CIRJE-F-715

Voluntarily Separable Repeated Games with Social Norms

Takako Fujiwara-Greve
Keio University

Masahiro Okuno-Fujiwara
University of Tokyo

Nobue Suzuki
Komazawa University

February 2010

Voluntarily Separable Repeated Games with Social Norms*

Takako Fujiwara-Greve	Masahiro Okuno-Fujiwara	Nobue Suzuki
Keio University	University of Tokyo	Komazawa University
Dept. of Economics	Graduate School of Economics	Faculty of Economics

This version: February 4, 2010

Abstract: We extend the voluntarily separable repeated Prisoner's Dilemma (Fujiwara-Greve and Okuno-Fujiwara, 2009) to continuous actions. We show that there is a (constrained) efficient bimorphic equilibrium which is robust under evolutionary pressure. It consists of a cooperative strategy and a myopic defection strategy so that our model provides a foundation to incomplete information models as well.

Key words: social norm, evolution, voluntary separation, repeated game.
JLE classification: C 73.

*This work was supported by JSPS Grant-in-Aid for Scientific Research (C), No.21530171. The usual disclaimer applies.

1 Introduction

We consider a model of voluntarily separable repeated games (VSRG) in which two players play prisoner's dilemma-like game repeatedly over time. VSRG is endogenous in that one can unilaterally end the partnership and start another with a randomly-matched new partner. Society is assumed to be large and anonymous, and one will get to know nothing about new partner's past conducts. In such a case, if you blindly seek for cooperation with the partner, you must risk the possibility that she takes advantage of anomymity and tries to realize the short-run profit. On the other hand, you might be able to establish a trusting relationship with her and enjoy repeated cooperation over time. For repeated cooperation to be incentive compatible, however, there must be some mechanism to sanction those players who betray the trust, by deviating to obtain the short-run profit and immediately breaking away from the partnership.

In the past literature, there are three known such sanctioning mechanisms. Matching friction (unemployment) induces cooperation among selfish players, as is well known from Shapiro and Stiglitz (1984) and Okuno-Fujiwara (1987). Even when there is no matching friction, trust-builiding equilibrium provides a sanction (see, Datta, 1996, Kranton, 1996a, and Fujiwara-Greve and Okuno-Fujiwara, 2009). In this mechanism, players are assumed to be homogenous but, whenever new parnership is formed, they play non-cooperation with a lower payoff for some periods before starting repeated cooperation to enjoy a higher payoff. After coopeataion is established, one would not defect because doing so reduces her payoff by forcing her to go through non-cooperation periods again with a new partner. The third mechanism is given by incomplete information models (see, Ghosh and Ray, 1996, Kranton, 1996b, and Rob and Yang, 2006). In this mechanism there are different types of players, far-sighted and myopic, whose population ratio is exogenously given. In this case, far-sighted type would not defect because she wants to signal that she is not a myopic type in order to induce cooperation from fellow far-sighted type players.

In this paper, we propose yet another mechcanism. The game is of complete information and all players are homogenous, but two different strategies constitute an equilibrium, *i.e.,* a bimorphic equilibrium. One strategy (called *cooperative strategy*) is to always play cooperation and continue the partnership if and only if the partner also cooperates. The other strategy (called *myopic strategy*) is to always play defect and always terminate the partnership immediately.

Population ratio of two strategies is endogenously determined where two strategies' payoffs are balanced. Under a bimorphic equilibrium, deviation is sanctioned by the possibility of matching with the myopic strategy. That is, the strategic diversity is the source of sanction mechanism. In addition, our result of bimorphic equilibrium may be thought of as providing a basis for the incomplete information models; two different types of players emerge as an evolutionary outcome among homogenous players.

Our result also contributes to the analysis of social norms. A social norm is a standard of behavior that people in a society feel obliged to follow, because social (moral and psychological) pressure exists that one should play according to the norm. It is often argued that, if a standard of behavior is a Nash Equilibrium of a prevailing game, it is a social norm. For example, driving on the right side is a Nash Equilibrium of a coordination game where you choose on which side of the road to drive. However in this case, the fact that the driving on the right side is a standard of behavior only means that you are better off driving on the right, rather than left. Thus, the right hand driving may be thought of as a convention but we can hardly think of it as a norm, because there is no feeling of obligation.

For a behavioral standard to be a norm, not only it is a part of equilibrium behavior (hence viewed as pro-social) but also it is used despite the existence of a minority of people who choose an opposite behavior (which is viewed as anti-social). In this sense, not only following the social norm is a "must" for pro-social players, but also pro-social players should not be exploited by anti-social players. Our bimorphic equilibria have both of these properties.

This paper is organized as follows. Section 2 describes the voluntarily separable repeated game model, which is an extension of VSRPD model of Fujiwara-Greve and Okuno-Fujiwara (2009) (henceforth Greve-Okuno). In Section 3 we show the range of bimorphic equilibria and in Section 4 we consider evolution of social norms to select among bimorphic equilibria. Section 5 concludes the paper.

2 Model

2.1 VSRG Model

A *voluntarily separable repeated game* (VSRG) is defined as follows. There is a large society of homogeneous players, with measure 1, who play the following dynamic game over the discrete

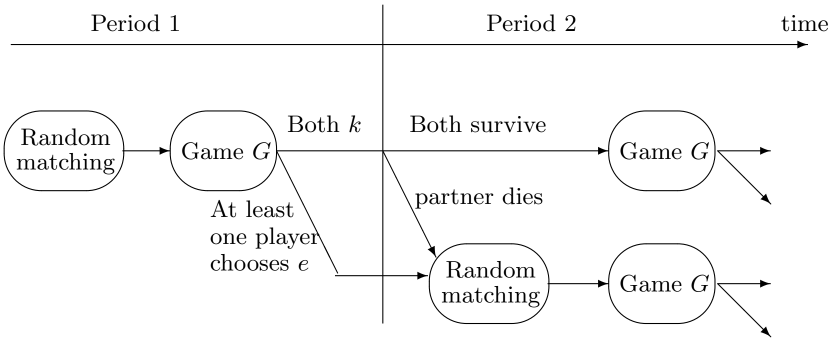

Figure 1: Outline of VSRG

horizon $1, 2, \ldots$.

At the beginning of a period, each player is either "matched" with another player or "unmatched". Unmatched players enter a random matching pool and find a new partner. There is no information flow across partnerships so that at the time of newly formed partnership, players do not know each other's past action histories. Matched players then play a symmetric two-person simultaneous-move game $G = (S, u)$ and observe each other's actions. After the action observation, each partner simultaneously decide whether to *keep* the partnership or *end* it. The partnership continues if and only if both partner choose to keep.

At the end of each period, each player may stochastically exit the dynamic game for an exogenous reason. We call this a "death" of a player. Let the probability of stochastic exit be $1 - \delta \in (0, 1)$. Once a player dies, a newborn player enters the population so that the population size is stationary over time. Those who lost partners for some reason (death or end-decision) become "unmatched" at the beginning of the next period. The outline of the dynamic game is depicted in Figure 1. The survival rate $\delta \in (0, 1)$ is the natural discount factor for each player's long-run payoffs.

The one-shot game G is specified as follows. The set of feasible actions for each player is $[0, \infty)$. An action $x \in [0, \infty)$ generates the joint utility level of $2x$ within the partnership. However the contribution comes with personal cost $c(x)$. Thus, if you choose x and your partner chooses y, you enjoy half of $2(x + y)$ minus the cost, so that the one-shot payoff is defined as

$$u(x, y) = x + y - c(x). \tag{1}$$

Assumption 1: $c(\cdot)$ is a strictly convex, C^1-function with the property: $c(0) = 0$, $\lim_{x\to\infty} c(x) =$

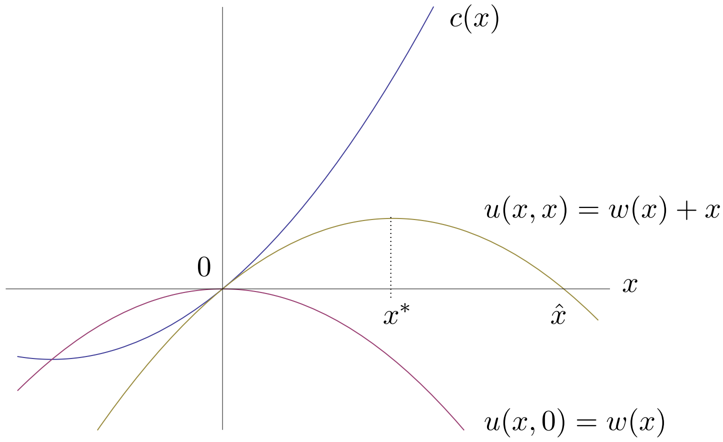

Figure 2: u, c and w functions.

∞, $c'(0) = 1$, $c'(x^*) = 2$ for some $x^* \in (0, \infty)$ and $\lim_{x\to\infty} c'(x) = \infty$.

For notational convenience, let us define $w(x) := x - c(x)$. Then $u(x, y) = w(x) + y$. From Assumption 1, we have the following observations regarding the functions w and u. (The proof is obvious and thus is omitted.)

Remark 1. *(1) $w(\cdot)$ is a strictly concave, C^1-function.*
(2) $w(0) = 0$, $w(x) \leqq 0$ for all $x \in [0, \infty)$, and $\lim_{x\to\infty} w(x) = -\infty$.
(3) $w'(0) = 0$, $w'(x^*) = -1$, and $\lim_{x\to\infty} w'(x) = -\infty$.
(4) $u(x, x) = w(x) + x$ is a strictly concave function.
(5) $u(0, 0) = 0$, $u(x, x)$ is maximized at (x^*, x^*), $\lim_{x\to\infty} u(x, x) = -\infty$, and there exists $\hat{x} > 0$ such that $u(\hat{x}, \hat{x}) = 0$.
(6) For any $y \in [0, \infty)$, $x = 0$ is a best response, i.e., $u(0, y) \geqq u(x, y)$ for any $x, y \in [0, \infty)$.
(7) $u(x, 0) = w(x) = x - c(x)$ and $u(0, y) = y$, for all $x, y \in [0, \infty)$.

The one-shot game is essentially a continuous action version of Prisoner's Dilemma. Although the symmetric payoff $u(x, x)$ is maximized at a positive mutual contribution (x^*, x^*), which gives a positive payoff $u(x^*, x^*)$ to both players, the unique Nash equilibrium of G is $(0, 0)$, which gives $u(0, 0) = 0$ to both players. Figure 2 shows how these functions look like.

In the VSRG model with no information flow, the well-known Tit-for-Tat and Trigger strategies do not induce a positive level of cooperation, because one can select $x = 0$ and run away immediately, without affecting one's future payoffs.

2.2 Strategies

Let $t = 1, 2, \ldots$ indicate the periods in a match, not the calendar time in the game. Under the no-information-flow assumption, we focus on match-independent strategies that only depend on t and the private history of actions in G within a match.[1] Let $H_t := [0, \infty)^{2(t-1)}$ be *the set of partnership histories at the beginning of* $t \geqq 2$ and let $H_1 := \{\emptyset\}$.

Definition: A *pure strategy* s of VSRG consists of $(x_t, z_t)_{t=1}^{\infty}$ where:
$x_t : H_t \to [0, \infty)$ specifies a contribution level in that period $x_t(h_t) \in [0, \infty)$ given the partnership history $h_t \in H_t$, and
$z_t : H_t \times [0, \infty)^2 \to \{k, e\}$ specifies whether to keep or end the partnership, depending on the partnership history $h_t \in H_t$ and the current period action profile.

The set of pure strategies of VSRG is denoted as $\boldsymbol{S}$ and the set of all strategy distributions in the population is denoted as $\mathcal{P}(\boldsymbol{S})$. We assume that each player uses a pure strategy, which is natural in an evolutionary game and simplifies the analysis.

We investigate the evolutionary stability of *stationary* strategy distributions in the matching pool. Although the strategy distribution in the matching pool may be different from the distribution in the entire society, if the former is stationary, the distribution of various states of matches is also stationary, thanks to the stationary death process. (For details see Greve-Okuno, 2009.)

2.3 Average Payoff

When a strategy $s \in \boldsymbol{S}$ is matched with another strategy $s' \in \boldsymbol{S}$, the *expected length* of the match is denoted as $L(s, s')$ and is computed as follows. Notice that even if s and s' intend to maintain the match, it will only continue with probability δ^2. Suppose that the planned length of the partnership of s and s' is $T(s, s')$ periods, if no death occurs. Then

$$L(s, s') := 1 + \delta^2 + \delta^4 + \cdots + \delta^{2\{T(s,s')-1\}} = \frac{1 - \delta^{2T(s,s')}}{1 - \delta^2}.$$

[1] The continuation decision is observable, but strategies cannot vary depending on combinations of $\{k, e\}$ since only (k, k) will lead to the future choice of actions.

The *expected total discounted value of the payoff stream of s within the match with* s' is denoted as $V(s, s')$. It is computed as

$$V(s, s') = \sum_{t=1}^{T(s,s')} \delta^{2(t-1)} u(x_t(h_t), x'_t(h_t)),$$

where $x_t(h_t)$ (resp. $x'_t(h_t)$) is the action specified by s (resp. s') as the partnership history h_t is generated according to (s, s').

Let *supp*(p) be the (countable) support of a strategy distribution p. Then the average payoff of $s \in \boldsymbol{S}$ under a stationary distribution p in the matching pool is

$$v(s; p) = \frac{\sum_{s' \in supp(p)} p(s')V(s, s')}{\sum_{s' \in supp(p)} p(s')L(s, s')}, \tag{2}$$

where $p(s')$ is the share of strategy s' in p, which is essentially the matching probability. For details of the derivation of $v(s; p)$, see Greve-Okuno (2009).

2.4 Stability Concepts

We define Nash equilibrium of VSRG, following Greve-Okuno (2009).

Definition: Given a stationary strategy distribution in the matching pool $p \in \mathcal{P}(\boldsymbol{S})$, $s \in \boldsymbol{S}$ is a *best reply against* p (denoted as $s \in BR(p)$) if for all $s' \in \boldsymbol{S}$,

$$v(s; p) \geqq v(s'; p).$$

Definition: A stationary strategy distribution in the matching pool $p \in \mathcal{P}(\boldsymbol{S})$ is a *Nash equilibrium* if, for all $s \in \mathrm{supp}(p)$, $s \in BR(p)$.

The idea is the same as ordinary Nash equilibrium, except that we use the average payoff given the population distribution p. In addition, we consider a stronger stability when there are more than one strategy in the matching pool.

Definition: A stationary strategy distribution $p \in \mathcal{P}(\boldsymbol{S})$ in the matching pool with at least two strategies in the support is *locally stable* if, for any $s' \in supp(p)$, there exists $\bar{\epsilon} \in (0, 1)$ such that for any $\epsilon \in (0, \bar{\epsilon})$ and any $s \in supp(p)$ such that $s \neq s'$,

$$v(s; (1-\epsilon)p + \epsilon p_{s'}) > v(s'; (1-\epsilon)p + \epsilon p_{s'}), \tag{3}$$

where $p_{s'}$ is the strategy distribution consisting only of s'.

The local stability selects Nash equilibria by requiring that small increase of the fraction of a strategy would make the strategy worse than other existing strategies. This is a weaker concept than neutral stability (see Greve-Okuno, 2009) because neutral stability requires that small invasion of any strategy would not upset the equilibrium. However, in our model, this concept is sufficient to make the bimorphic equilibrium unique, which will be shown in the next section.

3 Bimorphic Norm Equilibria

We focus on the following two types of strategies.

Myopic strategy: for any $t = 1, 2, \ldots$ and any partnership history h_t, $x_t(h_t) = 0$ and $z_t(h_t, a_t, a'_t) = e$ for any observation $(a_t, a'_t) \in [0, \infty)^2$ in period t.

Myopic strategy (denoted as 0-strategy) does not contribute after any partnership history and ends the partnership immediately. Clearly, if the population consists only of myopic strategies, it is a Nash equilibrium, because any strategy against the 0-strategy must play the one-shot game G.

Cooperative strategy with norm $\bar{x}$: for any $t = 1, 2, \ldots$ and any partnership history h_t, play $x_t(h_t) = \bar{x}$ for some $\bar{x} \in (0, \hat{x})$ and keep the partnership if and only if the partner's current period contribution is not less than $\bar{x}$.

The contribution level $\bar{x}$ is interpreted as an "obliged level of contribution". A player using a cooperative strategy may know that there are players not using it but (s)he follows the cooperative strategy because it is an obligation. As we discussed in the Introduction, this strategy together with the myopic strategy embodies our notion of social norm equilibrium. Depending on $\bar{x}$, a cooperative strategy is a different strategy. Let us denote a cooperative strategy under the norm $\bar{x}$ as $c(\bar{x})$-strategy.

We show that there is a continuum of bimorphic Nash equilibria (i.e., norm equilibria) such that α of the players in the matching pool are $c(\bar{x})$-strategy for some $\bar{x} \in (0, \hat{x})$ and $1 - \alpha$ are the 0-strategy. In such a bimorphic population, the average payoffs of the two strategies are as follows.

$$v(c(\bar{x});\alpha) = \frac{\alpha\frac{u(\bar{x},\bar{x})}{1-\delta^2} + (1-\alpha)u(\bar{x},0)}{\alpha \cdot \frac{1}{1-\delta^2} + (1-\alpha)}, \tag{4}$$

$$v(0;\alpha) = \alpha u(0,\bar{x}) + (1-\alpha)u(0,0). \tag{5}$$

To explain, take the $c(\bar{x})$-strategy. With probability α, it meets another $c(\bar{x})$. In this case the partnership continues with probability δ^2 in every period and the one-shot payoff is $u(\bar{x},\bar{x}) > 0$. Therefore the total expected payoff in a match with another $c(\bar{x})$-strategy is $\frac{u(\bar{x},\bar{x})}{1-\delta^2}$. With probability $1-\alpha$, the $c(\bar{x})$-strategy meets a 0-strategy, in which case the partnership lasts only one period and the one-shot payoff (as well as the total payoff of the partnership) is $u(\bar{x},0)$. These constitute the numerator of $v(c(\bar{x});\alpha)$. The denominator of $v(c(\bar{x});\alpha)$ is the expected length of partnerships for a $c(\bar{x})$-strategy.

As for the 0-strategy, it meets a $c(\bar{x})$-strategy with probability α and earns $u(0,\bar{x})$ for one period and ends the partnership. It meets another 0-strategy with probability $1-\alpha$ and gets $u(0,0)$ for one period. The expected length of partnerships is 1, regardless of whom it meets.

Lemma 1. *For any $\bar{x} \in (0,\hat{x})$, a stationary bimorphic strategy distribution $\alpha \cdot c(\bar{x}) + (1-\alpha)\cdot 0$ (which means that α of the players are the $c(\bar{x})$-strategy and $1-\alpha$ are the 0-strategy) in the matching pool is a Nash equilibrium if and only if $(\alpha,\bar{x})$ satisfies*

$$v(c(\bar{x});\alpha) = v(0;\alpha). \tag{6}$$

Proof: It suffices to prove that (6) implies that no other strategy earns higher average payoff. By the dynamic programming logic, it is necessary and sufficient to prove that no one-step deviation from $c(\bar{x})$-strategy earns higher average payoff than $c(\bar{x})$-strategy does. Notice that (4) and (5) imply that (6) is equivalent to, for some $\bar{u}$,

$$\alpha u(\bar{x},\bar{x}) + (1-\alpha)[(1-\delta^2)u(\bar{x},0) + \delta^2\bar{u}] = \bar{u}, \tag{7}$$

$$\alpha u(0,\bar{x}) + (1-\alpha)u(0,0) = \bar{u}. \tag{8}$$

No one-step deviation from $c(\bar{x})$-strategy earns a higher average payoff if and only if an optimal deviation in G (to contribute 0), when the partner is going to contribute $\bar{x} > 0$ does not earn a higher average payoff. This is equivalent to

$$u(\bar{x},\bar{x}) \geqq (1-\delta^2)u(0,x) + \delta^2\bar{u}. \tag{9}$$

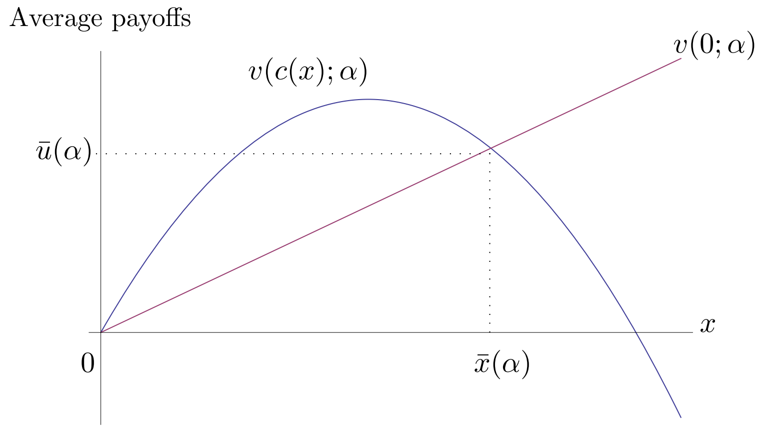

Figure 3: Existence of a Nash equilibrium given α.

From (7), (8) and $u(0,0)=0$, we have that

$$\begin{aligned}\alpha u(\bar{x},\bar{x}) &= \bar{u}-(1-\alpha)[(1-\delta^2)u(\bar{x},0)+\delta^2\bar{u}] \\ &= \alpha u(0,\bar{x})-(1-\alpha)[(1-\delta^2)u(\bar{x},0)+\delta^2\alpha u(0,\bar{x})] \\ &= [1-(1-\alpha)\delta^2]\alpha u(0,\bar{x})-(1-\alpha)(1-\delta^2)u(\bar{x},0) \\ &\geqq [1-\delta^2+\alpha\delta^2]\alpha u(0,\bar{x}) \\ &= \alpha[(1-\delta^2)u(0,\bar{x})+\delta^2\bar{u}]. \quad \square\end{aligned}$$

Hence we only need to find combinations of $(\alpha,\bar{x})$ such that (6) holds. Using Remark 1, let us rewrite (4) and (5) as follows.

$$v(c(\bar{x});\alpha) = \frac{\alpha\frac{w(\bar{x})+\bar{x}}{1-\delta^2}+(1-\alpha)w(\bar{x})}{\alpha\cdot\frac{1}{1-\delta^2}+(1-\alpha)} = w(\bar{x})+\frac{\alpha\bar{x}}{\alpha+(1-\alpha)(1-\delta^2)}, \tag{10}$$

$$v(0;\alpha) = \alpha\bar{x}. \tag{11}$$

Given α, the average payoff of the 0-strategy is linear in $\bar{x}$. By differentiation,

$$\frac{\partial v(c(\cdot);\alpha)}{\partial x} = w'(x)+\frac{\alpha}{\alpha+(1-\alpha)(1-\delta^2)},$$

and recall that $w'(0)=0$ and $\lim_{x\to\infty}w'(x)=-\infty$ from Remark 1. Hence the derivative $\frac{\partial v(c(\cdot);\alpha)}{\partial x}$ is positive at $\bar{x}=0$ but will eventually become negative, so that the average payoff of $c(\bar{x})$-strategy is concave in $\bar{x}$. At $\bar{x}=0$, the derivatives are $\frac{\partial v(c(\cdot);\alpha)}{\partial x}(0)=\frac{\alpha}{\alpha+(1-\alpha)(1-\delta^2)}>\alpha=\frac{\partial v(0;\alpha)}{\partial x}(0)$ for any $\alpha\in(0,1)$. Therefore there exists an intersection $\bar{x}(\alpha)>0$ of the two average payoffs. See Figure 3.

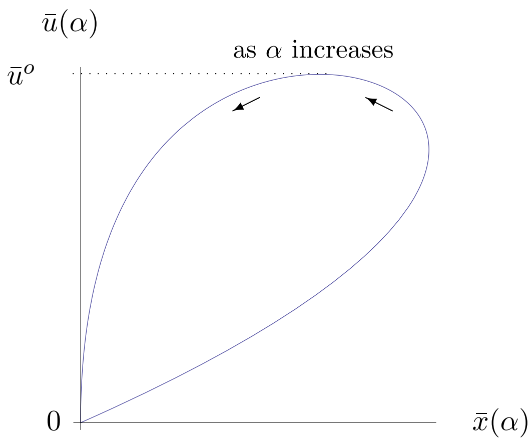

Figure 4: Nash equilibrium norm and equilibrium payoff

This $\bar{x}(\alpha)$ is the "Nash equilibrium norm" that makes cooperative players with fraction α stable in the presence of myopic players.

In summary we have the following existence result of bimorphic Nash equilibria.

Proposition 1. *For any $\alpha \in (0, 1)$, there exists $\bar{x}(\alpha) \in (0, \hat{x})$ such that a stationary bimorphic distribution $\alpha \cdot c(\bar{x}(\alpha)) + (1 - \alpha) \cdot 0$ in the matching pool is a Nash equilibrium.*

As α changes from 0 to 1, the Nash norm $\bar{x}(\alpha)$ and the equilibrium average payoff $\bar{u}(\alpha) = v(c(\bar{x}(\alpha)); \alpha) = v(0; \alpha)$ display a closed curve as in Figure 4. This is because, as α increases, $v(0; \alpha)$ becomes steeper but $v(c(\bar{x}(\alpha); \alpha)$ continues to be concave so that eventually the intersection $\bar{x}(\alpha)$ becomes decreasing in α.

Interestingly, the most efficient bimorphic equilibrium (the one with the highest average equilibrium payoff $\bar{u}^o$, i.e., the top of the closed curve in Figure 4) is not the one that uses the norm x as high as possible (the right-most point of the closed curve). This is because the equilibrium norm is determined in combination with α and larger fraction of the cooperative strategy gives greater average payoff.

To look at the figure vertically, we see that given x, there can be at most two α that makes (α, x) a bimorphic Nash equilibrium. Among such α, only the larger one makes a locally stable bimorphic equilibrium.

Let $X = \{x \in (0, \hat{x}) \mid$ there exist two distinct $\alpha_1 < \alpha_2 \in (0, 1)$ such that (α_i, x) satisfies (6) for both $i = 1, 2\}$ be the range of x such that two bimorphic Nash equilibria exist. As

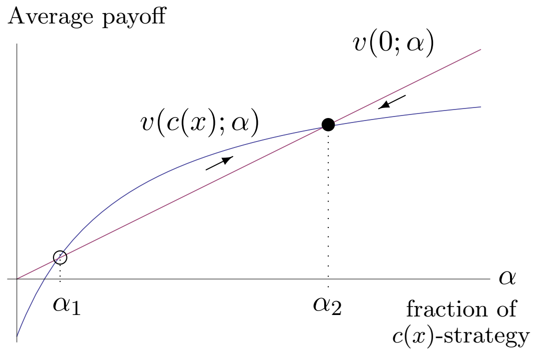

Figure 5(a): when $x \in X$

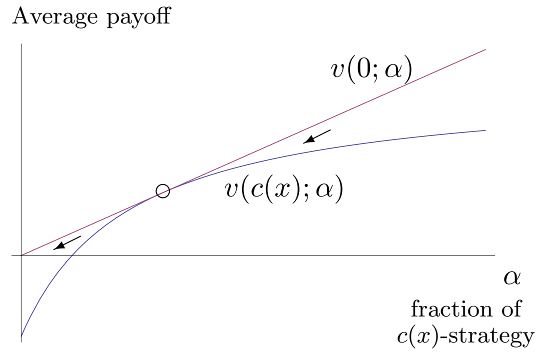

Figure 5(b): unique α is not locally stable

Figure 5: Local stability

Figure 5(a) shows, for such x, one of the bimorphic Nash equilibira (namely with the larger share of $c(x)$-strategy) is locally stable.

Corollary 1. *For any $x \in X$, only the larger α_2 such that (α_2, x) satisfies (6) constitutes a locally stable bimorphic Nash equilibrium.*

Proof: By differentiation of (4), it is straightforward to show that given $x \in X$, the average payoff function of $c(x)$-strategy is concave in α. It is also obvious from (5) that the average payoff function of the 0-strategy is linear in α. Therefore there exists a neighborhood U of the larger intersection α_2 such that for any $\alpha \in U$ (see Figure 5(a)),

$$v(c(x);\alpha) \gtreqless v(0;\alpha) \iff \alpha_2 \gtreqless \alpha$$

with equality holding only at α_2, so that local stability condition is satisfied. □

By contrast, if there is a unique $(\alpha, \bar{x}(\alpha))$ that makes the bimorphic distribution $\alpha \cdot c(\bar{x}(\alpha)) + (1-\alpha) \cdot 0$ a Nash equilibrium, it is not locally stable, because the increase of the 0-strategy makes it fare better than the $c(x)$-strategy. See Figure 5(b).

We have shown that in the VSRG model, there is a continuum of locally stable equilibria with myopic strategy and a cooperative strategy. This gives a rationale for incomplete information models with two types of players.

Moreover, the locally stable bimorphic equilibria are stable in a stronger sense. They are robust against invasion of other strategies that share the same norm as the cooperative strategy.

Note that $c(x)$-strategy always contribute x, but there are other strategies that may shift between myopic 0 contribution and the norm x. A prominent example is a class of "trust-builiding" strategies (see Greve-Okuno, 2009) such that in the first T periods of a partnership it does not contribute but keeps the partnership and after T periods of trust-building is done, it starts contributing $x > 0$ and keeps the partnership if and only if the partner also contributes $y \geqq x$. This strategy can in fact invade the monomorphic population of 0-strategy.

Let us define the set of pure strategies that has norm $x \in (0, \hat{x})$ as

$$S(x) := \{s = \{(x_t, z_t); t = 1, 2, \ldots\} \in \boldsymbol{S} \mid x_t(h_t) \in \{0, x\} \ \ \forall \ h_t \in H_t, \ \forall \ t = 1, 2, \ldots\}.$$

That is, a strategy which has norm x contributes x whenever it contributes a positive amount, but it may not contribute at all for some cases.

Definition: A stationary strategy distribution $p \in \mathcal{P}(\boldsymbol{S})$ in the matching pool with the support $\{0, c(x)\}$ for some $x \in (0, \hat{x})$ is *norm stable* if for any $s \in S(x)$, there exists $\bar{\epsilon} > 0$ such that for any $\epsilon \in (0, \bar{\epsilon})$,

$$v(c(x); (1-\epsilon)p + \epsilon p_s) > v(s; (1-\epsilon)p + \epsilon p_s).$$

Proposition 2. *For sufficiently large δ, any locally stable bimorphic equilibrium is norm stable.*

Proof: Let $p = \alpha \cdot c(\bar{x}) + (1-\alpha) \cdot 0$ be a locally stable equilibrium with $\bar{x} \in (0, \hat{x})$. It suffices to show that the following "one-period trust-building" strategy (denoted as $c(0, \bar{x})$) cannot invade the population.

$c(0, \bar{x})$-strategy: in $t = 1$, contribute 0 but keep the partnership for any observation. In $t \geqq 2$, contribute $\bar{x}$ and keep the partnership if and only if the partner contributes $y \geqq \bar{x}$.

Among strategies in $S(\bar{x})$, this $c(0, \bar{x})$-strategy earns highest payoff when it meets itself.

Suppose that ϵ of $c(0, \bar{x})$ enters the population. Then the average payoff of the three strategies are as follows.

$$v(c(\bar{x}); (1-\epsilon)p + \epsilon p_{c(0,\bar{x})}) = \frac{(1-\epsilon)\{\alpha \frac{w(\bar{x})+\bar{x}}{1-\delta^2} + (1-\alpha)w(\bar{x})\} + \epsilon w(\bar{x})}{(1-\epsilon)\{\alpha \cdot \frac{1}{1-\delta^2} + (1-\alpha)\} + \epsilon}, \tag{12}$$

$$v(0; (1-\epsilon)p + \epsilon p_{c(0,\bar{x})}) = (1-\epsilon)\alpha\bar{x}, \tag{13}$$

$$v(c(0,\bar{x}); (1-\epsilon)p + \epsilon p_{c(0,\bar{x})}) = \frac{(1-\epsilon)\alpha\bar{x} + \epsilon \frac{\delta^2(w(\bar{x})+\bar{x})}{1-\delta^2}}{1-\epsilon+\epsilon \cdot \frac{1}{1-\delta^2}}. \tag{14}$$

Clearly, when $\epsilon = 0$, all three strategies earn the same average payoff since $v(c(0,\bar{x});p) = \alpha\bar{x}$. We show that the average payoff difference between $c(\bar{x})$ and $c(0,\bar{x})$ is essentially quadratic and concave in ϵ and for small $\epsilon > 0$, it is positive. From (12) and (14), we have

$$\begin{aligned} & v(c(\bar{x});(1-\epsilon)p + \epsilon p_{c(0,\bar{x})}) - v(c(0,\bar{x});(1-\epsilon)p + \epsilon p_{c(0,\bar{x})}) \\ & = \frac{(1-\delta^2)f(\epsilon)}{[1-\delta^2\{1-\alpha(1-\epsilon)\}]\{1-\delta^2(1-\epsilon)\}} \end{aligned}$$

where

$$f(\epsilon) := \left[1-\delta^2\{1-\alpha(1-\epsilon)\}\right] w(\bar{x}) - \delta^2\left\{\alpha^2(1-\epsilon)^2 + \epsilon - \alpha(1-\epsilon^2)\right\}\bar{x}$$

Therefore the average payoff difference is positive if and only if the quadratic function $f(\epsilon)$ is positive. As we have checked above, $f(0) = 0$. By differentiation

$$f'(0) = \delta^2\{(2\alpha^2 - 1)\bar{x} - \alpha w(\bar{x})\}.$$

Since $w(\bar{x}) < 0$, this is an increasing function of α and δ. Note that the locally stable fraction α is increasing in δ, since $v(0;p)$ is constant in δ but $v(c(\bar{x});p)$ is increasing in δ as (10) shows. Then as Figure 5(a) shows, the larger intersection increases as δ shifts $v(c(\bar{x});p)$ upwards. Note also that when δ is close to 1, α exceeds $\frac{1}{\sqrt{2}}$, which is sufficient for $f'(0) > 0$. Therefore for sufficiently large δ, $f'(0) > 0$. □

In summary, the locally stable bimorphic equilibrium is a quite stable distribution given a norm $\bar{x}$. If there is a norm of $\bar{x}$ level of contribution, no strategy that shifts between 0 and $\bar{x}$ can invade the population.[2] In the next section we consider an extension of the model in which some players try to change the norm.

4 Social Norm Evolution

In this section we extend the model to allow mutations of strategies with initial message exchange in order to change the contribution level from the prevailing norm. The idea is similar to Robson (1990) and Matsui (1991). Let us assume that when two players are newly matched, they simultaneously send a message from a countable set M to the partner. M is common to all players. The messages do not directly alter the payoff and thus are cheap-talk. The message choice is observed only by the partners and not known by any other palyer.

[2] We are not saying that the distribution is neutrally stable (Greve-Okuno, 2009), which requires that not only one but all strategies in the support earn not less than the entrant. This it not true for our 0-$c(\bar{x})$ equilibrium, unfortunately.

Definition: A pure strategy s^{CT} of VSRG with cheap talk consists of (m, σ) such that:

1. $m \in M$ specifies the message the player sends to any new partner, and
2. $\sigma : M \to \boldsymbol{S}$ specifies a VSRG strategy $\sigma(m')$ the player chooses to play for each message $m' \in M$ (s)he receives from the partner.

Let $\boldsymbol{S}^{CT}$ be the set of all pure strategies of VSRG with cheap talk. A strategy s in the original VSRG can be extended to a strategy in $\boldsymbol{S}^{CT}$ such that it sends an arbitrary message and plays s regardless of the partner's message. Such a strategy is called a *babbling* strategy.

Definition: Given a strategy $s \in \boldsymbol{S}$ of VSRG, a strategy $s^B(s) = (m, \sigma^s) \in \boldsymbol{S}^{CT}$ of the cheap talk game is an *associated babbling strategy* of s if $\sigma^s(m') = s$ for all $m' \in M$.

Note that there is a class of associated babbling strategies of the same $s \in \boldsymbol{S}$ depending on the initial message m, but, if all players use associated babbling strategies of the same $s \in \boldsymbol{S}$, then the initial message does not matter. Given a strategy distribution $p \in \mathcal{P}(\boldsymbol{S})$, a class of associated babbling strategy distributions is similarly defined. As is well-known, any babbling extension of a Nash equilibrium is always a Nash equilibrium of the cheap talk model because the initial message exchange does not matter.

Lemma 2. *For any Nash Equilibrium $p \in \mathcal{P}(\boldsymbol{S})$ of VSRG, any associated babbling strategy distribution is a Nash Equilibrium of the cheap talk model.*

Proof: Obvious.

Suppose that, if there are two societies with different locally stable bimorphic equilibrium, the one with higher in-match average payoff within cooperative strategies can influence the other. That is, the following "pair-wise" invasion is possible. Let two bimorphic equilibria be $p = \alpha \cdot c(\bar{x}(\alpha)) + (1-\alpha) \cdot 0$ and $p' = \alpha' \cdot c(\bar{x}(\alpha)) + (1-\alpha') \cdot 0$ such that $w(\bar{x}(\alpha)) + \bar{x}(\alpha) < w(\bar{x}(\alpha')) + \bar{x}(\alpha')$. That is, within a pair of cooperative strategies, the norm $\bar{x}(\alpha')$ is more efficient. For notational simplicity, let $\bar{x} = \bar{x}(\alpha)$ and $\bar{x}' = \bar{x}(\alpha')$.

The entrants use a neologism $\zeta \in M$ so that they can recognize whether the randomly matched opponent is an incumbent from distribution p or not. If a cooperative entrant from p' recognized the opponent as an incumbent, it plays as if $c(\bar{x})$ while if it recognized the opponent as an entrant it plays as if $c(\bar{x}')$. Let us write this neologism strategy in S^{CT} as $s'^{CT} = (\zeta, \sigma')$, where $\sigma'(\zeta) = c(\bar{x}')$ and $\sigma'(m) = c(\bar{x})$ for other $m \neq \zeta$.

Then the babbling $s^B(c(\bar{x}))$-strategy's average payoff is

$$\begin{aligned} v(s^B(c(\bar{x})); (1-\epsilon)p + \epsilon p') &= \frac{(1-\epsilon)\{\alpha \frac{w(\bar{x})+\bar{x}}{1-\delta^2} + (1-\alpha)w(\bar{x})\} + \epsilon\{\alpha' u(\bar{x},\bar{x}) + (1-\alpha')w(\bar{x})\}}{(1-\epsilon)\alpha\frac{1}{1-\delta^2} + 1 - (1-\epsilon)\alpha} \\ &= \frac{(1-\epsilon)\{\alpha \frac{w(\bar{x})+\bar{x}}{1-\delta^2} + (1-\alpha)w(\bar{x})\} + \epsilon\{\alpha'\{w(\bar{x}) + \bar{x}\} + (1-\alpha')w(\bar{x})\}}{(1-\epsilon)\alpha\frac{1}{1-\delta^2} + 1 - (1-\epsilon)\alpha} \\ &= \frac{(1-\epsilon)V(c(\bar{x}); p) + \epsilon\{w(\bar{x}) + \alpha'\bar{x}\}}{(1-\epsilon)\alpha\frac{1}{1-\delta^2} + 1 - (1-\epsilon)\alpha}, \end{aligned}$$

while s'^{CT}'s average payoff is

$$\begin{aligned} v(s'^{CT}; (1-\epsilon)p + \epsilon p') &= \frac{(1-\epsilon)V(c(\bar{x}); p) + \epsilon\{\alpha' \frac{w(\bar{x}')+\bar{x}'}{1-\delta^2} + (1-\alpha')w(\bar{x}')\}}{\{(1-\epsilon)\alpha + \epsilon\alpha'\}\frac{1}{1-\delta^2} + 1 - (1-\epsilon)\alpha - \epsilon\alpha'} \\ &= \frac{(1-\epsilon)V(c(\bar{x}); p) + \epsilon V(c(\bar{x}'); p')}{\{(1-\epsilon)\alpha\frac{1}{1-\delta^2} + 1 - (1-\epsilon)\alpha\} + \epsilon\alpha'\frac{1}{1-\delta^2} - \epsilon\alpha'} \\ &= \frac{(1-\epsilon)V(c(\bar{x}); p) + \epsilon V(c(\bar{x}'); p')}{\{(1-\epsilon)\alpha\frac{1}{1-\delta^2} + 1 - (1-\epsilon)\alpha\} + \epsilon\alpha'\frac{\delta^2}{1-\delta^2}} \end{aligned}$$

Let $L(\epsilon) = (1-\epsilon)\alpha\frac{1}{1-\delta^2} + 1 - (1-\epsilon)\alpha$. Then we can rewrite

$$v(s^B(c(\bar{x})); (1-\epsilon)p + \epsilon p') = \frac{(1-\epsilon)V(c(\bar{x}); p) + \epsilon\{w(\bar{x}) + \alpha'\bar{x}\}}{L(\epsilon)}, \tag{15}$$

$$v(s'^{CT}; (1-\epsilon)p + \epsilon p') = \frac{(1-\epsilon)V(c(\bar{x}); p) + \epsilon V(c(\bar{x}'); p')}{L(\epsilon) + \epsilon\alpha'\frac{\delta^2}{1-\delta^2}}. \tag{16}$$

By computation

$$\begin{aligned} &\Delta(\epsilon) := \{v(s'^{CT}; (1-\epsilon)p + \epsilon p') - v(s^B(c(\bar{x})); (1-\epsilon)p + \epsilon p')\}L(\epsilon)\{L(\epsilon) + \epsilon\alpha'\frac{\delta^2}{1-\delta^2}\}\frac{1}{\epsilon} \\ = \; & L(\epsilon)V(c(\bar{x}'); p') - \big[L(\epsilon)\{w(\bar{x}) + \alpha'\bar{x}\} + \frac{\alpha'\delta^2(1-\epsilon)}{1-\delta^2}V(c(\bar{x}); p) + \frac{\epsilon\alpha'\delta^2}{1-\delta^2}\{w(\bar{x}) + \alpha'\bar{x}\}\big], \end{aligned}$$

so that as $\epsilon \to 0$, the payoff difference converges to

$$
\begin{aligned}
& \lim_{\epsilon \to 0} \Delta(\epsilon) \\
= \;& (1-\alpha)[V(c(\bar{x}'); p') - \{w(\bar{x}) + \alpha' \bar{x}\}] + \frac{1}{1-\delta^2}\big[\alpha V(c(\bar{x}'); p') - \alpha\{w(\bar{x}) + \alpha'\bar{x}\} - \alpha'\delta^2 V(c(\bar{x}); p)\big] \\
= \;& (1-\alpha)\big[\alpha' \frac{w(\bar{x}') + \bar{x}'}{1-\delta^2} + (1-\alpha')w(\bar{x}') - \{w(\bar{x}) + \alpha'\bar{x}\}\big] \\
& + \frac{1}{1-\delta^2}\big[\alpha\big\{\alpha' \frac{w(\bar{x}') + \bar{x}'}{1-\delta^2} + (1-\alpha')w(\bar{x}')\big\} - \alpha\{w(\bar{x}) + \alpha'\bar{x}\} - \alpha'\delta^2\big\{\alpha \frac{w(\bar{x}) + \bar{x}}{1-\delta^2} + (1-\alpha)w(\bar{x})\big\}\big] \\
= \;& (1-\alpha)\big[\{\alpha' \frac{w(\bar{x}') + \bar{x}'}{1-\delta^2} + (1-\alpha')w(\bar{x}')\} - \{w(\bar{x}) + \alpha'\bar{x}\}\big] \\
& + \frac{1}{1-\delta^2}\Big[\frac{\alpha\alpha'}{1-\delta^2}\big[\{w(\bar{x}') + \bar{x}'\} - \delta^2\{w(\bar{x}) + \bar{x}\}\big] \\
& + \alpha(1-\alpha')w(\bar{x}') - \alpha'\delta^2(1-\alpha)w(\bar{x}) - \alpha w(\bar{x}) - \alpha\alpha'\bar{x}\Big].
\end{aligned}
$$

Therefore, for sufficiently large δ, only the term $\frac{\alpha\alpha'}{1-\delta^2}\big[\{w(\bar{x}') + \bar{x}'\} - \delta^2\{w(\bar{x}) + \bar{x}\}\big]$ matters and this is positive by the assumption. In sum, we have proved the following.

Proposition 3. *For sufficiently large δ, the associated babbling strategy distribution of a bimorphic Nash equilibrium $p = \alpha \cdot c(\bar{x}(\alpha)) + (1-\alpha) \cdot 0$ is robust against small fraction of entrants of a neologism strategy distribution of another bimorphic Nash equilibrium $p' = \alpha' \cdot c(\bar{x}(\alpha')) + (1-\alpha') \cdot 0$ if and only if $w(\bar{x}(\alpha)) + \bar{x}(\alpha) > w(\bar{x}(\alpha')) + \bar{x}(\alpha')$.*

Therefore, the most efficient norm, within a match of cooperative strategies, is selected under cheap talk. This result is similar to Robson (1990) as well.

5 Concluding Remarks

In Proposition 2, we showed norm stability of the bimorphic distribution of a cooperative strategy $c(x)$ (that starts contributing immediately) and the myopic 0-strategy. The norm stability is warranted when all entrants are restricted to contribute the same amount if they contribute at all, as well as the post-entry distribution keeps the balance of the initial combination of the incumbent $c(x)$-strategy and 0-strategy, i.e., of the form $(1-\epsilon)p + \epsilon p_s$, where $p = \alpha \cdot c(x) + (1-\alpha) \cdot 0$ is the Nash equilibrium distribution. However, if some of the myopic strategies mutate to a different strategy, say $c(0, x)$-strategy, then the post-entry distribution will be $\alpha \cdot c(x) + (1-\alpha)(1-\epsilon)0 + (1-\alpha)\epsilon c(0, x)$. Under this distribution, the incumbent $c(x)$ no longer earns higher average payoff than the entrant $c(0, x)$. Therefore, the form of post-entry distribution, or how strategies mutate, makes a big difference in the stability.

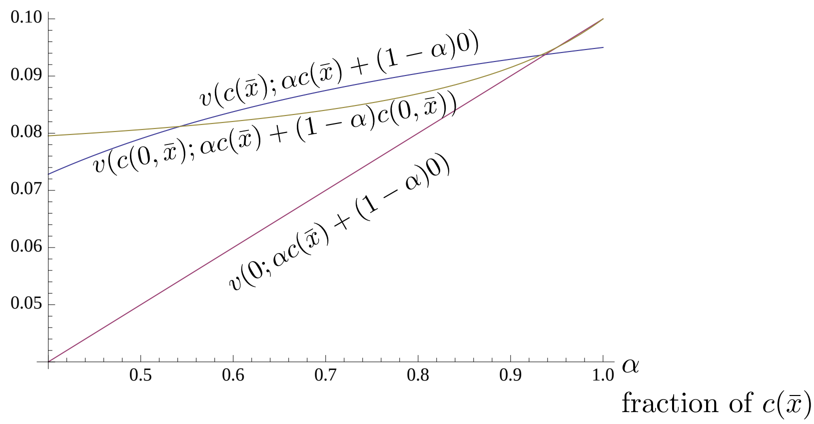

Figure 6: Comparison of different bimorphic equilibria

Nonetheless, even if the "trust-building" strategy $c(0,x)$ may upset the population of $c(x)$-strategy and 0-strategy, the new bimorphic equilibrium of $c(x)$ and $c(0,x)$ will have lower average payoff than that of $c(x)$-strategy and 0-strategy. This is because, given the fraction α of the $c(x)$-strategy, the $c(0,x)$-strategy earns strictly higher payoff than 0-strategy does. The logic is obvious, since $c(0,x)$-strategy plays the same way as 0-strategy when it meets $c(x)$-strategy but it earns higher payoff when it meets itself, than when 0-strategy meets itself. However, the higher payoff of $c(0,x)$ implies smaller equilibrium fraction α of the cooperative strategy, yielding a lower equilibrium average payoff. See Figure 6.

The mutation of the myopic 0-strategy to $c(0,x)$-strategy is, however, not so plausible. The myopic strategy is "anti-social", while $c(0,x)$-strategy is not. The $c(0,x)$-strategy does contribute a positive amount x if the partnership continues to the second period. Thus the mutation of the myopic 0-strategy to $c(0,x)$-strategy is a mutation from an "anti-social" strategy to a "social" strategy, which is unlikely.

We also note that the range of parameters of the game that allows bimorphic equilibria of $c(x)$-strategy and 0-strategy is much larger than that of $c(x)$ and $c(0,x)$. Therefore the social vs. anti-social distribution is robust in this sense as well. It is thus quite reasonable that many authors focused on ordinary incomplete information games with rational and myopic types.

REFERENCES

Datta, S. (1996), "Building Trust", Working Paper, London School of Economics.

Fujiwara-Greve, T. and Okuno-Fujiwara, M. (2009), "Voluntarily Seperable Repeated Prisoner's Dilemma", *Review of Economic Studies*, 76, 993-1021.

Ghosh, P. and Ray D. (1996), "Cooperation in Community Interaction without Information Flows", *Review of Economic Studies*, **63**, 491-519.

Kranton, R. (1996a), "The Formation of Cooperative Relationships", *Journal of Law, Economics, & Organization*, **12**, 214-233.

Kranton, R. (1996b), "Reciprocal Exchange: A Self-Sustaining System", *American Economic Review*, **86**, 830-851.

Matsui, A. (1991), "Cheap Talk and Cooperation in a Society", *Journal of Economic Theory*, **54**, 245-258.

Okuno-Fujiwara, M. (1987), "Monitoring Cost, Agency Relationship, and Equilibrium Modes of Labor Contract", *Journal of the Japanese and International Economies*, **1**, 147-167.

Rob, R. and Yang, H. (2006) "Long-term relationships as Safeguards", forthcoming in *Economic Theory.*

Robson, A. (1990), "Efficiency in Evolutionary Games: Darwin, Nash and the Secret Handshake", *Journal of Theoretical Biology*, **144**, 379-396.

Shapiro, C. and Stiglitz J. (1984), "Equilibrium Unemployment as a Worker Discipline Device", *American Economic Review*, **74**, 433-444.

Swinkels, J. (1992): "Evolutionary Stability with Equilibrium Entrants", *Journal of Economic Theory*, 57, 306-332.