# Analyzing and Forecasting Volatility Spillovers and Asymmetries in Major Crude Oil Spot, Forward and Futures Markets

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# Abstract

Crude oil price volatility has been analyzed extensively for organized spot, forward and futures markets for well over a decade, and is crucial for forecasting volatility and Value-at-Risk (VaR). There are four major benchmarks in the international oil market, namely West Texas Intermediate (USA), Brent (North Sea), Dubai/Oman (Middle East), and Tapis (Asia-Pacific), which are likely to be highly correlated. This paper analyses the volatility spillover and asymmetric effects across and within the four markets, using three multivariate GARCH models, namely the constant conditional correlation (CCC), vector ARMA-GARCH (VARMA-GARCH) and vector ARMA-asymmetric GARCH (VARMA-AGARCH) models. A rolling window approach is used to forecast the 1-day ahead conditional correlations. The paper presents evidence of volatility spillovers and asymmetric effects on the conditional variances for most pairs of series. In addition, the forecast conditional correlations between pairs of crude oil returns have both positive and negative trends. Moreover, the optimal hedge ratios and optimal portfolio weights of crude oil across different assets and market portfolios are evaluated in order to provide important policy implications for risk management in crude oil markets.

## 1. Introduction

Over the past 20-30 years, oil has become the biggest traded commodity in the world. In the crude oil market, oil is sold under a variety of contract arrangements and in spot transactions, and is also traded in futures markets which set the spot, forward and futures prices. Crude oil is usually sold close to the point of production, and is transferred as the oil flows from the loading terminal to the ship FOB (free on board). Thus, spot prices are quoted for immediate delivery of crude oil as FOB prices. Forward prices are the agreed upon price of crude oil in forward contracts. Futures price are prices quoted for delivering in a specified quantity of crude oil at a specified time and place in the future in a particular trading centre.

The four major benchmarks in the world of international trading today are: 1) West Texas Intermediate (WTI), the reference crude for USA, (2) Brent, the reference crude oil for the North Sea, (3) Dubai, the benchmark crude oil for the Middle East and Far East, and (4) Tapis, the benchmark crude oil for the Asia-Pacific region. Volatility (or risk) is important in finance and is typically unobservable, and volatility spillovers appear to be widespread in financial markets (Milunovich and Thorp, 2006), including energy futures markets (Lin and Tamvakis, 2001). These results hold even when markets do not necessarily trade at the same time. Consequently, a volatility spillover occurs when changes in volatility in one market produce a lagged impact on volatility in other markets, over and above local effects. Volatility spillovers and asymmetries among those four major benchmarks are likely to be important for constructing hedge ratios and optimal portfolios. As research has typically focused on oil spot and futures prices to the neglect of forward prices, this paper analyses all three oil prices.

Accurate modelling of volatility is crucial in finance and for commodity. Shocks to returns can be divided into predictable and unpredictable components. The most frequently analyzed predictable component in shocks to returns is the volatility in the time-varying conditional variance. The success of the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model of Engle (1982) and Bollerslev (1986) has subsequently led to a family of univariate and multivariate GARCH models which can capture different behavior in financial returns, including time-varying volatility, persistence and clustering of volatility, and the asymmetric effects of positive and negative shocks of equal magnitude. In modelling multivariate returns, such as spot, forward and futures returns, shocks to returns not only have dynamic interdependence in risks, but also in the conditional correlations

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which are key elements in portfolio construction and the testing of unbiasedness and the efficient market hypothesis. The hypothesis of efficient markets is essential for understanding optimal decision making, especially for hedging and speculation.

Substantial research has been conducted on spillover effects in energy futures markets. Lin and Tamvakis (2001) investigated volatility spillover effects between NYMEX and IPE crude oil contracts in both non-overlapping and simultaneous trading hours. They found that substantial spillover effects exist when both markets are trading simultaneously, although IPE morning prices seem to be affected considerably by the close of the previous day on NYMEX. Ewing et al (2002) examined the transmission of volatility between the oil and natural gas markets using daily returns data, and found that changes in volatility in one market may have spillovers to the other market. Sola et al (2002) analyzed volatility links between different markets based on a bivariate Markov switching model, and discovered that it enables identification of the probabilistic structure, timing and the duration of the volatility transmission mechanism from one country to another.

Hammoudeh et al. (2003) examined the time series properties of daily spot and futures prices for three petroleum types traded at five commodity centres within and outside the USA by using multivariate vector error-correction models, causality models and GARCH models. They found that WTI crude oil NYMEX 1-month futures prices involves causality and volatility spillovers, NYMEX gasoline has bi-directional causality relationships among all the gasoline spot and futures prices, spot prices produce the greatest spillovers, and NYMEX heating oil for 1- and 3-month futures are particularly strong and significant. Chang et al. (2009) examined multivariate conditional volatility and conditional correlation models of spot, forward, and futures returns from three crude oil markets, namely Brent, WTI and Dubai, and provided evidence of significant volatility spillovers and asymmetric effects in the conditional volatilities across returns for each market.

Of the four major crude oil markets, only the most well known oil markets, namely WTI and Brent, the light sweet grade category, have spot, forward and futures prices, while the Dubai and Tapis markets, the heavier and less sweet grade category, have only spot and forward prices. It would seem that no research has yet tested the spillover effects for each of the spot, forward and futures crude oil prices in and across all markets, or estimated the optimal portfolio weights and optimal hedge ratios for purposes of risk diversification.

Several multivariate GARCH models specify risk for one asset as depending dynamically on its own past and on the past of other assets (see McAleer, 2005). da Veiga, Chan and McAleer (2008) analyzed the multivariate vector ARMA-GARCH (VARMA-

GARCH) model of Ling and McAleer (2003) and vector ARMA-asymmetric GARCH (VARMA-AGARCH) model of McAleer, Hoti and Chan (2009), and found that they were superior to the GARCH model of Bollerslev (1986) and the GJR model of Glosten, Jagannathan and Runkle (1992).

This paper has two main objectives, as follows: (1) We investigate the importance of volatility spillovers and asymmetric effects of negative and positive shocks of equal magnitude on the conditional variance for modelling crude oil volatility in the returns of spot, forward and futures prices within and across the Brent, WTI, Dubai and Tapis markets, using multivariate conditional volatility models. The spillover effects between returns in and across markets are also estimated. A rolling window is used to forecast 1-day ahead conditional correlations, and to explain the conditional correlations movements, which are important for portfolio construction and hedging. (2) We apply the estimated results to compute the optimal hedge ratios and optimal portfolio weights of the crude oil portfolio, which provides important policy implications for risk management in crude oil markets.

The plan of the paper is as follows. Section 2 discusses the univariate and multivariate GARCH models to be estimated. Section 3 explains the data, descriptive statistics and unit root tests. Section 4 describes the empirical estimates and some diagnostic tests of the univariate and multivariate models, and forecasts of 1-day ahead conditional correlations. Section 5 presents the economic implications for optimal hedge ratios and optimal portfolio weights. Section 6 provides some concluding remarks.

## 2. Econometric Models

This section presents the constant conditional correlation (CCC) model of Bollerslev (1990), the VARMA-GARCH model of Ling and McAleer (2003) and VARMA-AGARCH model of McAleer, Hoti and Chan (2009). These models assume constant conditional correlations, and do not suffer from the problem of dimensionality, as compared with the VECH and BEKK models, and also possess regularity and statistical properties, unlike the DCC model (see McAleer et al. (2008) and Carporin and McAleer (2009, 2010) for detailed explanations of these issues).

In explaining a vector of oil prices, $Y$, the VARMA-GARCH model of Ling and McAleer (2003), assumes symmetry in the effects of positive and negative shocks of equal magnitude on the conditional volatility, and is given by

$$Y_t = E\left(Y_t \middle| F_{t-1}\right) + \varepsilon_t \tag{1}$$

$$\Phi(L)\left(Y_t - \mu\right) = \Psi(L)\varepsilon_t \tag{2}$$

$$\varepsilon_t = D_t \eta_t \tag{3}$$

$$H_t = W_t + \sum_{l=1}^{r} A_l \vec{\varepsilon}_{t-l} + \sum_{l=1}^{s} B_l H_{i,t-j} \tag{4}$$

where (1) denotes the decomposition of *Y* into its predictable (conditional mean) and random components, $D_t = \mathrm{diag}\left(h_{i,t}^{1/2}\right)$, $H_t = \left(h_{1t},...,h_{mt}\right)'$, $W_t = \left(\omega_{1t},...,\omega_{mt}\right)'$, $\eta_t = \left(\eta_{1t},...,\eta_{mt}\right)'$ is a sequence of independently and identically (iid) random vectors, $\vec{\varepsilon}_t = \left(\varepsilon_{it}^2,...,\varepsilon_{mt}^2\right)'$, $A_t$ and $B_l$ are $m \times m$ matrices with typical elements $\alpha_{ij}$ and $\beta_{ij}$, respectively, for $i, j = 1,...,m$, $I(\eta_t) = diag\left(I\left(\eta_{it}\right)\right)$ is an $m \times m$ matrix. $\Phi(L) = I_m - \Phi_1 L - ... \quad -\Phi_p L^p$ and $\Psi(L) = I_m - \Psi_1 L - ... - \Psi_q L^q$ are polynomials in *L*, the lag operator, and $F_t$ is the past information available to time *t*. $\alpha_l$ represents the ARCH effect, and $\beta_l$ represents the GARCH effect.

Spillover effects, or the dependence of conditional variances across crude oil returns, are given in the conditional volatility for each asset in the portfolio. Based on equation (3), the VARMA-GARCH model also assumes that the matrix of conditional correlations is given by $E\left(\eta_t \eta_t'\right) = \Gamma$. If $m = 1$, equation (4) reduces to the univariate GARCH model of Bollerslev (1986):

$$h_t = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_i h_{t-i}^2 \tag{5}$$

The VARMA-GARCH model assumes that negative and positive shocks of equal magnitude have identical impacts on the conditional variance. An extension of the VARMA-GARCH model to accommodate asymmetric impacts of positive and negative shocks is the VARMA-AGARCH model of McAleer, Hoti and Chan (2009), which captures asymmetric spillover effects from other crude oil returns. An extension of (4) to accommodate asymmetries with respect to $\varepsilon_{it}$ is given by

$$H_t = W + \sum_{l=1}^{r} A_l \vec{\varepsilon}_{t-l} + \sum_{l=1}^{r} C_l I_{t-l} \vec{\varepsilon}_{t-l} + \sum_{l=1}^{s} B_l H_{t-l} \tag{6}$$

in which $\varepsilon_{it} = \eta_{it}\sqrt{h_{it}}$ for all $i$ and $t$, $C_l$ are $m \times m$ matrices, and $I(\eta_{it})$ is an indicator variable distinguishing between the effects of positive and negative shocks of equal magnitude on conditional volatility, such that

$$I(\eta_{it}) = \begin{cases} 0, & \varepsilon_{it} > 0 \\ 1, & \varepsilon_{it} \leq 0 \end{cases} \tag{7}$$

When $m = 1$, equation (4) reduces to the asymmetric univariate GARCH, or GJR, model of Glosten et al. (1992):

$$h_t = \omega + \sum_{j=1}^{r} \left( \alpha_j + \gamma_j I(\varepsilon_{t-j}) \right) \varepsilon_{t-j}^2 + \sum_{j=1}^{s} \beta_j h_{t-j} \tag{8}$$

For the underlying asymptotic theory, see McAleer et al. (2007) and, for an alternative asymmetric GARCH model, namely EGARCH, see Nelson (1991).

If $C_l = 0$, with $A_l$ and $B_l$ being diagonal matrices for all $l$, then VARMA-AGARCH reduces to:

$$h_{it} = \omega_i + \sum_{l=1}^{r} \alpha_l \varepsilon_{i,t-l} + \sum_{l=1}^{s} \beta_l h_{i,t-l} \tag{9}$$

which is the CCC model of Bollerslev (1990). As given in equation (7), the CCC model does not have volatility spillover effects across different financial assets, and hence is intrinsically univariate in nature. In addition, CCC also does not capture the asymmetric effects of positive and negative shocks on conditional volatility.

The parameters in model (1), (4), (6) and (9) can be obtained by maximum likelihood estimation (MLE) using a joint normal density, namely

$$\hat{\theta} = \arg\min_{\theta} \frac{1}{2} \sum_{t=1}^{n} \left( \log|Q_t| + \varepsilon_t' Q_t^{-1} \varepsilon_t \right) \tag{10}$$

where $\theta$ denotes the vector of parameters to be estimated on the conditional log-likelihood function, and $|Q_t|$ denotes the determinant of $Q_t$, the conditional covariance matrix. When $\eta_t$ does not follow a joint multivariate normal distribution, the appropriate estimators are defined as the Quasi-MLE (QMLE).

In order to forecast 1-day ahead conditional correlation, we use rolling windows technique and examine the time-varying nature of the conditional correlations using VARMA-GARCH and VARMA-AGARCH. Rolling windows are a recursive estimation procedure whereby the model is estimated for a restricted sample, then re-estimated by adding one observation at the end of the sample and deleting one observation from the beginning of the sample. The process is repeated until the end of the sample. In order to strike a balance between efficiency in estimation and a viable number of rolling regressions, the rolling window size is set at 2008 for all data sets.

## 3. Data

The univariate and multivariate GARCH models are estimated using 3,009 observations of daily data on crude oil spot, forward and futures prices in the Brent, WTI, Dubai and Tapis markets for the period 30 April 1997 to 10 November 2008. All prices are expressed in US dollars. In the WTI market, prices are crude oil-WTI spot cushing price ($/BBL), crude oil-WTI one-month forward price ($/BBL), and NYMEX one-month futures prices. The prices in the Brent market are crude oil-Brent spot price FOB ($/BBL), crude oil-Brent one-month forward price ($/BBL), and one-month futures prices. In the Dubai market, the prices are crude oil-Arab Gulf Dubai spot price FOB ($/BBL) and crude oil-Dubai one-month forward price ($/BBL). In the Tapis market, the prices are crude oil-Malaysia Tapis spot price FOB ($/BBL) and crude oil-Tapis one-month forward price ($/BBL). Three series are obtained from DataStream database service, while the series for Tapis are collected from Reuters.

The synchronous price returns $i$ for each market $j$ are computed on a continuous compounding basis as the logarithm of closing price at the end of the period minus the logarithm of the closing price at the beginning of the period, which is defined as

$$r_{ij,t} = \log\left(P_{ij,t}/P_{ij,t-1}\right).$$

Table 1 presents the descriptive statistics for the returns series of crude oil prices. The average return of spot, forward and futures in Brent, WTI and Dubai are similar, while Tapis has the lowest average returns. The normal distribution has a skewness statistic equal to zero and a kurtosis statistic of 3, but these crude oil returns series have high kurtosis, suggesting the presence of fat tails, and negative skewness statistics, signifying the series has a longer left tail (extreme losses) than right tail (extreme gain). The Jarque-Bera Lagrange multiplier statistics of the crude oil returns in each market are statistically significant, thereby signifying that the distributions of these prices are not normal, which may be due to the presence of extreme observations. Brent and WTI returns are more volatile than those of Dubai/Oman and Tapis, as shown by the estimates of their respective standard errors. This may be explained by the fact that light sweet crude oil is less plentiful and in greater demand than the more sour and heavier grades, or due to the presence of different regulatory restrictions in these markets. It also seems that the forward returns are less volatile than those of spot and futures (if they exist) prices, with the exception of Tapis. This has to do with the nature and characteristics of the forward contracts relative to those of the spot and futures contracts

Figure 1 presents the plot of synchronous crude oil price returns. These indicate volatility clustering or period of high volatility followed by periods of tranquility, such that crude oil returns oscillate in a range smaller than the normal distribution. However, there are some circumstances where crude oil returns fluctuate in a much wider scale than is permitted under normality.

The unit root tests for all crude oil returns in each market are summarized in Table 2. The Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests are used to test the null hypothesis of a unit root against the alternative hypothesis of stationarity. The tests yield large negative values in all cases for levels, such that the individual returns series reject the null hypothesis at the 1% significance level, so that all returns series are stationary.

Since the univariate ARMA-GARCH is nested in the VARMA-GARCH model, and ARMA-GJR is nested in VARMA-AGARCH, with conditional variance specified in (5) and (8), the univariate ARMA-GARCH and ARMA-GJR models are estimated. It is sensible to extend univariate models to their multivariate counterparts if the regularity conditions of

univariate models are satisfied, so that the QMLE will be consistent and asymptotically normal. All estimation is conducted using the EViews 6 econometric software package.

## 4. Empirical Results

From Tables 3 and 4, the univariate ARMA(1,1)-GARCH(1,1) and ARMA (1,1)-GJR(1,1) models are estimated to check whether the conditional variance follows the GARCH process. In Table 3, not all the coefficients in mean equations of ARMA(1,1)-GARCH(1,1) are significant, whereas all the coefficients in the conditional variance equation are statistically significant. Table 4 shows that the long-run coefficients are all statistically significant in the variance equation, but rbrefu (brent futures return), rwtisp (WTI spot return), rwtifor (WTI forward return), rtapsp (Tapis spot return), and rtapfor (Tapis forward return) are only significant in the short run. In addition, the asymmetric effects of negative and positive shocks on the conditional variance are generally statistically significant.

### [**Insert Tables 3-5 here**]

In order to check the sufficient condition for consistency and asymptotic normality of the QMLE for GARCH and GJR model, the second moment conditions are $\alpha_1 + \beta_1 < 1$ and $\alpha_1 + (\gamma/2) + \beta_1 < 1$, respectively. Table 5 shows that all of the estimated second moment conditions are less than one. In order to derive the statistical properties of the QMLE, Lee and Hausen (1997) derived the log-moment condition for GARCH(1,1) as $E\left(\log\left(\alpha_1\eta_t^2 + \beta_1\right)\right) < 0$, while McAleer et al. (2007) established the log-moment condition for GJR(1,1) as $E\left(\log\left(\alpha_1 + \gamma_1 I(\eta_t)\eta_t^2 + \beta_1\right)\right) < 0$. Table 5 shows that the estimated log-moment condition for both models is satisfied for all returns. The high persistence of volatility shown in Table 5 can be explained be the reinforcing mechanism between oil inventories and the oil basis = (futures – spot).

For the spot, forward and futures returns in the four crude oil markets, there are ten series of returns to be analyzed. Consequently, 45 bivariate models need to be estimated. The calculated constant conditional correlations between the volatility of two returns within and across markets using the CCC model and the Bollerslev and Wooldridge (1992) robust *t*-ratios are presented in Table 6. The highest estimated constant conditional correlation is

0.935, namely between the standardized shocks in Brent spot returns (rbresp) and Brent forward returns (rbrefor).

[**Insert Tables 6 here**]

Corresponding multivariate estimates of the conditional variances from the VARMA(1,1)-GARCH(1,1) and VARMA(1,1)-AGARCH(1,1) models are also estimated. The estimates of volatility and asymmetric spillovers are presented in Table 7, which shows that volatility spillovers for VARMA-GARCH and VARMA-AGARCH are evident in 32 and 31 of 45 cases, respectively. The significant interdependences in the conditional volatility among returns hold for 3 of 45 cases for both VARMA-GARCH and VARMA-AGARCH. In addition, asymmetric effects are evident in 27 of 45 cases. Consequently, the evidence of volatility spillovers and asymmetric effects of negative and positive shocks on the conditional variance suggest that VARMA-AGARCH is superior to the VARMA-GARCH and CCC models.

[**Insert Tables 7 here**]

The estimates of the conditional variances based on the VARMA-GARCH and VARMA-AGARCH models reported in Table 7 suggest the presence of volatility spillovers between Brent and WTI returns, namely volatility spillovers from Brent futures returns to Brent spot and forward returns, from Brent spot returns to WTI spot returns, and from WTI futures returns to Brent spot returns. In addition, the results show that most of the Dubai and Tapis returns have volatility spillover effects from Brent and WTI returns. This evidence is in agreement with the knowledge that the Brent and WTI markets are two "marker" crudes that set crude oil prices and influence the other crude oil markets.

[**Insert Figure 2 here**]

The conditional correlation forecasts are obtained from a rolling window technique. Figure 2 plots the dynamic paths of the conditional correlations derived from VARMA-GARCH and VARMA-AGARCH. All the conditional correlations display significant variability, which suggests that the assumption of constant conditional correlation is not valid. It is interesting to note that the correlations are positive for all pairs of crude oil returns,

and rtapsp_rtapfor has the highest correlation, at 0.98. In addition, the conditional correlation forecasts of some pairs of crude oil returns exhibit an upward trend in 22 of 45 cases, and a downward trend in 20 of 45 cases. This evidence should also be considered in diversifying a portfolio containing these assets.

## 5. Implications for Portfolio Design and Hedging Strategies

This section presents optimal hedge ratios and optimal portfolio weights among crude oil returns and across markets. Theoretically, hedging involves the determination of the optimal hedge ratio. One of the most widely used hedging strategies is based on the minimization of the variance of the portfolio, the so-called minimum variance hedge ratio (see, for example, Kroner and Sultan (1993), Lien and Tse (2002), and Chen et al. (2003)). In order to minimize risk, the dynamic hedge ratio, based on conditional information available at $t$, is given by:

$$\beta_{12,t} = \frac{h_{12,t}}{h_{22,t}}$$

(11)

where $\beta_{12,t}$ is the risk-minimizing hedge ratio for two crude oil assets, $h_{12,t}$ is the conditional covariance between crude oil assets 1 and 2, and $h_{22,t}$ is the conditional variance of crude oil asset 2. In order to minimize risk, a long position of one dollar taken in one crude oil asset should be hedged by a short position of $\$\beta_t$ in another crude oil asset at time $t$ (Hammoudeh et al. (2009)).

The average values of the optimal hedge ratio ($\beta_t$) using estimates from the VARMA-GARCH model are presented in the first column of Table 8. By following the estimated hedge strategy, the highest average optimal hedge ratio is 0.956 (rwtisp/rwtifu), meaning one dollar long in WTI spot should be shorted by 95 cents in WTI futures. The lowest average optimal hedge ratio is 0.125 (rtapfor/rwtifor), meaning one dollar long in Tapis forward should be shorted by 12 cents in WTI forward. Interestingly, we find that the average optimal hedge ratio across markets, namely Dubai and WTI, Tapis and Brent, and Tapis and WTI, are very low, signifying one dollar long in the first market should be shorted by only a few cents in the second market.

In the case of optimal portfolio weights, the estimated covariance matrices from the VARMA-GARCH model are used to compute the optimal portfolio holdings that minimize portfolio risk, assuming the expected returns are zero. Applying the methods of Kroner and Ng (1998), the optimal portfolio weight of crude oil asset 1/2 holding ($w_{12,t}$) is given by:

$$w_{12,t} = \frac{h_{22,t} - h_{12,t}}{h_{11,t} - 2h_{12,t} + h_{22,t}} \tag{12}$$

and

$$w_{12,t} = \begin{cases} 0, & \text{if } w_{12,t} < 0 \\ w_{12,t}, & \text{if } 0 < w_{12,t} < 0 \\ 1, & \text{if } w_{12,t} > 0 \end{cases} \tag{13}$$

where $w_{12,t}$, is the portfolio weight of the first asset relative to the second asset at time $t$. The average of the weights $w_{12,t}$ means the optimal portfolio holdings for the first asset should be $w_{12,t}$ cents to a dollar. Obviously, the optimal portfolio holding for the second asset would be $(1 - w_{12,t})$ to a dollar.

The average values of $w_{12,t}$ based on the VARMA-GARCH estimates are presented in the second column of Table 8. For instance, the highest average optimal hedge ratio is 0.968 (rbrefor/rbresp), suggesting that the optimal holding of Brent forward in one dollar of forward/spot for Brent market is 97 cents, compared with 3 cents for Brent spot. These optimal portfolio weights suggest that investors should have much more Brent forward than Brent spot in their portfolio. Surprisingly, the average optimal portfolio weights across markets, namely Dubai and Brent, Dubai and WTI, Tapis and Brent, and Tapis and WTI, suggest that investors should own WTI and Brent (the light sweet grade category) in greater proportions than Dubai and Tapis (the heavier and less sweet grade category).

## 6. Conclusion

The empirical analysis in the paper examined the spillover effects in the returns on spot, forward and futures prices of four major benchmarks in the international oil market, namely West Texas Intermediate (USA), Brent (North Sea), Dubai/Oman (Middle East) and

Tapis (Asia-Pacific), for the period 30 April 1997 to 10 November 2008. Alternative multivariate conditional volatility models were used, namely the CCC model of Bollerslev (1990), VARMA-GARCH model of Ling and McAleer (2003), and VARMA-AGARCH model of McAleer et al (2009). Both the ARCH and GARCH estimates were significant for all returns in the ARMA(1,1)-GARCH(1,1) models. However, in case of the ARMA(1,1)-GJR(1,1) models, only the GARCH estimates were statistically significant, and most of the estimates of the asymmetric effects were significant. Based on the asymptotic standard errors, the VARMA-GARCH and VARMA-AGARCH models showed evidence of volatility spillovers and asymmetric effects of negative and positive shocks on the conditional variances, which suggested that VARMA-AGARCH was superior to both VARMA-GARCH and CCC.

The paper also presented some volatility spillover effects from Brent and WTI returns, and from the Brent and WTI crude oil markets to the Dubai and Tapis markets, which confirmed that the Brent and WTI crude oil markets are the world references for crude oil. The paper also compared 1-day ahead conditional correlation forecasts from the VARMA-GARCH and VARMA-AGARCH models using the rolling window approach, and showed that the conditional correlation forecasts exhibited both upward trend and downward trends. In order to design optimal portfolio holdings across two crude oil grade categories, the optimal portfolio weights suggest holding the light sweet grade category (WTI and Brent) in a greater proportion than the heavier and less sweet grade category (Dubai and Tapis). In the case of minimizing risk by using a hedge, a long position of one dollar in the light sweet grade category (WTI) should be shorted by only a few cents in the heavier and less sweet grade category (Dubai and Tapis).

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# Table 1

## Descriptive Statistics for Crude Oil Price Returns

| Returns | Mean | Max | Min | S.D. | Skewness | Kurtosis | Jarque-Bera |
|---------|------|-----|-----|------|----------|----------|-------------|
| rbresp | 0.043 | 15.164 | -12.601 | 2.347 | -0.0007 | 5.341 | 686.6157 |
| rbrefor | 0.043 | 12.044 | -12.534 | 2.146 | -0.141 | 4.939 | 480.941 |
| rbrefu | 0.043 | 12.898 | -10.946 | 2.212 | -0.124 | 4.934 | 476.538 |
| rwtisp | 0.043 | 15.873 | -13.795 | 2.412 | -0.129 | 6.479 | 1524.764 |
| rwtifor | 0.042 | 13.958 | -12.329 | 2.316 | -0.182 | 5.204 | 625.414 |
| rwtifu | 0.043 | 14.546 | -12.939 | 2.349 | -0.151 | 6.318 | 1390.425 |
| rdubsp | 0.043 | 14.705 | -12.943 | 2.199 | -0.179 | 5.844 | 1029.861 |
| rdubfor | 0.040 | 13.767 | -12.801 | 2.115 | -0.308 | 5.718 | 973.0103 |
| rtapsp | 0.038 | 11.081 | -10.483 | 2.000 | -0.183 | 5.373 | 722.053 |
| rtapfor | 0.038 | 12.071 | -12.869 | 2.076 | -0.289 | 5.567 | 867.187 |

# Table 2

## Unit Root Tests for Returns

| Returns | ADF test | | | Phillips-Perron test | | |
|---------|----------|----------|---------------------|----------|----------|---------------------|
| | None | Constant | Constant and Trend | None | Constant | Constant and Trend |
| rbresp | -54.264$^*$ | -54.274$^*$ | -54.265$^*$ | -54.301$^*$ | -54.298$^*$ | -54.291$^*$ |
| rbrefor | -57.076$^*$ | -57.092$^*$ | -57.083$^*$ | -57.088$^*$ | -57.100$^*$ | -57.091$^*$ |
| rbrefu | -57.944$^*$ | -57.958$^*$ | -57.949$^*$ | -57.901$^*$ | -57.919$^*$ | -57.909$^*$ |
| rwtisp | -41.065$^*$ | -41.079$^*$ | -41.073$^*$ | -55.652$^*$ | -55.677$^*$ | -55.667$^*$ |
| rwtifor | -56.618$^*$ | -56.626$^*$ | -56.617$^*$ | -56.697$^*$ | -56.715$^*$ | -56.705$^*$ |
| rwtifu | -55.872$^*$ | -55.881$^*$ | -55.872$^*$ | -56.011$^*$ | -56.030$^*$ | -56.020$^*$ |
| rdubsp | -59.130$^*$ | -59.145$^*$ | -59.135$^*$ | -59.090$^*$ | -59.129$^*$ | -59.119$^*$ |
| rdubfor | -59.664$^*$ | -59.677$^*$ | -59.667$^*$ | -59.542$^*$ | -59.573$^*$ | -59.564$^*$ |
| rtapsp | -59.059$^*$ | -59.072$^*$ | -59.062$^*$ | -58.955$^*$ | -58.956$^*$ | -58.947$^*$ |
| rtapfor | -59.949$^*$ | -59.961$^*$ | -59.951$^*$ | -59.747$^*$ | -59.775$^*$ | -59.766$^*$ |

Note: * denotes significance at the 1% level.

**Table 3**

**Univariate ARMA(1,1)-GARCH(1,1)**

| Returns | Mean equation | | | Variance equation | | |
|---|---|---|---|---|---|---|
| | C | AR(1) | MA(1) | $\omega$ | $\hat{\alpha}$ | $\hat{\beta}$ |
| rbresp | 0.088 | -0.981 | 0.988 | 0.069 | 0.039 | 0.949 |
| | 2.179* | -95.091* | 119.046* | 2.585* | 4.292* | 83.066* |
| rbrefor | 0.084 | 0.236 | -0.277 | 0.084 | 0.042 | 0.940 |
| | 2.407* | 0.596 | -0.707 | 2.708* | 4.281* | 68.425* |
| rbrefu | 0.081 | 0.092 | -0.141 | 0.062 | 0.042 | 0.946 |
| | 2.281* | 0.259 | -0.399 | 2.396* | 4.451* | 77.153* |
| rwtisp | 0.072 | -0.949 | 0.955 | 0.101 | 0.046 | 0.938 |
| | 1.698 | -18.055* | 19.298* | 2.502* | 3.698* | 58.264* |
| rwtifor | 0.078 | 0.350 | -0.387 | 0.144 | 0.055 | 0.919 |
| | 2.063 | 0.888 | -0.998 | 2.731* | 4.448* | 48.541* |
| rwtifu | 0.085 | -0.971 | 0.969 | 0.189 | 0.065 | 0.902 |
| | 2.142* | -32.149* | 30.750* | 2.971* | 3.633* | 36.669* |
| rdubsp | 0.090 | 0.019 | -0.099 | 0.048 | 0.049 | 0.942 |
| | 2.771* | 0.083 | -0.434 | 2.303* | 5.355* | 85.548* |
| rdubfor | 0.086 | 0.052 | -0.134 | 0.061 | 0.048 | 0.939 |
| | 2.696* | 0.227 | -0.593 | 2.571* | 4.331* | 69.601* |
| rtapsp | 0.067 | 0.153 | -0.211 | 0.076 | 0.047 | 0.935 |
| | 2.217* | 0.493 | -0.687 | 2.419* | 3.818* | 53.855* |
| rtapfor | 0.058 | 0.173 | -0.246 | 0.056 | 0.041 | 0.946 |
| | 1.856 | 0.742 | -1.072 | 2.618* | 4.314* | 80.476* |

Notes: (1) The two entries for each parameter are their respective parameter estimate and the Bollerslev and Wooldridge (1992) robust $t$- ratios.

(2) * denotes significance at the 1% level.

**Table 4**

**Univariate ARMA(1,1)-GJR (1,1)**

| Returns | Mean equation | | | Variance equation | | | |
|---|---|---|---|---|---|---|---|
| | C | AR(1) | MA(1) | $\omega$ | $\hat{\alpha}$ | $\hat{\gamma}$ | $\hat{\beta}$ |
| rbresp | 0.054 | -0.981 | 0.988 | 0.069 | 0.0116 | 0.042 | 0.955 |
| | 1.367 | -91.730* | 114.293* | 2.5514* | 0.974 | 2.792* | 85.638* |
| rbrefor | 0.063 | 0.178 | -0.224 | 0.086 | 0.019 | 0.035 | 0.944 |
| | 1.814 | 0.454 | -0.573 | 2.687* | 1.498 | 2.419* | 68.125* |
| rbrefu | 0.069 | 0.059 | -0.111 | 0.059 | 0.029 | 0.017 | 0.951 |
| | 1.942 | 0.169 | -0.318 | 2.349* | 2.329* | 1.252 | 79.661* |
| rwtisp | 0.059 | 0.954 | -0.963 | 0.597 | 0.064 | 0.059 | 0.802 |
| | 1.730 | 17.911* | -19.727* | 3.814* | 2.104* | 1.782 | 18.291* |
| rwtifor | 0.058 | 0.3439 | -0.385 | 0.137 | 0.029 | 0.035 | 0.927 |
| | 1.560 | 0.9369 | -1.068 | 2.772* | 2.046* | 2.069 | 53.349* |
| rwtifu | 0.060 | -0.9709 | 0.969 | 0.187 | 0.039 | 0.042 | 0.905 |
| | 1.521 | -30.237* | 29.056* | 3.054* | 1.812 | 1.964* | 37.680* |
| rdubsp | 0.064 | 0.034 | -0.117 | 0.052 | 0.022 | 0.036 | 0.949 |
| | 1.970* | 0.154 | -0.539 | 2.579* | 1.797 | 2.445* | 89.095* |
| rdubfor | 0.065 | 0.049 | -0.135 | 0.069 | 0.023 | 0.034 | 0.944 |
| | 2.031* | 0.221 | -0.616 | 2.699* | 1.566 | 2.229* | 63.537* |
| rtapsp | 0.052 | 0.1438 | -0.199 | 0.072 | 0.019 | 0.037 | 0.944 |
| | 1.661 | 0.445 | -0.628 | 2.886* | 2.037* | 2.665* | 70.250* |
| rtapfor | 0.043 | 0.169 | -0.242 | 0.055 | 0.017 | 0.032 | 0.953 |
| | 1.372 | 0.724 | -1.053 | 3.132* | 2.045* | 2.457* | 107.102* |

Notes: (1) The two entries for each parameter are their respective parameter estimates and Bollerslev and Wooldridge (1992) robust *t*- ratios.

(2) * denotes significance at the 1% level.

## Table 5

## Log-moment and Second Moment Conditions for the

## ARMA(1,1)-GARCH(1,1) and ARMA(1,1)-GJR(1,1) models

| Returns | ARMA-GARCH | | ARMA-GJR | |
|---|---|---|---|---|
| | Log-Moment | Second moment | Log-Moment | Second moment |
| rbresp | -0.0060 | 0.988 | -0.0058 | 0.987 |
| rbrefor | -0.0087 | 0.982 | -0.0084 | 0.980 |
| rbrefu | -0.0061 | 0.988 | -0.0050 | 0.988 |
| rwtisp | -0.0089 | 0.984 | -0.0492 | 0.895 |
| rwtifor | -0.0131 | 0.974 | -0.0114 | 0.973 |
| rwtifu | -0.0173 | 0.967 | -0.0153 | 0.965 |
| rdubsp | -0.0051 | 0.991 | -0.0048 | 0.989 |
| rdubfor | -0.0068 | 0.987 | -0.0069 | 0.984 |
| rtapsp | -0.0093 | 0.982 | -0.0082 | 0.982 |
| rtapfor | -0.0063 | 0.987 | -0.0056 | 0.986 |

**Table 6**

**Constant Conditional Correlation for CCC-GARCH(1-1) Model**

| Returns | rbresp | rbrefor | rbrefu | rwtisp | rwtifor | rwtifu | rdubsp | rdubfor | rtapsp | rtapfor |
|---|---|---|---|---|---|---|---|---|---|---|
| rbresp | 1.000 | **0.935** (126.157) | **0.762** (74.699) | **0.696** (57.939) | **0.756** (87.222) | **0.713** (61.139) | **0.576** (45.118) | **0.586** (57.787) | **0.259** (13.994) | **0.254** (14.047) |
| rbrefor | | 1.000 | **0.778** (75.679) | **0.723** (66.055) | **0.786** (99.892) | **0.740** (64.702) | **0.740** (64.702) | **0.609** (44.895) | **0.263** (16.679) | **0.253** (14.199) |
| rbrefu | | | 1.000 | **0.824** (148.267) | **0.839** (90.429) | **0.843** (104.926) | **0.430** (37.236) | **0.443** (22.395) | **0.187** (11.102) | **0.176** (10.188) |
| rwtisp | | | | 1.000 | **0.873** (108.318) | **0.920** (199.900) | **0.390** (22.564) | **0.398** (18.390) | **0.176** (9.418) | **0.161** (8.286) |
| rwtifor | | | | | 1.000 | **0.902** (160.272) | **0.421** (20.303) | **0.437** (24.507) | **0.126** (6.294) | **0.115** (6.329) |
| rwtifu | | | | | | 1.000 | **0.403** (19.881) | **0.410** (21.240) | **0.176** (10.239) | **0.164** (9.031) |
| rdubsp | | | | | | | 1.000 | **0.958** (169.158) | **0.466** (19.442) | **0.455** (20.383) |
| ubfor | | | | | | | | 1.000 | **0.468** (22.445) | **0.457** (16.468) |
| rtapsp | | | | | | | | | 1.000 | **0.930** (139.082) |
| rtapfor | | | | | | | | | | 1.000 |

Notes:  (1) The two entries for each parameter are their respective estimated conditional correlation and Bollerslev and Wooldridge (1992) robust *t*- ratios.

  (2) Bold denotes significance at the 5% level.

**Table 7**

**Summary of Volatility Spillovers and Asymmetric Effects of Negative and Positive Shocks**

| No. | Returns | Number of volatility spillovers | | Number of Asymmetric effects |
|---|---|---|---|---|
| | | VARMA-GARCH | VARMA-GJR | |
| 1 | rbresp_rbrefor | 0 | 0 | 1 |
| 2 | rbresp_rbrefu | 1($\leftarrow$) | 1($\leftarrow$) | 0 |
| 3 | rbrefor_rbrefu | 1($\leftarrow$) | 1($\leftarrow$) | 0 |
| 4 | rbresp_rwtisp | 1($\rightarrow$) | 1($\rightarrow$) | 1 |
| 5 | rbrefor_rwtisp | 0 | 0 | 1 |
| 6 | rbrefu_rwtisp | 0 | 0 | 0 |
| 7 | rbresp_rwtifor | 0 | 0 | 1 |
| 8 | rbrefor_rwtifor | 0 | 0 | 1 |
| 9 | rbrefu_rwtifor | 0 | 0 | 0 |
| 10 | rwtisp_rwtifor | 0 | 0 | 0 |
| 11 | rbresp_rwtifu | 1($\leftarrow$) | 1($\leftarrow$) | 1 |
| 12 | rbrefor_rwtifu | 0 | 0 | 1 |
| 13 | rbrefu_rwtifu | 0 | 0 | 0 |
| 14 | rwtisp_rwtifu | 0 | 0 | 0 |
| 15 | rwtifor_rwtifu | 1($\leftarrow$) | 0 | 0 |
| 16 | rbresp_rdubsp | 0 | 0 | 2 |
| 17 | rbrefor_rdubsp | 1($\rightarrow$) | 1($\rightarrow$) | 1 |
| 18 | rbrefu_rdubsp | 0 | 1($\rightarrow$) | 0 |
| 19 | rwtisp_rdubsp | 2($\leftrightarrow$) | 2($\leftrightarrow$) | 1 |
| 20 | rwtifor_rdubsp | 1($\rightarrow$) | 1($\rightarrow$) | 1 |
| 21 | rwtifu_rdubsp | 1($\rightarrow$) | 1($\rightarrow$) | 1 |
| 22 | rbresp_rdubfor | 1($\rightarrow$) | 1($\rightarrow$) | 0 |
| 23 | rbrefor_rdubfor | 1($\rightarrow$) | 1($\rightarrow$) | 0 |
| 24 | rbrefu_rdubfor | 1($\rightarrow$) | 1($\rightarrow$) | 0 |
| 25 | rwtisp_rdubfor | 1($\leftarrow$) | 1($\leftarrow$) | 1 |
| 26 | rwtifor_rdubfor | 1($\rightarrow$) | 1($\rightarrow$) | 0 |
| 27 | rwtifu_rdubfor | 1($\rightarrow$) | 1($\rightarrow$) | 0 |
| 28 | rdubsp_rdubfor | 1($\rightarrow$) | 0 | 1 |
| 29 | rbresp_rtapsp | 1($\rightarrow$) | 1($\rightarrow$) | 2 |
| 30 | rbrefor_rtapsp | 1($\rightarrow$) | 1($\rightarrow$) | 2 |
| 31 | rbrefu_rtapsp | 1($\rightarrow$) | 1($\rightarrow$) | 1 |
| 32 | rwtisp_rtapsp | 2($\leftrightarrow$) | 2($\leftrightarrow$) | 1 |
| 33 | rwtifor_rtapsp | 1($\rightarrow$) | 1($\rightarrow$) | 1 |
| 34 | rwtifu_rtapsp | 1($\rightarrow$) | 1($\rightarrow$) | 1 |
| 35 | rdubsp_rtapsp | 1($\rightarrow$) | 1($\rightarrow$) | 2 |
| 36 | rdubfor_rtapsp | 1($\rightarrow$) | 1($\rightarrow$) | 2 |
| 37 | rbresp_rtapfor | 1($\rightarrow$) | 1($\rightarrow$) | 1 |
| 38 | rbrefor_rtapfor | 1($\rightarrow$) | 1($\rightarrow$) | 1 |
| 39 | rbrefu_rtapfor | 1($\rightarrow$) | 1($\rightarrow$) | 0 |
| 40 | rwtisp_rtapfor | 2($\leftrightarrow$) | 2($\leftrightarrow$) | 0 |
| 41 | rwtifor_rtapfor | 0 | 0 | 0 |
| 42 | rwtifu_rtapfor | 1($\rightarrow$) | 1($\rightarrow$) | 0 |
| 43 | rdubsp_rtapfor | 1($\rightarrow$) | 1($\rightarrow$) | 1 |
| 44 | rdubfor_rtapfor | 1($\rightarrow$) | 1($\rightarrow$) | 1 |
| 45 | rtapsp_rtapfor | 1($\rightarrow$) | 1($\rightarrow$) | 1 |

Notes: The symbols $\rightarrow$ ($\leftarrow$) indicate the direction of volatility spillovers from A returns to B returns (B returns to A returns), $\leftrightarrow$ means they are interdependent, and 0 means there are no volatility spillovers between pairs of returns.

## Table 8

### Summary of Volatility Spillovers and Asymmetric Effects of Negative and Positive Shocks

| No. | Portfolio | Average Optimal Hedge Ratio $(\gamma_t)$ | Optimal Portfolio Weights $(w_{12,t})$ of first crude oil return in 1\$ portfolio |
|---|---|---|---|
| 1 | rbrefor/rbresp | 0.870 | 0.968 |
| 2 | rbresp/rbrefu | 0.864 | 0.342 |
| 3 | rbrefor/rbrefu | 0.806 | 0.601 |
| 4 | rbresp/rwtisp | 0.726 | 0.519 |
| 5 | rwtisp/rbrefor | 0.859 | 0.299 |
| 6 | rwtisp/rbrefu | 0.917 | 0.301 |
| 7 | rbresp/rwtifor | 0.808 | 0.463 |
| 8 | rbrefor/rwtifor | 0.769 | 0.714 |
| 9 | rbrefu/rwtifor | 0.817 | 0.661 |
| 10 | rwtisp/rwtifor | 0.917 | 0.409 |
| 11 | rbresp/rwtifu | 0.761 | 0.476 |
| 12 | rbrefor/rwtifu | 0.722 | 0.671 |
| 13 | rbrefu/rwtifu | 0.818 | 0.662 |
| 14 | rwtisp/rwtifu | 0.956 | 0.383 |
| 15 | rwtifor/rwtifu | 0.920 | 0.514 |
| 16 | rdubsp/rbresp | 0.537 | 0.725 |
| 17 | rdubsp/rbrefor | 0.643 | 0.650 |
| 18 | rdubsp/rbrefu | 0.436 | 0.676 |
| 19 | rdubsp/rwtisp | 0.354 | 0.685 |
| 20 | rdubsp/rwtifor | 0.387 | 0.705 |
| 21 | rdubsp/rwtifu | 0.375 | 0.688 |
| 22 | rdubfor/rbresp | 0.794 | 0.773 |
| 23 | rdubfor/rbrefor | 0.633 | 0.698 |
| 24 | rdubfor/rbrefu | 0.420 | 0.707 |
| 25 | rdubfor/rwtisp | 0.341 | 0.715 |
| 26 | rdubfor/rwtifor | 0.379 | 0.733 |
| 27 | rdubfor/rwtifu | 0.356 | 0.713 |
| 28 | rdubsp/rdubfor | 0.932 | 0.818 |
| 29 | rtapsp/rbresp | 0.220 | 0.819 |
| 30 | rtapsp/rbrefor | 0.266 | 0.812 |
| 31 | rtapsp/rbrefu | 0.192 | 0.853 |
| 32 | rtapsp/rwtisp | 0.152 | 0.828 |
| 33 | rtapsp/rwtifor | 0.136 | 0.845 |
| 34 | rtapsp/rwtifu | 0.157 | 0.836 |
| 35 | rtapsp/rdubsp | 0.553 | 0.732 |
| 36 | rtapsp/rdubfor | 0.572 | 0.714 |
| 37 | rtapfor/rbresp | 0.462 | 0.737 |
| 38 | rtapfor/ rbrefor | 0.272 | 0.712 |
| 39 | rtapfor/rbrefu | 0.197 | 0.770 |
| 40 | rtapfor/rwtisp | 0.151 | 0.755 |
| 41 | rtapfor/rwtifor | 0.125 | 0.759 |
| 42 | rtapfor/rwtifu | 0.155 | 0.762 |
| 43 | rdubsp/tapfor | 0.487 | 0.640 |
| 44 | rtapfor/rdubfor | 0.506 | 0.617 |
| 45 | rtapsp/rtapfor | 0.746 | 0.689 |

Notes: Average $(\gamma_t)$ is the risk-minimizing hedge ratio for two crude oil assets. $(w_{12,t})$ is the portfolio weight of two assets at time $t$.

# Figure 1
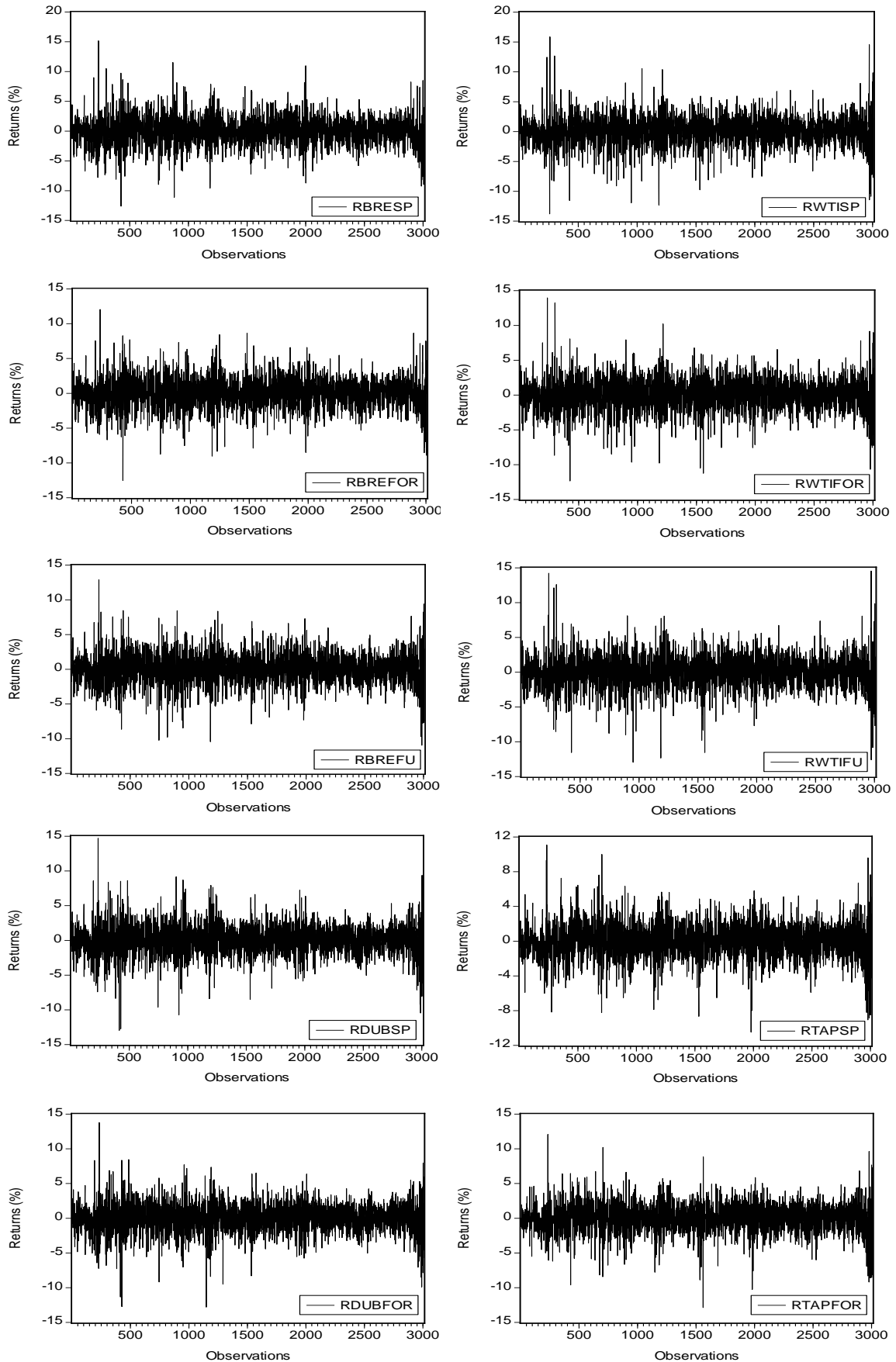## Logarithm of daily spot, forward and futures of Brent, WTI, Dubai and Tapis

**Figure 2**
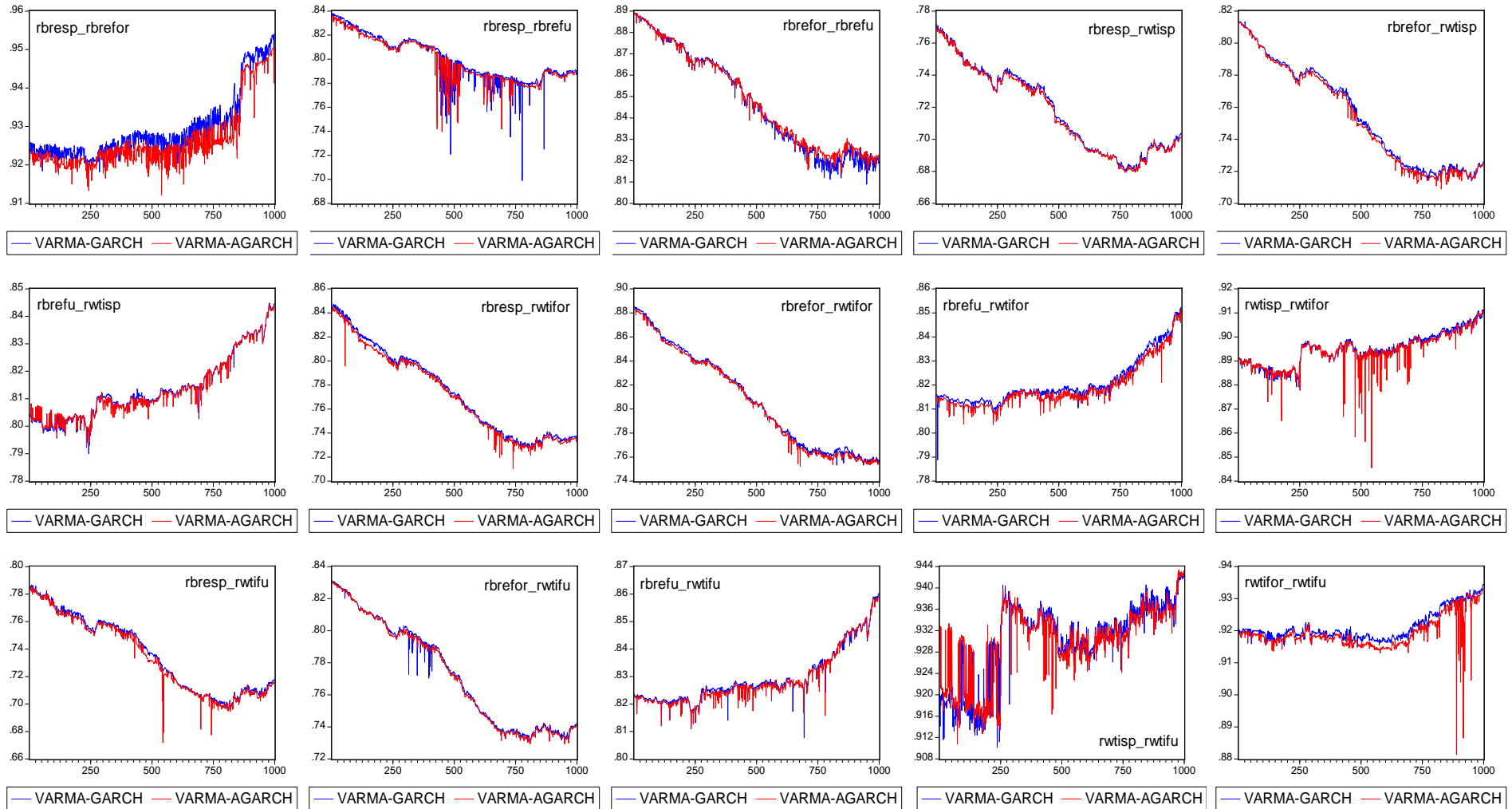**Forecasts of the conditional correlations between pair of returns from the VARMA-GARCH and VARMA-AGARCH**
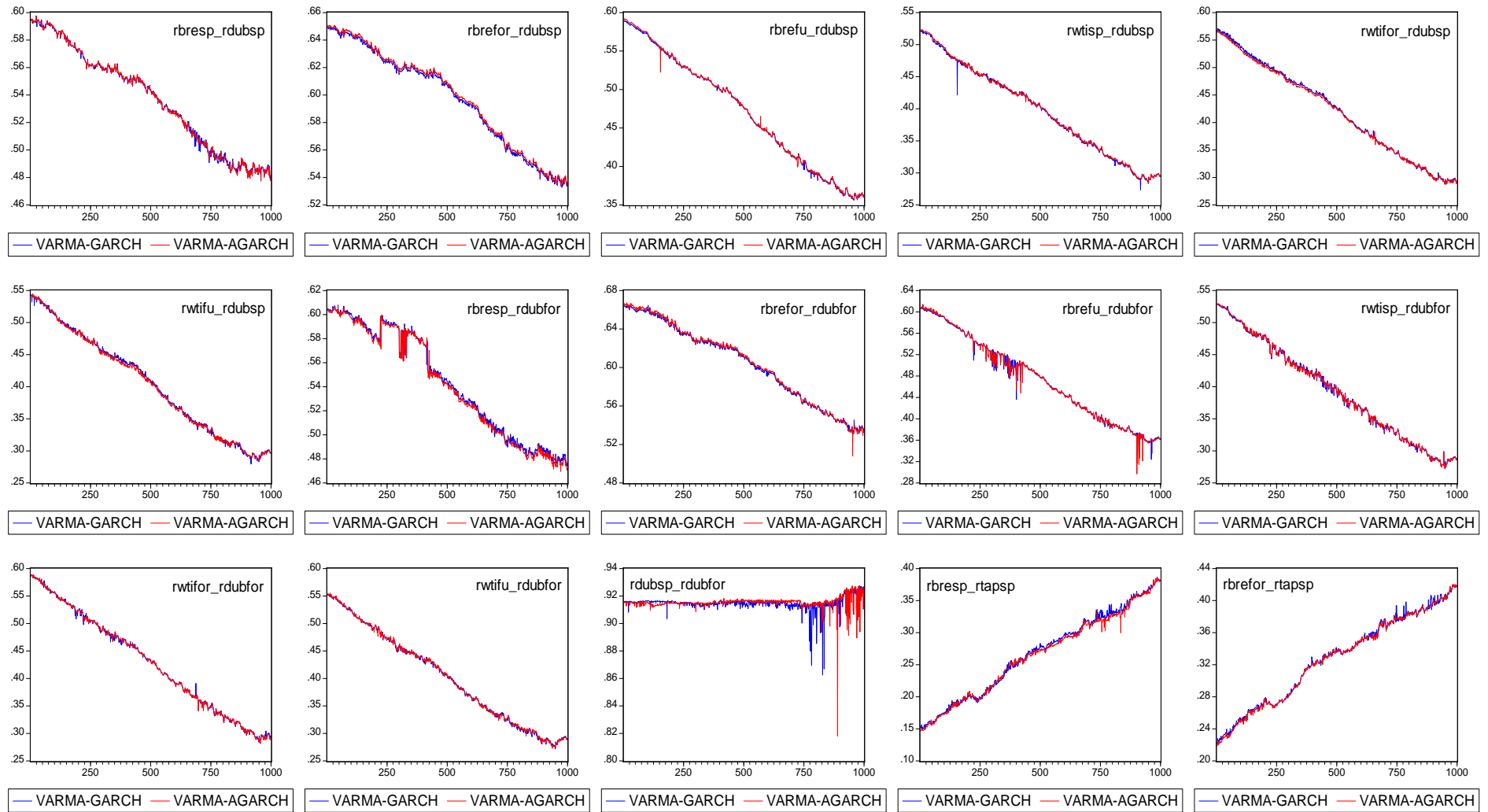
25

**Figure 2 (*continued*)**

**Figure 2 (*continued*)**