CIRJE-F-607

Self-organizing Marketplaces

Takatoshi Tabuchi
University of Tokyo

January 2009

Self-organizing Marketplaces*

Takatoshi Tabuchi†

21 January 2009

Abstract

Dynamics of retail firms in marketplaces is analyzed, assuming that firms compete within a marketplace as well as between marketplaces under monopolistically competition. The number, size, and location of marketplaces or edge cities are analytically obtained, which is hardly done in the previous literature. Furthermore, extending the model to a two-dimensional space, Christaller-Lösch system of hexagonal market areas is analytically derived.

Keywords: edge cities, agglomeration, Christaller-Lösch system, spatial competition, monopolistic competition.

J.E.L. Classification: L13, R12, R39.

---

*I wish to thank Rainald Borck and Matthias Wrede for helpful comments and discussions. This research was supported by the Japanese Ministry of Eduction and Science (Grant-in-Aid for Science Research 18203012).

†Faculty of Economics, University of Tokyo

1 Introduction

Ever since the seminal work of Hotelling (1929), spatial competition has been extended in a number of ways within the framework of oligopoly. When firms compete in location and price of a homogeneous good, Hotelling (1929) conjectured that they agglomerate at the market center in order to obtain a larger market area. However, this is proved to be false by d'Aspremont, Gabszewicz, and Thisse (1979), who showed that firms always locate apart in order to relax price competition. Otherwise, they are involved in Bertrand price competition, which pushes their profits down to zero.[1] In brief, the main message in spatial competition is that keen competition always leads to dispersion of firms over space.

It is true that some retail firms such as gas stations and convenience stores tend to locate apart, but it is also true that they often form clusters. Casual empiricism suggests that shopping centers and malls are prevalent and have been increasing in size and number everywhere in recent years. One could think of Broadway or Champs d'Elysees, where hundreds of shops and restaurants provide a wide array of differentiated goods and services.

Such a stylized fact of agglomeration of retail firms is in sharp contrast to the results obtained in the above literature on spatial competition. The crucial reason for the contrast is substitutability of goods. If goods are homogeneous, it is no doubt that firms avoid Bertrand price competition in spite of the attractiveness of the market center. However, if the goods sold by firms are heterogenous, such competition would be relaxed. It is possible that the repulsion due to the fierce price competition may be outweighed by the attractiveness of the center in the case of heterogeneous goods.

As a matter of fact, agglomeration of retail firms is shown to be a Nash equilibrium by

[1]The same can be said when firms compete in different strategies. For example, Peng and Tabuchi (2007) show that firms never locate back to back when they compete in location and variety.

introducing heterogeneity of goods in the literature. de Palma, Ginsburgh, Papageorgiou, and Thisse (1985) first showed that agglomerated configuration at the market center is a Nash equilibrium (which is the so-called the principle of minimum differentiation in the theory of product differentiation) when goods are sufficiently differentiated and/or the transport costs are sufficiently low. De Fraja and Norman (1993) showed the same result in the case of duopoly with the linear demand under several pricing schemes. Henkel, Stahl, and Walz (2000) and Ago (2008) also showed the same result in the case of monopolistic competition.

Although firms choose to agglomerate at the market center under sufficient differentiation and/or low transport costs, this is not the only possible equilibrium. In reality, firms often locate in the suburbs of large cities due to easy access buy cars for the benefit of consumers as well as low land rent for firms. It is therefore more appropriate to consider the case that in large cities there are multiple marketplaces, where many firms enter freely and sell differentiated goods and services under a monopolistically competitive market.

Table 1 describes the declining shares of retail employment in the central district, along with the increasing share in the suburbs during the postwar Tokyo Metropolitan Area (TMA). Such a tendency is also similarly found in the value of retail sales during the study period. Two definitions of the center are taken into account: the smaller center is CBD1 consisting of four wards located around Tokyo Station, and the larger one is CBD2 comprising twenty-four wards that include CBD1. TMA is composed of the center and suburbs, which encompasses four prefectures with a population greater than 17 million in 1960 and 34 million in 2007. Table 1 evidently shows how retail employment is growing much faster in the suburbs than in the center of the TMA during the postwar period.

Based on the foregoing observations, I build on Henkel et al. (2000). Self-organizing marketplaces across space, firms compete in price and location in order to attract consumers under a monopolistically competitive market, whey each firm has a negligible impact on other firms in terms of their price and location strategies in the markets. They compete not only within the marketplace in which they locate but also between marketplaces. The competition within a marketplace is keener as the number of firms at the same marketplace increases. However, such an agglomeration is not necessarily undesirable for firms because it can attract more consumers relative to other marketplaces.

This paper differs from Henkel et al. (2000) in three respects. First, stability of equilibrium is defined by dynamics with a mass of firms rather than by a strong Nash equilibrium with a discrete number of firms. This is because each firm has no strategic impact on others under monopolistic competition, so that the dynamics with a mass of firms should be more appropriate than a strong Nash equilibrium in the case of monopolistically competitive markets.

Second, unlike Henkel et al. (2000), some consumers may not be served consumers when transport costs are high. This is because the income net of transport costs of consumers located at a distance from the marketplace may become negative as the geographical space gets sufficiently large. In this case, new marketplaces, which are often called *edge cities*, would emerge in peripheral areas when cities grow sufficiently large in size. It should be noted that in spite of the fact that edge cities are shown to be prevalent in the real world (Garreau, 1991; McMillen and Smith, 2003), few analytical models of edge cities and subcenters have been developed in the literatures to the best of my knowledge. Exceptions are Fujita and Ogawa (1982), Henderson and Mitra (1996); Fujita, Thisse, and Zenou (1997); and Cavailhes, Gaigne, Tabuchi, and Thisse (2007). However, they consider two edge cities at the most, where firms produce rather than sell goods.

Third, the space is extended from one-dimensional to two-dimensional in order to meet a more realistic urban structure. Interestingly it is shown that the two-dimensional extension yields Christaller's (1933) and Lösch's (1940) hexagonal systems of marketplaces as a market outcome. Note that Löschian polygonal systems are investigated by Eaton and Lipsey (1976). However, firms produce rather than sell goods in their model.

The rest of the paper is organized as follows. Section 2 sketches the model by Henkel et al. (2000) briefly. Section 3 characterizes the agglomerated equilibrium and its stability, and section 4 analyzes the symmetric equilibrium and its stability. Section 5 then studies an evolutionary process of urban structures and show how edge cities emerge successively. Section 6 extends it to the two-dimensional space and obtains the hexagonal configuration of marketplaces. Section 7 concludes.

2 The model

Consumers are uniformly distributed over space with the density normalized to 1. For a moment, they are assumed to be distributed on a line segment $x \in [-L/2, L/2]$, where L is the mass of consumers. They have the same CES utility with respect to a continuum n of varieties of a horizontally differentiated good:

$$U = \left[\int_0^n q(v,x)^{\frac{\sigma-1}{\sigma}} \mathrm{d}v \right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where $q(v,x)$ is the consumption of variety v at location x and $\sigma > 1$ is the elasticity of substitution between any two varieties. The income constraint is given by

$$y = \int_0^n p(v)q(v,x)\mathrm{d}v + tx, \tag{2}$$

where t is the unit transport cost for visiting a marketplace and x is the distance to a marketplace. Assume for a moment that

$$y - tL > 0 \tag{3}$$

so that visiting any marketplace is possible throughout the line segment.

Each consumer maximizes (1) with respect to $q(v,x)$ subject to the income constraint (2). The consumer demand for variety $v \in [0,n]$ is given by

$$q(v) = \frac{p(v)^{-\sigma}}{\int_0^n p(v')^{1-\sigma}\mathrm{d}v'} \left(y - tx\right). \tag{4}$$

The profit of firm v is:

$$\pi(v) = \int_{x \in X} (p(v) - c)\, q(v, x) \mathrm{d}x - f. \tag{5}$$

where X is the set of consumers who purchase a variety from firm v at a marketplace. Firm v maximizes its profit (5) with respect to its mill price given demand (4). The equilibrium mill price is given by

$$p^*(v) = \frac{c\sigma}{\sigma - 1},$$

which turns out to be constant for any variety v and for any location. Because each firm can be treated symmetrically, we drop v hereafter.

Under free entry and exit of firms, the equilibrium profit should be equal to zero:

$$\pi = \frac{1}{n\sigma} \int_{x \in X} (y - tx)\, \mathrm{d}x - f = 0 \tag{6}$$

so that the number of firms entering a marketplace is given by

$$n^* = \frac{1}{f\sigma} \int_{x \in X} (y - tx)\, \mathrm{d}x. \tag{7}$$

Then, the indirect utility of a consumer located at x from a marketplace is

$$V = \frac{\sigma - 1}{c\sigma} (n^*)^{\frac{1}{\sigma - 1}} (y - tx)\,.$$

When there are multiple marketplaces, consumers are assumed to visit only one that yields the highest utility and consume all varieties available at the marketplace.

3 Agglomerated equilibrium

In order to examine stability of equilibrium, one has to define dynamics of firm behavior. When there are m marketplaces at locations x_1, x_2,..., and x_m, I assume the following dynamics

$$\dot{n}_i = \pi_i \left(n_1, n_2, ..., n_m\right), \tag{8}$$

where the dot denotes the time derivative and the subscript i denotes the marketplace number. Dynamics (8) implies that firms are more attracted to marketplaces having higher profits and they do not enter a marketplace if the anticipating profit is negative.

Given the dynamics, the *spatial equilibrium* is such that

$$\pi_i \leq 0 \qquad \text{and} \qquad \pi_i n_i = 0 \qquad \forall x_i \in [-L/2, L/2].$$

The dynamics (8) is *stable* if any infinitesimal perturbations in the distribution of firms result in a movement back toward the equilibrium. This can be checked by computing the eigenvalues of Jacobian of the RHS in (8).

Suppose there are two marketplaces at x_1 and x_2 with $-L/2 \leq x_1 < x_2 \leq L/2$. Let $\widehat{x}$ be the market boundary, i.e., the location of the marginal consumer, who is indifferent toward visiting either of them. Equating the indirect utilities of visiting both marketplaces yields the market boundary:

$$\widehat{x} = \begin{cases} -L/2 & \text{if } x_{\text{int}} \leq x_1 \\ x_{\text{int}} & \text{if } x_1 < x_{\text{int}} < x_2 \\ L/2 & \text{if } x_{\text{int}} \geq x_2, \end{cases}$$

where

$$x_{\text{int}} \equiv \frac{y\,(r-1) + t\,(x_1 r + x_2)}{t\,(r+1)}$$

is the interior market boundary and $r \equiv (n_1/n_2)^{\frac{1}{\sigma-1}}$.

I show that agglomerated configuration is a stable equilibrium as follows. If an infinitesimal mass of firms is located at $x_2 \in]x_1, L/2]$ while a all remaining mass of firms is located at x_1, then r goes to infinity. From (3), one gets

$$x_{\text{int}} = \frac{y + t x_1}{t} > L + x_1 > x_2.$$

Hence, $\widehat{x} = L/2$, implying that no consumers visit marketplace 2, and that π_2 is necessarily equal to 0 for any small increase in n_2. In other words, the agglomerated configuration is always a stable equilibrium. This suggests the lock-in effect in the location of the marketplace.

In order to upset the agglomerated equilibrium, the nonnegligible number of firms should simultaneously move from x_1 to x_2. This is possible if a coalition is formed among the firms, which is however not allowed in the above dynamics with a mass of firms under the monopolistically competitive market.[2]

From (7), the number of firms in the agglomerated equilibrium is computed as

$$n_1^* = \frac{1}{f\sigma}\left[yL - t\left(x_1^2 + \frac{L^2}{4}\right)\right].$$

This is maximized when the marketplace is located at the center of the line segment $x_1 = 0$. This is due to the elastic demand for the differentiated goods.

Stability of the agglomerated equilibrium is guaranteed because $\partial\pi_1/\partial n_1 < 0$ and $\pi_2 < 0$ hold when they are evaluated at $(n_1, n_2) = (n_1^*, 0)$. The foregoing argument may then be summarized as follows:

Proposition 1 *The agglomerated configuration is always a stable equilibrium irrespective of its location.*

De Fraja and Norman (1993) and Ago (2008) show that the equilibrium location of the agglomerated marketplace is at the center under sufficiently low transport costs. However, it is not necessarily at the center in the case of nonnegligible transport costs.

4 Symmetric equilibrium

Next, consider the case of two marketplaces symmetrically located about the center of the line segment such that $x_1 + x_2 = 0$. Making use of symmetry, the equilibrium number

[2] Note that this is possible under the oligopolistic market with a finite number of firms, which can build coalition as shown in Henkel et al. (2000).

of firms, (7), in each marketplace is readily computed as

$$n_1^* = n_2^* = \frac{1}{2f\sigma}\left[yL - t\left(\left(\frac{L}{2} - x_2\right)^2 + x_2^2\right)\right].$$

This is maximized when $x_2 = L/4$, where the sum of the consumer demand is the largest. Again, this is attributed to the elastic demand for differentiated goods.

Checking the signs of Jacobian of the RHS of (8), one obtains the stability condition of the symmetric equilibrium as follows:

$$y < \frac{t}{4}\left[(\sigma - 1)L + 4x_2 + \sqrt{(\sigma - 1)\left[(\sigma + 1)L^2 - 16(L/2 - x_2)^2\right]}\right]. \qquad (9)$$

Examining (9), one can say that the symmetric equilibrium is stable when goods are close substitutes (σ large), the transport costs are high (t large), the consumer demand is large (L large), and the marketplaces are located far apart (x_2 large).

When goods are close substitutes, consumers do not care for product variety, and hence, the agglomeration force is weak. This is in agreement with the result in new economic geography (Krugman, 1991) as well as that in spatial competition under product heterogeneity (de Palma et al., 1985).

When the transport costs are high, competition between the marketplaces is softened because the market boundary is not sensitive to changes in the size of marketplaces (the integral part in (6)). However, competition within a marketplace does matter (n in the denominator in (6)). Because the latter effect dominates the former, the symmetric equilibrium turns out to be stable. As before, this agrees with the result in new economic geography as well as that in spatial competition.

Finally, when the marketplaces are located at a distance, the demand at the market boundary is small due to elastic demand with respect to distance to be covered for shopping. Because competition between the marketplaces is localized only at the market boundary in this model, small demand at the market boundary implies weak competition, which ensures stability of the symmetric equilibrium.[3]

[3] Arakawa (2006) shows that this is not necessarily true if consumers can visit both marketplaces.

5 Evolution of spatial structure

Thus far, the consumer demand has been spatially fixed. However, it is of interest to consider endogenous locations of consumers together with those of marketplaces in a growing city in the following way. Each consumer resides on a plot of land, the length of which is normalized to 1.[4] In order to receive the fixed income y, each consumer has to commute to the central business district, which is located at $x = 0$ and is assumed to be spaceless. Because commuting involves costs, consumers eventually locate on the interval $[-L/2, L/2]$ in equilibrium, where L is the population size as given by the length of the line segment.

The population is initially small and is steadily growing exogenously. It is inferred from (9) that configurations with multiple marketplaces are unlikely to be a stable equilibrium for sufficiently small L. This would suggest that the initial equilibrium with a sufficiently small city is the agglomerated configuration. Once the agglomeration is formed, it is necessarily stable from Proposition 1. Although the location of the marketplace can be anywhere in the city, it is natural to assume that it coincides with the location of the central business district $x = 0$, to which all consumers commute.

Such an agglomerated equilibrium continues insofar as condition (3) is met. More precisely, because the maximum distance between the marketplace and consumer locations is now $L/2$, the condition is replaced with

$$y - tL/2 > 0. \tag{10}$$

Stated differently, if the population size L grows and exceeds $2y/t$, then condition (10) is violated. In this case, since some consumers are unable to visit a marketplace with distance greater than y/t, firms in this single marketplace no longer can serve all consumers in the linear city. Consequently, the agglomerated configuration is no more an equilibrium, and hence, new marketplaces would emerge at both edges of the city, $x = \pm L/2$. They may be

[4] We assume away the land rent for analytical simplicity.

called the subcenters or edge cities, whereas the initial marketplace is called the center. Note that the unit transport cost t consists of the unit shopping trip cost t_s and the unit commuting cost t_c, i.e., $t = t_s + t_c$ unlike the preceding sections.

Consumers living close to the marketplace in the city center would visit it for shopping as well as working on the way back from commuting. Consumers living in the suburbs may also do the same at the city center. However, they are more likely to go to a marketplace at the subcenter near their residence because shopping is often done on weekends and shopping trips are more elastic than commuting trips with respect to distance costs.

In the sequel, I consider several stages according to the change in the number of subcenters. For this purpose, define the thresholds of city size as

$$L_i \equiv \frac{2y}{t_s} \sum_{j=1}^{i} \left(\frac{t_s}{t_s + t_c} \right)^j, \qquad i \geq 1. \tag{11}$$

(i) The first stage with $L \in \left]0, L_1\right]$

The spatial structure of the right part of the city is illustrated in Figure 1 (the left part is its mirror image). The city develops from the center $x = 0$ to the right (and left) gradually as population increases. As the city increases in size, consumers at the city edges entail more costs of shopping trip and commuting and their net income $y - (t_s + t_c) L/2$ reduces and finally equals zero when $L = L_1$.

Due to the existence of the commuting cost to the center, agglomeration at the center should be a unique equilibrium configuration in the first stage as mentioned above. The equilibrium number of firms is computed as

$$n_1^* = \frac{yL (L_1 - L/2)}{f \sigma L_1}.$$

The indirect utility of a consumer living at location x is given by

$$V = \frac{\sigma - 1}{c\sigma} (n^*)^{\frac{1}{\sigma-1}} \left[y - (t_s + t_c) x \right]. \tag{12}$$

The number n^* of varieties in (12) is increasing in $L \in \left]0, L_1\right]$, but the average distance of x is decreasing in L. It can be readily shown that as the city develops, the welfare of all consumers in the city increases for a small L. However, as the city grows further, their welfare near the center $x \approx 0$ still increases, but may decrease near the city edges $x \approx \pm L/2$ due to high commuting costs. Therefore, the welfare on average initially increases, but may or may not decrease when the city increases in size. This is because the benefit from the product variety may or may not be dominated by the commuting costs. Because the welfare is not transferable across consumers, I do not go into the welfare analysis any more.

(ii) The second stage with $L \in\,]L_1, L_2]$

When L exceeds L_1, consumers living in the interval of $]L_1, L]$ are unable to go to the city center for shopping because their net income is negative. However, they can instead visit one of the two new subcenters that emerge at $x = \pm L_1/2$. Since they never visit the center, firms located at the center cannot take over the whole demand. Put differently, firms locating at the subcenters always have a positive demand by consumers at $]L_1, L]$. This implies that *even if the marketplaces at the subcenters are very small, they are always protected from the large marketplace at the center.*

When L is not much larger than L_1, all consumers residing in $]-L_1/2, L_1/2\,[$ go to the center for shopping and the rest of consumers residing in $[-L/2, -L_1/2]$ and $[L_1/2, L/2]$ visit the nearest subcenter. However, as L gets larger, the number of firms at the subcenters increases, and hence, the market boundaries between the center and subcenters would move inside the interval of $]-L_1/2, L_1/2\,[$. As a result, although the two subcenters become smaller than the center when L is close to L_1, they may become larger when L approaches L_2.

Finally, when L becomes equal to L_2, the net income of a consumer located at the city edges is equal to zero, which leads to the emergence of additional subcenters at $x = \pm L_2/2$.

(iii) The i-th stage with $L \in\,]L_{i-1}, L_i]$

The evolutionary process (ii) is repeated for each stage. That is, there are $2i - 1$ marketplaces at the center and subcenters in the i-th stage. The interval between subcenters $i-1$ and i is computed as

$$\frac{2y}{t_s}\left(\frac{t_s}{t_s+t_c}\right)^i. \tag{13}$$

Because the location of each subcenter is defined by the sum of the above intervals, it is given by (11). Thus, we have obtained the following.

Proposition 2 *As the subcenters are farther away from the city center, their intervals get narrower.*

This proposition suggests that the marketplaces are smaller in size depending on the distance from the center because their hinterlands get smaller. This is consistent with casual observations that the sizes of marketplaces in commuter towns and exurbs are small in size as they are far away from the city center. Note however that subcenters may become larger than the center as demonstrated in the above second stage. This may correspond to the prosperous shopping malls in the suburbs versus the stagnant central cities often observed in Japan's small cities.

In order to gain further insight, I impose an assumption that the commuting cost t_c is sufficiently low but not zero hereafter. Setting $t_c = 0$ in (13), it can be easily verified that each interval between the neighboring marketplaces is equal. This implies that each marketplace would be of equal size except for the two edges. Such a difference is ascribed to the length of the hinterlands near the city edges. As we saw above, the two edge marketplaces can be larger or smaller in size than the others. In sum, we have the following.

Proposition 3 *When the commuting cost is sufficiently small, all the marketplaces are of the same size except for the two subcenters near the edges. The two edge marketplaces are smaller (resp. larger) if* $L \in \,]L_{i-1}, (L_{i-1}+L_i)/2\,[$ *(resp.* $L \in \,](L_{i-1}+L_i)/2, L_i]$*).*

Because the intervals of marketplaces are identical, there is no locational difference between the marketplaces but for the two edge marketplaces. This is the first statement of the proposition. The second statement implies locational disadvantage and advantage of the edge marketplaces depending upon the size of the hinterlands. When $L \in \,]L_{i-1}, (L_{i-1} + L_i)/2\,[$, consumers outside the edge locations $x = \pm L_{i-1}/2$ are relatively few. Because of the locational disadvantage, the size of the edge marketplaces should be smaller than that of the others. This corresponds to the early development stages of edge cities. On the other hand, when $L \in \,](L_{i-1} + L_i)/2, L_i]$, consumers outside the edge locations are relatively many, and therefore the edges have better access, i.e., locational advantage. This may correspond to large shopping centers and malls, which are often observed in the suburbs of large cities. Thus, the expansion of the hinterlands changes locational disadvantage to locational advantage according to development stages.

6 Two-dimensional extension

Thus far, the analysis has been confined to the one-dimensional space. This is extended to the two-dimension in this section because the geographical space in the real world is better approximated by a two-dimensional space, while keeping the assumption that the commuting cost t_c is sufficiently low but not zero. Rather than involving complicated and detailed analyses with a two-dimensional space, this section is more or less intuitively described.

Consider a featureless plane with a city center at $(x, y) = (0, 0)$, where consumers are uniformly distributed around the city center over the two-dimensional disc. The first stage is not very different from the one-dimensional case. The equilibrium number of firms, n_1^*, is somewhat larger than before. Comparative statics and so on are qualitatively similar. The agglomerated configuration continues to be a stable equilibrium for all $L \in \,]0, L_1]$.

When the city size slightly exceeds L_1, there emerges a continuum of equilibrium location candidates for edge cities unlike the case of the one-dimensional space. That is, the location candidates are any locations on the circumference of a circle with radius L_1 in the two-dimensional space, whereas they are confined to the two locations $x = \pm L/2$ in the one-dimensional space. Suppose many marketplaces emerge simultaneously on the circumference of a circle when $L = L_1$. Then, distances between many marketplaces would be short, which would destroy the configuration. This is inferred from the stability condition of the symmetric equilibrium that when marketplaces are located close (x_2 large in (9)), the configuration is unstable. Hence, it is more likely that only a few edge marketplaces are viable simultaneously when $L = L_1$. Because the smaller number of marketplaces implies a more stable equilibrium configuration, which is likely to survive in the long-term evolutionary process, we assume the most stable configuration when L reaches L_1.We therefore assume that *the number of edge marketplaces is three when* $L = L_1$ since the minimum number of edge marketplaces that can serve the entire consumers is readily shown to be three. The edge marketplaces are drawn as three **I** located symmetrically around the center in Figure 2.

When L exceeds L_1 further, but still slightly, consumers located near the three **I'** are unserved. Therefore, three more edge marketplaces are to be immediately established. In sum, when L slightly exceeds L_1, there are one center at $(x, y) = (0, 0)$ and six subcenters **I+I'** equidistantly located around the circumference of a circle with radius L_1, thus constituting a regular hexagon. Since distances between any pair of the seven marketplaces are no less than L_1, each marketplace is always protected, i.e., the hexagonal configuration is stable due to the similar logic as (ii) in section 5.

As the population of the city keeps growing, consumers are continuously spreading around the city center with one central marketplace and six edge marketplaces **I+I'** while $L \in]L_1, \sqrt{3}L_1]$. When L slightly exceeds$\sqrt{3}L_1$, exactly six location points, as marked by **II** in Figure 2, are not served. Their locations are on the circumference of a circle with radius $\sqrt{3}L_1$. Thus, adding the first group of six marketplaces **I+I'** to the second group of six new marketplaces **II**, there are thirteen marketplaces for $L \in]\sqrt{3}L_1, 2L_1]$, and then there are nineteen marketplaces with an addition of the third group of six new marketplaces **III** as illustrated in Figure 2.

Continuing these processes of the emergence of new edge cities, the hexagonal configuration similar to Figure 6 in chapter B of part I of Christaller (1933) or Figure 24 in chapter 10 of Lösch (1940) can be depicted. Note that it differs from Christaller (1933) in that the hexagons are not nested because there is only one good here. It is expected that Christaller's hierarchical system of nested hexagons is self-organized by introducing multiple goods having different parameters.[5] For example, goods with high transport costs (t_s large) form marketplaces with short intervals, whereas those with low transport costs (t_s small) form marketplaces with long intervals. As a result, large marketplaces with long intervals would emerge, offering a wide array of goods and small marketplaces with short intervals offering few goods.

In sum, we arrive at the following conclusion.

Proposition 4 *Christaller-Lösch's hexagonal configuration is self-organized endogenously and stable.*

It should be noted that the configuration is self-organized without any presence of social planners. Finally, when the commuting cost t_c is not negligible, the equilibrium configuration is expected to be a two-dimensional version of Figure 1. Namely, it would consist of successive hexagons, but the sizes of hexagons would gradually shrink according to the distance from the center. This would be also true when the population density is decreasing in the distance from the center.

[5] To be more precise, multipurpose shopping is carried out by consumers in order to obtain the nested hexagonal configuration (Quinzii and Thisse, 1990).

7 Conclusion

I have extended the model of Henkel et al. (2000), where firms compete not only within a marketplace, but also between marketplaces in order to uncover the number, size, and locations of marketplaces that constitute central and edge cities. Empirical evidence in Japan's large cities shows that the retail share in the suburban areas has been rising as compared to that in the central areas in these years. The results in this paper agree with the evidence. Furthermore, this paper has shown that Christaller-Lösch's hexagonal configuration self-organizes endogenously in the monopolistically competitive retail market. Thus, it may be safely concluded that the model in this paper is able to describe the real world as well as Christaller-Lösch's ideal world.

In this paper, I have chosen to focus on population increase as an exogenous driving force of emerging subcenters. The population increase accompanies not only suburbanization of residential areas, but also job decentralization. The latter is getting common in recent U.S. metropolitan areas (Lee, 2007), but is not taken into account in this paper. The land rent is also beyond the scope of this paper although it is of importance in urban economic theory (Alonso, 1964). The next line of research to be addressed may be incorporating job decentralization and land rent, although such extensions will not be an easy task.

References

[1] Ago, T. (2008) Central agglomeration of monopolistically competitive firms, *Journal of Economic Geography* 8, 811–823.

[2] Alonso, W. (1964) *Location and Land Use*, Cambridge, Mass.: Harvard University Press.

[3] Arakawa, K. (2006) A model of shopping centers, *Journal of Regional Science* 46, 969-990.

[4] Cavailhes, J., C. Gaigne, T. Tabuchi, and J.-F. Thisse (2007) Trade and the structure of cities, *Journal of Urban Economics* 62, 383-404.

[5] Christaller, W. (1933) *Die Zentralen Orte in Süddeutschland*, Jena: Gustav Fischer Verlag. Japanese translation: *Christaller's Location and Development of Cities*, Tokyo: Taimeidō (1969).

[6] d'Aspremont, C., J. Gabszewicz and J.-F. Thisse (1979) On Hotelling's "Stability in competition", *Econometrica* 47, 1145-1150.

[7] De Fraja, G. and G. Norman (1993) Product differentiation, pricing policy and equilibrium, *Journal of Regional Science* 33, 343-363.

[8] de Palma, A., V. Ginsburgh, Y.Y. Papageorgiou, and J.-F. Thisse (1985) The principle of minimum differentiation holds under sufficient heterogeneity, *Econometrica* 53, 767-781.

[9] Eaton, B.C. and R.G. Lipsey (1976) The non-uniqueness of equilibrium in the Löschian Location Model B, *American Economic Review* 66, 77-93.

[10] Fujita, M. and H. Ogawa (1982) Multiple equilibria and structural transition of non-monocentric urban configurations, *Regional Science and Urban Economics* 12, 161-196.

[11] Fujita, M., J.-F. Thisse, and Y. Zenou (1997) On the endogenous formation of secondary employment centers in a city, *Journal of Urban Economics* 41, 337–357.

[12] Garreau, J. (1991) *Edge City: Life on the New Frontier*, New York: Doubleday.

[13] Henderson, V. and A. Mitra (1996) The new urban landscape: Developers and edge cities, *Regional Science and Urban Economics* 26, 613–643.

[14] Henkel, J., K. Stahl, and U. Walz (2000) Coalition building in a spatial economy, *Journal of Urban Economics,* 47, 136-163.

[15] Hotelling, H. (1929) Stability in competition, *Economic Journal* 39, 41-57.

[16] Krugman, P. (1991) Increasing returns and economic geography, *Journal of Political Economy* 99, 483-499.

[17] Lee, B. (2007) "Edge" or "edgeless" cities? Urban spatial structure in U.S. metropolitan areas, 1980 to 2000, *Journal of Regional Science* 47, 479–515.

[18] Lösch, A. (1940) *Die Räumliche Ordnung der Wirtschaft*, Jena: Gustav Fischer. Japanese translation: *Lösch's Keizai Ricchiron*, Tokyo: Taimeidō (1968).

[19] McMillen, D.P. and S.C. Smith (2003) The number of subcenters in large urban areas, *Journal of Urban Economics* 53, 321–338.

[20] Peng, S.-K. and T. Tabuchi (2007) Spatial competition in variety and number of stores, *Journal of Economics & Management Strategy* 16, 227-250.

[21] Quinzii, M. and J.-F. Thisse (1990) On the optimality of central places, *Econometrica* 58, 1101-1119.

Table 1: Employment in retail industry in Tokyo Metropolitan Area

year	CBD1	CBD2	TMA	share CBD1	share CBD2
1960	92577	402616	757557	12.2	53.1
1964	98045	426749	831309	11.8	51.3
1968	113123	478628	1018142	11.1	47.0
1972	113771	478604	1129108	10.1	42.4
1974	113051	490908	1181980	9.6	41.5
1976	114541	500262	1244476	9.2	40.2
1979	118666	518210	1343315	8.8	38.6
1982	125677	547023	1460039	8.6	37.5
1985	116509	517986	1465911	7.9	35.3
1988	119072	545337	1623835	7.3	33.6
1991	123102	548495	1681536	7.3	32.6
1994	123524	555005	1797830	6.9	30.9
1997	116887	538283	1791467	6.5	30.0
1999	122540	590935	2000639	6.1	29.5
2002	125462	588989	1998351	6.3	29.5
2004	125423	581870	1956295	6.4	29.7
2007	126444	562501	1935927	6.5	29.1

Notes:
CBD1 consists of four wards of Chiyoda, Chuo, Minato and Shinjuku.
CBD2 consists of twenty-three wards including CBD1.
TMA consists of four prefectures of Tokyo, Kanagawa, Chiba and Saitama.

Figure 1:　Spatial structure of a city

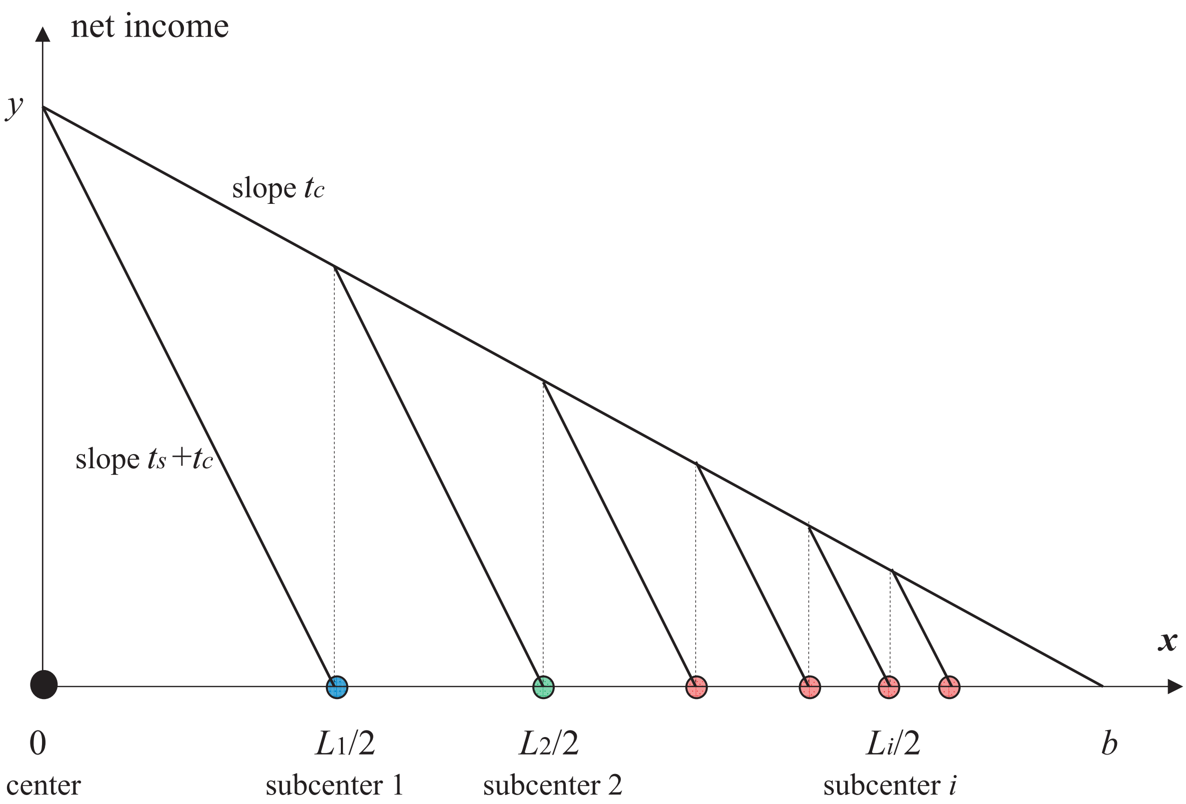

Figure 2:　Christaller's hexagonal configuration with 1 center, 6 subcenter I, 6 subcenter II, and 6 subcenter III