of this paper is to apply our estimation method for the analysis of Nikkei-225 Spot-Index and Nikkei Futures. Unlike some estimates of the realized volatility, the realized covariance and the hedging ratio by some traditional methods, our estimates can be calculated in a simple way. Also the resulting estimates on these important quantities in the actual trading are stable over different frequency periods and thus they are reliable for practical purposes. There are some interesting findings on the Nikkei-225 futures from our data analysis. Since the Nikkei-225 futures at Osaka have been the most important futures contracts in the Japanese financial sector, there would be a number of important implications of our results for finance and the financial industries.

In Section 2 we discuss some aspects of the high frequency data of the Nikkei-225 futures. Then we shall explain the Separating Information Maximum Likelihood (SIML) estimator of the realized volatility and the realized covariance with micro-market noise in Section 3. In Section 4 we shall report some empirical results on the high frequency data of Nikkei-225 futures and then some brief remarks will be given in Section 5. In Appendix we shall report the results of simulations we have conducted on the SIML estimation.

2. High Frequency Data of Nikkei-225 Spot and Futures Markets

The most important futures market in Japan was formally started in September 1987 at the Osaka Securities Exchanges (OSE), which is the second largest securities exchange after Tokyo Securities Exchange and it has been developed in the trading size and scale over the past 20 years. The Nikkei-225 futures, the successful products of OSE [1], correspond to the Nikkei-225 Spot-Index as its future contracts. The Nikkei-225 spot index has been the most important stock index in the Japanese financial sector. The trading volume of the Nikkei225 futures at OSE has been heavy and there have been usually trades occurred within one second in most days. There are several important features on the high frequency data of Nikkei-225 Futures, which we have analyzed.

(i) Heavily Traded Data :

The Nikkei-225 Futures have been the major financial tool in the financial industry because the Nikkei-225 is the major index in Japan. We have high frequency data less than 1 second of Nikkei-225 Futures. In our analysis we have been using 1 second, 5 seconds, 10 seconds, 30 seconds and 60 seconds. Although we have high frequency data on the Nikkei-225 Futures within less than one second, we only have the Nikkei-225 Spot Index at every minute. Then we have an interesting new problem in the high frequency data analysis.

(ii) Intra-day Volatility Movements

When we analyze the tick data over a day, there has been an observation that the volatility of asset price changes over time within a day. Thus it is important to develop the method of measurements on the realized volatility, the realized covariance and the realized hedging ratio, which are free from these movements within a day.

(iii) Tick Size of Nikkei-225

In the standard finance theory the continuous time stochastic processes are often assumed for dynamic behaviors of securities prices. The typical example is the Black-Scholes theory. On the other hand, the Nikkei-225 Futures have the mini-

[1] It has been well-known in finance that futures of rice called Cho-Go-Mai were actively traded in the early 18th century at the Do-Jima-Rice Market in Osaka.

mum tick size and thus the observation of prices cannot be continuous over time. We may interpret the underlying price process as the efficient price and the tick seize effects as a kind of the micro-market noise.

(iv) Spot Market and Futures Market

Because the Nikkei-225 Futures are the major derivatives for Nikkei-225 Spot, it is important to measure the realized covariance and correlation between the spot-futures and the realized hedging ratio.

It has been known that the standard way of hedging is to use the covariance and variance. (See Duffie (1998), for the details of its explanation.) Thus it has been important to estimate the realized covariance and variance of the Nikkei-225 spot and Nikkei-225 futures.

3. The SIML Estimation of Realized Volatility, Covariance and Hedging Coefficient with Micro-Market Noise

Let y_{is} and y_{if} be the $i-$th observation of the $j-$th (log) spot price and the $j-$th (log) futures price at t_i^n for $j = 1, \cdots, p;\ 0 = t_0^n \leq t_1^n \leq \cdots \leq t_n^n = 1$. We set $\mathbf{y}_i = (y_{is}, y_{if})$ be a 2×1 vector and $\mathbf{Y}_n = (\mathbf{y}_i^{'})$ be an $n \times 2$ matrix of observations. The underlying continuous process $\mathbf{x}_i = (x_{is}, x_{if})^{'}$ is not necessarily the same as the observed prices and let $\mathbf{v}_i^{'} = (v_{is}, v_{if})$ be the vector of the micro-market noise. Then we have

$$\mathbf{y}_i = \mathbf{x}_i + \mathbf{v}_i \tag{3.1}$$

where $\mathcal{E}(\mathbf{v}_i) = \mathbf{0}$ and

$$\mathcal{E}(\mathbf{v}_i\mathbf{v}_i^{'}) = \mathbf{\Sigma}_v = \begin{pmatrix} \sigma_{ss}^{(v)} & \sigma_{sf}^{(v)} \\ \sigma_{fs}^{(v)} & \sigma_{ff}^{(v)} \end{pmatrix} .$$

We assume that

$$\mathbf{x}_t = \mathbf{x}_0 + \int_0^t \mathbf{C}_x(s) d\mathbf{B}_s \ \ (0 \leq t \leq 1), \tag{3.2}$$

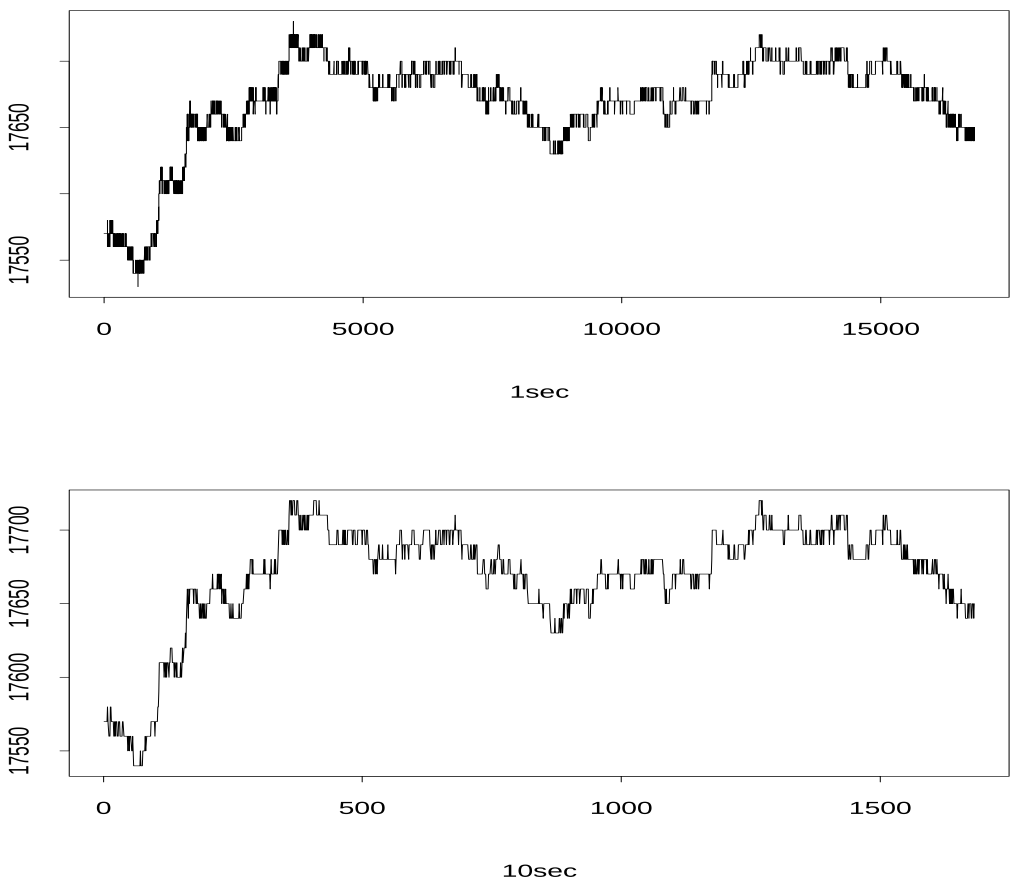

Figure 1: Nikkei 225F High-frequency-I

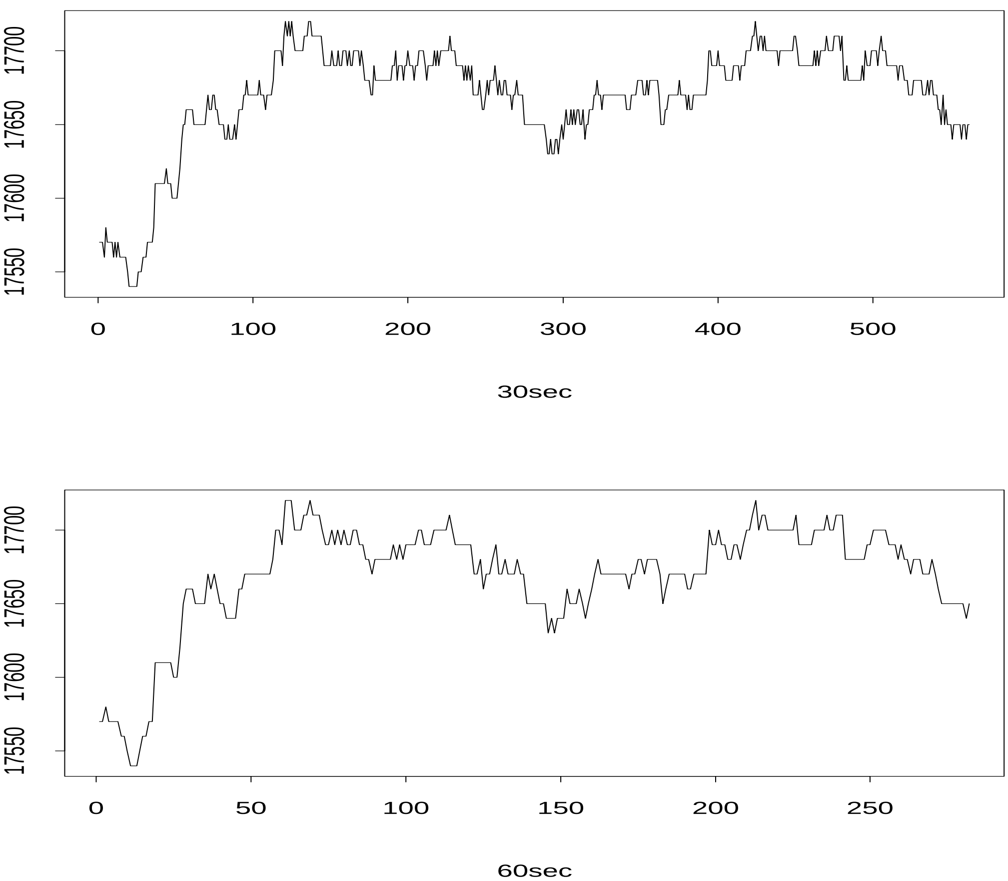

Figure 2: Nikkei 225F High-frequency-II

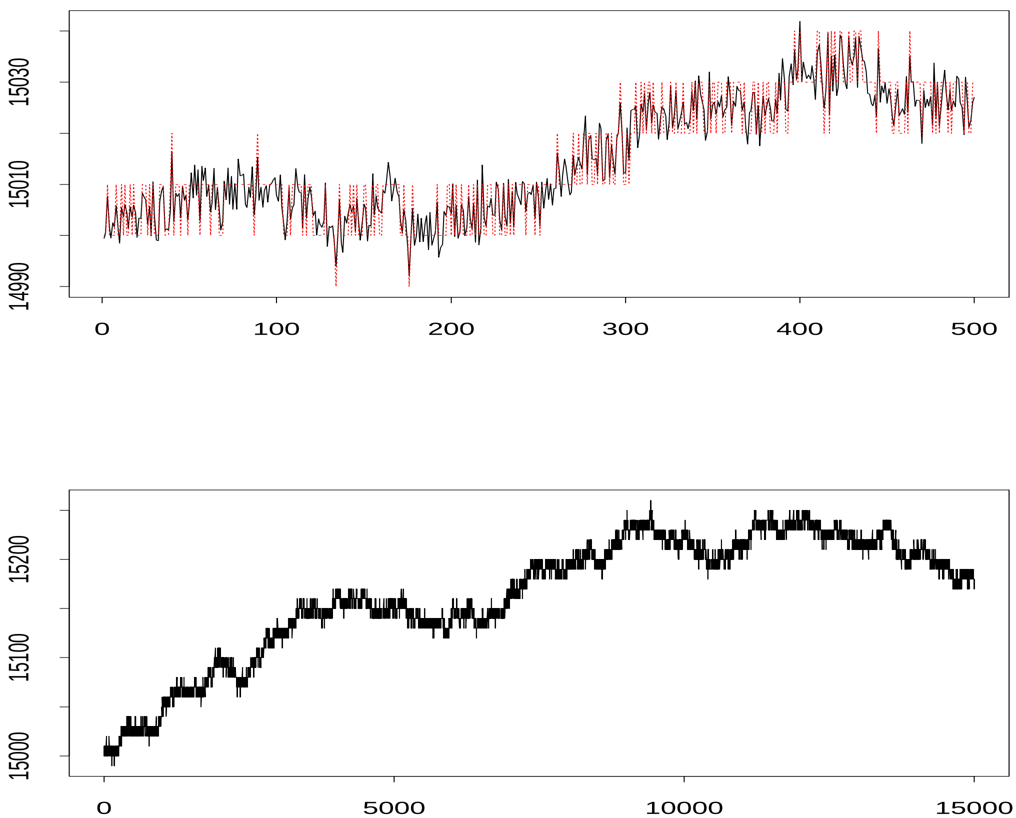

Figure 3: Effects of Tick-Size

where $\mathbf{B}_s$ is a $p \times 1$ vector of the standard Brownian motions and we write $\mathbf{\Sigma}_x(s) = \mathbf{C}_x(s)\mathbf{C}_x(s)^{'}$. Then the main statistical problem is to estimate the quadratic variations and co-variations

$$\mathbf{\Sigma}_x = \int_0^1 \mathbf{\Sigma}_x(s)ds = \begin{pmatrix} \sigma_{ss}^{(x)} & \sigma_{sf}^{(x)} \\ \sigma_{fs}^{(x)} & \sigma_{ff}^{(x)} \end{pmatrix} \tag{3.3}$$

of the underlying continuous process $\{\mathbf{x}_t\}$ and also the variance-covariance $\mathbf{\Sigma}_v = (\sigma_{ij}^{(v)})$ of the noises from the observed $\mathbf{y}_i$ $(i = 1, \cdots, n)$. Although we assume the Gaussian processes in order to derive the SIML estimation in this section, the asymptotic results do not depend on the Gaussianity of the underlying processes as we have discussed in Kunitmo and Sato (2008).

We consider the standard situation when $\mathbf{\Sigma}(s) = \mathbf{\Sigma}_x$ and $\mathbf{v}_i$ $(i = 1, \cdots, n)$ are independently and identically distributed with $\mathcal{E}(\mathbf{v}_i) = \mathbf{0}$ and $\mathcal{E}(\mathbf{v}_i\mathbf{v}_i^{'}) = \mathbf{\Sigma}_v$. We transform $\mathbf{Y}_n$ to $\mathbf{Z}_n$ $(= (\mathbf{z}_k^{'}))$ $(k = 1, \cdots, n)$ by

$$\mathbf{Z}_n = h_n^{-1/2}\mathbf{P}_n^{'}\mathbf{C}_n^{-1}\left(\mathbf{Y}_n - \bar{\mathbf{Y}}_0\right) \tag{3.4}$$

where

$$\mathbf{C}_n^{-1} = \begin{pmatrix} 1 & 0 & \cdots & 0 & 0 \\ -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & 0 & \cdots \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & \cdots & 0 & 0 \\ 1 & 1 & 0 & \cdots & 0 \\ 1 & 1 & 1 & \cdots & 0 \\ 1 & \cdots & 1 & 1 & 0 \\ 1 & \cdots & 1 & 1 & 1 \end{pmatrix}^{-1} . \tag{3.5}$$

and

$$\mathbf{P}_n = (p_{jk}) \ , \ p_{jk} = \sqrt{\frac{2}{n+\frac{1}{2}}} \cos\left[\pi(\frac{2k-1}{2n+1})(j-\frac{1}{2})\right] \ , \tag{3.6}$$

$$\bar{\mathbf{Y}}_0 = \mathbf{1}_n \cdot \mathbf{y}_0^{'} \ . \tag{3.7}$$

By considering the information on $\mathbf{\Sigma}_x$ and $\mathbf{\Sigma}_v$ in the Gaussian-likelihood function, Kunitomo and Sato (2008) defined the SIML estimator of $\hat{\mathbf{\Sigma}}_v$ by

$$\hat{\mathbf{\Sigma}}_x = \frac{1}{m_n} \sum_{k=1}^{m_n} \mathbf{z}_k \mathbf{z}_k^{'} \tag{3.8}$$

and also they defined the SIML estimator of $\hat{\mathbf{\Sigma}}_v$ by

$$\hat{\mathbf{\Sigma}}_v = \frac{1}{l_n} \sum_{k=n+1-l_n}^{n} a_{kn}^{-1} \mathbf{z}_k \mathbf{z}_k^{'} \ , \tag{3.9}$$

where

$$a_{kn} = 4n \sin^2 \left[\frac{\pi}{2} \left(\frac{2k-1}{2n+1} \right) \right] \ . \tag{3.10}$$

For both $\hat{\mathbf{\Sigma}}_v$ and $\hat{\mathbf{\Sigma}}_x$, the number of terms m and l should be dependent on n. Then we only need the order requirements that $m_n = O(n^{\alpha})$ $(0 < \alpha < \frac{1}{2})$ and $l_n = O(n^{\beta})$ $(0 < \beta < 1)$ for $\mathbf{\Sigma}_x$ and $\mathbf{\Sigma}_v$, respectively.

Although the SIML estimation was introduced under the Gaussian processes and the standard model, it has reasonable finite sample properties as well as asymptotic properties under the non-Gaussian processes and the volatility models. Let the conditional covariance matrix of the (underlying) returns without noise be

$$\mathbf{\Sigma}_i = \mathcal{E}\left[n \ \mathbf{r}_i \mathbf{r}_i^{'} | \mathcal{F}_{n,i-1} \right] \ , \tag{3.11}$$

where $\mathbf{r}_i = \mathbf{x}_i - \mathbf{x}_{i-1}$ is a sequence of martingale differences and $\mathcal{F}_{n,i-1}$ is the $\sigma-$field generated by $\mathbf{x}_s$ $(s \leq t_{i-1})$ and $\mathbf{v}_s$ $(s \leq t_{i-1})$. In this setting it is natural to impose the condition

$$\frac{1}{n} \sum_{i=1}^{n} \mathbf{\Sigma}_i \xrightarrow{p} \mathbf{\Sigma}_x = \int_0^1 \mathbf{\Sigma}_x(s) ds \ . \tag{3.12}$$

When the realized volatility and covariance $\mathbf{\Sigma}_x = (\sigma_{ij}^{(x)})$ is a constant (positive definite) matrix, we summarize the asymptotic properties of the SIML estimator under some regularity conditions [2].

Proposition 1 : We assume that $\mathbf{x}_i$ and $\mathbf{v}_i$ $(i = 1, \cdots, n)$ are mutually independent in (2.1), $\mathbf{r}_i = \mathbf{x}_i - \mathbf{x}_{i-1}$ and $\mathbf{v}_i$ are a sequence of martingale differeces with (3.1) and (3.2), and $\sup_{1 \le i \le n} \mathcal{E}(\|\mathbf{v}_i\|^4) < \infty$.
(i) As $n \longrightarrow \infty$,

$$\hat{\mathbf{\Sigma}}_x - \mathbf{\Sigma}_x \xrightarrow{p} \mathbf{O} \tag{3.13}$$

with $m_n = n^{\alpha}$ $(0 < \alpha < 1/2)$ and

$$\sqrt{m_n}\left[\hat{\sigma}_{ij}^{(x)} - \sigma_{ij}^{(x)}\right] \xrightarrow{w} N(0, \sigma_{ii}^{(x)}\sigma_{jj}^{(x)} + \left[\sigma_{ij}^{(x)}\right]^2) \tag{3.14}$$

with $m_n^5/n^2 \to 0$ for $i, j = s$ or f.
(ii) As $n \longrightarrow \infty$,

$$\hat{\mathbf{\Sigma}}_v - \mathbf{\Sigma}_v \xrightarrow{p} \mathbf{O} \tag{3.15}$$

and

$$\sqrt{l_n}\left[\hat{\sigma}_{ij}^{(v)} - \sigma_{ij}^{(v)}\right] \xrightarrow{w} N(0, \sigma_{ii}^{(v)}\sigma_{jj}^{(v)} + \left[\sigma_{ij}^{(v)}\right]^2) \tag{3.16}$$

with $l_n = n^{\beta}$ $(0 < \beta < 1)$ for $i, j = s$ or f.

When $\mathbf{\Sigma}_x$ is a random (positive definite) matrix, we need the concept of stable convergence, which has been explained by Hall and Heyde (1980) and Barndorff-Nielsen et al. (2006) in the details. In this situation (3.16) should be replaced by

$$\sqrt{m_n}\left[\frac{\hat{\sigma}_{ii}^{(x)} - \sigma_{ii}^{(x)}}{\sigma_{ii}^{(x)}}\right] \xrightarrow{w} N(0, 2) \tag{3.17}$$

as $n \to \infty$ for $i, j = s$ or f, for instance.

Choice of m and l

[2]It is a special case of Theorem 2 of Kunitomo and Sato (2008). See Kunitomo ans Sato (2008) for the proof of the results.

Because the properties of the SIML estimation method crucially depends on the choice of m_n and l_n, which are dependent on n, we have investigated the small sample effects of several possibilities by using a number of simulations. (See Tables A-1 and A-2 in the Appendix.) As we had expected from Proposition 1, we have found that we have more efficiency as α increases and the bias becomes significant when α is greater than 1/2. Currently, we are using $\alpha = 0.3, 0.45$ and $\beta = 0.8$.

By using Proposition 1, it is possible to evaluate the SIML estimators of the realized volatility, covariance, correlation and the hedging ratio, which will be useful for empirical analysis.

Hedging Ratio

The SIML estimator of the hedging ratio $H = \sigma_{sf}^{(x)}/\sigma_{ff}^{(x)}$ can be defined by

$$\hat{H} = \frac{\hat{\sigma}_{sf}^{(x)}}{\hat{\sigma}_{ff}^{(x)}} \ . \tag{3.18}$$

Then by using Proposition 1 we can derive the limiting distribution of the hedging ratio estimator, which is given by

$$\sqrt{m_n}\left[\hat{H} - H\right] \xrightarrow{w} N(0, \omega_H) \tag{3.19}$$

as $m_n^5/n^2 \to 0$, where

$$\omega_H = \frac{\sigma_{ss}^{(x)}}{\sigma_{ff}^{(x)}}\left[1 - \frac{\sigma_{sf}^{(x)2}}{\sigma_{ss}^{(x)}\sigma_{ff}^{(x)}}\right] \ . \tag{3.20}$$

Correlation Coefficient

The SIML estimator of the correlation coefficient $\rho_{sf} = \sigma_{sf}^{(x)}/[\sigma_{ss}^{(x)}\sigma_{ff}^{(x)}]$ is defined by

$$\hat{\rho}_{sf} = \frac{\hat{\sigma}_{sf}^{(x)}}{\sqrt{\hat{\sigma}_{ss}^{(x)}\hat{\sigma}_{ff}^{(x)}}} \ . \tag{3.21}$$

Then by using Proposition 1 the limiting distribution of the hedging ratio estimator is given by

$$\sqrt{m_n}\left[\hat{\rho}_{sf} - \rho_{sf}\right] \xrightarrow{w} N(0, \omega_\rho) \tag{3.22}$$

as $m_n^5/n^2 \to 0$, where

$$\omega_\rho = \left[1 - \frac{\sigma_{sf}^{(x)2}}{\sigma_{ss}^{(x)}\sigma_{ff}^{(x)}}\right] . \tag{3.23}$$

This formula agrees with the standard one known in the statistical multivariate analysis (see Theorem 4.2.4 of Anderson (2003) for instance) except the fact that we use m_n instead of n.

4. Estimation Results

Realized Volatility

We have picked one day in April 2007 and estimated the realized volatility with different time intervals in Table 4.1 by both the traditional historical volatility estimation and the SIML estimation as a typical example. Then we found that the estimated HI heavily depends on the observation intervals while our estimation does not depend on them very much. The problem of significant biases of the estimated HI has been pointed out recently by several researchers.

We also have conducted a number of simulations and the details of our results of simulations are summarized as from Tables A-1 to Table A-8. The number of data is 300, 5000 and 20000 and the first case corresponds to one minute data while the third case correspond to one second data. To examine the effects of the non-Gaussian distributions we have conducted some simulations with t-distributions. See Table A-4 for the details. To summarize our simulations, we have confirmed that the SIML estimates are stable and reliable over different observation periods.

Table 4.1 : Estimation of Realized Volatility :

	Σ_x	Σ_v	HI
1s	5.252E-05	9.853E-09	4.946E-04
10s	4.513E-05	4.168E-08	1.764E-04
30s	5.099E-05	7.217E-08	9.449E-05
60s	6.151E-05	8.976E-08	6.964E-05

Realized Covariance and Correlation

The SIML estimators of the realized covariance and the realized correlation can be defined as the realized variance. We give some estimates of the realized covariance of the Nikkei-225 spot-future by high frequency data, which are summarized as Tables A-6 and Table A-7. (We have used both the SIML estimation and the historical estimation.) We have found that the effects of micro-market noise should not be ignored and the correlation between the spot and futures is quite high based on the high frequency data, which agree with the standard arguments in the standard financial theory. Our method gives stable estimation results on the realized covariance and the realized correlation.

In addition to the simulation reported in Kunitomo and Sato (2008), we have examined some properties of the estimation of realized variance and covariance by using simulations, which are reported in Appendix.

Realized Hedging

We have obtained the estimates of the hedging ratio by the SIML estimation. Unlike other methods, we have found that our estimates are stable and reliable. The estimated values of the historical hedging estimates (HI) vary day-by-day and often deviate significantly different from 1. On the other hand, the estimated values of the SIML estimates are often near to one, which agrees with the intuitive reasoning among the market participants. The most important finding is the fact that the estimates of the hedging ratio from high frequency historical data are not reliable while we have reasonable estimates of the hedging ratio by the SIML estimation. (See Table A-8.)

We also have conducted the hedging simulations. By using the high frequency data, each day we estimate the hedging strategy with the optimal hedging ratio and then determine the hedging strategy in the next day. By using the historical data, we have poor performance in the hedging practice. However, we have found that the hedging strategy with the SIML estimates is reasonable well in the sense it is very close to the one by using the pure futures hedging (i.e. it is one).

Effects of Tick Size and the Rounding-Error model

The tick size of the Nikkei-225 futures have small impact on the realized volatility. It may be because the effects of tick size have been treated as the micro-market noise in our formulation. For instance, we can simulate a quasi-continuous path and then generate the realizations of the rounding error process (i.e. the continuous sample path plus the rounding errors) which is Figure 3. We may observ the fact that the resulting realization in Figure 3 is very similar the actual high frequency data.

Also we have examined some properties of the rounding error model by a set of simulations.(See Table A-3 in the Appendix.) The rounding error model with a finite tick size we considered is

$$y_i = \log\left[10 \times \mathrm{floor}\left(\frac{1}{10}\exp(x_{t_i} + v_i) + 0.5\right)\right] , \tag{4.1}$$

where x_t follows (3.2) with $p = 1$.

While the estimated values of the historical volatility estimates (HI) have some impacts by the tick size effects, the SIML estimates of the realized volatility are quite robust against the contamination of Tick-Size effects, which is reported as Table A-3. Since the tick size has been 10 yen in the Nikkei225 futures, its effects do not have major impact once we use the SIML estimation.

On Estimates of the Realized Volatility

In order to remove some unstable movements in the markets, we have estimated the realized volatility by deleting the first 10 minutes after several trials. We compare the SIML estimation and the historical volatility calculations for the realized volatility, correlation and the hedging coefficient from 1 minute data, which are reported in Table 4.2 in the period from March 1, 2007 to April 27, 2007 [3].

[3] We have excluded some data observed in the first 10 minutes mainly because there can be some other factors influencing the wild movements and fluctuations in this particular time period.

In order to make a comparison of the Nikkei-225 spot and futures, we also picked one day and give a figure on the Nikkei-225 Futures and the spot index in Figure 4. We observed the similarity of two time series data. The important use of the Nikkei-225 futures is to hedge risks involving the Nikkei-225 Spot market. We have done some simulation by using the estimates of the hedging coefficient by the historical method and the SIML estimation. We have found that the realized volatility, covariance and the hedging ration by the historical method heavily depend on the time scales or time intervals we measure the high frequency data. On the other hand, the estimates by the SIML estimation are robust on the time scale and time intervals. We definitely find that the SIML estimation is useful in this respect.

Table 4.2 : Realized Volatility and Correlation of Nikkei 225 Index
(Spot and futures of NIKKEI225 index)

1min. data	SIML				Historical			
date	Var(Spot)	Var(Future)	cor	Hedge	Var(Spot)	Var(Future)	cor	Hedge
20070301	6.84E-05	5.59E-05	1.00	1.10	6.23E-05	8.88E-05	0.60	0.50
20070302	8.13E-05	8.83E-05	0.99	0.95	7.34E-05	9.94E-05	0.64	0.55
20070305	7.57E-05	7.08E-05	0.99	1.03	9.11E-05	1.19E-04	0.59	0.52
20070306	5.40E-05	5.05E-05	1.00	1.03	7.91E-05	1.13E-04	0.68	0.56
20070307	1.06E-04	1.02E-04	0.99	1.02	7.45E-05	1.22E-04	0.71	0.56
20070308	9.32E-05	1.05E-04	0.99	0.94	6.92E-05	1.13E-04	0.61	0.48
20070309	4.64E-05	3.72E-05	0.99	1.11	6.80E-05	1.01E-04	0.62	0.51
20070312	3.83E-05	3.77E-05	0.99	1.00	4.22E-05	6.81E-05	0.51	0.40
20070313	3.94E-05	3.87E-05	1.00	1.01	4.38E-05	6.91E-05	0.52	0.42
20070314	5.44E-05	6.10E-05	1.00	0.94	6.00E-05	9.45E-05	0.62	0.49
20070315	3.35E-05	3.35E-05	0.99	0.99	4.61E-05	8.28E-05	0.58	0.43
20070316	1.20E-04	1.26E-04	1.00	0.98	7.81E-05	9.85E-05	0.64	0.57
20070319	9.60E-05	9.29E-05	0.97	0.99	6.95E-05	8.74E-05	0.58	0.52
20070320	3.32E-05	3.24E-05	0.99	1.00	3.94E-05	7.01E-05	0.65	0.49
20070322	1.14E-05	1.06E-05	0.97	1.01	1.61E-05	4.61E-05	0.41	0.24
20070323	1.44E-05	1.37E-05	0.96	0.98	3.01E-05	5.90E-05	0.51	0.37
20070326	3.60E-05	2.86E-05	0.99	1.11	2.99E-05	5.59E-05	0.50	0.37
20070327	5.63E-05	5.25E-05	0.99	1.02	3.82E-05	6.01E-05	0.54	0.43
20070328	5.96E-05	5.45E-05	1.00	1.04	5.94E-05	9.88E-05	0.55	0.42
20070329	6.36E-05	5.60E-05	0.96	1.03	6.13E-05	9.40E-05	0.56	0.45
20070330	6.03E-05	6.26E-05	0.99	0.97	3.28E-05	7.34E-05	0.60	0.40
20070402	1.10E-04	1.10E-04	1.00	1.00	7.16E-05	9.72E-05	0.56	0.49
20070403	3.62E-05	4.14E-05	0.96	0.90	5.41E-05	8.01E-05	0.51	0.42
20070404	3.04E-05	2.97E-05	0.97	0.98	2.84E-05	6.96E-05	0.56	0.36
20070405	3.13E-05	3.11E-05	0.97	0.98	3.14E-05	6.59E-05	0.53	0.37
20070406	1.62E-05	1.29E-05	0.96	1.07	2.04E-05	5.11E-05	0.40	0.25
20070409	2.77E-05	2.77E-05	0.97	0.97	2.66E-05	4.60E-05	0.41	0.31
20070410	3.01E-05	2.22E-05	0.95	1.11	2.79E-05	5.23E-05	0.32	0.23
20070411	1.04E-05	7.10E-06	0.91	1.11	2.70E-05	4.36E-05	0.33	0.26
20070412	3.35E-05	2.63E-05	0.99	1.11	3.19E-05	5.33E-05	0.40	0.31
20070413	6.84E-05	6.25E-05	0.99	1.04	5.58E-05	7.27E-05	0.52	0.46
20070416	6.82E-05	6.68E-05	1.00	1.01	3.67E-05	6.56E-05	0.56	0.42
20070417	6.58E-05	5.80E-05	1.00	1.06	3.97E-05	7.61E-05	0.53	0.38
20070418	7.83E-05	6.69E-05	1.00	1.08	3.57E-05	6.11E-05	0.60	0.46
20070419	4.77E-05	3.66E-05	0.99	1.13	7.50E-05	8.69E-05	0.60	0.56
20070420	4.30E-05	3.65E-05	0.99	1.08	3.81E-05	7.57E-05	0.63	0.45
20070423	3.78E-05	3.65E-05	1.00	1.01	4.70E-05	6.53E-05	0.59	0.50
20070424	5.29E-05	4.56E-05	0.99	1.07	5.23E-05	8.70E-05	0.60	0.47
20070425	3.18E-05	2.32E-05	0.98	1.14	4.69E-05	6.24E-05	0.52	0.45
20070426	3.03E-05	2.82E-05	0.99	1.02	2.91E-05	5.35E-05	0.48	0.35
20070427	4.59E-05	4.27E-05	0.99	1.02	7.26E-05	9.66E-05	0.60	0.52

* use data after 9:10 for everyday.

2007/04/16

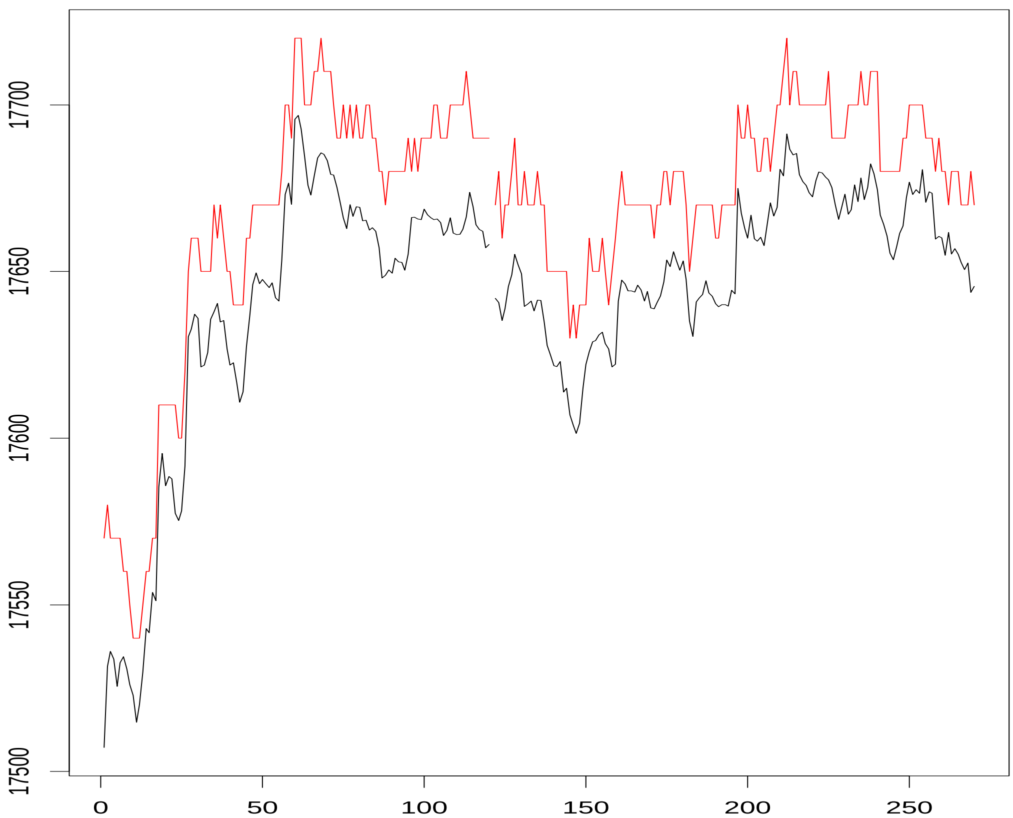

Figure 4: Nikkei-225 Spot-Futures

hedge results (20070301-20070427)

	SIML	Historical	Hedge ratio =1
hedge error ratio	0.247%	0.66%	0.244%

5. Concluding Remarks

In this paper we have applied the Separating Information Maximum Likelihood (SIML) estimation method to estimate the realized volatility, the realized covariance, and the realized hedging coefficient by using high-frequency financial data of Nikkei-225 futures with possibly micro-market noise. The SIML estimator is so simple that it can be practically used not only for the single high frequency data, but also for the multivariate high frequency series with micro-market noise. This has an important aspect because we want to estimate the hedging ratio from high-frequency data for instance.

We have found several important observations by analyzing a set of high frequency data of Nikkei-225 Futures (on the Nikkei-225 Spot-Index), which has been actively traded at the Osaka Securities Exchange in the past twenty years. There are some important features in our high frequency data. Although we have high frequency data on the Nikkei 225 Futures within less than one second, we only have the Nikkei-225 Spot Index at every minute. The other features is the fact that the tick size of the Nikkei-225 futures is much larger (100 times) than its spot counterpart and it has been 10 yen.

In conclusion, our analysis of high frequency data on the Nikkei-225 futures and Nikkei-225 spot suggest that the SIML estimation can handle these problem easily and properly while there are significant bias of the estimates obtained by the traditional historical method. Also we can treat the tick size effect as the micro-market noise and the data length problem properly by the SIML estimation method. These findings may have a number of implications on the derivative pricing and the risk management in practice.

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APPENDIX : Simulations

We have reported several simulation results on the SIML estimation of the realized volatility in Kunitomo and Sato (2008). Also we have conducted a number of additional simulations on the effects of the estimation problem of the realized volatility, the realized covariance, the realized hedging ratio and the effects of tick size or the rounding error model. We are summarizing our results of simulations.

Table A-1 : Estimation of Realized Volatility (constant volatility, $\alpha = 0.3$)

n=300	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol
True	2.00E-04	2.00E-06		2.00E-04	2.00E-07		2.00E-04	2.00E-09	
Mean	2.00E-04	2.20E-06	1.41E-03	2.03E-04	3.84E-07	3.21E-04	1.92E-04	1.85E-07	2.01E-04
SD	1.28E-04	3.10E-07	1.31E-04	1.32E-04	5.64E-08	2.79E-05	1.24E-04	2.60E-08	1.65E-05
MSE	1.64E-08	1.34E-13		1.73E-08	3.70E-14		1.55E-08	3.40E-14	
AVAR	1.45E-08	8.34E-14		1.45E-08	8.34E-16		1.45E-08	8.34E-20	
n=5000	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol
True	2.00E-04	2.00E-06		2.00E-04	2.00E-07		2.00E-04	2.00E-09	
Mean	2.07E-04	2.01E-06	2.02E-02	2.02E-04	2.10E-07	2.20E-03	2.01E-04	1.23E-08	2.20E-04
SD	8.63E-05	9.13E-08	4.83E-04	8.53E-05	1.01E-08	5.28E-05	8.16E-05	5.88E-10	4.47E-06
MSE	7.51E-09	8.46E-15		7.28E-09	2.08E-16		6.67E-09	1.06E-16	
AVAR	6.21E-09	8.79E-15		6.21E-09	8.79E-17		6.21E-09	8.79E-21	
n=20000	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol
True	2.00E-04	2.00E-06		2.00E-04	2.00E-07		2.00E-04	2.00E-09	
Mean	2.04E-04	2.00E-06	8.02E-02	2.03E-04	2.02E-07	8.20E-03	2.01E-04	4.55E-09	2.80E-04
SD	6.62E-05	5.55E-08	1.02E-03	6.40E-05	5.67E-09	1.01E-04	6.58E-05	1.24E-10	2.86E-06
MSE	4.39E-09	3.08E-15		4.11E-09	3.75E-17		4.33E-09	6.50E-18	
AVAR	4.10E-09	2.90E-15		4.10E-09	2.90E-17		4.10E-09	2.90E-21	

Data generating process:

$$y_t = x_t + v_t$$
$$x_t = x_{t-1} + u_t$$
$$u_t \sim i.i.d.N(0, \sigma_x^2/n), v_t \sim i.i.d.N(0, \sigma_v^2)$$

Table A-2 : Estimation of Realized Volatility (constant volatility, $\alpha = 0.45$)

n=300	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol
True	2.00E-04	2.00E-06		2.00E-04	2.00E-07		2.00E-04	2.00E-09	
Mean	2.06E-04	2.19E-06	1.41E-03	1.99E-04	3.82E-07	3.21E-04	1.96E-04	1.84E-07	2.01E-04
SD	7.96E-05	3.10E-07	1.35E-04	7.55E-05	5.62E-08	2.72E-05	7.52E-05	2.83E-08	1.71E-05
MSE	6.37E-09	1.32E-13		5.70E-09	3.63E-14		5.67E-09	3.40E-14	
AVAR	6.14E-09	8.34E-14		6.14E-09	8.34E-16		6.14E-09	8.34E-20	
n=5000	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol
True	2.00E-04	2.00E-06		2.00E-04	2.00E-07		2.00E-04	2.00E-09	
Mean	2.05E-04	2.01E-06	2.02E-02	1.99E-04	2.11E-07	2.20E-03	2.01E-04	1.24E-08	2.20E-04
SD	4.22E-05	9.33E-08	4.80E-04	3.94E-05	9.30E-09	5.21E-05	3.84E-05	5.69E-10	4.46E-06
MSE	1.81E-09	8.88E-15		1.55E-09	1.98E-16		1.47E-09	1.08E-16	
AVAR	1.73E-09	8.79E-15		1.73E-09	8.79E-17		1.73E-09	8.79E-21	
n=20000	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol
True	2.00E-04	2.00E-06		2.00E-04	2.00E-07		2.00E-04	2.00E-09	
Mean	2.04E-04	2.00E-06	8.03E-02	2.02E-04	2.02E-07	8.20E-03	1.99E-04	4.54E-09	2.80E-04
SD	3.17E-05	5.27E-08	9.31E-04	3.01E-05	5.25E-09	9.93E-05	2.88E-05	1.28E-10	2.76E-06
MSE	1.02E-09	2.80E-15		9.10E-10	3.28E-17		8.28E-10	6.49E-18	
AVAR	9.28E-10	2.90E-15		9.28E-10	2.90E-17		9.28E-10	2.90E-21	

Table A-3 : Round Error Model ($\alpha = 0.3$)

n=300	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol
True	5.00E-05	5.00E-07		5.00E-05	5.00E-08		5.00E-05	0.00E+00	
Mean	5.06E-05	5.84E-07	3.74E-04	4.82E-05	1.32E-07	1.02E-04	4.87E-05	8.29E-08	7.24E-05
SD	3.25E-05	8.31E-08	3.71E-05	3.04E-05	1.96E-08	8.89E-06	3.23E-05	1.18E-08	5.81E-06
MSE	1.06E-09	1.40E-14		9.24E-10	7.14E-15		1.04E-09	7.01E-15	
AVAR	9.03E-10	5.22E-15		9.03E-10	5.22E-17		9.03E-10	0.00E+00	
n=5000	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol
True	5.00E-05	5.00E-07		5.00E-05	5.00E-08		5.00E-05	0.00E+00	
Mean	5.08E-05	5.41E-07	5.42E-03	4.86E-05	8.91E-08	9.18E-04	4.92E-05	2.03E-08	2.65E-04
SD	2.10E-05	2.47E-08	1.26E-04	1.86E-05	3.98E-09	2.20E-05	1.92E-05	1.59E-09	1.34E-05
MSE	4.44E-10	2.27E-15		3.49E-10	1.54E-15		3.69E-10	4.15E-16	
AVAR	1.08E-10	5.49E-16		1.08E-10	5.49E-18		3.88E-10	0.00E+00	
n=20000	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol
True	5.00E-05	5.00E-07		5.00E-05	5.00E-08		5.00E-05	0.00E+00	
Mean	5.06E-05	5.38E-07	2.15E-02	5.09E-05	8.73E-08	3.52E-03	5.08E-05	1.01E-08	5.28E-04
SD	1.75E-05	1.40E-08	2.53E-04	1.62E-05	2.29E-09	4.69E-05	1.63E-05	7.43E-10	2.91E-05
MSE	3.07E-10	1.67E-15		2.63E-10	1.40E-15		2.66E-10	1.02E-16	
AVAR	5.80E-11	1.81E-16		5.80E-11	1.81E-18		2.56E-10	0.00E+00	

Data generating process: $y_t = \log(y'_t)$

$$y'_t = 10 \times floor(exp(y''_t)/10 + 0.5)$$

$$y''_t = x_t + v_t$$

$$x_t = x_{t-1} + u_t, x_0 = \log(15000)$$

$$u_t \sim i.i.d.N(0, \sigma_x^2/n), v_t \sim i.i.d.N(0, \sigma_v^2)$$

* $floor(x)$ is a function whose value is the largest integer less than or equal to x.

Table A-4 : T-Error-Distribution ($df_x = 5$, $df_v = 7$, $\alpha = 0.3$)

n=300	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol
True	8.33E-05	7.00E-07		8.33E-05	7.00E-08		8.33E-05	7.00E-10	
Mean	8.27E-05	7.79E-07	5.07E-04	8.11E-05	1.45E-07	1.25E-04	8.25E-05	7.67E-08	8.39E-05
SD	5.20E-05	1.27E-07	5.90E-05	5.21E-05	2.33E-08	1.49E-05	5.64E-05	1.55E-08	1.38E-05
MSE	2.71E-09	2.23E-14		2.72E-09	6.12E-15		3.18E-09	6.02E-15	
AVAR	2.51E-09	1.02E-14		2.51E-09	1.02E-16		2.51E-09	1.02E-20	
n=5000	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol
True	8.33E-05	7.00E-07		8.33E-05	7.00E-08		8.33E-05	7.00E-10	
Mean	8.51E-05	7.04E-07	7.08E-03	8.28E-05	7.41E-08	7.83E-04	8.41E-05	5.00E-09	9.05E-05
SD	3.50E-05	3.53E-08	2.21E-04	3.62E-05	3.81E-09	2.34E-05	3.49E-05	2.92E-10	3.42E-06
MSE	1.23E-09	1.26E-15		1.31E-09	3.15E-17		1.22E-09	1.86E-17	
AVAR	1.08E-09	1.08E-15		1.08E-09	1.08E-17		1.08E-09	1.08E-21	
n=20000	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol
True	8.33E-05	7.00E-07		8.33E-05	7.00E-08		8.33E-05	7.00E-10	
Mean	8.46E-05	7.01E-07	2.81E-02	8.36E-05	7.10E-08	2.88E-03	8.25E-05	1.76E-09	1.11E-04
SD	2.81E-05	1.99E-08	4.28E-04	2.69E-05	2.08E-09	4.64E-05	2.68E-05	5.18E-11	1.81E-06
MSE	7.93E-10	3.96E-16		7.25E-10	5.23E-18		7.20E-10	1.12E-18	
AVAR	7.12E-10	3.55E-16		7.12E-10	3.55E-18		7.12E-10	3.55E-22	

Data generating process:

$$y_t = x_t + \sqrt{\sigma_v^2} v_t$$
$$x_t = x_{t-1} + \sqrt{\sigma_x^2/n} u_t$$
$$u_t \sim i.i.d.T(df_x), v_t \sim i.i.d.T(df_v)$$

Table A-5 : Estimation of Realized Volatility (U-shaped volatility, $\alpha = 0.3$)

n=300	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol
True	1.67E-04	2.00E-06		1.67E-04	2.00E-07		1.67E-04	2.00E-09	
Mean	1.71E-04	2.16E-06	1.37E-03	1.68E-04	3.51E-07	2.88E-04	1.65E-04	1.55E-07	1.68E-04
SD	1.08E-04	3.12E-07	1.35E-04	1.09E-04	5.10E-08	2.43E-05	1.07E-04	2.34E-08	1.41E-05
MSE	1.16E-08	1.22E-13		1.19E-08	2.56E-14		1.14E-08	2.39E-14	
AVAR	1.00E-08	8.34E-14		1.00E-08	8.34E-16		1.00E-08	8.34E-20	
n=5000	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol
True	1.67E-04	2.00E-06		1.67E-04	2.00E-07		1.67E-04	2.00E-09	
Mean	1.68E-04	2.01E-06	2.02E-02	1.68E-04	2.09E-07	2.17E-03	1.65E-04	1.06E-08	1.87E-04
SD	6.95E-05	9.15E-08	4.79E-04	7.04E-05	1.02E-08	5.20E-05	6.73E-05	5.13E-10	3.87E-06
MSE	4.83E-09	8.48E-15		4.95E-09	1.80E-16		4.53E-09	7.43E-17	
AVAR	4.32E-09	8.79E-15		4.32E-09	8.79E-17		4.32E-09	8.79E-21	
n=20000	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol
True	1.67E-04	2.00E-06		1.67E-04	2.00E-07		1.67E-04	2.00E-09	
Mean	1.70E-04	2.00E-06	8.01E-02	1.65E-04	2.02E-07	8.17E-03	1.64E-04	4.12E-09	2.47E-04
SD	5.61E-05	5.38E-08	9.72E-04	5.24E-05	5.39E-09	1.00E-04	5.19E-05	1.16E-10	2.50E-06
MSE	3.16E-09	2.89E-15		2.75E-09	3.34E-17		2.70E-09	4.49E-18	
AVAR	2.85E-09	2.90E-15		2.85E-09	2.90E-17		2.85E-09	2.90E-21	

Data generating process:

$$y_t = x_t + v_t,\ x_t = x_{t-1} + u_t$$
$$u_t \sim i.i.d.N(0, (1 - s + s^2)\sigma_x^2/n), v_t \sim i.i.d.N(0, \sigma_v^2), s = t/n$$

Table A-6 : Correlation ($\alpha = 0.45$,$corv = 0$)

n=300	Σ_x	Σ_v	corx	Σ_x	Σ_v	corx	Σ_x	Σ_v	corx
True	5.00E-05	5.00E-07	9.00E-01	5.00E-05	5.00E-08	9.00E-01	5.00E-05	5.00E-10	9.00E-01
	corx	corv	H-vol	corx	corv	H-vol	corx	corv	H-vol
Mean	8.56E-01	7.42E-02	1.27E-01	8.88E-01	4.30E-01	5.62E-01	8.92E-01	8.90E-01	8.94E-01
SD	8.61E-02	9.81E-02	6.39E-02	6.27E-02	8.57E-02	4.43E-02	6.25E-02	2.17E-02	1.18E-02
MSE	9.36E-03			4.06E-03			3.97E-03		
AVAR	1.46E-02			1.46E-02			1.46E-02		
n=5000	Σ_x	Σ_v	corx	Σ_x	Σ_v	corx	Σ_x	Σ_v	corx
True	5.00E-05	5.00E-07	9.00E-01	5.00E-05	5.00E-08	9.00E-01	5.00E-05	5.00E-10	9.00E-01
	corx	corv	H-vol	corx	corv	H-vol	corx	corv	H-vol
Mean	8.68E-01	4.05E-03	9.52E-03	8.96E-01	4.52E-02	8.24E-02	8.99E-01	7.53E-01	8.18E-01
SD	4.62E-02	3.21E-02	1.72E-02	2.99E-02	3.29E-02	1.64E-02	2.93E-02	1.44E-02	4.93E-03
MSE	3.19E-03			9.13E-04			8.59E-04		
AVAR	4.11E-03			4.11E-03			4.11E-03		
n=300	Σ_x	Σ_v	corx	Σ_x	Σ_v	corx	Σ_x	Σ_v	corx
True	5.00E-05	5.00E-07	-5.00E-01	5.00E-05	5.00E-08	-5.00E-01	5.00E-05	5.00E-10	-5.00E-01
	corx	corv	H-vol	corx	corv	H-vol	corx	corv	H-vol
Mean	-4.60E-01	-4.01E-02	-6.89E-02	-4.73E-01	-2.36E-01	-3.12E-01	-4.78E-01	-4.97E-01	-4.96E-01
SD	2.16E-01	1.03E-01	6.74E-02	2.25E-01	9.79E-02	5.67E-02	2.21E-01	7.77E-02	4.19E-02
MSE	4.83E-02			5.12E-02			4.93E-02		
AVAR	5.76E-02			5.76E-02			5.76E-02		
n=5000	Σ_x	Σ_v	corx	Σ_x	Σ_v	corx	Σ_x	Σ_v	corx
True	5.00E-05	5.00E-07	-5.00E-01	5.00E-05	5.00E-08	-5.00E-01	5.00E-05	5.00E-10	-5.00E-01
	corx	corv	H-vol	corx	corv	H-vol	corx	corv	H-vol
Mean	-4.71E-01	-4.15E-03	-5.51E-03	-5.01E-01	-2.31E-02	-4.47E-02	-4.98E-01	-4.19E-01	-4.55E-01
SD	1.23E-01	3.37E-02	1.73E-02	1.10E-01	3.30E-02	1.67E-02	1.13E-01	2.75E-02	1.11E-02
MSE	1.59E-02			1.20E-02			1.28E-02		
AVAR	1.62E-02			1.62E-02			1.62E-02		

Data generating process:

$$y_{i,t} = x_{i,t} + v_{i,t}, i = 1, 2$$

$$x_{i,t} = x_{i,t-1} + u_{i,t}$$

$$u_{i,t} \sim i.i.d.N(0, \sigma_x^2/n), v_{i,t} \sim i.i.d.N(0, \sigma_v^2)$$

$$corr(u_{1,t}, u_{2,t}) = corx$$

$$corr(v_{1,t}, v_{2,t}) = corv$$

Table A-7 : Correlation ($\alpha = 0.45$,$corv = 0.5$)

n=300	Σ_x	Σ_v	corx	Σ_x	Σ_v	corx	Σ_x	Σ_v	corx
True	5.00E-05	5.00E-07	9.00E-01	5.00E-05	5.00E-08	9.00E-01	5.00E-05	5.00E-10	9.00E-01
	corx	corv	H-vol	corx	corv	H-vol	corx	corv	H-vol
Mean	8.75E-01	5.33E-01	5.56E-01	8.88E-01	6.88E-01	7.49E-01	8.93E-01	8.95E-01	8.98E-01
SD	7.17E-02	7.38E-02	4.81E-02	6.24E-02	5.30E-02	2.60E-02	6.22E-02	2.16E-02	1.19E-02
MSE	5.77E-03			4.05E-03			3.92E-03		
AVAR	1.46E-02			1.46E-02			1.46E-02		
n=5000	Σ_x	Σ_v	corx	Σ_x	Σ_v	corx	Σ_x	Σ_v	corx
True	5.00E-05	5.00E-07	9.00E-01	5.00E-05	5.00E-08	9.00E-01	5.00E-05	5.00E-10	9.00E-01
	corx	corv	H-vol	corx	corv	H-vol	corx	corv	H-vol
Mean	8.88E-01	5.02E-01	5.04E-01	8.97E-01	5.20E-01	5.36E-01	8.97E-01	8.35E-01	8.64E-01
SD	3.36E-02	2.42E-02	1.28E-02	2.95E-02	2.47E-02	1.25E-02	2.99E-02	1.01E-02	3.64E-03
MSE	1.28E-03			8.76E-04			9.01E-04		
AVAR	4.11E-03			4.11E-03			4.11E-03		

Data generating process: same as Table A-6.

Table A-8 : Hedge Ratio ($\alpha = 0.45$, $corv = 0$)

n=300	Σ_x	Σ_v	corx	Σ_x	Σ_v	corx	Σ_x	Σ_v	corx
True	5.00E-05	5.00E-07	9.00E-01	5.00E-05	5.00E-08	9.00E-01	5.00E-05	5.00E-10	9.00E-01
	Hx		Hh	Hx		Hh	Hx		Hh
Mean	8.67E-01		1.28E-01	8.94E-01		5.60E-01	8.97E-01		8.93E-01
SD	1.48E-01		6.84E-02	1.26E-01		5.17E-02	1.33E-01		2.59E-02
MSE	2.30E-02			1.59E-02			1.76E-02		
AVAR	1.46E-02			1.46E-02			1.46E-02		
n=5000	Σ_x	Σ_v	corx	Σ_x	Σ_v	corx	Σ_x	Σ_v	corx
True	5.00E-05	5.00E-07	9.00E-01	5.00E-05	5.00E-08	9.00E-01	5.00E-05	5.00E-10	9.00E-01
	Hx		Hh	Hx		Hh	Hx		Hh
Mean	8.74E-01		9.81E-03	8.99E-01		8.18E-02	9.01E-01		8.19E-01
SD	7.73E-02		1.66E-02	6.80E-02		1.56E-02	6.75E-02		8.29E-03
MSE	6.66E-03			4.62E-03			4.55E-03		
AVAR	4.11E-03			4.11E-03			4.11E-03		

Data generating process: same as Table A-6.

* Hx and Hh mean the estimated hedge ratios based on SIML and historical estimator, respectively.

Note : In tables, Mean and SD are the sample mean and the standard deviation of the SIML estimator and the historical estimator(H-vol) in the simulation. AVAR corresponds to the asymptotic variance in Proposition 1.