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A Dynamic Multitask Model: Fixed Wages, No Externalities, and Holdup Problems

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A Dynamic Multitask Model: Fixed Wages, No Externalities, and Holdup Problems*

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Abstract

This article develops a multitask model in which the agent has to produce both verifiable and unverifiable outputs in a dynamic framework as observed in actual labor markets and practices. The model derives an important result regarding the timing and the length of a wage contract. A short-term wage contract allows for greater holdup, but it motivates the agent to engage in a task whose output is unverifiable. In contrast, a long-term wage contract does not allow for holdup and induces the first-best level effort for verifiable outputs, but it removes the incentive for unverifiable outputs. By studying an optimal wage profile and finding the optimal timing to sign a contract in a multitask framework, our model explains why individuals paid fixed wages have more frequent opportunities for wage negotiation compared with those paid through incentive pay. Furthermore, our model predicts that in industries where unverifiable outputs are valued, wage contracts are renewed more frequently.

Keywords: Multitask; No Externality; Holdup; Unverifiability; Incentive Pay; Fixed Pay; Short-term Contracts; Long-term Contracts

JEL Codes: D86; J41; J31

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1 Introduction

This article addresses an effort allocation problems in multitask principal–agent analysis when an agent must engage in multitasking of verifiable and unverifiable outputs. To do so, we analyze the contract environment in which the principal determines the length of contract depending on how important to the principal one output is relative to the other. The existing literature on combining different-length contracts is limited (Fudenberg, Holmstrom and Milgrom 1990, Ray and Salanie 1990). Moreover, these studies tend to confine their attention to how and when the principal can achieve the utility level of a long-term contract by repeating short-term contracts.[1] This paper shows that the principal is better off by offering a repetition of short-term contracts than a long-term contract when she values unverifiable outputs more than verifiable outputs or when the agent has large firm-specific knowledge. In contrast, the principal is better off if she offers a long-term contract when she values verifiable output more than unverifiable outputs. This paper is the first to show that repeating short-term contracts can be strictly better than offering a long-term contract.[2]

Most real-world jobs require agents to engage in multiple tasks. We typically observe that most office workers and bureaucrats receive *fixed pay* because it is difficult to clearly identify the amount of work each individual devotes to *unverifiable* outputs. For instance, academics are commonly engaged in multitask behavior where any unverifiable outputs are just as important as the verifiable outputs.[3] The general expectation is that academics provide administrative service, teach and raise students, and engage in collegial work such as attending seminars and providing useful comments (*unverifiable outputs*) along with publishing in journals (*a verifiable output*).[4] Even though we can readily quantify reports

[1] Dutta and Reichelstein (1996) show that short-term contracts can be better than a long-term contract in a different context. That is, in their model, the agents get fired on the equilibrium path, and hence they allow agents to change sequentially. Their model setting is different from the context in which the principal wishes to motivate one agent in a dynamic framework.

[2] Kamiya and Sato (2013) also show that repeating short-term contracts can be better than a long-term contract in a different context. See Section 2.

[3] As the focus of this analysis is the frequency of wage renewal and not dismissal, we do not consider the possibility of an agent (here academics) being fired or dismissed on the equilibrium path. For example, in the case of academics, we consider the wage negotiation of "tenured" associate/full professors rather than "untenured" assistant professors.

[4] Though the number of publication can be easily verified, the quality of the publication may not be easily

and publications for each agent, the monthly wage does not fluctuate with the number of reports or publications the agent produces each month. In short, the wage for verifiable outputs is not separated from the wage for unverifiable outputs, hence they receive a single fixed wage.[5]

Conversely, taxi drivers, pension and mutual fund managers, and car salespersons are typical examples of agents receiving *incentive pay* because they are expected to produce more *verifiable outputs*. Nonetheless, they are still expected to consider some unverifiable factors, including the safety of their customers or their customer's satisfaction.[6] Agents receiving incentive pay do not have many opportunities for wage negotiation as they are already given an incentive by wage fluctuation. For example, the wage or wage profile of a salesperson may not be reviewed very often during the period of employment, but the wage may nonetheless fluctuate a great deal according to performance.

In order to explore the efficient timing and design of a contract when an agent must engage in multitasking to produce both verifiable and unverifiable outputs, we first examine the similarities and differences between the production of verifiable and unverifiable outputs in a dynamic framework. The similarity is that both the verifiable and unverifiable outputs require a certain investment (effort) in their production, and the principal is therefore required to motivate the agent to undertake these two investments. Investment the agent makes is observable only to the principal. This makes the effort of the agent firm-specific to some extent. Hence in our model, we assume that the agent stays in one firm on-the-equilibrium path as in Levin (2002).[7] The difference between the production of verifiable and unverifiable outputs is that the wage can be written to reflect the verifiable output but not the unverifiable output.

Naturally, the trade-off that arises from the given similarity and difference is that if

verified. In this sense, we can say that academics are constantly expected to produce both verifiable and unverifiable outputs even if our sole role were to conduct research.

[5]Hellmann and Thiele (2011) examine the case in which the verifiable output can be separated from the unverifiable output (innovation).

[6]For instance, in many cases, investors in mutual and pension funds do not have sufficient knowledge about the risks of their investment products, and hence it is expected that fund managers handle their customer's assets safely.

[7]In this article, the agent may be dismissed if the bargaining crashes at the beginning of the second period under the short-term wage contract. In Levin (2002) the agent is not dismissed in the main analysis but he considers the case in which the agent is dismissed with exogenous shock in the extension. Osano (2011) considers the incentive problem when the agent has the possibility of quitting in the middle of the contracting relationship.

wages for all periods are agreed at the beginning of the initial period, it deprives the agent of the incentive to expend effort in the production of the unverifiable outputs in later stages. By contrast, if there will be wage negotiation after the agent has made an effort, the agent is provided with an incentive to produce both verifiable and unverifiable outputs. However, this process exhibits a holdup problem. In sum, there is a trade-off between *ex ante commitment* and *ex post bargaining* in inducing the effort of the agent.

In order to attain the objective set for this analysis, we develop a simple dynamic principal–agent model in which the principal engages in the trade-off of ex ante commitment and ex post bargaining. The outline of our model is as follows. The model has two periods, and there are two possibilities for wage contracts: a short-term wage contract that determines the second-period wage at the beginning of the second period (ex post bargaining), and a long-term wage contract that determines the second-period wage at the beginning of the first period (ex ante commitment).[8] [9] Note that the difference between a short- and long-term wage contract is when and how to contract the second-period wage.[10] We introduce two outputs: 'observable and verifiable' output x (contractible) and 'observable but unverifiable' output y (noncontractible).[11] We assume that task A produces verifiable output x and task B produces unverifiable output y, where both x and y are observable. In the first period, the agent needs to make an effort (a firm-specific human capital investment) to obtain the skills to produce x and y. We assume the skills to produce x and y are observable only to the principal, hence some what firm specific to some extent. In short, we consider the situation in which the agent does not move to the other firm after the investment, unless the principal and the agent reaches disagreement in wage negotiation. As in practice, we assume the agent obtains bargaining power after working in the firm for a while.[12]

[8] The length of contracts is not about how often the agent is fired; rather, it is about the frequency of renewing the wage contract. In this article, the agent remains employed under both contracts.

[9] We show later that a long-term contract is renegotiation-proof because the effort is observable.

[10] The first-period wage does not affect the choice of offering a short- or long-term contract as it is determined before the agent undertakes effort under both contracts.

[11] The observability of unverifiable output does not affect the result. That is, unverifiable output can either be observable or unobservable. Only the expected value of the unverifiable output must be observable. Therefore, for the sake of consistency, we write 'observable but unverifiable' output in this analysis.

[12] The agent obviously obtains bargaining power after making some investments/effort in human capital. However, we can consider that the agent obtains some bargaining power if he has stayed in one firm for a certain period. In our model, whether or not the agent has made investment in human capital, the agent gains some

In this model setting, we demonstrate that whether the principal offers a short- or long-term wage contract depends on how important to the principal one output is relative to the other and/or the relative efficiency of effort for the two tasks. The effort needed to complete one task is more efficient than the effort needed to complete the other task if both tasks yield the same amount of output but the output from the first task is produced with less effort. If the principal values the verifiable output x over the unverifiable output y, the principal decides to offer incentive pay for the second-period wage at the beginning of the first period (a long-term wage contract). Alternatively, if the principal values the unverifiable output y over the verifiable output x, a fixed wage is offered for the second-period wage at the beginning of the second period (a short-term wage contract).[13]

The logic behind this result is as follows. If the principal offers a wage contract before the agent makes effort (long-term wage contracts), the agent has no incentive to engage in producing the unverifiable output y, as his wage will only depend on the verifiable output x in the second period. However, the agent will have an incentive to engage in producing the verifiable output x. In fact, the principal can write a contract that can induce the agent to achieve the first-best level effort in producing the verifiable output x. On the other hand, if the principal is going to offer the agent a wage contract after the agent has made effort (short-term wage contracts), the agent has an incentive to make the effort (during the first period) needed in the production of x and y as he will wish to obtain a larger bargaining surplus.[14] The more the agent invests, the larger the bargaining surplus. However, as the principal will also obtain a bargaining surplus, the agent will not be granted the entire wage he wishes and hence the holdup problem arises. Therefore, the production of the verifiable output x becomes smaller than the first-best level.[15]

bargaining power at the beginning of the second stage. Therefore, if the short-term wage contract is agreed at the beginning of the first period, the agent always negotiates his second period wage at the beginning of the second period. Furthermore, this implies that the change in the bargaining power at the beginning of the first period and the second period is not crucial in deriving our main results.

[13] We show in Section 3 that if the agent is risk neutral, it can be either a fixed wage or incentive pay. We further show in Section 4 that if the agent is risk averse, the principal offers a fixed wage contract.

[14] The bargaining power of the agent naturally increases at the beginning of the second period if he has invested during the first period. We emphasize that the change in the bargaining power of the agent between the first and second periods is not crucial in obtaining our main results. In other words, we would derive our main results even if it was assumed the agent had bargaining power at the beginning of the first stage, and hence the principal and the agent Nash bargained in the first stage.

[15] If the agent is risk neutral and there is no holdup, the first-best level is achieved.

In sum, if the principal values the verifiable output x relatively more than the unverifiable output y, the principal offers a long-term wage contract, and thereby induces a stronger incentive to engage in the task that produces x by suppressing the agent's incentive to engage in the task that produces y. Conversely, if the principal values the unverifiable output y relatively more than the verifiable output x, the principal offers a short-term contract and the opposite holds true. These results generally hold in the environment in which the agent must produce both verifiable and unverifiable outputs (see Kamiya and Sato 2013). In other words, any sophisticated or complex contract cannot perform better than the simple wage contract developed in the present analysis.

Our results predict that the more the agent is expected to produce unverifiable outputs, such as leadership, popularity, and collegial work, the more often they will renew their wage contracts or have opportunities for promotion. For example, bureaucrats are not motivated by frequent wage fluctuations that reflect their outputs. Rather, they constantly receive fixed wage and are motivated by wage increase through promotions, which can be interpreted as wage negotiation. Potentially, these theoretical results could be empirically tested in industries where employees that renew their wage contracts may often be expected to produce more unverifiable outputs than employees of other industries that do not get the opportunity to renew their contracts as often. Our results further show that the more firm-specific knowledge the agent has, the more likely that the principal offers the agent a repetition of short-term contracts. This is because the distortion from the hold-up problem reduces when the agent has strong bargaining power. In deed, both verifiable and unverifiable outputs can achieve the level that is close to the first-best level when the agent has strong bargaining power. This finding is interesting in that a long-term contract can achieve the first- best level for the verifiable output, but the repetition of short-term contracts can achieve the level that is close to the first-best for both verifiable and unverifiable outputs.

The remainder of the article is organized as follows. Section 2 discusses the related literature. Section 3 analyzes the case of a risk-neutral agent. We also discuss the case with a limited liability constraint. Section 4 discusses the case in which the agent is risk averse. The final section concludes.

2 Literature

Our paper is closely related to Holmstrom and Milgrom (1991). In their paper, among their other important and novel findings, developed a multitask principal–agent model that i) "can account for paying fixed wages, even when good, objective output measures are available and agents are highly responsive to incentive pay" (Holmstrom and Milgrom 1991, p. 24) and ii) "can shed light on how tasks are allocated to different jobs" (Holmstrom and Milgrom 1991, p. 25). Here, a multitask problem is where the principal's utility is determined by output(s) produced through several tasks the agent engages in. In Holmstrom and Milgrom (1991), the externalities between tasks play an important role in deriving their key findings. Another crucial assumption used by Holmstrom and Milgrom (1991) in obtaining their results is that the agent's effort can be negative and the disutility function satisfies the following properties: $f'(x) < 0$ for $0 < x < \overline{x}$, $f'(x) > 0$ for $\overline{x} < x$, and $f''(x) > 0$ for all x. This implies that they assume that making some effort increases the agent's utility to some extent.[16]

We aim to contribute to the multitask literature by deriving similar results as Holmstrom and Milgrom (1991) as outlined in i) and ii) above in a simple dynamic multiperiod principal–agent model where externality need not necessarily exists between tasks.[17] That is, we assume that the cost functions are additive separable and the outputs are stochastically independent. We can therefore derive the trade-off mechanism in this paper *with* or *without* externalities between the tasks. While the former setting has been studied a lot (for example, Itoh 1992, Hemmer 1995, Dewatripont and Tirole 1999, MacDonald and Marx 2001, Akai, Mizuno, and Osano 2010, and Hellmann and Thiele 2011 all assume externalities in costs and/or production), the literature on the latter setting is limited (Baker 2001).[18] Therefore, in this paper we use the latter setting. We also assume $f'(x) > 0$ and

[16] In Holmstrom and Milgrom's (1991) model, similar results can be obtained using the following assumptions: $f'(x) = 0$ for $0 < x < \overline{x}$, $f'(x) > 0$ for $\overline{x} < x$, and $f''(x) > 0$ for all x. That is, the agent may make an effort even when the wage is fixed, as he is indifferent up to $\overline{x}$. However, in order to enforce $\overline{x}$, the principal must use some device, such as monitoring.

[17] This situation corresponds to, for example, a baseball player who is expected to make many hits (the verifiable output) and to also exercise leadership within the team (the unverifiable output). In this case, there may be some externalities in the production function. That is, a baseball player's popularity may increase if he makes a lot of hits. However, unlike other studies that require a large amount of externalities, our results hold in both the case in which the externality is small and the case where there is no externality at all.

[18] Baker (2001) considers the effect of a distorted performance measure (contractible) in inducing the effort of

$f''(x) > 0$ for all x, overriding their assumptions regarding effort. Consequently, in our model, ex post bargaining gives the agent an incentive to undertake effort in producing noncontractible outputs, whereas in Holmstrom and Milgrom (1991) the effort to induce outputs with too much noise (which corresponds to our noncontractible output) is derived instead as a result of the externality and assumptions that the effort can be negative and enhances the agent's utility to some extent.

Our model is also related to Kamiya and Sato (2013). Kamiya and Sato study the length and the optimal timing of wage contracts using specific utility functions in three-period and five-period models. Their main finding is that they find a combination of a medium-term contract and short-term contract (under three-period model), or a repetition of medium-term contracts (under five-period model) can be an optimal contract under a certain environment. They further develop a general model to show that any other contracting forms, such as option contracts or menu contracts, cannot achieve any better utility than a simple wage contract form. Their result from the general model that simple wage contract cannot be over-ridden by any other form of contract, can also be applied to this paper. However, this paper is different from Kamiya and Sato (2013) in that this paper studies the optimal timing and the efficient length of contract using a general utility function in two-period model. In other words, it effectively shows the trade-off between ex ante commitment and ex post bargaining in a simple but general model. Furthermore, the present analysis also shows that the similar results hold for the case in which the agent's utility is risk averse.

We further briefly discuss that complex and sophisticated contracts (such as those considered in Edlin and Reichelstein 1996, Maskin and Tirole 1999a, 1999b, and Moore and Repullo 1988), would not attain the first-best outcome, even if they were to be introduced into the same environment as discussed in this article. For example, Maskin and Tirole (1999b) and Noldeke and Schmidt (1995) introduce "option contracts," while the former demonstrates that an option to sell contracts can induce the correct incentive to invest

the agent when the output is unobservable (noncontractible). In this model, the agent makes an effort to enhance the distorted performance measure, and hence there is deviation from what the principal truly needs. Though Baker's (2001) analysis is basically a special case of that in Holmstrom and Milgrom (1991), Baker's (2001) goal is markedly different from Holmstrom and Milgrom (1991), and hence from the present analysis. In our paper, the utility of the principal is directly affected by both contractible and noncontractible outputs.

under some environments. If we introduced an option contract into our environment, the outcome would be that the principal has a right to sell her property to the agent and the agent buys the entire property (the project). The principal will then exercise the option if the investment is not efficient, but will not exercise the option if the investment is efficient. However, under a limited liability constraint the agent cannot purchase the project, and hence this is not a feasible contract.[19]

Farrell and Shapiro (1989) and Bernheim and Whinston (1998) present models with verifiable and unverifiable attributes, where it is better not to contract or to contract incompletely over even verifiable attributes. Our paper may somewhat seem similar to theirs. However, their logics are quite different from ours. Indeed in Proposition 1 in Farrell and Shapiro, the seller does not prefer signing a contract on verifiable attributes, because doing so becomes a constraint in optimizing unverifiable attributes. This argument has nothing to do with an ex post bargaining, and cannot be applied to our case. This is because the principal does not choose any variable to optimize his utility after signing a contract. Bernheim and Whinston demonstrate that if there are some unverifiable actions and if agents' actions are sequential, there are cases in which an efficient outcome is obtained only by the incomplete contracting of verifiable actions. In their argument, by an incomplete contract, the shape of the second mover's best-response function is modified such that the first mover chooses an (unverifiable) action that leads to an efficient outcome. This is very different from our argument on trade-off between ex ante commitment and ex post bargaining in inducing the effort of the agent.

3 The Model: The Case of a Risk-neutral Agent

There is a principal and an agent. We assume both are risk neutral. There are two types of outputs: an observable and contractible output $x \geq 0$ and an observable but noncontractible output $y \geq 0$. There are two contractible output levels, x^H and x^L, where $x^H > x^L > 0$. The probabilities of x^H and x^L are denoted by $P^H \in [0, 1]$ and $P^L = 1 - P^H$. There are two noncontractible output levels, θy^H and θy^L, where $y^H > y^L > 0$. Note that $\theta \geq 0$ is a parameter introduced for later use. The probabilities of y^H and y^L are denoted

[19] If there is no limited liability constraint, the first-best outcome is attained when the agent is risk neutral. When the agent is risk averse, however, the first-best outcome is not attained.

by $Q^H \in [0,1]$ and $Q^L = 1 - Q^H$. In the first period, the agent makes two types of effort (which we henceforth call investments), $I_c \geq 0$ and $I_n \geq 0$ to obtain the skill for producing x and y, respectively. We assume that both I_c and I_n are observable but noncontractible, and that P^H and Q^H in the second period are functions of I_c and I_n, denoted by $P^H(I_c)$ and $Q^H(I_n)$, respectively. As is formally stated in Assumptions 1 and 2, we assume that the random variables x and y are stochastically independent and that $P^H = Q^H = 0$ in the first period. That is, we assume that the investments in human capital made in the first period will increase the agent's skills from the second period onwards. The agent incurs disutilities in making investments, denoted $D_c(I_c)$ and $D_n(I_n)$. Let $\delta \in (0,1)$ be the discount factor. The wages for each period are paid at the end of each period, or after the realization of the outputs in each period.[20] As x is the only contractible output, the wage depends only on the realization of x: the wages for x^H and x^L are denoted by w^H and w^L, respectively. w^i, $i = H, L$, in period t is denoted by w_t^i, $t = 1, 2$. Because of risk neutrality, w_2^i need not depend on the realization of an output in the first period. We first investigate the model without a limited liability constraint, and later provide similar results after imposing this constraint.

Note that there is no externality between I_c and I_n, as x and y are stochastically independent and the total cost of the investments is additively separable, i.e., $D_c(I_c) + D_n(I_n)$.

Throughout this section, we make the following three assumptions. The assumptions on D_c, D_n, P^H, and Q^H are standard.

Assumption 1

1. $\frac{dD_i}{dI_i} > 0$, $\frac{d^2D_i}{dI_i^2} > 0$, $D_i(0) = 0$, and $\frac{d^2D_i(0)}{dI_i^2} = 0$, $i = c, n$.
2. $\frac{dP^H}{dI_c} > 0$ and $\frac{d^2P^H}{dI_c^2} < 0$.
3. $\frac{dQ^H}{dI_c} > 0$ and $\frac{d^2Q^H}{dI_c^2} < 0$.
4. The random variables x and y are stochastically independent.

For simplicity, we make the following assumption.[21]

[20] As the agent is risk neutral, we can consider a model in which the wages of both periods are paid together at the end of the second period. This is, however, a special case of a long-term contract.

[21] We can obtain what we would like to achieve in this analysis without this assumption. The assumption is imposed purely for the sake of simplifying the analysis.

Assumption 2 In the first period, the probabilities of x^H and y^H are zero.

Using this assumption, the principal need not determine w_1^H in the first period.

We assume that the market for workers without firm-specific skills is competitive. We also assume that the agent obtains some firm-specific skills in the first period, and hence has bargaining power to negotiate his wage at the beginning of the second period.[22] Therefore, when the principal hires an agent without firm-specific skills, she posts a take-it-or-leave-it wage offer. After the agent has obtained the skills, the principal and the agent bargain over the wage at the beginning of the second period. For simplicity, we adopt Nash bargaining with a threat point set at $(0, 0)$. That is, we assume that their bargaining power is equal and that if they lose a partner, they cannot find a new partner, i.e., they can access the labor market only once and their reservation utilities are zero. It is worth noting that we can obtain similar results even if they have different bargaining power or their reservation utilities are nonzero in the second period. Note that in the discussion of how renegotiation-proof the contracts are in the following Propositions, we consider bargaining where the status quo is the wage contract signed in the previous period.

Assumption 3 When a contract is signed at the beginning of the first period, the principal posts a take-it-or-leave-it wage offer. When a contract is signed at the beginning of the second period, they Nash bargain over wages with the threat point held at $(0, 0)$.

We consider two types of wage contract: a short-term wage contract and a long-term wage contract. Under the short-term wage contract, the wages are determined at the beginning of each period and paid at the end of each period. Under the long-term wage contract, the wages for both periods are determined at the beginning of the first period but paid at the end of each period. As shown later, the equilibrium contracts are renegotiation-proof. We also discuss the limited liability constraint at the end of this section, and demonstrate that similar results are obtained under the constraint.

[22] Alternatively, we assume that the agent that made investments in the first period, i.e., the agent with $I_c > 0$ or $I_n > 0$ has bargaining power. We could assume that bargaining power is only given to the agent with $I_c > 0$ or $I_n > 0$, but we can obtain a similar result even when we give bargaining power to the agent that did not make any investments.

3.1 A Short-term Wage Contract

Under the short-term wage contract, the principal and the agent determine the first-period wage at the beginning of the first period, and they bargain over the second-period wage at the beginning of the second period. The agent can make investments in human capital during the first period. By Assumption 3, the contracting problem of the short-term wage contract in the first period is a take-it-or-leave-it offer on the first-period wage, subject to the individual rationality constraint and the incentive compatibility constraint on investments:

$$\max_{w_1^L, I_c, I_n} x^L - w_1^L + \theta y^L + \delta V_2^p(I_c, I_n) \tag{1}$$

$$\text{s.t. } w_1^L - D_c(I_c) - D_n(I_n) + \delta V_2^a(I_c, I_n) \geq u, \tag{2}$$

$$\begin{aligned} & w_1^L - D_c(I_c) - D_n(I_n) + \delta V_2^a(I_c, I_n) \\ & \geq w_1^L - D_c(I_c') - D_n(I_n') + \delta V_2^a(I_c', I_n'), \ {}^\forall I_c', I_n', \end{aligned} \tag{3}$$

where $u > 0$ is the reservation utility determined in the competitive market, and $V_2^p(I_c, I_n)$ and $V_2^a(I_c, I_n)$ are the principal's and the agent's values when the investments are I_c and I_n. The individual rationality constraint is given by (2) and the incentive compatibility constraint is given by (3). Note that $V_2^p(I_c, I_n)$ and $V_2^a(I_c, I_n)$ are determined by the backward induction given below. The agent has bargaining power at the beginning of the second period. Applying Assumption 3, the principal and the agent Nash bargain over wages: for a given (I_c, I_n),

$$\max_{w_2^H, w_2^L} \left\{ \sum_{j=H,L} P^j(I_c)(x^j - w_2^j) + g(I_n, \theta) \right\} \left\{ \sum_{j=H,L} P^j(I_c) w_2^j \right\},$$

where $g(I_n, \theta) = \sum_{i=H,L} Q^i(I_n)\theta y^i$. As both players are risk neutral, they obtain the same utilities from the Nash bargaining solution and this equals half of the total utility. Formally, their utilities are expressed as

$$\frac{1}{2} \left\{ \sum_{j=H,L} P^j(I_c) x^j + g(I_n, \theta) \right\},$$

and this is equal to $V_2^p(I_c, I_n)$ and $V_2^a(I_c, I_n)$.

3.2 A Long-term Wage Contract

Under the long-term wage contract, the principal and the agent agree on the wages for both periods at the beginning of the first period. The agent can make investments during the first period. By Assumption 3, the contracting problem is a take-it-or-leave-it offer on the first- and second-period wages, subject to the individual rationality constraint and the incentive compatibility constraint on investments:

$$\max_{w_1^L, I_c, I_n, w_2^H, w_2^L} x^L - w_1^L + \theta y^L + \delta \left(\sum_{j=H,L} P^j(I_c)(x^j - w_2^j) + g(I_n, \theta) \right) \tag{4}$$

$$\text{s.t.} \qquad w_1^L - D_c(I_c) - D_n(I_n) + \delta \sum_{j=H,L} P^j(I_c) w_2^j \geq u, \tag{5}$$

$$\begin{aligned} & w_1^L - D_c(I_c) - D_n(I_n) + \delta \sum_{j=H,L} P^j(I_c) w_2^j \\ \geq \; & w_1^L - D_c(I_c') - D_n(I_n') + \delta \sum_{j=H,L} P^j(I_c') w_2^j, \; {}^\forall I_c', I_n'. \end{aligned} \tag{6}$$

The principal's utility is (4), and the agent's utility is the left-hand side of (5). (5) and (6) are expressions satisfying the individual rationality and the incentive compatibility of the agent.

3.3 A Comparison of the Two Types of Wage Contract

We explain below the mechanism through which the principal decides the wage profile and the frequency with which to renew the wage contract. We show that it depends on several important parameters, including the relative efficiency of investment. Under the long-term wage contract, at the beginning of the first period, the principal can write a second-period wage depending on the output x the agent is going to produce in the second period. However, she cannot write a second-period wage to reflect the amount of y the agent is going to produce in the second period, as y may be observable but unverifiable. Then, it is clear that the long-term wage contract deprives the agent of investing in I_n, an effort to improve his skill to produce y. The benefit of the long-term wage contract, however, is that the principal can motivate the agent to invest significantly more in I_c than under the short-term wage contract. More specifically, the long-term wage contract can induce the first-best level of I_c. As shown in Appendix A, the agent's incentive to invest in I_c under the short-term wage contract is smaller than the first-best level. Note that in order to motivate the agent to invest in I_c, w_2^H must be larger than w_2^L.

Under the short-term wage contract, the bargaining position/surplus of the agent at the beginning of the second period depends on his skill in producing y as well as on his skill in producing x. Therefore, the agent has an incentive to invest in I_n during the first period, which is also beneficial for the principal. However, the agent has less incentive to invest in I_c under the short-term wage contract than under the long-term wage contract. This is because the principal obtains half of the benefit generated from the agent's investment in I_c, through Nash bargaining. Note that there is no need to motivate the agent to invest in I_c (i.e., w_2^H can equal w_2^L), as both parties sign the second-period wage contract after the agent has made an investment.

Hence, the principal chooses whether to offer a short- or long-term wage contract depending on the relative efficiency of investment. That is, if the principal values x relatively more than y, and if the principal expects that the investment the agent makes in x is efficient, she prefers a long-term to a short-term wage contract. Otherwise, she prefers the short-term wage contract. In other words, if the principal wishes the agent to invest a great deal in I_c, the principal chooses the long-term wage contract where the agent has no incentive to invest in I_n. If the principal wishes the agent to invests in both I_n and I_c but values I_n relatively to I_c, the principal chooses the short-term wage contract.

Proposition 1

1. *The investment for the contractible output x is the first-best under the long-term wage contract and it is larger than that under the short-term wage contract.*

2. *Under the long-term wage contract, w_2^H is strictly larger than w_2^L. Under the short-term wage contract, the fixed wage, i.e., $w_2^H = w_2^L$, can be offered.*

3. *There exists a $\bar{\theta} > 0$ such that the principal prefers a long-term to a short-term wage contract at the beginning of the first period for $\theta \in [0, \bar{\theta})$, and prefers a short-term to a long-term wage contract for $\theta \in (\bar{\theta}, \infty)$. Moreover, the equilibria are renegotiation-proof.*

Proof:
See Appendix A. ∎

Proposition 1 has several intuitions. First, a long-term wage contract is more likely to motivate the agent to make an effort to produce verifiable outputs by offering the agent incentive pay and infrequent wage negotiations. Moreover, the equilibrium effort level for the verifiable output is the first-best under the long-term wage contract. Second, a short-term wage contract motivates the agent to make effort in producing both verifiable and unverifiable outputs but the effort for the verifiable output is not the first-best. Furthermore, as another intuition of Proposition 1, a fixed wage *can* be used to motivate the agent under the short-term wage contract. Later in Proposition 3, we prove that a fixed wage must be used under the short-term wage contract if the agent is risk averse.

The implication of Proposition 1 is that it is better not to hold wage negotiations too often in an industry or firm where verifiable outputs are valued. In addition, even in the same firm, if one agent is expected to produce more of the verifiable outputs whereas the other agent is expected to produce more of the unverifiable outputs, then future wages for the former agent should be agreed at the beginning of the initial contract whereas future wages should be negotiated more often for the latter agent. Examples of this (encompassing infrequent wage negotiations, incentive pay, and verifiable outputs) are the wages of taxi drivers; pension and mutual fund managers; and car and insurance salespersons; gratuities for waiters and waitresses in the US; and book, music, film, and software royalties.

The examples of agents expected to produce more unverifiable outputs and hence receiving fixed pay while motivated by promotions , or wage renewal by promotion, are bureaucrats; tenured academics; and office workers. The tendency is the strongest in bureaucrats where the experience in one ministry may not necessarily be useful to the other ministry, hence making it difficult for them to move to the other ministry. As for tenured academics, we can consider that their bargaining power is stronger (at least the threat point is higher) than bureaucrats or regular office workers because their experience in research(investments) is transferrable. As they can more easily move to the other university if the negotiation crashes (although this is off-the-path of equilibrium in our model), our conjecture is that negotiations held between academics and their principals are more likely to crash with a smaller exogenous shock (e.g., a vacancy in the post of associate professor at other universities) than negotiations held between bureaucrats and their principals. As the purpose of this article is the comparison of different length contract, we do not formally model exogenous shock into our model. Interested readers are referred to Levin (2002) in which he analyzes the effect of the exogenous shock in the extension.

Our model assumes that the agent stays in one firm till the end of the second period. If we consider this to be the result of the firm-specificity in the effort of the agent, it implies that market wage is lower than the wage in the firm, hence on the equilibrium path, the principal can motivate the agent to expend the effort she wishes by adjusting the length of contracts. Holmstrom (1999) shows that it is inefficient if the wage the agent receives from the firm is equal to the market wage (fixed wage). He discusses that market wage could induce the efficient level of effort only when some strong assumptions are imposed.

Next, we consider the effect of firm-specificity on the choice of contracts. There are two ways to investigate the effects of firm-specificity of investments: one is to consider that (i) it is reflected in the threat point. The other is to consider that (ii) it is reflected in the bargaining power. In (i), even if the threat point changes it does not affect the choice of the contracts, because the contract with larger total utility would be chosen and this has nothing to do with the threat point. On the other hand, in (ii), the change in the bargaining power does affect the choice between the short and the long-term contracts.

Proposition 2 *If the agent had more firm-specific knowledge (bargaining power), the principal is likely to offer the agent short-term contracts.*

Proof:

Let the bargaining power of the principal and the agent be $1-\beta$ and β. Then the second period bargaining becomes:

$$\max_{w_2^H, w_2^L} \left\{ \sum_{j=H,L} P^j(I_c)(x^j - w_2^j) + g(I_n, \theta) \right\}^{1-\beta} \left\{ \sum_{j=H,L} P^j(I_c) w_2^j \right\}^{\beta}.$$

The first order condition for I_c and I_n for short-term contract are as follows:

$$\frac{dD_c(I_c)}{dI_c} = \beta\delta\frac{dP^H(I_c)}{dI_c}(x^H - x^L),$$

$$\frac{dD_n(I_n)}{dI_n} = \beta\delta\theta\frac{dQ^H(I_n)}{dI_n}(y^H - y^L).$$

Then Ω is defined as in the case of $\beta = \frac{1}{2}$ and

$$\frac{\partial\Omega}{\partial\beta} = -\left(\delta(1-\beta)P^{H\prime}(I_c^*)\frac{\partial I_c^*}{\partial\beta}(x^H - x^L) + \delta(1-\beta)Q^{H\prime}(I_n^*)\frac{\partial I_n^*}{\partial\beta}(y^H - y^L)\right)$$

holds. From the first order conditions, $\frac{\partial I_c^*}{\partial\beta}$ and $\frac{\partial I_n^*}{\partial\beta}$ are positive, and thus $\frac{\partial\Omega}{\partial x^H} < 0$, i.e., the short-term contract is likely to be chosen.

Intuitively, the agent with strong bargaining power does not have a fear of hold-up. Therefore, the agent has an incentive to exert his effort for both verifiable and unverifiable efforts in order to achieve large bargaining surplus. As a result, the principal offers short-term contracts, instead of a long-term contract.

The implication of this proposition is that if the agent has strong bargaining power, the more likely the repetition of short-term contracts are chosen and hence, nearly the optimal level are produced for both verifiable and unverifiable outputs. If there is an agent in a company who cannot be substituted or replaced by another agent easily, the company can minimize the distortion of the agent's effort level by choosing short-term contracts or frequently giving opportunities to renew the wage contracts of the agent.

3.4 Limited Liability Constraints

We discuss below the limited liability constraints. We consider two types of constraints: (i) all wages are nonnegative, and (ii) $w_1^L + \delta w_2^i \geq 0, i = H, L$. For the short-term wage contract, we can set $w_2^H = w_2^L = V_2^a(I_c^*, I_n^*) \geq 0$. Then

$$w_1^L + \delta w_2^i = D_c(I_c^*) + D_n(I_n^*) + u \geq 0, i = H, L$$

holds, where I_c^* and I_n^* are investments chosen under the short-term wage contract (see Appendix A). Therefore, the limited liability constraint of type (ii) is always satisfied. Moreover, if

$$D_c(I_c^*) + D_n(I_n^*) - \delta V_2^a(I_c^*, I_n^*) + u > 0, \tag{7}$$

holds, w_1^L can be nonnegative, i.e., (i) is satisfied.

For the long-term wage contract, we can set $w_2^H = x^H - r$ and $w_2^L = x^L - r$, where r is the principal's utility in period two (see Appendix A). Then

$$\delta r = w_1^L - D_c(I_c^{**}) + \delta \sum_{j=H,L} P^j(I_c^{**})x^j - u$$

holds, where I_c^{**} is the investment chosen under the long-term wage contract. Hence

$$w_1^L + \delta w_2^H > w_1^L + \delta w_2^L = \delta x^L + D_c(I_c^{**}) - \delta \sum_{j=H,L} P^j(I_c^{**})x^j + u.$$

The right-hand side is positive for a sufficiently large u, as I_c^{**} does not depend on u. Therefore, the limited liability constraint of type (ii) is not binding for a sufficiently large u. Note that we can also find a sufficiently large u such that (i) is also satisfied. If we consider the case in which u is not sufficiently large, the limited liability constraint is binding under the long-term wage contract, hence the total utility is smaller than the case without the constraint.

We show below that Proposition 1.3 holds for a type (ii) limited liability constraint. When $\theta = 0$, the principal's utility is larger under the long-term wage contract than under the short-term wage contract. Indeed, setting $w_2^L = \frac{1}{2}x^L > 0$, $w^H = \frac{1}{2}x^H > 0$ and

$$w_1^L = D_c(I_c^*) + D_n(I_n^*) - \delta V_2^a(I_c^*, I_n^*) + u, \tag{8}$$

we can show that the agent chooses I_c^* and the principal obtains the same utility as she does under the short-term wage contract (see Appendix A). Moreover, the principal can choose the wage differences $(w_2^H - w_2^L)$ larger than $\frac{1}{2}(x^H - x^L)$. The principal can also keep the expected wages constant. Therefore, she can obtain a larger gain. For a type (ii) limited liability constraint, Proposition 1.3 still holds with smaller $\bar{\theta}$. This is because if we consider the short-term wage contract, the principal obtains the same gain as in the case without limited liability constraints. Alternatively, if we consider the long-term wage contract, the principal's gain is smaller.

Moreover, for a type (i) limited liability constraint, if (7) is satisfied, the same results are obtained by using the same argument as a type (ii) limited liability constraint.

Proposition 3 *If a limited liability constraint is imposed, contracts satisfy the following properties.*

1. *Under the long-term wage contract, w_2^H is larger than w_2^L. Under the short-term wage contract, a fixed wage, i.e., $w_2^H = w_2^L$, can be offered.*

2. *For a type (ii) limited liability constraint, there exists a $\bar{\theta} > 0$ such that the principal prefers a long-term to a short-term wage contract at the beginning of the first period for $\theta \in [0, \bar{\theta})$, and prefers a short-term to a long-term wage contract for $\theta \in (\bar{\theta}, \infty)$. Next, if the condition for limited liability constraint (i) for a short-term wage contract, expressed as :*

$$D_c(I_c^*) + D_n(I_n^*) - \delta V_2^a(I_c^*, I_n^*) + u > 0,$$

is satisfied, the same results can be obtained.

4 The Case of a Risk-averse Agent

In this section, we adopt the same model as in the previous section, except that the agent's utility regarding his wage, w, is expressed as $U(w) = w^{1-\rho}$, where $0 \leq \rho < 1$, i.e., the case of constant relative risk aversion, and the domain of w is the set of nonnegative real numbers, i.e., we adopt the limited liability constraint of type (i).[23] We can show that the same results as found in the previous section hold for ρ close to zero, as all equilibrium values can be shown to be continuous functions of (ρ, θ). Note that the risk-neutral case with a type (i) limited liability constraint corresponds to the case of $\rho = 0$.

Proposition 4 *Suppose*

$$D_c(I_c^*) + D_n(I_n^*) - \delta V_2^a(I_c^*, I_n^*) + u > 0$$

and $P^H(I_c) \in (0, 1)$ for all I_c, where I_c^* and I_n^* are investments chosen under the short-term wage contract when $\rho = 0$. Then, there exists a $\bar{\rho} \in (0, 1)$ such that the following properties hold for all $\rho \in (0, \bar{\rho}]$:

1. *Under the long-term wage contract, w_2^H is larger than w_2^L, and under the short-term wage contract, the fixed wage, i.e., $w_2^H = w_2^L$, is offered.*

[23] As we can derive similar results for the case with savings and that without savings, we do not consider savings for simplicity. Note that there is no need to consider savings for the risk-neutral agent, as there is no need for the agent to save because of the linearity of the utility function.

2. *There exists a* $\bar{\theta} > 0$ *such that the principal prefers a long-term to a short-term wage contract at the beginning of the first period for* $\theta \in [0, \bar{\theta})$, *and prefers a short-term to a long-term wage contract for* $\theta \in (\bar{\theta}, \infty)$.

Proof:
See Appendix B. ∎

Even if we consider renegotiation under the long-term wage contract, the principal offers the same wages and the agent invests the same amount of I_c as in the case without renegotiation. Furthermore, the parties would agree to have a fixed wage contract in the renegotiation, as the agent is risk averse. That is, the agent accepts a fixed wage larger than the certainty equivalent, and the parties share the gain (which is the difference between the expected wage and the certainty equivalent) from the renegotiation.

5 Conclusion

In this article, we have shown that incentive contracting (a long-term wage contract) and holdup (a short-term wage contract) are alternative ways to motivate the effort (investment) of the agent. That is, a long-term wage contract does not allow for holdup and induces the agent's effort for contractible outputs, but also removes the incentive for noncontractible outputs. A short-term wage contract allows for greater holdup and reduces the incentive for contractible outputs, but motivates the agent's effort for noncontractible outputs. Hence, an appropriate use of different length contracts can mitigate the inefficiency caused by the trade-off. These findings have been derived using a model where: i) dynamics exist, ii) the principal benefits directly from both contractible (verifiable) and noncontractible (unverifiable) outputs, iii) the agent needs to make investments of his human capital (which is firm specific) to produce both outputs, and iv) incentive pay contracts can be made only for a task that produces a verifiable outcome.

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Appendices

A The Proof of Proposition 1

A Short-term Wage Contract

In the first period, the agent chooses I_c and I_n satisfying the incentive compatibility constraint:

$$\max w_1^L - D_c(I_c) - D_n(I_n) + \frac{1}{2}\delta \left\{ \sum_{j=H,L} P^j(I_c)x^j + g(I_n, \theta) \right\}. \tag{9}$$

The first-order condition yields

$$\frac{dD_c(I_c)}{dI_c} = \frac{1}{2}\delta \frac{dP^H(I_c)}{dI_c}(x^H - x^L), \tag{10}$$

and

$$\frac{dD_n(I_n)}{dI_n} = \frac{1}{2}\delta \frac{\partial g(I_n, \theta)}{\partial I_n}.$$

Note that by Assumption 1 the second-order condition is satisfied. Let the solutions of the above equation be I_c^* and I_n^*. On the other hand, by the individual rationality constraint, the principal must set

$$w_1^L = D_c(I_c^*) + D_n(I_n^*) - \delta V_2^a(I_c^*, I_n^*) + u. \tag{11}$$

Then, the principal's utility is obtained as follows:

$$\begin{aligned} x^L - w_1^L + \theta y^L + \delta V_2^p(I_c^*, I_n^*) &= x^L + \theta y^L - D_c(I_c^*) - D_n(I_n^*) + 2\delta V_2^p(I_c^*, I_n^*) - u \\ &= x^L + \theta y^L - D_c(I_c^*) - D_n(I_n^*) \\ &\quad + \delta \left\{ \sum_{j=H,L} P^j(I_c^*)x^j + g(I_n^*, \theta) \right\} - u. \end{aligned} \tag{12}$$

Finally, we can see that the principal can choose a fixed wage, i.e.,

$$w_2^H = w_2^L = V_2^a(I_c^*, I_n^*) = \frac{1}{2} \left\{ \sum_{j=H,L} P^j(I_c^*)x^j + g(I_n^*, \theta) \right\}.$$

A Long-term Wage Contract

By (6), $I_n^{**} = 0$ is chosen. As both the principal and the agent are risk neutral, the joint utility

$$x^L + \theta y^L - D_c(I_c) + \delta \left(\sum_{j=H,L} P^j(I_c) x^j + g(0,\theta) \right) \tag{13}$$

is maximized with respect to I_c. Indeed, setting $w_2^j = x^j - r, j = H, L$, where r is the principal's utility in period two, (6) yields the following first-order condition for maximizing (13):

$$\frac{dD_c(I_c)}{dI_c} = \delta \frac{dP^H(I_c)}{dI_c}(x^H - x^L). \tag{14}$$

Let I_c^{**} be the solution. Then by the individual rationality constraint,

$$w_1^L = D_c(I_c^{**}) - \delta \sum_{j=H,L} P^j(I_c^{**}) w_2^j + u. \tag{15}$$

Then, the principal's utility is expressed as follows:

$$\begin{aligned} & x^L - w_1^L + \theta y^L + \delta \left(\sum_{j=H,L} P^j(I_c^{**})(x^j - w_2^j) + g(0,\theta) \right) \\ & = x^L - D_c(I_c^{**}) + \theta y^L + \delta \left(\sum_{j=H,L} P^j(I_c^{**}) x^j + g(0,\theta) \right) - u. \end{aligned} \tag{16}$$

Finally, from $w_2^j = x^j - r, j = H, L$, w_2^H is larger than w_2^L.

A Comparison of Two Types of Contract

First, comparing (10) and (14), the agent undertakes more investment in I_c under the long-term wage contract than under the short-term wage contract, i.e., $I_c^* < I_c^{**}$.

When $\theta = 0$, the principal prefers the long-term wage contract to the short-term wage contract, i.e., (16) is larger than (12). Indeed, when $\theta = 0$, $I_n^* = 0$ is chosen even in the short-term wage contract and thus

$$(16) - (12) = -D_c(I_c^{**}) + \delta \sum_{j=H,L} P^j(I_c^{**}) x^j - \left(-D_c(I_c^*) + \delta \sum_{j=H,L} P^j(I_c^*) x^j \right) > 0.$$

The last inequality follows from (14), i.e., I_c^{**} satisfies the first-order condition for maximizing $-D_c(I_c) + \delta \sum_{j=H,L} P^j(I_c) x^j$.

In order to investigate the effect of θ on the choice of contracts, we only need to investigate

$$-D_n(I_n) + \frac{1}{2}\delta g(I_n, \theta)$$

in (9), as I_c^* does not depend on θ.

Let

$$\kappa(\theta) = \max_{I_n} -D_n(I_n) + \frac{1}{2}\delta g(I_n, \theta)$$

and

$$h(\theta) = \arg\ \max_{I_n} -D_n(I_n) + \frac{1}{2}\delta g(I_n, \theta).$$

Then, by the envelope theorem

$$\kappa'(\theta) = \frac{1}{2}\delta \frac{\partial g(h(\theta), \theta)}{\partial \theta}.$$

Therefore, κ is a strictly increasing function of θ, and κ goes to $+\infty$ as θ goes to $+\infty$, because $\frac{\partial g(h(\theta),\theta)}{\partial \theta} = \sum_{i=H,L} Q^i(h(\theta))y^i \geq y^L > 0$. This implies that the principal's utility under the short-term wage contract (12) also goes to $+\infty$ as θ goes to $+\infty$. When $\theta = 0$, the principal strictly prefers the long-term wage contract to the short-term wage contract, and hence there exists a $\bar{\theta} > 0$ such that the principal prefers the long-term wage contract to the short-term wage contract for $\theta \in [0, \bar{\theta})$, and prefers the short-term wage contract to the long-term wage contract for $\theta \in (\bar{\theta}, \infty)$.

B The Proof of Proposition 4

A Short-term Wage Contract

The contracting problem in period two is as follows: for a given (I_c, I_n),

$$\max_{w_2^H, w_2^L} \left\{ \sum_{j=H,L} P^j(I_c)(x^j - w_2^j) + g(I_n, \theta) \right\} \left\{ \sum_{j=H,L} P^j(I_c)U(w_2^j) \right\},$$

where $g(I_n, \theta) = \sum_{i=H,L} Q^i(I_n)\theta y^i$. Note that $w_2^H, w_2^L \geq 0$ is shown later. The first-order conditions with respect to w_2^H and w_2^L are as follows:

$$\sum_{j=H,L} P^j(I_c)U(w_2^j) = U'(w_2^i) \left(\sum_{j=H,L} P^j(I_c)(x^j - w_2^j) + g(I_n, \theta) \right)$$

for $i = H, L$. This yields

$$w_2^H = w_2^L.$$

That is, a fixed wage is offered. On the other hand, from $U'(w) = (1-\rho)w^{-\rho}$,

$$w_2 = w_2^H = w_2^L = \frac{1-\rho}{2-\rho}\left(\sum_{j=H,L} P^j(I_c)x^j + g(I_n,\theta)\right) \geq 0$$

holds. Thus, the value for the agent in the second period, denoted by $V_2^a(I_c, I_n, \rho, \theta)$, is equal to $w_2^{1-\rho}$. Note that $V_2^a(I_c, I_n, 0, \theta)$ is equal to the value for the risk-neutral agent in the second period obtained in Section 2. The value for the principal is obtained as follows:

$$V_2^p(I_c, I_n, \rho, \theta) = \frac{1}{2-\rho}\left(\sum_{j=H,L} P^j(I_c)x^j + g(I_n,\theta)\right).$$

In the first period, the agent chooses I_c and I_n satisfying the incentive compatibility constraint:

$$\max w_1^L - D_c(I_c) - D_n(I_n) + \delta\left[\frac{1-\rho}{2-\rho}\left\{\sum_{j=H,L} P^j(I_c)x^j + g(I_n,\theta)\right\}\right]^{1-\rho}. \tag{17}$$

The first-order condition yields

$$\frac{dD_c(I_c)}{dI_c} = \delta\frac{1-\rho}{2-\rho}\frac{dP^H(I_c)}{dI_c}(x^H - x^L)(1-\rho)\left[\frac{1-\rho}{2-\rho}\left\{\sum_{j=H,L} P^j(I_c)x^j + g(I_n,\theta)\right\}\right]^{-\rho}, \tag{18}$$

and

$$\frac{dD_n(I_n)}{dI_n} = \delta\frac{1-\rho}{2-\rho}\frac{\partial g(I_n,\theta)}{\partial I_n}(1-\rho)\left[\frac{1-\rho}{2-\rho}\left\{\sum_{j=H,L} P^j(I_c)x^j + g(I_n,\theta)\right\}\right]^{-\rho}.$$

Note that by Assumption 1, the second-order condition is satisfied and the solutions of the above equation, denoted $I_c^*(\rho)$ and $I_n^*(\rho,\theta)$, are continuous functions. On the other hand, by the individual rationality constraint, the principal must set

$$w_1^L = D_c(I_c^*(\rho)) + D_n(I_n^*(\rho,\theta)) - \delta V_2^a(I_c^*(\rho), I_n^*(\rho,\theta)) + u. \tag{19}$$

Then, the principal's value,

$$x^L - w_1^L + \theta y^L + \delta V_2^p(I_c^*(\rho), I_n^*(\rho,\theta)), \tag{20}$$

is a continuous function of (ρ, θ), because I_c^* is a continuous function of ρ and I_n^* is a continuous function of (ρ, θ). Note that w_1^L is positive for ρ sufficiently close to 0, because $I_c^*(0)$ and $I_n^*(0, \theta)$ are the investments for a risk-neutral agent and wages are positive. That is, the limited liability constraint is satisfied.

A Long-term Wage Contract

The principal's problem is as follows:

$$\max_{w_1^L \geq 0, I_c \geq 0, I_n \geq 0, w_2^H \geq 0, w_2^L \geq 0} x^L - w_1^L + \theta y^L + \delta \left(\sum_{j=H,L} P^j(I_c)(x^j - w_2^j) + g(I_n, \theta) \right) \tag{21}$$

$$\text{s.t.} \quad (w_1^L)^{1-\rho} - D_c(I_c) - D_n(I_n) + \delta \sum_{j=H,L} P^j(I_c)(w_2^j)^{1-\rho} \geq u, \tag{22}$$

$$\begin{aligned} & (w_1^L)^{1-\rho} - D_c(I_c) - D_n(I_n) + \delta \sum_{j=H,L} P^j(I_c)(w_2^j)^{1-\rho} \\ \geq \; & (w_1^L)^{1-\rho} - D_c(I_c') - D_n(I_n') + \delta \sum_{j=H,L} P^j(I_c')(w_2^j)^{1-\rho}, \; {}^\forall I_c', I_n'. \end{aligned} \tag{23}$$

Clearly, $I_n = 0$ is chosen.

Below, we show by Berge's maximum Proposition (see, for example, Hildenbrand (1974)) that the value of the above problem is a continuous function of ρ. Let $B = D_c(I_c^{**} + 1) + u + 1$, where I_c^{**} is the first-best investment obtained in the case of a risk-neutral agent. Then, we can restrict the domain of investments and wages in the compact set $\Omega = \{(I_c, w_1^L, w_2^H, w_2^L) \mid 0 \leq I_c \leq I_c^{**} + 1, \; 0 \leq (w_1^L)^{1-\rho}, \delta(w_2^H)^{1-\rho}, \delta(w_2^L)^{1-\rho} \leq B\}$. Below, we show that the feasible set in the above problem is a continuous correspondence of ρ. Then, because the objective function is continuous, the continuity of the maximum value in ρ follows from Berge's maximum Proposition. Let $\Gamma(\rho)$ be the feasible set of the principal's problem, i.e., the set of $(I_c, w_1^L, w_2^H, w_2^L)$ satisfying (22) and (23). Let $\Pi(\rho) = \Gamma(\rho) \cap \Omega$. Below, we show that Π is a continuous correspondence of ρ. The upper hemicontinuity clearly follows from the continuity of the functions in the constraints. The lower hemicontinuity can be obtained as follows.

First, note that by the strict concavity of P^H and strict convexity of D_c, the optimal I_c in (23) is a continuous function of (ρ, w_2^H, w_2^L), denoted by $I_c(\rho, w_2^H, w_2^L)$. For $\hat{\rho} \in [0, 1)$, let $(\hat{I}_c, \hat{w}_1^L, \hat{w}_2^H, \hat{w}_2^L) \in \Pi(\hat{\rho})$ and $\rho^k \in [0,1), k = 1,2\ldots,$ be a sequence converging to $\hat{\rho}$. Suppose $(\hat{w}_1^L)^{1-\rho}, \delta(\hat{w}_2^H)^{1-\rho}$, and $\delta(\hat{w}_2^L)^{1-\rho}$ are larger than 0 and smaller than B, it is easy to find a sequence $(w_1^{Lk}, w_2^{Hk}, w_2^{Lk}), k = 1,2\ldots,$ satisfying (22) with $\rho = \rho^k$ and $I_c = I_c(\rho^k, w_2^{Hk}, w_2^{Lk})$, and converging to $(\hat{w}_1^L, \hat{w}_2^H, \hat{w}_2^L)$. Suppose some of $(\hat{w}_1^L)^{1-\rho}, \delta(\hat{w}_2^H)^{1-\rho}$, and $\delta(\hat{w}_2^L)^{1-\rho}$ are equal to 0 or to B. If all such wages are equal to zero, then (22) is not satisfied because $u > 0$. Thus, some of these wages must be positive. If at least one is less than B, it is easy to find a sequence $(w_1^{Lk}, w_2^{Hk}, w_2^{Lk}), k = 1,2\ldots,$ satisfying (22) with $\rho = \rho^k$ and $I_c = I_c(\rho^k, w_2^{Hk}, w_2^{Lk})$, and converging to $(\hat{w}_1^L, \hat{w}_2^H, \hat{w}_2^L)$. If all are equal to B, then (22) is satisfied with strict inequality, and thus it is easy to find $(w_1^{Lk}, w_2^{Hk}, w_2^{Lk})$ satisfying (22) with $\rho = \rho^k$ and $I_c = I_c(\rho^k, w_2^{Hk}, w_2^{Lk})$, and converging to $(\hat{w}_1^L, \hat{w}_2^H, \hat{w}_2^L)$. Clearly, $I_c(\rho^k, w_2^{Hk}, w_2^{Lk})$ converges to $\hat{I}_c$. Thus, Π is a lower hemicontinuous correspondence. Then, together with the continuity of the objective function, the continuity of the maximum value in ρ follows from Berge's maximum Proposition. Moreover, because $I_n = 0$ always holds and $g(0, \theta)$ is a continuous function of θ, the maximum value for the principal in the long-term wage contract is a continuous function of (ρ, θ).

A Comparison of Two Types of Contract

Therefore, the values for the principal under the short-term wage contract and the long-term contract are continuous functions of (ρ, θ), and they coincide with those in the case of a risk-neutral agent at $\rho = 0$. Hence ${}^{\exists}\bar{\rho} \in (0,1), {}^{\forall}\rho \in (0, \bar{\rho}]$ satisfying the same property as in the case of a risk-neutral agent.