CIRJE-F-767

Financial Institution, Asset Bubbles and Economic Performance

Tomohiro Hirano
Financial Services Agency

Noriyuki Yanagawa
University of Tokyo

October 2010

Financial Institution, Asset Bubbles and Economic Performance

Tomohiro Hirano*and Noriyuki Yanagawa†

October 15, 2010

Abstract

This paper explores the relation between the quality of financial institution and asset bubbles. In this paper, we will show that bubbles can improve the macro performance even if the quality of financial institution is very poor and the financial market does not work well. In this sense, the high quality of financial institution and bubbles are substitutes. We will explore, however, that they are not perfect substitutes. Bubbles may burst. If bubbles burst, the economic performance must go down if the quality of financial institution is low. Hence, we will show that not relaying on bubbles, but improving the quality of financial institution is important for long run macro performance.

Key words: Asset Bubbles, Financial Institution and burst of bubbles

JEL code: E44

*Tomohiro Hirano, Financial Research and Training Center, Financial Services Agency, The Japanese Government Email: tomohih@gmail.com

†Noriyuki Yanagawa, Faculty of Economics, The University of Tokyo, Email: yanagawa@e.u-tokyo.ac.jp

1 Introduction

This paper explores the relation between the quality of financial institution and asset bubbles. It is now well-known that the quality of financial institution is important for macro economic performance, but it is not yet obvious how bubbles affect the relation between the quality of financial institution and economic performance. Do those bubbles may deter the macro performance even when the quality of financial institution is better or do they improve the macro performance even if the quality of financial institution is relatively low? In this paper, we are going to show that bubbles can improve the macro performance even if the quality of financial institution is very poor and the financial market does not work well. In this sense, the high quality of financial institution and bubbles are substitutes. We will explore, however, that they are not perfect substitutes. Bubbles may burst. If bubbles burst, the economic performance must go down if the quality of financial institution is low. Hence, we will show that not relaying on bubbles, but improving the quality of financial institution is important for long run macro performance.

It is recognized that quality of financial institution is an important factor for macro economic performance. More theoretically speaking, if there are asymmetric information problems or enforcement problems, the financial market does not work well. If the quality of financial institution is low and financial market is imperfect, the resource of this economy is not allocated appropriately and macro performance must not work well. There are many papers which treated this relation. For example, Pagano (1993), Levine (1997) are the survey papers which examined this relation.

Those papers, however, do not examine the roles of asset bubbles explicitly. The purpose of this paper is to explore how asset bubbles work when the financial institution is poor. In order to capture the effects on bubbles explicitly, instead we assume an extreme situation in which credit market does not work entirely. We will show that even if credit market does not work, bubbles enhance the efficient allocation and macro performance. Moreover, the inefficiency of credit market is crucial for the existence of bubbles. In this paper, we use an infinite horizon model. It is well-known in the literature (Tirole:1995) that if financial market is perfect, bubbles cannot exist in the model. There are several papers which examined how the financial market conditions affect the existence conditions of (rational) asset bubbles. For example, Kocherlakota (1992) has shown that bubbles can exist even in an infinite horizon model if financial market condition is not working well.

In other words, the low quality of financial institution is an enhancing factor for the existence of asset bubbles[1] Hence, our setting is consistent with those papers.

The novel point of this paper is that we assume that there two types of investment opportunity and not all agents have the same investment opportunity. Only some of the agents are able to access the high productive investment opportunity and the other agents only have a chance to access low productive investment opportunity. One crucial assumption of this paper is that even if an agent does not face the high productive investment opportunity at one time, she may have a chance to face the high investment in the future. Hence, by purchasing bubbles instead of investing to the low productive investment, the bubbles can be used to sell when she gets the high investment opportunity. Presence of financial market imperfections, enough resources cannot be transferred to those who have investment from those who do not. As a result, underinvestment occurs. Bubbles help to transfer resources between them.

From this result, we can interpret that such transitivity of investment opportunity is important for the crowed-in effect of asset bubbles. if the investment opportunity changes frequently, the existence of bubbles enhance the macro performance. In this sense, we can say that the institutional environment which realizes such transitivity is important for the positive effects of bubbles[2].

We also examine stochastic bubbles. In the case of stochastic bubbles, bubbles burst at each period with some probabilities. We examine the existence condition of such stochastic bubbles and characterize the effects of stochastic bubbles. We will show that if bubbles are stochastic, the crowed in effect must be lower than the case of deterministic bubbles. The intuitive reason is simple. If bubbles are stochastic and an agent faces a low productive investment opportunity, she invests to both the low productive investment and bubbles in order to hedge the risk of bubbles' burst. Furthermore, we will show that the burst of bubbles has a negative impact on the economy. Hence, it is risky to relay on the asset bubbles for the appropriate transfer

[1] The possibility of bubbles in infinite horizon economies with borrowing constraints has been recognized even in several previous papers, including Scheinkman and Weiss (1986), Kocherlakota (1992), Santos and Woodford (1997), and Hellwig and Lorenzoni (2009).

[2] In this sense, our model is related to Matsuyama (2007, 2008), in which Matsuyama shows that a better credit market might be more prone to financing what he calls bad investments that do not have positive spillover effects on future generations.

of resources. Improving the quality of financial institution is important for long run macro performance.

The rest of this paper is organized as follows. In the following subsection, we present the literature in this field. In section 2, we present our basic model and describe the economy without bubbles. In section 3, we introduce bubbles to this economy. We examine the existence conditions of bubbles and the effects of bubbles. In section 4, we examine the effects of stochastic bubbles. We examine the effects of bubbles bursting and policies after the burst. In section 6, we conclude our argument.

1.1 Related Work in the Literature

The conventional wisdom (Samuelson, 1958; Tirole, 1985) suggests that bubbles crowd investment out and lower output. In the traditional view, the financial market is perfect and all the savings in the economy flow to investment. In such a situation, once bubbles appear in the economy, they crowd savings away from investment. Saint-Paul (1992), Grossman and Yanagawa (1993), and King and Ferguson (1993) extend the Samuelson-Tirole model to economies with endogenous growth, and show that bubbles reduce investment and retard long run economic growth.

Recently, some papers such as Caballero and Krishnamurthy (2006), Kiyotaki and Moore (2008), Kocherlakota (2009), Farhi and Tirole (2010) developed a model with financial frictions, and showed that bubbles crowd investment in and increase output as shown in this paper. Those papers, however, do not treat the transitivity of investment opportunities.

In Caballero and Krishnamurthy (2006), they mainly focused on capital flight and the international financial market works. In Kocherlakota (2009), agents can borrow against bubbles in land prices. Thus, bubbles directly enhance the financial imperfection in the Kocherlakota (2009). In the theory by Kiyotaki and Moore (2008), since fiat money (bubble) facilitates exchange for its high liquidity, people hold money even though the rate of return on it is low, that is money (bubble) works as a medium of exchange. In our model, however, we focus on the role of bubbles as a store of value.

Martin and Ventura (2010) is closely related to our paper. They investigated whether bubbles are expansionary or contractionary in an overlapping generations framework, and have found that bubbles are expansionary under a wide range of the parameter values of investment technology. In Martin and Ventura (2010), however, new bubbles must be created at each period for the expansionary effect. This paper show such new creation of bubbles is not necessary for the crowed in effect. Farhi and Tirole (2010) examined the existence of bubbles and they found that bubbles can exist when the pledgeability level is low, however, their main focus was the effects of outside liquidity. Woodford(1990) has shown that government debt (bubble) crowds investment in an endowment economy, while our model examine production economy. In Woodford (1990), the entrepreneurs have investment opportunities in alternating periods. This setup is related to our setting about the transitivity of investment opportunities. Hence, our model can be seen as a generalization of Woodford (1990).

2 The Model

Consider a discrete-time economy with two types of goods, consumption goods and capital goods, and two types of a continuum of agents, entrepreneurs and workers. Let us start with the entrepreneurs, who are the central agents in the model. A typical entrepreneur has the following expected discounted utility,

$$E_0\left[\sum_{t=0}^{\infty}\beta^t \log c_t^i\right], \tag{1}$$

where i is the index for each entrepreneur, and c_t^i is the consumption of him at date t. $\beta \in (0,1)$ is the subjective discount factor, and $E_0[x]$ is the expected value of x conditional on information at date 0.

At each date, each entrepreneur meets high productive investment projects (hereinafter H-projects) with probability p, and low productive investments (L-projects) with probability $1-p$.[3] The investment technologies are as follows:

$$k_{t+1}^i = \alpha_t^i z_t^i, \tag{2}$$

where $z_t^i (\geq 0)$ is the investment level at date t, and k_{t+1}^i is the capital produced at date $t+1$. α_t^i is the marginal productivity of investment at date t. $\alpha_t^i = \alpha^H$ if the entrepreneur has H-projects, and $\alpha_t^i = \alpha^L$ if he has L-projects. We assume that capital fully depreciates in one period. We also

[3] A similar setting is used in Gertler and Kiyotaki (2010), Kiyotaki (1998), Kiyotaki and Moore (2008), and Kocherlakota (2009). In Woodford (1990), the entrepreneurs have investment opportunities in alternating periods.

assume $\alpha^H > \alpha^L$. The probability p is exogenous, and independent across entrepreneurs and over time. At the beginning of each date t, the entrepreneur knows his own type at date t, whether he has H-projects or L-projects. Assuming that the initial population measure of each type of the entrepreneur is one at date 0, the population measure of each type after date 1 is $2p$ and $2 - 2p$, respectively. We call the entrepreneurs with H-projects (L-projects) "H-entrepreneurs" ("L-entrepreneurs").

In the present paper, to emphasize the role of bubbles in transferring resources, we assume that a credit market is completely shut down. This implies that it is impossible to transfer resources between agents through the credit market. However, as we will show later, bubbles make it possible to transfer resources between agents, even if the credit market is completely closed. To show this point clearly, we make the extreme assumption.[4]

The entrepreneur's flow of funds constraint is given by

$$c_t^i + z_t^i = q_t k_t^i, \tag{3}$$

where q_t is the relative price of capital to consumption goods. The left hand side of (3) is expenditure on consumption and investment. The right hand side is financing which comes from the returns from investment in the previous period. We define the net worth (wealth) of the entrepreneur in the bubbleless economy as $e_t^i \equiv q_t k_t^i$.

Now, let's turn to workers. In this economy, there are workers with a unit measure. Each worker has the same expected discounted utility as the entrepreneur shown in equation (1). Each worker is endowed with one unit of labor force at each period, which is supplied inelastically in a labor market, and earns the wage rate, w_t.

The worker's flow of funds constraint is given by

$$c_t = w_t. \tag{4}$$

That is, each worker consumes the wage rate at each period.

In this economy, there are competitive firms which produce consumption goods using capital and labor.[5] The aggregate production function follows

[4] Of course, this assumption can be relaxed to allow the agents to borrow as long as debts are secured by collateral. For example, we can consider the situation that creditors can seize some fraction of entrepreneurial capital. That faction can be a collateral in borrowing.

[5] Here, we suppose that each firm is operated by workers. Since the net profit of each

$$Y_t = K_t^{\sigma} N_t^{1-\sigma},$$

where K_t and N_t are the aggregate capital stock and labor input at date t. Y_t is the aggregate output at date t.

Markets are competitive and factors of production are paid their marginal product:

$$q_t = \sigma K_t^{\sigma-1} \text{ and } w_t = (1-\sigma) K_t^{\sigma}. \tag{5}$$

We see that the relative price of capital to consumption goods is a decreasing function of the aggregate capital stock, and the wage rate is an increasing function of it.

2.1 Equilibrium

Let us denote the aggregate consumption of H-and L-entrepreneurs, and workers at date t as $\sum_{i \in H_t} c_t^i \equiv C_t^H$, $\sum_{i \in L_t} c_t^i \equiv C_t^L$, and C_t^{wk}, where H_t and L_t mean a family of H-and L-entrepreneurs at date t. Similarly, let $\sum_{i \in H_t} z_t^i \equiv Z_t^H$, and $\sum_{i \in L_t} z_t^i \equiv Z_t^L$ be the aggregate investment of each type. Then, the market clearing condition for goods, and the market clearing condition for labor are

$$C_t^H + C_t^L + C_t^{wk} + Z_t^H + Z_t^L = Y_t, \tag{6}$$

$$N_t = 1. \tag{7}$$

The competitive equilibrium is defined as a set of prices $\{q_t, w_t\}_{t=0}^{\infty}$ and quantities $\left\{c_t, c_t^i, z_t^i, k_{t+1}^i, C_t^{wk}, C_t^H, C_t^L, Z_t^H, Z_t^L, K_t, Y_t\right\}_{t=0}^{\infty}$, such that (i) the market clearing conditions, (6) and (7), are satisfied, and (ii) each worker chooses consumption to maximize his expected discounted utility (1) under the constraint (4), and (iii) each entrepreneur chooses consumption, investment, and capital to maximize his expected discounted utility (1) under the constraints (2), and (3).

We are now in a position to characterize the equilibrium behaviour of the entrepreneurs. As is well-known, since the utility function is log-linear, each entrepreneur consumes a fraction $1-\beta$ of the net worth at every period,

firm is zero in equilibrium, the flow of funds constraint of the workers does not change, and is the same as (4).

that is, $c_t^i = (1-\beta)e_t^i$.[6] Then, from the flow of funds constraint, (3), the investment function of each type of the entrepreneur becomes

$$z_t^i = \beta e_t^i.$$

Since the credit market is shut down, each entrepreneur is forced to self-finance his projects.

Now we consider the aggregate economy. Since the investment function of the entrepreneur is a linear function of the net worth, we can aggregate across the entrepreneurs to obtain the aggregate investment function of H- and L-entrepreneurs, respectively:

$$Z_t^H = \beta E_t^H,$$

$$Z_t^L = \beta E_t^L,$$

where $\sum_{i \in H_t} e_t^i \equiv E_t^H$ and $\sum_{i \in L_t} e_t^i \equiv E_t^L$ are the aggregate net worth (wealth) of H-entrepreneurs and L-entrepreneurs at date t, respectively.

Hence, the law of motion of the aggregate capital stock becomes

$$K_{t+1} = \alpha^H \beta E_t^H + \alpha^L \beta E_t^L. \tag{8}$$

The first and the second term of (8) represent the capital stock produced by H-and L-entrepreneurs at date $t+1$, respectively.

The movement of the aggregate net worth of H-entrepreneurs evolves according to the following equation.

$$E_t^H = pq_t \alpha^H \beta E_{t-1}^H + pq_t \alpha^L \beta E_{t-1}^L = pq_t K_t, \tag{9}$$

where $q_t K_t$ is the aggregate wealth of the entrepreneurs. The first term of (9) represents the aggregate net worth of the entrepreneurs who continue to have H-projects from the previous period (we call these H-H entrepreneurs). The second term represents the aggregate net worth of the entrepreneurs who switch from the state with L-projects to the state with H-projects (we call these L-H entrepreneurs). Since every entrepreneur has the same opportunity to invest in H-projects at each period, the aggregate net worth of H-entrepreneurs at date t is a fraction p of the aggregate wealth of the entrepreneurs at date t.

[6]See, for example, chapter 1.7 of Sargent (1988).

In the same way, we have the movement of the aggregate net worth of L-entrepreneurs:

$$E_t^L = (1-p)q_t\alpha^H\beta E_{t-1}^H + (1-p)q_t\alpha^L\beta E_{t-1}^L = (1-p)q_tK_t. \qquad (10)$$

We see that a fraction $1-p$ of the aggregate wealth of the entrepreneurs is the aggregate net worth of L-entrepreneurs at date t.

Then, by using (9) and (10), (8) can be rewritten as

$$K_{t+1} = \left[\alpha^H p + \alpha^L(1-p)\right]\beta q_t K_t,$$

where $\beta q_t K_t$ is the aggregate savings of the entrepreneurs. We see that a fraction p of them flows to H-projects, while a fraction $1-p$ of them flows to L-projects.

By using the relation $q_t = \sigma K_t^{\sigma-1}$, the above equation becomes

$$K_{t+1} = \left[\alpha^H p + \alpha^L(1-p)\right]\beta\sigma K_t^{\sigma}. \qquad (11)$$

Given the initial value of K_0, this economy converges to a steady state. In the steady-state equilibrium, the aggregate capital stock, the relative price of capital to consumption goods, and the wage rate become

$$K = \left\{\left[\alpha^H p + \alpha^L(1-p)\right]\beta\sigma\right\}^{\frac{1}{1-\sigma}},$$

$$q = \frac{1}{\left[\alpha^H p + \alpha^L(1-p)\right]\beta},$$

$$w = (1-\sigma)\left\{\left[\alpha^H p + \alpha^L(1-p)\right]\beta\sigma\right\}^{\frac{\sigma}{1-\sigma}}.$$

In this economy, since the credit market is completely shut down, L-entrepreneurs cannot lend their savings to H-entrepreneurs and end up with investing all of their savings in their own projects with low returns. Resource allocation is inefficient. As a result, the aggregate capital stock and the wage rate become low, and they are indeed lower than that under the perfect credit market, where all the savings in the economy flow to H-projects.

3 Existence of Asset Bubbles

Now we describe the economy with asset bubbles (we call this a "bubble economy"). We define bubble assets as the assets that produce no real return, i.e., the fundamental value of the assets is zero. Let x_t^i be the level of bubble assets purchased by type i entrepreneur at date t, and let P_t be the per unit price of bubble assets at date t in terms of consumption goods. In the bubble economy, each entrepreneur faces the following two constraints: flow of funds condition, and the short-sale constraint [7]:

$$c_t^{*i} + z_t^{*i} + P_t x_t^i = q_t k_t^{*i} + P_t x_{t-1}^i, \tag{12}$$

$$x_t^i \geq 0, \tag{13}$$

where * represents the case of the bubble economy. Both sides of (12) include bubble assets. $P_t x_{t-1}^i$ in the right hand side is the sales of the bubble assets, and $P_t x_t^i$ in the left hand side is the new purchase of them. We define the net worth of the entrepreneur in the bubble economy as $e_t^{*i} \equiv q_t k_t^{*i} + P_t x_{t-1}^i$.

Concerning workers, the worker's flow of funds constraint is

$$P_t x_t^i + c_t^* = w_t^* + P_t x_{t-1}^i. \tag{14}$$

The worker also faces the short sale constraint:

$$x_t \geq 0. \tag{15}$$

In the bubble economy, an equilibrium process of prices $\{q_t^*, w_t^*, P_t\}_{t=0}^{\infty}$ and quantities $\left\{c_t^*, c_t^{*i}, z_t^{*i}, k_{t+1}^{*i}, C_t^{*wk}, C_t^{*H}, C_t^{*L}, Z_t^{*H}, Z_t^{*L}, K_t^*, Y_t^*\right\}_{t=0}^{\infty}$, such that (i) each entrepreneur chooses the levels of consumption, investment, capital, and bubble assets to maximize the expected discounted utility (1) subject to (12), and (13), (ii) each worker chooses the levels of consumption, and bubble

[7] We should add a few remarks about the short-sale constraint (18). As Kocherlakota (1992) has shown, the short-sale constraint is important for the existence of bubbles in deterministic economies with a finite number of infinitely lived agents. Without the constraint, bubbles always represent an arbitrage opportunity for an infinitely lived agent; he can gain by permanently reducing his holdings of the asset. However, it is well known that in such economies, equilibria can only exist if agents are constrained not to engage in Ponzi schemes. Kocherlakota (1992) has demonstrated that the short-sale constraint is one of no-Ponzi-game conditions and hence, it can support bubbles by eliminating the agent's ability to permanently reduce his holdings of the asset. See Kocherlakota (1992) for details.

assets to maximize the expected discounted utility (1) subject to (14), and (15), and (iii) the markets for goods, labor, and bubbles all clear.

Now, we are in a position to characterize the equilibrium behaviour of the entrepreneurs and the workers in the bubble economy. Here we use a method of guess and verify. We consider the case where the equilibrium rate of return on bubbles is strictly lower than the rate of return on H-projects, that is, L-entrepreneurs purchase bubbles, and the short sale constraint is binding for H-entrepreneurs and the workers. Since the entrepreneur consumes a fraction $1-\beta$ of the net worth at every period, the investment function of the entrepreneurs who have H-projects at date t is

$$z_t^* = \beta e_t^{*i},$$

where $i \in H_t$, $e_t^{*i} = P_t x_{t-1}^i$ for L-H entrepreneurs, and $e_t^{*i} = q_t k_t^{*i}$ for H-H entrepreneurs. For L-H entrepreneurs, since they purchased bubbles in the previous period, they are able to sell bubbles at the time they encounter H-projects. As a result, their net worth increases (compared to the bubbleless case) and boosts their investments, that is, the "wealth effect" works.[8] H-H entrepreneurs, however, are not able to take advantage of this merit, because they did not buy bubbles in the previous period.

For those entrepreneurs who have L-projects at date t, they buy bubble assets with all of their savings:

$$P_t x_t^i = \beta e_t^{*i},$$

where $i \in L_t$.

Next, we describe the aggregate economy. When we aggregate the investment function of each H-entrepreneur, we obtain the aggregate investment function at date t:

$$Z_t^{*H} = \beta E_t^{*H}.$$

On the other hand, when we aggregate the bubbles' demand function of each L-entrepreneur, we get the aggregate demand function for bubble assets at date t:

[8]In Kiyotaki and Moore (1997), the rise in land price increases the entrepreneurs' net worth, which results in increasing investment. In this paper, bubbles play a similar role as the land in Kiyotaki and Moore's paper. Also this wealth effect is simillar to balance sheet effects developed by Bernanke and Gertler(1989), and Bernanke et al.(1999).

$$\sum_{i \in L_t} P_t x_t^i \equiv \beta E_t^{*L}$$

Then, the market clearing condition for bubble assets is written as

$$P_t X = \beta E_t^{*L}, \tag{16}$$

where X is the aggregate quantity of bubbles, which is exogenously fixed. The left hand side of (16) is the aggregate supply of bubble assets, and the right hand side is the aggregate demand of them.

Next, we consider how the aggregate net worth of H-and L-entrepreneurs evolves. The aggregate net worth of H-entrepreneurs in the bubble economy evolves according to the following equation:

$$E_t^{*H} = pq_t^* K_t^* + pP_t X = p(q_t^* K_t^* + P_t X), \tag{17}$$

where $q_t K_t^* + P_t X$ is the aggregate wealth of the entrepreneurs in the bubble economy. The first term in equation (17) is the aggregate net worth of H-H entrepreneurs and the second term represents the one of L-H entrepreneurs. (17) suggests that a fraction p of the aggregate wealth of the entrepreneurs is the aggregate net worth of H-entrepreneurs.

In the same way, the movement of the aggregate net worth of L-entrepreneurs follows

$$E_t^{*L} = (1-p)q_t^* K_t^* + (1-p)P_t X = (1-p)(q_t^* K_t^* + P_t X). \tag{18}$$

We see that a fraction $1-p$ of the aggregate wealth of the entrepreneurs is the aggregate net worth of L-entrepreneurs.

From (16) and (18), we can derive the per unit price of bubble assets as a function of the aggregate capital stock:

$$P_t X = \frac{\beta(1-p)}{1-\beta+p\beta} q_t^* K_t^*. \tag{19}$$

We observe that the price of bubble assets at date t is an increasing function of the aggregate capital stock at date t.

On the other hand, by using (17), the aggregate investment function can be written as

$$Z_t^{*H} = \beta E_t^{*H} = \beta p(q_t^* K_t^* + P_t X).$$

Hence, the aggregate capital stock can be written as

$$K_{t+1}^* = \alpha^H Z_t^{*H} = \alpha^H \beta p (q_t^* K_t^* + P_t X). \tag{20}$$

By substituting (19) into (20), we can derive the law of motion of the aggregate capital stock in the bubble economy.

$$K_{t+1}^* = \frac{\alpha^H \beta p}{1-\beta+p\beta} q_t^* K_t^* = \frac{\sigma \alpha^H \beta p}{1-\beta+p\beta} K_t^{*\sigma}. \tag{21}$$

From (21), there is a unique stationary equilibrium of the bubble economy. In the steady-state equilibrium, the aggregate capital stock becomes

$$K^* = \left(\frac{\sigma \alpha^H \beta p}{1-\beta+p\beta}\right)^{\frac{1}{1-\sigma}}.$$

And then, from (5), once K^* is determined, we obtain the relative price of capital to consumption goods, and the wage rate:

$$q^* = \frac{1-\beta+p\beta}{\alpha^H \beta p},$$

$$w^* = (1-\sigma)\left(\frac{\sigma \alpha^H \beta p}{1-\beta+p\beta}\right)^{\frac{\sigma}{1-\sigma}}.$$

3.1 Existence Condition of Bubbles

In this subsection, we examine the existence condition of bubbles. For the existence of bubbles at the steady-state economy, the following conditions must be satisfied. First, the growth rate of bubbles (in this model, that is P_{t+1}/P_t) must be equal or lower than the economic growth rate (in this model, that is 1) since this economy cannot sustain the bubbles if the growth rate of bubbles is higher that the economic growth rate. Moreover, if the growth rate of bubbles is strictly lower than the economic growth rate, the economy converges to the asymptotically bubbleless economy. Hence, as usual, we focus on the case where the growth rate of bubbles is equal to the economic growth rate. Second, in order that the L-entrepreneurs are willing to buy bubbles, the equilibrium rate of return on bubbles must not be lower than the rate of return on L-projects, $q^*\alpha^L$, at each period. This means,

$$q^*\alpha^L = \frac{\alpha^L(1-\beta+p\beta)}{\alpha^H\beta p} \leq \frac{P_{t+1}}{P_t} = 1.$$

The left hand side is the rate of return on L-projects at the steady state, and the right hand side is the one on bubbles. We can solve for $\frac{\alpha^H}{\alpha^L}$, and get the following condition:

$$\frac{\alpha^H}{\alpha^L} \geq 1 + \frac{1-\beta}{\beta p}.$$

As long as the above condition is satisfied, L-entrepreneurs are willing to buy bubbles in equilibrium instead of investing in their L-projects. This is the necessary condition for the existence of bubbles.[9] Here we summarize the result in the following proposition.

Proposition 1 *Bubbles can exist as long as $\frac{\alpha^H}{\alpha^L}$ satisfies the following condition,*

$$\frac{\alpha^H}{\alpha^L} \geq 1 + \frac{1-\beta}{\beta p}.$$

We can verify here that H-entrepreneurs never buy bubbles. To verify this, we need to check that the rate of return on H-projects is strictly greater than the rate of return on bubbles at the steady state. We know that the rate of return on H-projects at the steady state is $q^*\alpha^H$, while the rate of return on bubbles is 1. It is obvious that $q^*\alpha^H = 1 + \frac{1-\beta}{\beta p} > 1$.[10]

From this proposition, we can understand that bubbles are likely to exist when the difference in the marginal productivity between H-projects and

[9] The reason this condition is called "necessary condition for the existence of bubbles" is that unless people expect to be able to pass bubbles on to other people, bubbles cannot arise in the economy. This expectation is the sufficient condition for the existence of bubbles. Here, we assume that the condition is satisfied when bubbles appear.

[10] Here we verify that the short sale constraint is binding for the workers in the neighborhood of the steady state. The constraint binds if and only if marginal utility of consumption exceeds the marginal benefit of buying bubbles. That is, $\frac{1}{c_t^*} > \beta E_t\left[\frac{1}{c_{t+1}^*}\frac{P_{t+1}}{P_t}\right]$ must hold. This inequality condition can be rewritten as $1 > \beta E_t\left[\frac{c_t^*}{c_{t+1}^*}\frac{P_{t+1}}{P_t}\right]$. In the steady-state equilibrium, the inequality condition is equivalent to $1 > \beta$, which is true. By continuity, this holds in the neighborhood of the steady state.

L-projects is large, the possibility of finding high productive investments is high, or the entrepreneurs are more patient.

Moreover, we can use the structure of the bubbleless economy to characterize the existence condition. The existence condition of the bubbles is that the growth rate, which is equal to one, is not lower than the rate of return on L-projects under the bubbleless economy. That is, $q\alpha^L = \frac{\alpha^L}{[\alpha^H p + \alpha^L(1-p)]\beta} \leq 1$. This condition is equivalent to $\frac{\alpha^H}{\alpha^L} \geq 1 + \frac{1-\beta}{\beta p}$.

Proposition 2 *The necessary condition for the existence of bubbles is that the equilibrium growth rate is not lower than the equilibrium rate of return on L-projects under the bubbleless economy.*

3.2 Macroeconomic Effects of Asset Bubbles

In this section, we examine how bubbles affect macroeconomic variables. We will show here that if the existence condition of bubbles is satisfied, bubbles are expansionary, i.e., bubbles increase the aggregate capital stock, the aggregate production level, the aggregate consumption level, and the wage rate. We summarize the result in the following Proposition.

Proposition 3 *If the existence condition of bubbles is satisfied, the aggregate capital stock, the aggregate production level, and the wage rate under the bubble economy are all higher than that under the bubbleless economy at each period.*

Proof. *The condition that the aggregate capital stock under the bubble economy is greater than that under the bubbleless economy is*

$$K^* = \left(\frac{\sigma\alpha^H\beta p}{1-\beta+p\beta}\right)^{\frac{1}{1-\sigma}} \geq K = \left\{\left[\alpha^H p + \alpha^L(1-p)\right]\beta\sigma\right\}^{\frac{1}{1-\sigma}}.$$

In other words,

$$\frac{\alpha^H}{\alpha^L} \geq 1 + \frac{1-\beta}{\beta p}.$$

Since the aggregate production level and the wage rate are increasing functions of the aggregate capital stock, they are higher in the bubble economy compared to the bubbleless economy. ■

Here we explain an intuitive reason of this result. Defining $A^* \equiv q^*K^* + PX$ and $A \equiv qK$, from (11) and (20), we get

$$K^* = \alpha^H \beta A^* - \alpha^H (1-p) \beta A^*, \tag{22}$$

$$K = \left[\alpha^H p + \alpha^L (1-p)\right] \beta A. \tag{23}$$

The second term in equation (22) implies that a fraction $1-p$ of the aggregate savings of the entrepreneurs flows to bubble assets, which crowds H-projects, resulting in a decrease in the aggregate capital stock.

More precisely, the difference in the aggregate capital stock between the bubble economy and the bubbleless economy can be written as follows.

$$K^* - K = \underbrace{\underbrace{\alpha^H \beta (A^* - A)}_{\text{saving volume effect}} + \underbrace{\left\{\alpha^H - \left[\alpha^H p + \alpha^L (1-p)\right]\right\} \beta A}_{\text{saving composition effect}}}_{\text{crowd-in effect}} - \underbrace{\alpha^H (1-p) \beta A^*}_{\text{crowd-out effect}} \tag{24}$$

Here, we will give intuitive explanations on the role of bubbles in allocating resources. In the bubbleless economy, since the credit market is shut down, it is impossible to transfer resources from L-entrepreneurs to H-entrepreneurs. However, in the bubble economy, all the savings of L-entrepreneurs can be transferred to H-entrepreneurs through the transaction of bubbles, even if the credit market is completely shut down. Bubbles complement the credit market. Moreover, bubbles improve efficiency in production by eliminating low-productive investments.

The first term in equation (24) is a saving volume effect. In other words, bubbles increase the aggregate savings of the economy, which improves the aggregate net worth of H-entrepreneurs. This expands high productive investments. The second term is a saving composition effect. For L-H entrepreneurs, they sell bubble assets to L-entrepreneurs. As a result, all the savings of L-entrepreneurs are transferred to H-entrepreneurs. These two effects generate a crowd-in effect on the aggregate capital stock, which in turn increases the aggregate production level, and the wage rate. On the other hand, there is a third effect, i.e., a traditional crowding-out effect, which is the third term in equation (24). Depending on which one of these competing effects dominates, the effect of bubbles on the aggregate capital stock is determined. Proposition 3 shows us that the crowd-in effect dominates the crowd-out effect. Therefore, bubbles are expansionary. Note that the aggregate consumption level for the entrepreneurs, which is a fraction $1-\beta$ of the aggregate wealth of the entrepreneurs, and for the workers expands, because the aggregate wealth and the wage rate increase.

4 Stochastic Bubbles

In this section, we analyze the effects of bubbles' bursting on the economy. In order to do so, we consider stochastic bubbles.[11] Here we assume the following Markov chain:

$$\Pr(P_t > 0 \mid P_{t-1} > 0) = \pi,$$
$$\Pr(P_t > 0 \mid P_{t-1} = 0) = 0.$$

This chain implies that bubbles continue with probability $\pi(< 1)$, and their prices are positive until they switch to being equal to zero forever.

Let ** represents the case of the stochastic bubble economy. Here we focus on the case where all macroeconomic variables are constant. As in the previous case, H-entrepreneurs act much like they do in the deterministic bubbly steady-state. They consume a fraction $1-\beta$ of the net worth at every period, and they invest a fraction β of the net worth in their H-projects.

L-entrepreneurs also consume a fraction $1-\beta$ of the net worth at every period, but their portfolio problem is more complicated than in the deterministic bubbly steady-state. Since bubble assets deliver no return with probability π, hence, L-entrepreneurs want to hedge themselves by investing some of their savings in their L-projects.

More specifically, L-entrepreneurs invest $\lambda\beta E_t^{**L}$ in their L-projects, and they also buy bubble assets with $(1-\lambda)\beta E_t^{**L}$. Here, λ satisfies the first-order condition:

$$\frac{\pi(1-q^{**}\alpha^L)}{q^{**}\alpha^L\lambda + 1 - \lambda} = \frac{1-\pi}{\lambda}.$$

It follows that:

$$\lambda = \frac{1-\pi}{1-q^{**}\alpha^L}, \text{ and } 1-\lambda = \frac{\pi - q^{**}\alpha^L}{1-q^{**}\alpha^L}.$$

[11] Weil (1987) is the first study which considers stochastic bubbles in a general equilibrium framework.

With these decision rule in hand, we can derive the evolution of the aggregate wealth:

$$A^{**} = q^{**}\alpha^H \beta p A^{**} + q^{**}\alpha^L \frac{1-\pi}{1-q^{**}\alpha^L}\beta(1-p)A^{**} + \frac{\pi - q^{**}\alpha^L}{1-q^{**}\alpha^L}\beta(1-p)A^{**}. \tag{25}$$

(25) suggests that the aggregate wealth is composed of three parts. The first term of the right hand side in equation (25) represents the returns from H-projects. The second and the third terms represent the returns from L-projects and the ones from bubble assets. Note that the rate of return on bubbles is equal to one.

Hence, we can solve for q^{**}:

$$q^{**} = \frac{1-\beta(1-p)\pi}{\alpha^H \beta p}.$$

As in the deterministic bubbly steady-state case, we can also solve for the other macroeconomic variables:

$$K^{**} = \left[\frac{\sigma\alpha^H \beta p}{1-\beta(1-p)\pi}\right]^{\frac{1}{1-\sigma}},$$

$$w^{**} = (1-\sigma)\left[\frac{\sigma\alpha^H \beta p}{1-\beta(1-p)\pi}\right]^{\frac{\sigma}{1-\sigma}}.$$

We should remember that unlike in the deterministic bubbly steady-state, L-entrepreneurs are investing some of their savings in low productive investments.

4.1 Existence Condition of Stochastic Bubbles

In this subsection, we investigate the existence of stochastic bubbles. The following condition must be satisfied in order that stochastic bubbles can exist in the equilibrium path where all variables become constant. The share of L-projects against the aggregate net worth of L-entrepreneurs must be strictly less than one, which is equivalent to the condition that the share of bubbles against the aggregate net worth of L-entrepreneurs must be strictly positive. In other words,

$$\lambda = \frac{1-\pi}{1-q^{**}\alpha^L} < 1.$$

From this condition, we obtain the following Proposition.

Proposition 4 *As long as $\frac{\alpha^H}{\alpha^L}$ satisfies the following condition, stochastic bubbles can exist in the equilibrium where all variables are constant.*

$$\frac{\alpha^H}{\alpha^L} > 1 + \frac{1-\beta\pi}{\pi\beta p}.$$

We see that compared to the deterministic bubbles, the existence condition gets tightened.

4.2 Macroeconomic Effects of Stochastic Bubbles

In this subsection, we examine how stochastic bubbles affect macroeconomic variables. We summarize the results in the following Propositions.

Proposition 5 *If the existence condition of stochastic bubbles is satisfied, the aggregate capital stock, the aggregate production level, and the wage rate under the stochastic bubble economy are all strictly higher than that under the bubbleless economy at each period.*

Proof. *The condition that the aggregate capital stock under the stochastic bubble economy is strictly greater than that under the bubbleless economy is*

$$K^{**} = \left[\frac{\sigma\alpha^H\beta p}{1-\beta(1-p)\pi}\right]^{\frac{1}{1-\sigma}} > K = \left\{\left[\alpha^H p + \alpha^L(1-p)\right]\beta\sigma\right\}^{\frac{1}{1-\sigma}}.$$

In other words,

$$\frac{\alpha^H}{\alpha^L} > 1 + \frac{1-\beta\pi}{\pi\beta p}.$$

Since the aggregate production level and the wage rate are increasing functions of the aggregate capital stock, they are higher in the stochastic bubble economy compared to the bubbleless economy. ■

Proposition 6 *The aggregate capital stock, the aggregate production level, and the wage rate under the stochastic bubble economy are all strictly lower than that under the deterministic bubble economy at each period.*

Proof. *If we compare K to K^{**}, it is obvious that K^{**} is strictly lower than K as long as $\pi < 1$.* $\blacksquare$

4.3 Bubble Bursts

Now, we are ready to discuss how bubbles' bursting affects the economy. Suppose that until date $s-1$, bubbles continue, and then, at date s, they collapse. The entrepreneurs who did not buy bubble assets at date $s-1$ (H-entrepreneurs at date $s-1$) are unaffected by the bubbles collapse, but the entrepreneurs who bought bubbles at date $s-1$ (L-entrepreneurs at date $s-1$) realize that they cannot sell the bubbles suddenly. As a result, the aggregate wealth of the entrepreneurs decreases. Hence, the aggregate consumption level of the entrepreneurs at date s also falls. Recall that the aggregate consumption is a fraction $1-\beta$ of the aggregate wealth.

Despite the immediate impact on the entrepreneurs' consumption level, there is no immediate impact on the aggregate output level and the wage rate at date s, because the aggregate capital stock at date s is pinned down by the investments at date $s-1$. However, at date $s+1$, all macroeconomic variables fall sharply. From date $s+1$ onwards, the evolution of the aggregate capital stock follows

$$K_{s+1} = \left[\alpha^H p + \alpha^L (1-p)\right] \beta\sigma K_s^{\sigma}. \tag{26}$$

Along the new path, the economy transits to a new, lower, steady-state level.

The interesting point is that until date $s-1$, some of the savings of L-entrepreneurs are transferred to H-entrepreneurs through bubbles, but once the economy is hit by the bubbles collapse at date s, it suddenly becomes impossible to transfer resources from L-entrepreneurs to H-entrepreneurs. L-H entrepreneurs from date $s-1$ to date s are forced to cut back on their investment, because they cannot sell the bubbles to L-entrepreneurs and as a result, their net worth decreases. This results in producing negative amplification effects on the aggregate investment of H-entrepreneurs at date s.

Moreover, the bubbles burst has a persistent effect on the aggregate variables. Figure 1 depicts the movement of the aggregate output before and after the bubbles' bursting.

4.4 Government Policy After Bubbles Collapse

As we discussed in the previous section, the collapse of the bubbles triggers a fall in all macroeconomic variables. The question is what can the government do to restore the economy?

In the model economy, the bubbles are useful because they provide a store of value whose rate of return is high. This high rate of return improves the entrepreneurs' net worth, which increases the investment of H-entrepreneurs and the aggregate output. Once the bubbles burst, there is no such a store of value, and the entrepreneurs have to reduce their investment. To help the economy, the government needs to provide another form of assets as a store of value into the economy.

Caballero and Krishnamurthy (2006) and Kocherlakota (2009) contemplate government debt as a store of value. They consider a policy that the government re-finances existing debt simply by rolling over it. In the case of bubbles, bubbles are not backed, and they simply depend upon the self-fulfilling beliefs of private agents. However, government debt is backed by taxes. If the government cannot roll over its debt, it can levy taxes on agents, and repay its debt.[12] The government debt is fundamentally different from bubble assets. Hence, we can think of such a government debt as a deterministic bubble asset.

Here we consider the effect of this policy. As before, suppose that at date s, the bubbles burst and then, at date $s + \tau(\tau \geq 0)$, the government hands out government bonds to entrepreneurs. The distribution of this handout across the entrepreneurs is irrelevant. After date $s + \tau + 1$, the government simply rolls over the debt.

Because of this policy, the dynamics of the aggregate capital stock is governed by the following equation:

[12] Caballero and Krishnamurthy (2006) and Kocherlakota (2009) argue that, in equilibrium, the government will never need to collect the taxes. Instead, the government can commit to a strategy under which it commits to rolling over the existing debt, and then levies taxes if agents fail to buy the issued debt.

$$K_{s+\tau+1} = \frac{\sigma\alpha^H \beta p}{1-\beta+p\beta} K^{\sigma}_{s+\tau}. \tag{27}$$

After date $s+\tau+1$, the economy rides on a new transitory path, which is exactly the same as the path in the deterministic bubble economy, and converges to a new, higher, steady-state level. Figure 2 describes the movement of the aggregate output after this policy.[13]

5 Conclusion

In this paper, we have shown that even if the quality of financial institution is very poor and credit market does not work at all, bubbles have a positive role for enhancing the efficiency of investments. In the bubble economy, all savings of entrepreneurs who have only less productive investment opportunities can be transferred to the entrepreneurs who have high productive investment opportunities by trading asset bubbles. As a result, bubbles improve efficiency in production by eliminating low-productive investments. This implies that bubble bursts results in productive inefficiency.

For this mechanism, the transition of investment opportunities is crucial. The agent who sell bubbles is the entrepreneur who did not have a good investment opportunity and purchased bubbles in the previous period but has a good investment opportunity now. Such agent gets the fund for the good investment by selling the purchased bubbles. On the other hand, the agent who lost a good investment opportunity becomes a buyer of the bubbles. By investing to the bubbles instead to a low productive investment, she prepare a good investment opportunity in the future.

In this sense, bubbles and financial institution is substitutes as long as the transition of investment opportunity is satisfied. If bubbles burst, however, the economy should go down as explored in the previous section. Thus, bubbles and financial institution are not perfect substitutes.

In order to avoid the crash risk, we have explained the possibility that government intervenes by issuing government bond, since government bond works almost similar to asset bubbles, Another possible policy is directly improving the quality of financial institution. When the mechanism by bubbles

[13]Note that the interest rate on the government bonds at the new steady state as well as on the transitory path is equal to the rate of return on deterministic bubble assets analyzed in section 2.

work well, there might be no incentive to improve the quality of financial institution. However, the result of this paper implies that it is important to improving the quality of financial institution before bursting of bubbles. If the quality is high and credit market works well, even if bubbles burst, the resource can be transferred to the high productive investments and economy does not go down. Hence, it is important to improving the quality of financial market for the stability of macro performance.

References

[1] Bernanke, Ben, and Mark Gertler. 1989. “Agency Costs, Net Worth, and Business Fluctuations,” *American Economic Review*, 79(1): 14–31.

[2] Bernanke, Ben, Mark Gerlter, and Simon Gilchrist. 1999. “The Financial Accelerator in a Quantitative Business Cycle Framework,” in J. Taylor and M. Woodford eds, *the Handbook of Macroeconomics*, 1341-1393. Amsterdam: North-Holland.

[3] Broner, Fernando, Alberto Martin, and Jaume Ventura. 2010. Forthcoming. “Sovereign Risk and Secondary Markets,” *American Economic Review.*

[4] Caballero, Ricardo. 2006. “On the Macroeconomics of Asset Shortages,” The Role of Money: Money and Monetary Policy in the Twenty-First Century The Fourth European Central Banking Conference 9-10 November 2006, Andreas Beyer and Lucrezia Reichlin, editors, 272-283.

[5] Caballero, Ricardo, and Arvind Krishnamurthy. 2006. “Bubbles and Capital Flow Volatility: Causes and Risk Management,” *Journal of Monetary Economics*, 53(1): 35-53.

[6] Farhi, Emmanuel, and Jean Tirole. 2009. “Bubbly Liquidity,” IDEI Working Paper 577.

[7] Gertler, Mark, and Nobuhiro Kiyotaki. 2010. “Financial Intermediation and Credit Policy in Business Cycle Analysis,” New York University.

[8] Grossman, Gene, and Noriyuki Yanagawa. 1993. “Asset Bubbles and Endogenous growth,” *Journal of Monetary Economics*, 31(1): 3-19.

[9] Hart, Oliver, and John Moore. 1994. "A Theory of Debt Based on the Inalienability of Human Capital," *Quarterly Journal of Economics*, 109(4): 841–879.

[10] Hellwig, Christian, and Guido Lorenzoni. 2009. "Bubbles and Self-Enforcing Debt," *Econometrica*, 77(4): 1137-1164.

[11] Holmstrom, Bengt. and Jean Tirole. 1997. "Financial Intermediation, Loanable Funds, and the Real Sector," *Quarterly Journal of Economics,* 112(3): 663-91.

[12] King, Ian, and Don Ferguson. 1993. "Dynamic Inefficiency, Endogenous Growth, and Ponzi Games," *Journal of Monetary Economics,* 32(1): 79-104.

[13] Kiyotaki, Nobuhiro, 1998. "Credit and Business Cycles," *The Japanese Economic Review*, 49(1): 18–35.

[14] Kiyotaki, Nobuhiro, and John Moore. 1997. "Credit Cycles," *Journal of Political Economy*, 105(2): 211-248.

[15] Kiyotaki, Nobuhiro, and John Moore. 2008. "Liquidity, Business Cycles and Monetary Policy," Princeton university.

[16] Kocherlakota, R. Narayana. 1992. "Bubbles and Constraints on Debt Accumulation," *Journal of Economic Theory*, 57(1): 245-256.

[17] Kocherlakota, R. Narayana. 2009. "Bursting Bubbles: Consequences and Cures," University of Minnesota.

[18] Levine, Ross (1997), "Financial Development and Economic Growth." Journal of Economic Literature, 35(2), pp. 688-726.

[19] Martin, Alberto, and Jaume Ventura. 2010. "Economic Growth with Bubbles," Universitat Pompeu Fabra.

[20] Matsuyama, Kiminori. 2007. "Credit Traps and Credit Cycles," *American Economic Review*, 97(1): 503-516.

[21] Matsuyama, Kiminori. 2008. "Aggregate Implications of Credit Market Imperfections," in D. Acemoglu, K. Rogoff, and M. Woodford, eds., *NBER Macroeconomics Annual 2007*, 1-60. University of Chicago Press.

[22] Olivier, Jacques. 2000. "Growth-Enhancing Bubbles," *International Economic Review*, 41(1): 133-51.

[23] Pagano, Marco (1993), "Financial Markets and Growth: An Overview." European Economic Review, 37, pp. 613-22.

[24] Samuelson, Paul. 1958. "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money," *Journal of Political Economy*, 66(6): 467-482.

[25] Santos, S. Manuel, and Michael Woodford. 1997. "Rational Asset Pricing Bubbles," *Econometrica*, 65(1): 19-58.

[26] Saint G. Paul. 1992. "Fiscal Policy in an Endogenous Growth Model," *Quarterly Journal of Economics*, 107(4): 1243-1260.

[27] Scheinkman, A. Jose, and Laurence Weiss. 1986. "Borrowing Constraints and Aggregate Economic Activity," *Econometrica*, 54(1): 23-45.

[28] Tirole, Jean. 1982. "On the Possibility of Speculation under Rational Expectations," *Econometrica*, 50(5): 1163-1182.

[29] Tirole, Jean. 1985. "Asset Bubbles and Overlapping Generations," *Econometrica*, 53(6): 1499-1528.

[30] Tirole, Jean. 2005. *The Theory of Corporate Finance*. Princeton, New Jersey: Princeton University Press.

[31] Woodford, Michael. 1990. "Public Debt as Private Liquidity," *American Economic Review*, 80(2): 382-388.

[32] Weil, Philippe. 1987. "Confidence and the Real Value of Money," *Quarterly Journal of Economics*, 102(1): 1-22.

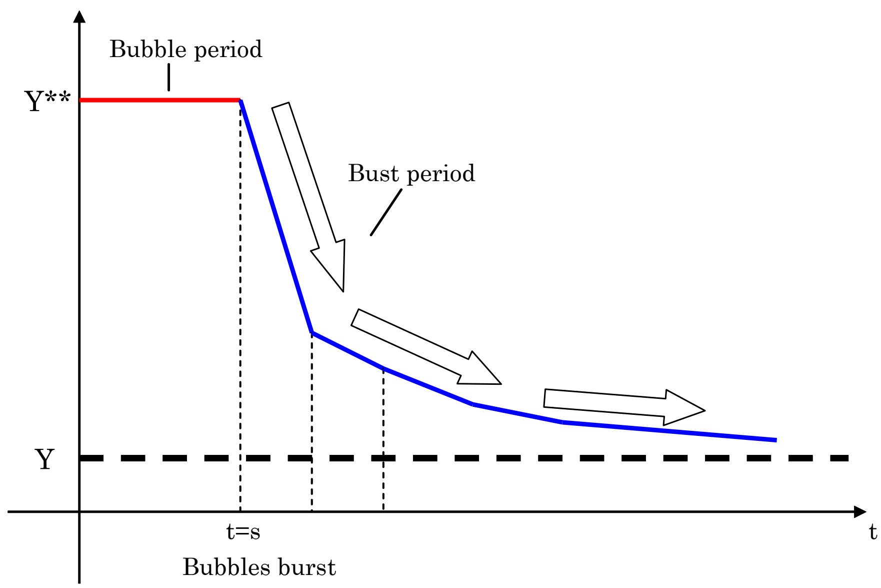

Figure 1 : Output movement after bubbles collapse

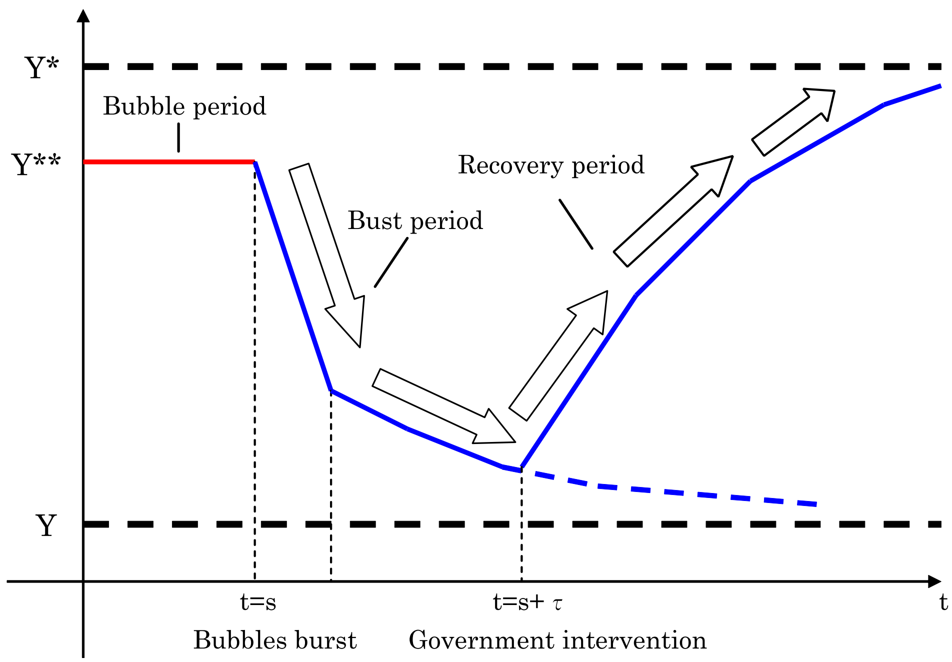

Figure 2: Output movement after government policy