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The SIML Estimation of Integrated Covariance and Hedging Coefficient under Micro-market noise and Random Sampling

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Abstract

For estimating the integrated volatility and covariance by using high frequency data, Kunitomo and Sato (2008, 2011) have proposed the Separating Information Maximum Likelihood (SIML) method when there are micro-market noises. The SIML estimator has reasonable finite sample properties and asymptotic properties when the sample size is large under general conditions with *non-Gaussian processes* or *volatility models*. We shall show that the SIML estimation is useful for estimating the integrated covariance and hedging coefficient when we have micro-market noise and financial high frequency data are randomly sampled. The SIML estimation is consistent and has the stable convergence (i.e. the asymptotic normality in the deterministic case) and it has reasonable finite sample properties with these effects.

Key Words

Integrated Covariance with Micro-Market Noise, Hedging Coefficient, High-Frequency Data, Separating Information Maximum Likelihood (SIML), Random Sampling.

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1. Introduction

Recently a considerable interest has been paid on the estimation problem of the integrated volatility and covariance by using high-frequency data of financial prices. Although the earlier studies often had ignored the presence of micro-market noises in financial markets, there have been arguments that the micro-market noises have important roles in high-frequency financial data, and then several new statistical estimation methods have been developed. See Ait-Sahalia, Mykland and Zhang (2005), Bandorff-Nielsen, Hansen, Lund and Shephard (2008), and Malliavin and Mancino (2009) as recent literatures on the related topics among many. In this respect Kunitomo and Sato (2008, 2011) have proposed a new statistical method called the Separating Information Maximum Likelihood (SIML) method for estimating the integrated volatility and the integrated covariance by using high frequency data under the presence of micro-market noises. The SIML estimator has reasonable asymptotic properties as well as finite sample properties ; it is consistent and it has the asymptotic normality (or the stable convergence in the more general case) when the sample size is large and the data frequency interval is small under a set of regularity conditions. Kunitomo and Sato (2008) have also shown that the SIML estimator has the robustness properties, that is, it is consistent and asymptotically normal even when the noise terms are autocorrelated and/or there are endogenous correlations between the market-noise terms and the (underlying) efficient market price process.

In this paper we shall investigate some properties of the SIML estimation when we have the micro-market noises and randomly sampled data. For actual high frequency financial data, they are recorded at random times and the effects of randomness could be significant when we have micro-market noises. When the effects of measurement errors are present, it seems that the standard statistical methods measuring the integrated volatility and covariance may have some problem. The non-synchronous sampling on the covariance estimation in high frequency financial data was investigated by Hayashi and Yoshida (2005, 2008) and they proposed the *so-called* Hayashi-Yoshida estimation (H-Y) method under the situation that there are no micro-market noises.

We shall consider the estimation problem of hedging coefficients and correlation coefficients when we have micro-market noises. Since we need the volatility estimation as we well as the covariance estimation in this problem, it may be difficult to use the H-Y method directly when we have micro-market noises. We show that the SIML estimator is asymptotically robust; that is, it is consistent and asymptotically normal as the sample size increases under a reasonable set of assumptions. The asymptotic property of the SIML method on the integrated volatility and covariance has desirable properties over other estimation methods for the underlying continuous stochastic process with micro-market noise. Because the SIML estimation is a simple method, it can be practically used for analyzing the multivariate (high frequency) financial time series. We shall focus on the estimation of covariance and the hedging coefficients in this paper, while as a companion paper Misaki and Kunitomo (2012) have investigated the problem of volatility estimation when we have randomly sampled observations.

In Section 2 we introduce the high frequency micro-market models with micro-market noise and the estimation methods including the Separating Information Maximum Likelihood (SIML) method. Then in Section 3 we give some numerical results of the SIML estimation in the basic simulation framework. In Section 4 we shall give the asymptotic properties of the SIML estimator. Then in Section 5 we shall report further results in a variety of micro-market models. Finally, in Section 6 some brief remarks will be given.

2. Micro-market noise models and the estimation of the integrated volatility

2.1 A General Formulation

Let $y_s(t_i^s)$ be the $i-$th observation of the (log-) price of the first asset at t_i^s for $0 = t_0^s < t_1^s < \cdots < t_{n_s^*}^s \leq t_{n_s}^s = 1$ and $y_f(t_j^f)$ be the $j-$th observation of the (log-) price of the second asset at t_j^f for $0 = t_0^f < t_1^f < \cdots < t_{n_f^*}^f \leq t_{n_f}^f = 1$. Let $t_{n_s^*}^s = \max_{i\geq 1, t_i^s \leq 1} t_i^s$, $t_{n_f^*}^f = \max_{i\geq 1, t_i^f \leq 1} t_i^f$ and we denote n as a constant index and n^* as a stochastic index. We consider the situation that the high-frequency data are observed at random times t_i^a ($a = s$ or f) under some conditions on random sampling.

Assumption 2.1 : There exist positive constants c_a ($a = s$ or f) such that

$$t_n^a \longrightarrow 1 \ , \ \frac{n_a^*}{n} \xrightarrow{p} c_a \tag{2.1}$$

and

$$\mathcal{E}\left[|t_i^a - t_{i-1}^a|\right] = O(n^{-1}) \tag{2.2}$$

as $n \to \infty$, where $a = s$ or f.

These conditions imply that n^{-1} corresponds to the average duration on intervals in $[0,1]$ when n is relatively large. Without loss of generality we take $c_s = c_f = 1$. A typical example is the Poisson Process Sampling on t_i^s and t_i^f with the intensity functions $\lambda_n^{(s)} = nc_s$ and $\lambda_n^{(f)} = nc_f$. In this case the sequence of random variables τ_i^a ($a = s, f$) are exponentially distributed with $\mathcal{E}(\tau_i^a) = 1/n$ ($\tau_i^a = t_i^a - t_{i-1}^a$) if we take $c_s = c_f = 1$. In this paper we make a further assumption on the independence of $X(t)$ and t_i^a ($i \geq 1$).

Assumption 2.2 : The stochastic process $X(t)$ ($0 \leq t \leq 1$) is independent of the random sequences t_i^s and t_j^f ($i, j \geq 1$).

The underlying two-dimensional continuous process $\mathbf{X}(t) = (X_s(t), X_f(t))'$ ($0 \leq t \leq 1$) is not necessarily the same as the observed (log-)price at t_i^s and t_j^f ($i, j \geq 1$) and

$$\mathbf{X}(t) = \mathbf{X}(0) + \int_0^t \mathbf{C}_x(s) d\mathbf{B}(s) \ \ (0 \leq t \leq 1), \tag{2.3}$$

where $\mathbf{B}(s)$ is the two-dimensional Brownian motion, $\mathbf{C}_x(s)$ is the 2×2 instantaneous volatility matrix adapted to the $\sigma-$field $\mathcal{F}(\mathbf{x}(r), \mathbf{B}(r), r \leq s)$. The main statistical objective is to estimate the quadratic variation or the integrated volatility matrix

$$\mathbf{\Sigma}_x = \int_0^1 \mathbf{\Sigma}_x(s)ds = \begin{pmatrix} \sigma_{ss}^{(x)} & \sigma_{sf}^{(x)} \\ \sigma_{sf}^{(x)} & \sigma_{ff}^{(x)} \end{pmatrix} \tag{2.4}$$

($\mathbf{\Sigma}_x(s) = \mathbf{C}_x(s)\mathbf{C}_x^{'}(s)$) of the underlying continuous process $\mathbf{X}(t)$ $(0 \leq t \leq 1)$ from the set of discrete observations on $(y_s(t_i^s), y_f(t_j^f))$ with the condition that $\mathbf{\Sigma}_x(s)$ is a progressively measurable matrix and $\sup_{0\leq s\leq 1} \mathbf{\Sigma}_x(s) < \infty$ $(a.s.)$.
We also consider the situation that the observed (log-)prices $y_s(t_i^s)$ and $y_f(t_j^f)$ are the sequence of discrete stochastic processes generated by

$$y_s(t_i^s) = h_s\left(\mathbf{X}(t_i^s), y_s(t_{i-1}^s), u_s(t_i^s)\right) \tag{2.5}$$

and

$$y_f(t_j^f) = h_f\left(\mathbf{X}(t_j^f), y_f(t_{j-1}^f), u_f(t_j^f)\right) \ , \tag{2.6}$$

where $h_s(\ \cdot\ )$ and $h_f(\ \cdot\ )$ are measurable functions, the (unobservable) continuous martingale process $\mathbf{X}(t)$ $(0 \leq t \leq 1)$ is defined by (2.3) and the micro-market noises $u_s(t_i^s)$ and $u_f(t_j^f)$ are the discrete stochastic processes. In particular, we assume that $u_s(t_i^s)$ and $u_f(t_j^f)$ are a sequence of independently and identically distributed random variables with $\mathcal{E}(u_s(t_i^s)) = 0, \mathcal{E}(u_f(t_j^f)) = 0$ and $\mathcal{E}(u_s(t_i^s)^2) = \sigma_{ss}^{(u)}, \mathcal{E}(u_f(t_j^f)^2) = \sigma_{ff}^{(u)}, \mathcal{E}(u_s(t_i^s)u_f(t_j^f)) = \delta(t_i^s, t_j^f)\sigma_{sf}^{(u)}$.

There are special cases of (2.3), (2.5) and (2.6), which reflect the important aspects on modeling financial markets and the high frequency financial data. The basic (high-frequency) financial model with micro market noises can be represented by

$$y_s(t_i^s) = X_s(t_i^s) + u_s(t_i^s)\ ,\ y_f(t_j^f) = X_f(t_j^f) + u_f(t_j^f)\ , \tag{2.7}$$

where the underlying process $\mathbf{X}(t) = (X_s(t), X_f(t))^{'}$ is given by (2.3). The synchronous sampling means $t_i^s = t_i^f$ and the fixed grid observation means $t_i^a - t_{i-1}^a =$

n^{-1}. This paper we considers the more general situation, that is, we have the non-synchronous observations as well as the random sampling.

The most important statistical aspect of (2.7) is the fact that it is an additive (signal-plus-noise) measurement error model. However, there are some reasons why the standard situation as (2.7) is not enough for applications. For instance, the high frequency financial models for micro-market price adjustments and the round-off-errors models for financial prices are not in the form of (2.7), but they can be represented as special cases of (2.3), (2.5) and (2.6). Sato and Kunitomo (2012) have discussed several important examples of (2.5) and (2.6) with one dimension.

Hedging Ratio and Correlation

For the financial risk management problems, the use of hedging coefficients and correlation coefficients has been often discussed in the literatures on financial futures. (See Duffie (1989) for instance.) Then it is important to estimate the hedging ratio and the correlation coefficient. The (integrated) hedging ratio based on high-frequency financial data can be defined by

$$H = \frac{\sigma_{sf}^{(x)}}{\sigma_{ff}^{(x)}} . \tag{2.8}$$

The (integrated) correlation coefficient between two prices can be defined by

$$\rho_{sf} = \frac{\sigma_{sf}^{(x)}}{\sqrt{\sigma_{ss}^{(x)}\sigma_{ff}^{(x)}}} . \tag{2.9}$$

2.2 The SIML Method

When the two dimensional financial data are synchronously observed and equally spaced, the derivation of the separating information maximum likelihood (SIML) estimation has been given by Kunitomo and Sato (2008, 2011). For non-synchronouly observed data, there have been several ways to handle the problem and we adopts *the Refreshing Time method* developed by Harris, Mcinish, Shoesmith and Wood (1995), and Bandorff-Nielsen, Hansen, Lund and Shepard (2011). There can be other ways to handle the problem.

We consider the standard situation when $\mathbf{\Sigma}_s(s) = \mathbf{\Sigma}_x$ and $\mathbf{X}(t)$ $(0 \le t \le 1)$ are independent of $u_s(t_i^s)$ and $u_f(t_j^f)$. Let $\mathbf{y}_{n_s^*}^s = (y_s(t_i^s))$ and $\mathbf{y}_{n_f^*}^f = (y_f(t_j^f))$ be $n_s^* \times 1$ and $n_f^* \times 1$ vectors of observations. We transform $\mathbf{y}_{n_1^*}^s$ and $\mathbf{y}_{n_2^*}^f$ to $\mathbf{z}^s$ $(= (\mathbf{z}_k^s))$ and $\mathbf{z}^f$ $(= (\mathbf{z}_k^f))$ by

$$(2.10) \qquad \mathbf{z}^s = \sqrt{n_s^*}\mathbf{P}'\mathbf{C}^{-1}\left(\mathbf{y}_{n_s^*}^s - \bar{\mathbf{y}}_{s0}\right) \ , \ \mathbf{z}^f = \sqrt{n_f^*}\mathbf{P}'\mathbf{C}^{-1}\left(\mathbf{y}_{n_f^*}^f - \bar{\mathbf{y}}_{f0}\right) \ ,$$

where

$$(2.11) \qquad \mathbf{C}^{-1} = \begin{pmatrix} 1 & 0 & \cdots & 0 & 0 \\ -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & 0 & \cdots \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & \cdots & 0 & 0 \\ 1 & 1 & 0 & \cdots & 0 \\ 1 & 1 & 1 & \cdots & 0 \\ 1 & \cdots & 1 & 1 & 0 \\ 1 & \cdots & 1 & 1 & 1 \end{pmatrix}^{-1} ,$$

$$(2.12) \qquad \mathbf{P} = (p_{jk}) \ , \ p_{jk} = \sqrt{\frac{2}{n_a^* + \frac{1}{2}}}\cos\left[\pi(\frac{2k-1}{2n_a^*+1})(j-\frac{1}{2})\right] \ ,$$

and

$$(2.13) \qquad \bar{\mathbf{y}}_{a0} = \mathbf{1}_{n_a^*} \cdot y_a(0)$$

for $a = s$ or $a = f$.

Under the assumption of the Gaussian distributions both on the signal terms and noise terms with equally spaced observation case, Kunitomo and Sato (2008) have introduced the SIML estimator. By using the similar argument, we define the SIML estimator of $\sigma_{ss}^{(x)}$ and $\sigma_{ss}^{(v)}$ by

$$(2.14) \qquad \hat{\sigma}_{ss}^{(x)} = \frac{1}{m_{n_s^*}}\sum_{k=1}^{m_{n_s^*}}[z_k^s]^2 \ ,$$

and

$$(2.15) \qquad \hat{\sigma}_{ss}^{(v)} = \frac{1}{l_{n_s^*}}\sum_{k=n_s^*+1-l_{n_s^*}}^{n_s^*} a_{k,n_s^*}^{-1}[z_k^s]^2 \ ,$$

where

$$(2.16) \qquad a_{k,n_s^*} = 4n_s^*\sin^2\left[\frac{\pi}{2}\left(\frac{2k-1}{2n_s^*+1}\right)\right]$$

and we shall use n instead of n_s^*. Similarly, we define the SIML estimator of $\sigma_{ff}^{(x)}$ and $\sigma_{ff}^{(v)}$ as $\hat{\sigma}_{ff}^{(x)}$ and $\hat{\sigma}_{ff}^{(v)}$, where n should be interpreted as n_f^*. Then the SIML covariance estimator with *the Refreshing Time scheme* can be defined by

$$\hat{\sigma}_{sf}^{(x)} = \frac{1}{m_n} \sum_{k=1}^{m_n} [z_k^s z_k^f] \ , \tag{2.17}$$

where n should be interpreted as n^* and delete extra terms when $n_s^* \neq n_f^*$. Although we have non-synchronous observations originally, we do not have any serious problem under Assumption 2.1.

For both $\hat{\sigma}_{ss}^{(x)}$ and $\hat{\sigma}_{ss}^{(x)}$, the number of terms m and l should be dependent on n. Then we only need the order requirements that $m_n = O(n^{\alpha})$ $(0 < \alpha < \frac{1}{2})$ and $l_n = O(n^{\beta})$ $(0 < \beta < 1)$ for $\sigma_{ss}^{(x)}$ and $\sigma_{ss}^{(v)}$, respectively.

By using the estimators of the integrated volatility and the integrated covariance, the SIML estimator of the hedging ratio $H = \sigma_{sf}^{(x)} / \sigma_{ff}^{(x)}$ can be defined by

$$\hat{H} = \frac{\hat{\sigma}_{sf}^{(x)}}{\hat{\sigma}_{ff}^{(x)}} \ . \tag{2.18}$$

When the two dimensional financial data are non-synchronously observed, there are several different ways to use the SIML estimation. (i) The first method of covariance estimation is to set the transaction time of the second asset is to synchronize the transaction time of the first asset. (ii) The second method is to use the Hayashi-Yoshida method of covariance estimation and then use the SIML method to obtain the hedging coefficient. Since there can be several estimation methods, we shall compare the performance of different methods in this paper.

3. Basic Simulation

We have investigated the robust properties of the SIML estimator for the integrated volatility based on a set of simulations with the number of replications being 1,000. We have taken the sample size $n = 1,800$ and $n = 18,000$, and we have chosen $\alpha = 0.4$ and $\beta = 0.8$. The details of the simulation procedure are similar to the corresponding ones reported by Misaki and Kunitomo (2013).

In our basic simulations we consider two cases when the observations are the sum of signal and micro-market noise. We use two Poisson Random Sampling as the basic stochastic sampling with the parameter $\lambda = n$. In the first example the signal is the Brownian motion with the instantaneous volatility function

$$\boldsymbol{\Sigma}_x(s) = \boldsymbol{\Sigma}_x(0)\left[a_0 + a_1 s + a_2 s^2\right] \tag{3.1}$$

where a_i $(i = 0, 1, 2)$ are constants and we have some restrictions such that $\boldsymbol{\Sigma}_x(s)$ is positive definite for $s \in [0, 1]$. In this case the integrated volatility is given by

$$\boldsymbol{\Sigma}_x = \int_0^1 \boldsymbol{\Sigma}_x(s)ds = \boldsymbol{\Sigma}_x(0)\left[a_0 + \frac{a_1}{2} + \frac{a_2}{3}\right] . \tag{3.2}$$

In this example we have taken several intra-day instantaneous volatility patterns including the flat (or constant) volatility, the monotone (decreasing or increasing) movements and the U-shaped movements.
In the second example let $n_s^* = \max_{1 \geq i, t_i^s \leq 1} t_i^s$ and the instantaneous volatility function follows the stochastic volatility model that

$$\sigma_{ss}^{(x)} = \frac{1}{n_s^*}\sum_{i=1}^{n_s^*} \sigma_{ss}^{(x)}(t_i^s) , \tag{3.3}$$

where $\sigma_{ss}^{(x)}(t_i^s) = \sigma_{ss}^{(x)}(0)e^{H(t_i^s)}$ $(0 < t_1^s < \cdots < t_{n_s^*}^s)$, $\sigma_{ff}^{(x)}(t_j^f) = \sigma_{ff}^{(x)}(0)e^{H(t_j^f)}$ $(0 < t_1^n < \cdots < t_{n_f^*}^n)$ and

$$h_{aa}(t_i^a) = \gamma\, h_{aa}(t_{i-1}^a) + \delta\, u_a(t_i^a) \tag{3.4}$$

for $a = s$ or f. In our experiments we have set $\gamma = 0.9, \delta = 0.2$ and each of $(u_s(t_i^s)$ and $u_f(t_j^f))$ are the white noise processes followed by $N(0, \sigma_{ss}^{(v)})$ and $N(0, \sigma_{ff}^{(v)})$ as the typical case.

We summarize our estimation results of the first example in Tables 3.1-3.4 and the second example in Tables 3.5-3.6, respectively. In each table we have also calculated the historical volatility (the realized volatility, RV), the historical covariance

(RCV) and the HY estimator, and the SIML estimator, in order to make comparisons. We have compared several estimation methods of the hedging coefficients, that is, the combinations of covariance-variance estimation methods of RCV-RV, HY-RV, HY-SIML and SIML-SIML.

When there are micro-market noise components with the martingale signal part, the value of RCV often differs substantially from the true integrated covariance of the signal part. However, we have found that it is possible to estimate the integrated variances, the integrated covariance and the noise variances when we have the signal-noise ratio as $10^{-2} \sim 10^{-6}$ by the SIML estimation method. Although we have omitted the details of the second example, the estimation results are similar in the stochastic volatility model.

For the estimation of the hedging coefficients, We have confirmed that by using the H-Y method we can improve the historical covariance method as was pointed out by Hayashi and Yoshida (2005). However, when we have micro-market noise terms, the SIML-SIML combination of the integrated variance and the integrated covariance. This point is vivid and important on the hedging coefficient estimation as Tables 3.1-3.4 have shown.

By our basic simulations we can conclude that we can estimate both the integrated covariance of the hidden martingale part and the (integrated) hedging coefficients reasonably in all cases we have examined by the SIML estimation.

Table 3.1 : Estimation of covariance and hedging coefficient :

Case 1 ($a_0 = 1, a_1 = a_2 = 0; \lambda = 1800$)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ_{x1}^2	2.00E-04	2.01E-04	2.21E-04	2.03E-04	2.00E-04	2.00E-04	2.05E-04
		6.60E-05	4.58E-05	6.51E-05	8.22E-05	9.53E-05	1.30E-04
σ_{v1}^2	2.00E-06	2.03E-06	1.05E-07	9.88E-07	1.92E-06	2.17E-06	3.01E-06
		1.49E-07	6.07E-09	8.64E-08	2.16E-07	3.19E-07	8.34E-07
RV1	2.00E-04	7.40E-03	7.06E-03	4.75E-03	2.48E-03	1.39E-03	4.39E-04
		3.53E-04	3.40E-04	2.54E-04	1.78E-04	1.38E-04	8.70E-05
σ_{x2}^2	2.00E-04	2.07E-04	2.22E-04	2.09E-04	2.06E-04	2.07E-04	2.07E-04
		6.74E-05	4.49E-05	6.63E-05	8.67E-05	1.01E-04	1.37E-04
σ_{v2}^2	2.00E-06	2.03E-06	1.05E-07	9.84E-07	1.92E-06	2.17E-06	3.00E-06
		1.43E-07	6.02E-09	8.51E-08	2.21E-07	3.06E-07	8.14E-07
RV2	2.00E-04	7.41E-03	7.06E-03	4.75E-03	2.48E-03	1.39E-03	4.41E-04
		3.42E-04	3.28E-04	2.54E-04	1.80E-04	1.33E-04	8.43E-05
σ_{x12}^2	1.00E-04	9.93E-05	9.98E-05	9.99E-05	1.00E-04	1.01E-04	1.02E-04
		5.59E-05	3.43E-05	5.04E-05	6.57E-05	7.66E-05	1.04E-04
σ_{v12}^2	0.00E+00	5.46E-09	-4.10E-11	2.09E-09	1.85E-08	5.86E-08	4.52E-07
		1.21E-07	2.11E-09	4.97E-08	1.48E-07	2.20E-07	5.82E-07
RCV	1.00E-04	6.75E-05	7.33E-06	4.30E-05	6.34E-05	8.03E-05	9.53E-05
		1.78E-04	5.34E-05	1.24E-04	1.19E-04	9.33E-05	6.15E-05
HY	1.00E-04	1.05E-04					
		1.25E-04					
RCV-RV	5.00E-01	9.08E-03	1.04E-03	8.98E-03	2.55E-02	5.77E-02	2.20E-01
		2.41E-02	7.57E-03	2.60E-02	4.83E-02	6.68E-02	1.39E-01
HY-RV	5.00E-01	1.42E-02	1.49E-02	2.21E-02	4.24E-02	7.59E-02	2.49E-01
		1.70E-02	1.78E-02	2.64E-02	5.06E-02	9.11E-02	3.04E-01
HY-SIML	5.00E-01	5.81E-01	4.87E-01	5.71E-01	6.31E-01	6.86E-01	8.72E-01
		7.54E-01	5.98E-01	7.42E-01	8.74E-01	1.06E+00	1.75E+00
SIML-SIML	5.00E-01	4.97E-01	4.51E-01	4.91E-01	5.01E-01	5.17E-01	5.05E-01
		2.42E-01	1.29E-01	2.05E-01	2.72E-01	3.30E-01	5.11E-01
1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ_{x1}^2	2.00E-04	1.96E-04	2.00E-04	1.97E-04	1.95E-04	1.95E-04	1.98E-04
		6.46E-05	4.14E-05	6.36E-05	8.01E-05	9.26E-05	1.27E-04
σ_{v1}^2	2.00E-08	4.92E-08	3.84E-09	3.85E-08	1.08E-07	2.01E-07	1.02E-06
		3.73E-09	1.83E-10	2.81E-09	1.20E-08	3.00E-08	2.84E-07
RV1	2.00E-04	2.72E-04	2.69E-04	2.45E-04	2.23E-04	2.11E-04	2.02E-04
		1.19E-05	1.18E-05	1.22E-05	1.43E-05	1.85E-05	3.86E-05
σ_{x2}^2	2.00E-04	2.02E-04	2.00E-04	2.02E-04	2.01E-04	2.02E-04	2.02E-04
		6.47E-05	4.03E-05	6.38E-05	8.37E-05	9.75E-05	1.33E-04
σ_{v2}^2	2.00E-08	4.90E-08	3.85E-09	3.85E-08	1.07E-07	2.02E-07	1.03E-06
		3.65E-09	1.86E-10	2.84E-09	1.24E-08	2.98E-08	2.77E-07
RV2	2.00E-04	2.72E-04	2.69E-04	2.46E-04	2.23E-04	2.12E-04	2.04E-04
		1.18E-05	1.18E-05	1.21E-05	1.43E-05	1.82E-05	3.63E-05
σ_{x12}^2	1.00E-04	9.95E-05	9.98E-05	1.00E-04	1.00E-04	1.02E-04	1.03E-04
		5.45E-05	3.14E-05	4.87E-05	6.43E-05	7.51E-05	1.02E-04
σ_{v12}^2	0.00E+00	7.94E-09	-2.45E-13	1.31E-09	1.91E-08	6.34E-08	4.70E-07
		3.97E-09	7.85E-11	2.00E-09	8.47E-09	2.22E-08	2.30E-07
RCV	1.00E-04	6.67E-05	4.92E-06	3.73E-05	6.84E-05	8.35E-05	9.76E-05
		8.61E-06	2.17E-06	6.10E-06	9.70E-06	1.37E-05	3.01E-05
HY	1.00E-04	1.00E-04					
		1.11E-05					
RCV-RV	5.00E-01	2.45E-01	1.83E-02	1.52E-01	3.08E-01	3.96E-01	4.82E-01
		2.96E-02	8.08E-03	2.40E-02	3.97E-02	5.52E-02	1.16E-01
HY-RV	5.00E-01	3.68E-01	3.73E-01	4.09E-01	4.51E-01	4.77E-01	5.12E-01
		3.76E-02	3.81E-02	4.17E-02	4.62E-02	5.22E-02	1.06E-01
HY-SIML	5.00E-01	5.69E-01	5.22E-01	5.65E-01	6.10E-01	6.65E-01	9.02E-01
		2.10E-01	1.18E-01	2.01E-01	2.98E-01	4.31E-01	1.21E+00
SIML-SIML	5.00E-01	5.10E-01	5.01E-01	5.11E-01	5.13E-01	5.29E-01	5.24E-01
		2.39E-01	1.23E-01	2.03E-01	2.71E-01	3.36E-01	5.02E-01

Table 3.2 : Estimation of covariance and hedging coefficient :

Case 1 ($a_0 = 1, a_1 = a_2 = 0; \lambda = 18000$)

18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ^2_{x1}	2.00E-04	2.04E-04	2.05E-04	2.05E-04	2.07E-04	2.08E-04	2.11E-04
		4.18E-05	4.17E-05	6.74E-05	8.56E-05	1.00E-04	1.33E-04
σ^2_{v1}	2.00E-06	2.00E-06	9.37E-07	2.02E-06	2.09E-06	2.19E-06	3.01E-06
		5.68E-08	3.03E-08	1.39E-07	2.25E-07	3.10E-07	8.43E-07
RV1	2.00E-04	7.22E-02	4.57E-02	7.39E-03	2.60E-03	1.41E-03	4.42E-04
		1.08E-03	8.07E-04	2.78E-04	1.77E-04	1.29E-04	8.66E-05
σ^2_{x2}	2.00E-04	2.05E-04	2.06E-04	2.06E-04	2.08E-04	2.09E-04	2.10E-04
		4.25E-05	4.26E-05	6.68E-05	8.73E-05	1.01E-04	1.31E-04
σ^2_{v2}	2.00E-06	2.00E-06	9.40E-07	2.04E-06	2.09E-06	2.20E-06	3.00E-06
		5.53E-08	3.06E-08	1.46E-07	2.29E-07	3.13E-07	8.41E-07
RV2	2.00E-04	7.22E-02	4.57E-02	7.41E-03	2.60E-03	1.41E-03	4.41E-04
		1.08E-03	7.74E-04	2.89E-04	1.81E-04	1.33E-04	8.64E-05
σ^2_{x12}	1.00E-04	1.00E-04	9.99E-05	1.02E-04	1.05E-04	1.07E-04	1.07E-04
		3.59E-05	3.35E-05	5.10E-05	6.61E-05	7.90E-05	1.05E-04
σ^2_{v12}	0.00E+00	-2.30E-10	-9.84E-10	1.40E-08	4.31E-08	9.76E-08	5.14E-07
		4.64E-08	1.89E-08	1.02E-07	1.57E-07	2.22E-07	5.85E-07
RCV	1.00E-04	8.27E-05	4.15E-05	1.02E-04	1.02E-04	1.03E-04	1.02E-04
		5.44E-04	3.83E-04	2.10E-04	1.27E-04	9.32E-05	6.32E-05
HY	1.00E-04	9.94E-05					
		3.86E-04					
RCV-RV	5.00E-01	1.14E-03	9.13E-04	1.38E-02	3.91E-02	7.36E-02	2.31E-01
		7.53E-03	8.39E-03	2.83E-02	4.87E-02	6.60E-02	1.37E-01
HY-RV	5.00E-01	1.38E-03	2.18E-03	1.35E-02	3.78E-02	7.09E-02	2.32E-01
		5.35E-03	8.45E-03	5.23E-02	1.49E-01	2.78E-01	9.25E-01
HY-SIML	5.00E-01	5.11E-01	5.07E-01	5.20E-01	5.31E-01	5.70E-01	9.00E-01
		2.00E+00	1.99E+00	2.24E+00	2.52E+00	2.89E+00	5.28E+00
SIML-SIML	5.00E-01	4.91E-01	4.86E-01	4.99E-01	5.12E-01	5.18E-01	5.19E-01
		1.44E-01	1.30E-01	2.02E-01	2.68E-01	3.17E-01	5.16E-01
18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ^2_{x1}	2.00E-04	1.99E-04	1.99E-04	2.00E-04	2.02E-04	2.02E-04	2.04E-04
		4.08E-05	4.06E-05	6.58E-05	8.36E-05	9.89E-05	1.30E-04
σ^2_{v1}	2.00E-08	2.28E-08	1.22E-08	4.88E-08	1.09E-07	2.02E-07	1.01E-06
		6.51E-10	3.76E-10	3.40E-09	1.20E-08	2.96E-08	2.92E-07
RV1	2.00E-04	9.20E-04	6.55E-04	2.72E-04	2.24E-04	2.12E-04	2.02E-04
		1.24E-05	1.03E-05	8.98E-06	1.28E-05	1.75E-05	3.76E-05
σ^2_{x2}	2.00E-04	2.00E-04	2.00E-04	2.01E-04	2.04E-04	2.03E-04	2.03E-04
		4.14E-05	4.13E-05	6.55E-05	8.48E-05	9.74E-05	1.26E-04
σ^2_{v2}	2.00E-08	2.28E-08	1.22E-08	4.93E-08	1.09E-07	2.01E-07	1.02E-06
		6.31E-10	3.68E-10	3.46E-09	1.22E-08	2.92E-08	2.92E-07
RV2	2.00E-04	9.20E-04	6.55E-04	2.73E-04	2.24E-04	2.11E-04	2.03E-04
		1.27E-05	1.01E-05	9.09E-06	1.27E-05	1.76E-05	3.83E-05
σ^2_{x12}	1.00E-04	1.01E-04	1.00E-04	1.03E-04	1.06E-04	1.07E-04	1.08E-04
		3.49E-05	3.21E-05	5.00E-05	6.48E-05	7.73E-05	1.03E-04
σ^2_{v12}	0.00E+00	7.25E-10	9.42E-11	1.18E-08	4.19E-08	8.79E-08	4.92E-07
		5.54E-10	2.41E-10	2.47E-09	9.23E-09	2.28E-08	2.24E-07
RCV	1.00E-04	6.67E-05	3.68E-05	9.00E-05	9.67E-05	9.82E-05	1.00E-04
		6.77E-06	5.28E-06	6.86E-06	9.94E-06	1.37E-05	2.98E-05
HY	1.00E-04	9.99E-05					
		6.36E-06					
RCV-RV	5.00E-01	7.25E-02	5.62E-02	3.31E-01	4.32E-01	4.63E-01	4.95E-01
		7.34E-03	8.04E-03	2.27E-02	3.70E-02	5.18E-02	1.18E-01
HY-RV	5.00E-01	1.09E-01	1.53E-01	3.68E-01	4.47E-01	4.74E-01	5.11E-01
		6.90E-03	9.59E-03	2.45E-02	3.62E-02	4.67E-02	1.00E-01
HY-SIML	5.00E-01	5.22E-01	5.23E-01	5.57E-01	5.93E-01	6.39E-01	8.14E-01
		1.13E-01	1.13E-01	1.98E-01	2.98E-01	4.04E-01	9.23E-01
SIML-SIML	5.00E-01	5.05E-01	5.02E-01	5.15E-01	5.29E-01	5.33E-01	5.40E-01
		1.40E-01	1.25E-01	1.99E-01	2.63E-01	3.12E-01	4.85E-01

Table 3.3 : Estimation of covariance and hedging coefficient :

Case 2 ($a_0 = 1, a_1 = -1, a_2 = 1; \lambda = 1800$)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ^2_{x1}	1.66E-04	1.68E-04	1.88E-04	1.71E-04	1.68E-04	1.67E-04	1.72E-04
		5.59E-05	3.92E-05	5.51E-05	6.95E-05	8.06E-05	1.10E-04
σ^2_{v1}	2.00E-06	2.03E-06	1.04E-07	9.83E-07	1.91E-06	2.14E-06	2.84E-06
		1.49E-07	6.06E-09	8.62E-08	2.14E-07	3.15E-07	7.89E-07
RV1	1.66E-04	7.37E-03	7.02E-03	4.72E-03	2.44E-03	1.36E-03	4.06E-04
		3.52E-04	3.40E-04	2.53E-04	1.76E-04	1.35E-04	8.12E-05
σ^2_{x2}	1.66E-04	1.74E-04	1.89E-04	1.75E-04	1.73E-04	1.74E-04	1.74E-04
		5.71E-05	3.85E-05	5.61E-05	7.38E-05	8.55E-05	1.16E-04
σ^2_{v2}	2.00E-06	2.02E-06	1.04E-07	9.79E-07	1.91E-06	2.14E-06	2.83E-06
		1.43E-07	6.00E-09	8.49E-08	2.19E-07	3.02E-07	7.68E-07
RV2	1.66E-04	7.38E-03	7.03E-03	4.72E-03	2.44E-03	1.36E-03	4.07E-04
		3.41E-04	3.27E-04	2.53E-04	1.78E-04	1.31E-04	7.88E-05
σ^2_{x12}	8.33E-04	8.27E-05	8.33E-05	8.33E-05	8.33E-05	8.45E-05	8.47E-05
		4.73E-05	2.93E-05	4.27E-05	5.57E-05	6.47E-05	8.80E-05
σ^2_{v12}	0.00E+00	4.09E-09	-3.95E-11	1.83E-09	1.53E-08	4.84E-08	3.73E-07
		1.20E-07	2.10E-09	4.94E-08	1.47E-07	2.17E-07	5.46E-07
RCV	8.33E-04	5.62E-05	6.49E-06	3.67E-05	5.21E-05	6.67E-05	7.91E-05
		1.77E-04	5.31E-05	1.23E-04	1.18E-04	9.17E-05	5.70E-05
HY	8.33E-04	8.83E-05					
		1.24E-04					
RCV-RV	5.00E-01	7.60E-03	9.30E-04	7.71E-03	2.12E-02	4.90E-02	1.98E-01
		2.40E-02	7.57E-03	2.61E-02	4.85E-02	6.73E-02	1.40E-01
HY-RV	5.00E-01	1.20E-02	1.26E-02	1.87E-02	3.61E-02	6.53E-02	2.26E-01
		1.69E-02	1.77E-02	2.63E-02	5.08E-02	9.24E-02	3.24E-01
HY-SIML	5.00E-01	5.83E-01	4.79E-01	5.72E-01	6.37E-01	6.96E-01	8.68E-01
		8.82E-01	6.95E-01	8.66E-01	1.02E+00	1.23E+00	1.98E+00
SIML-SIML	5.00E-01	4.94E-01	4.43E-01	4.88E-01	4.98E-01	5.14E-01	5.02E-01
		2.44E-01	1.31E-01	2.08E-01	2.74E-01	3.31E-01	5.13E-01
1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ^2_{x1}	1.66E-04	1.64E-04	1.66E-04	1.64E-04	1.63E-04	1.62E-04	1.66E-04
		5.42E-05	3.47E-05	5.33E-05	6.72E-05	7.77E-05	1.07E-04
σ^2_{v1}	2.00E-08	4.43E-08	3.37E-09	3.37E-08	9.28E-08	1.70E-07	8.52E-07
		3.35E-09	1.61E-10	2.46E-09	1.03E-08	2.55E-08	2.38E-07
RV1	1.66E-04	2.39E-04	2.35E-04	2.12E-04	1.89E-04	1.78E-04	1.69E-04
		1.03E-05	1.02E-05	1.04E-05	1.22E-05	1.56E-05	3.23E-05
σ^2_{x2}	1.66E-04	1.68E-04	1.67E-04	1.68E-04	1.68E-04	1.69E-04	1.69E-04
		5.42E-05	3.37E-05	5.34E-05	7.04E-05	8.20E-05	1.12E-04
σ^2_{v2}	2.00E-08	4.41E-08	3.37E-09	3.37E-08	9.25E-08	1.71E-07	8.58E-07
		3.26E-09	1.63E-10	2.50E-09	1.07E-08	2.54E-08	2.32E-07
RV2	1.66E-04	2.39E-04	2.36E-04	2.12E-04	1.89E-04	1.78E-04	1.71E-04
		1.02E-05	1.02E-05	1.03E-05	1.21E-05	1.54E-05	3.04E-05
σ^2_{x12}		8.29E-05	8.33E-05	8.36E-05	8.36E-05	8.48E-05	8.57E-05
		4.58E-05	2.63E-05	4.09E-05	5.41E-05	6.30E-05	8.56E-05
σ^2_{v12}	0.00E+00	6.60E-09	-6.09E-13	1.10E-09	1.58E-08	5.28E-08	3.92E-07
		3.50E-09	6.87E-11	1.75E-09	7.31E-09	1.89E-08	1.93E-07
RCV	8.33E-04	5.56E-05	4.10E-06	3.11E-05	5.70E-05	6.96E-05	8.14E-05
		7.39E-06	1.88E-06	5.26E-06	8.24E-06	1.15E-05	2.51E-05
HY	8.33E-04	8.35E-05					
		9.40E-06					
RCV-RV	5.00E-01	2.33E-01	1.74E-02	1.47E-01	3.01E-01	3.92E-01	4.81E-01
		2.91E-02	8.02E-03	2.40E-02	3.98E-02	5.54E-02	1.16E-01
HY-RV	5.00E-01	3.50E-01	3.55E-01	3.94E-01	4.42E-01	4.71E-01	5.10E-01
		3.66E-02	3.71E-02	4.11E-02	4.62E-02	5.23E-02	1.07E-01
HY-SIML	5.00E-01	5.69E-01	5.22E-01	5.65E-01	6.11E-01	6.67E-01	9.04E-01
		2.11E-01	1.18E-01	2.03E-01	3.02E-01	4.34E-01	1.21E+00
SIML-SIML	5.00E-01	5.09E-01	5.01E-01	5.10E-01	5.12E-01	5.28E-01	5.23E-01
		2.40E-01	1.24E-01	2.04E-01	2.72E-01	3.35E-01	5.04E-01

Table 3.4 : Estimation of covariance and hedging coefficient :

Case 2 ($a_0 = 1, a_1 = -1, a_2 = 1; \lambda = 18000$)

18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ^2_{x1}	1.66E-04	1.71E-04	1.72E-04	1.72E-04	1.73E-04	1.74E-04	1.76E-04
		3.54E-05	3.53E-05	5.69E-05	7.22E-05	8.41E-05	1.12E-04
σ^2_{v1}	2.00E-06	2.00E-06	9.37E-07	2.02E-06	2.08E-06	2.16E-06	2.84E-06
		5.68E-08	3.02E-08	1.39E-07	2.23E-07	3.06E-07	7.94E-07
RV1	1.66E-04	7.21E-02	4.56E-02	7.36E-03	2.57E-03	1.37E-03	4.08E-04
		1.08E-03	8.07E-04	2.78E-04	1.76E-04	1.26E-04	8.08E-05
σ^2_{x2}	1.66E-04	1.72E-04	1.73E-04	1.73E-04	1.74E-04	1.75E-04	1.76E-04
		3.60E-05	3.61E-05	5.61E-05	7.36E-05	8.49E-05	1.10E-04
σ^2_{v2}	2.00E-06	2.00E-06	9.39E-07	2.03E-06	2.07E-06	2.17E-06	2.83E-06
		5.52E-08	3.06E-08	1.46E-07	2.27E-07	3.08E-07	7.94E-07
RV2	1.66E-04	7.22E-02	4.57E-02	7.38E-03	2.57E-03	1.37E-03	4.08E-04
		1.08E-03	7.73E-04	2.89E-04	1.79E-04	1.31E-04	8.06E-05
σ^2_{x12}		8.36E-05	8.31E-05	8.50E-05	8.69E-05	8.85E-05	8.91E-05
		3.04E-05	2.84E-05	4.30E-05	5.57E-05	6.65E-05	8.77E-05
σ^2_{v12}	0.00E+00	-3.52E-10	-9.98E-10	1.20E-08	3.59E-08	8.31E-08	4.31E-07
		4.64E-08	1.89E-08	1.02E-07	1.56E-07	2.19E-07	5.51E-07
RCV	8.33E-04	7.16E-05	3.55E-05	8.70E-05	8.57E-05	8.71E-05	8.51E-05
		5.44E-04	3.83E-04	2.09E-04	1.25E-04	9.14E-05	5.88E-05
HY	8.33E-04	8.28E-05					
		3.86E-04					
RCV-RV	5.00E-01	9.90E-04	7.81E-04	1.18E-02	3.33E-02	6.35E-02	2.09E-01
		7.53E-03	8.39E-03	2.84E-02	4.89E-02	6.64E-02	1.39E-01
HY-RV	5.00E-01	1.15E-03	1.81E-03	1.13E-02	3.18E-02	6.04E-02	2.09E-01
		5.35E-03	8.45E-03	5.26E-02	1.51E-01	2.85E-01	1.00E+00
HY-SIML	5.00E-01	5.09E-01	5.04E-01	5.13E-01	5.20E-01	5.60E-01	9.17E-01
		2.39E+00	2.37E+00	2.69E+00	3.02E+00	3.47E+00	6.06E+00
SIML-SIML	5.00E-01	4.89E-01	4.83E-01	4.96E-01	5.08E-01	5.15E-01	5.18E-01
		1.47E-01	1.32E-01	2.05E-01	2.72E-01	3.21E-01	5.20E-01
18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ^2_{x1}	1.66E-04	1.66E-04	1.66E-04	1.66E-04	1.68E-04	1.69E-04	1.70E-04
		3.42E-05	3.39E-05	5.51E-05	7.01E-05	8.26E-05	1.09E-04
σ^2_{v1}	2.00E-08	2.24E-08	1.17E-08	4.40E-08	9.44E-08	1.72E-07	8.47E-07
		6.38E-10	3.63E-10	3.05E-09	1.04E-08	2.54E-08	2.46E-07
RV1	1.66E-04	8.86E-04	6.21E-04	2.38E-04	1.91E-04	1.79E-04	1.69E-04
		1.21E-05	9.87E-06	7.91E-06	1.09E-05	1.49E-05	3.15E-05
σ^2_{x2}	1.66E-04	1.67E-04	1.67E-04	1.68E-04	1.70E-04	1.69E-04	1.69E-04
		3.47E-05	3.46E-05	5.45E-05	7.07E-05	8.10E-05	1.04E-04
σ^2_{v2}	2.00E-08	2.24E-08	1.17E-08	4.44E-08	9.40E-08	1.71E-07	8.52E-07
		6.16E-10	3.57E-10	3.13E-09	1.05E-08	2.48E-08	2.44E-07
RV2	1.66E-04	8.86E-04	6.22E-04	2.39E-04	1.91E-04	1.78E-04	1.70E-04
		1.23E-05	9.71E-06	8.00E-06	1.08E-05	1.49E-05	3.22E-05
σ^2_{x12}		8.40E-05	8.34E-05	8.54E-05	8.78E-05	8.87E-05	8.94E-05
		2.93E-05	2.69E-05	4.18E-05	5.43E-05	6.46E-05	8.52E-05
σ^2_{v12}	0.00E+00	6.03E-10	7.63E-11	9.82E-09	3.49E-08	7.33E-08	4.09E-07
		5.38E-10	2.32E-10	2.23E-09	7.94E-09	1.94E-08	1.88E-07
RCV	8.33E-04	5.56E-05	3.06E-05	7.50E-05	8.06E-05	8.19E-05	8.34E-05
		6.50E-06	5.03E-06	6.02E-06	8.48E-06	1.16E-05	2.51E-05
HY	8.33E-04	8.32E-05					
		5.90E-06					
RCV-RV	5.00E-01	6.27E-02	4.93E-02	3.15E-01	4.23E-01	4.58E-01	4.94E-01
		7.32E-03	8.08E-03	2.29E-02	3.72E-02	5.20E-02	1.19E-01
HY-RV	5.00E-01	9.39E-02	1.34E-01	3.50E-01	4.38E-01	4.68E-01	5.10E-01
		6.65E-03	9.40E-03	2.56E-02	3.80E-02	4.84E-02	1.02E-01
HY-SIML	5.00E-01	5.23E-01	5.23E-01	5.58E-01	5.94E-01	6.40E-01	8.17E-01
		1.14E-01	1.14E-01	2.00E-01	3.00E-01	4.04E-01	9.37E-01
SIML-SIML	5.00E-01	5.05E-01	5.02E-01	5.15E-01	5.28E-01	5.32E-01	5.40E-01
		1.42E-01	1.26E-01	2.01E-01	2.65E-01	3.15E-01	4.87E-01

Table 3.5 : Estimation of covariance and hedging coefficient :

Case 3 (Stochastic volatility; $\lambda = 1800$)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ^2_{x1}		2.03E-04	2.22E-04	2.05E-04	2.02E-04	2.02E-04	2.02E-04
		6.83E-05	4.59E-05	6.76E-05	8.58E-05	9.74E-05	1.27E-04
σ^2_{v1}	2.00E-06	2.02E-06	1.05E-07	9.83E-07	1.92E-06	2.14E-06	2.97E-06
		1.45E-07	5.78E-09	8.69E-08	2.15E-07	3.08E-07	7.95E-07
RV1		7.40E-03	7.06E-03	4.75E-03	2.47E-03	1.39E-03	4.37E-04
		3.44E-04	3.36E-04	2.59E-04	1.75E-04	1.34E-04	8.49E-05
σ^2_{x2}		2.07E-04	2.24E-04	2.09E-04	2.06E-04	2.05E-04	2.09E-04
		6.69E-05	4.48E-05	6.61E-05	8.35E-05	9.59E-05	1.38E-04
σ^2_{v2}	2.00E-06	2.03E-06	1.04E-07	9.80E-07	1.92E-06	2.17E-06	3.03E-06
		1.48E-07	6.11E-09	8.50E-08	2.14E-07	3.21E-07	8.52E-07
RV2		7.40E-03	7.05E-03	4.74E-03	2.48E-03	1.40E-03	4.44E-04
		3.63E-04	3.47E-04	2.67E-04	1.80E-04	1.35E-04	8.58E-05
σ^2_{x12}		1.01E-04	1.01E-04	1.02E-04	1.02E-04	1.02E-04	1.02E-04
		5.46E-05	3.51E-05	5.05E-05	6.52E-05	7.76E-05	1.03E-04
σ^2_{v12}	0.00E+00	1.40E-08	-3.55E-11	3.81E-09	1.19E-08	7.02E-08	4.96E-07
		1.19E-07	2.08E-09	4.93E-08	1.49E-07	2.13E-07	6.14E-07
RCV		7.23E-05	4.27E-06	3.79E-05	6.44E-05	8.49E-05	9.93E-05
		1.74E-04	5.33E-05	1.24E-04	1.20E-04	9.15E-05	6.27E-05
HY		9.83E-05					
		1.22E-04					
RCV-RV	5.00E-01	9.78E-03	6.24E-04	7.99E-03	2.61E-02	6.18E-02	2.28E-01
		2.35E-02	7.57E-03	2.61E-02	4.85E-02	6.64E-02	1.40E-01
HY-RV	5.00E-01	1.33E-02	1.40E-02	2.07E-02	3.99E-02	7.16E-02	2.31E-01
		1.65E-02	1.73E-02	2.58E-02	4.97E-02	8.97E-02	3.01E-01
HY-SIML	5.00E-01	5.34E-01	4.58E-01	5.23E-01	5.86E-01	6.25E-01	8.47E-01
		7.17E-01	5.96E-01	6.97E-01	8.44E-01	9.46E-01	1.76E+00
SIML-SIML	5.00E-01	5.03E-01	4.53E-01	5.01E-01	5.11E-01	5.08E-01	5.02E-01
		2.41E-01	1.30E-01	2.06E-01	2.85E-01	3.46E-01	5.11E-01
1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ^2_{x1}		1.97E-04	2.00E-04	1.98E-04	1.95E-04	1.96E-04	1.96E-04
		6.65E-05	4.09E-05	6.55E-05	8.30E-05	9.41E-05	1.23E-04
σ^2_{v1}	2.00E-08	4.89E-08	3.84E-09	3.84E-08	1.07E-07	2.02E-07	1.03E-06
		3.75E-09	1.79E-10	3.00E-09	1.21E-08	3.05E-08	2.85E-07
RV1		2.72E-04	2.68E-04	2.45E-04	2.22E-04	2.11E-04	2.02E-04
		1.18E-05	1.17E-05	1.21E-05	1.40E-05	1.82E-05	3.72E-05
σ^2_{x2}		2.02E-04	2.01E-04	2.01E-04	2.00E-04	1.99E-04	2.02E-04
		6.39E-05	3.89E-05	6.25E-05	8.08E-05	9.22E-05	1.33E-04
σ^2_{v2}	2.00E-08	4.89E-08	3.84E-09	3.86E-08	1.08E-07	2.03E-07	1.03E-06
		3.73E-09	1.75E-10	2.88E-09	1.24E-08	2.97E-08	2.91E-07
RV2		2.72E-04	2.69E-04	2.46E-04	2.23E-04	2.13E-04	2.04E-04
		1.14E-05	1.14E-05	1.17E-05	1.39E-05	1.77E-05	3.63E-05
σ^2_{x12}		1.00E-04	1.00E-04	1.02E-04	1.01E-04	1.01E-04	1.02E-04
		5.33E-05	3.24E-05	4.88E-05	6.31E-05	7.56E-05	1.01E-04
σ^2_{v12}	0.00E+00	8.20E-09	-3.43E-12	1.26E-09	1.89E-08	6.34E-08	4.73E-07
		3.88E-09	8.06E-11	1.94E-09	8.54E-09	2.23E-08	2.29E-07
RCV		6.69E-05	4.79E-06	3.67E-05	6.83E-05	8.35E-05	9.73E-05
		8.82E-06	2.22E-06	6.09E-06	9.65E-06	1.32E-05	2.83E-05
HY		1.00E-04					
		1.13E-05					
RCV-RV	5.00E-01	2.46E-01	1.78E-02	1.50E-01	3.07E-01	3.96E-01	4.81E-01
		3.06E-02	8.24E-03	2.42E-02	3.99E-02	5.39E-02	1.13E-01
HY-RV	5.00E-01	3.68E-01	3.73E-01	4.08E-01	4.51E-01	4.76E-01	5.09E-01
		3.76E-02	3.81E-02	4.15E-02	4.58E-02	5.26E-02	1.02E-01
HY-SIML	5.00E-01	5.68E-01	5.20E-01	5.64E-01	6.18E-01	6.50E-01	8.19E-01
		2.17E-01	1.15E-01	2.09E-01	3.25E-01	3.83E-01	8.85E-01
SIML-SIML	5.00E-01	5.15E-01	4.99E-01	5.17E-01	5.21E-01	5.17E-01	5.16E-01
		2.39E-01	1.23E-01	2.01E-01	2.74E-01	3.34E-01	5.08E-01

Table 3.6 : Estimation of covariance and hedging coefficient :

Case 3 (Stochastic volatility; $\lambda = 18000$)

18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ^2_{x1}		2.05E-04	2.06E-04	2.07E-04	2.07E-04	2.07E-04	2.08E-04
		4.11E-05	4.11E-05	6.70E-05	8.46E-05	9.43E-05	1.29E-04
σ^2_{v1}	2.00E-06	2.00E-06	9.40E-07	2.03E-06	2.08E-06	2.18E-06	3.03E-06
		5.48E-08	3.02E-08	1.45E-07	2.33E-07	3.31E-07	8.33E-07
RV1		7.22E-02	4.57E-02	7.41E-03	2.60E-03	1.40E-03	4.42E-04
		1.05E-03	7.70E-04	3.01E-04	1.75E-04	1.38E-04	8.47E-05
σ^2_{x2}		2.07E-04	2.07E-04	2.09E-04	2.08E-04	2.07E-04	2.07E-04
		4.24E-05	4.22E-05	6.71E-05	8.48E-05	9.72E-05	1.35E-04
σ^2_{v2}	2.00E-06	2.00E-06	9.40E-07	2.03E-06	2.09E-06	2.19E-06	3.01E-06
		5.65E-08	3.03E-08	1.45E-07	2.34E-07	3.26E-07	8.68E-07
RV2		7.22E-02	4.57E-02	7.40E-03	2.60E-03	1.40E-03	4.41E-04
		1.10E-03	8.16E-04	3.01E-04	1.82E-04	1.38E-04	8.86E-05
σ^2_{x12}		1.01E-04	1.00E-04	1.02E-04	1.02E-04	1.02E-04	1.03E-04
		3.43E-05	3.14E-05	4.93E-05	6.47E-05	7.31E-05	1.05E-04
σ^2_{v12}	0.00E+00	1.54E-09	-1.12E-09	6.37E-09	2.93E-08	8.62E-08	4.70E-07
		4.61E-08	1.86E-08	1.02E-07	1.60E-07	2.25E-07	5.98E-07
RCV		8.23E-05	4.50E-05	7.87E-05	8.71E-05	9.35E-05	9.86E-05
		5.36E-04	3.80E-04	2.10E-04	1.25E-04	9.46E-05	6.15E-05
HY		1.31E-04					
		3.86E-04					
RCV-RV	5.00E-01	1.14E-03	9.88E-04	1.07E-02	3.37E-02	6.71E-02	2.25E-01
		7.42E-03	8.31E-03	2.84E-02	4.86E-02	6.79E-02	1.37E-01
HY-RV	5.00E-01	1.81E-03	2.86E-03	1.76E-02	5.05E-02	9.24E-02	3.11E-01
		5.34E-03	8.43E-03	5.20E-02	1.49E-01	2.78E-01	9.22E-01
HY-SIML	5.00E-01	6.79E-01	6.76E-01	7.17E-01	7.83E-01	8.27E-01	1.04E+00
		2.01E+00	2.01E+00	2.18E+00	2.51E+00	2.83E+00	4.02E+00
SIML-SIML	5.00E-01	4.93E-01	4.89E-01	4.98E-01	4.98E-01	4.95E-01	4.78E-01
		1.40E-01	1.26E-01	1.97E-01	2.68E-01	3.21E-01	4.85E-01
18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ^2_{x1}		2.00E-04	2.00E-04	2.02E-04	2.01E-04	2.01E-04	2.01E-04
		3.95E-05	3.94E-05	6.48E-05	8.21E-05	9.17E-05	1.27E-04
σ^2_{v1}	2.00E-08	2.28E-08	1.22E-08	4.92E-08	1.08E-07	2.02E-07	1.03E-06
		6.36E-10	3.80E-10	3.46E-09	1.17E-08	2.87E-08	2.90E-07
RV1		9.20E-04	6.55E-04	2.72E-04	2.24E-04	2.12E-04	2.04E-04
		1.26E-05	1.04E-05	9.28E-06	1.31E-05	1.76E-05	3.71E-05
σ^2_{x2}		2.01E-04	2.01E-04	2.03E-04	2.03E-04	2.02E-04	2.02E-04
		4.07E-05	4.04E-05	6.51E-05	8.32E-05	9.46E-05	1.31E-04
σ^2_{v2}	2.00E-08	2.28E-08	1.22E-08	4.89E-08	1.09E-07	2.03E-07	1.01E-06
		6.50E-10	3.83E-10	3.53E-09	1.18E-08	2.91E-08	2.88E-07
RV2		9.20E-04	6.55E-04	2.72E-04	2.24E-04	2.13E-04	2.03E-04
		1.26E-05	1.06E-05	8.92E-06	1.31E-05	1.75E-05	3.74E-05
σ^2_{x12}		1.01E-04	1.01E-04	1.03E-04	1.03E-04	1.02E-04	1.03E-04
		3.34E-05	3.03E-05	4.79E-05	6.31E-05	7.17E-05	1.02E-04
σ^2_{v12}	0.00E+00	7.50E-10	9.85E-11	1.17E-08	4.14E-08	8.82E-08	4.95E-07
		5.77E-10	2.38E-10	2.48E-09	8.95E-09	2.24E-08	2.29E-07
RCV		6.68E-05	3.70E-05	8.99E-05	9.64E-05	9.84E-05	1.01E-04
		7.08E-06	5.24E-06	6.89E-06	9.77E-06	1.34E-05	2.86E-05
HY		1.00E-04					
		6.32E-06					
RCV-RV	5.00E-01	7.26E-02	5.65E-02	3.31E-01	4.31E-01	4.65E-01	4.94E-01
		7.74E-03	8.00E-03	2.29E-02	3.65E-02	5.14E-02	1.11E-01
HY-RV	5.00E-01	1.09E-01	1.53E-01	3.70E-01	4.50E-01	4.77E-01	5.09E-01
		6.89E-03	9.63E-03	2.45E-02	3.55E-02	4.75E-02	9.81E-02
HY-SIML	5.00E-01	5.21E-01	5.21E-01	5.51E-01	5.92E-01	6.28E-01	8.22E-01
		1.11E-01	1.10E-01	1.90E-01	2.75E-01	3.66E-01	7.92E-01
SIML-SIML	5.00E-01	5.05E-01	5.03E-01	5.12E-01	5.12E-01	5.11E-01	5.04E-01
		1.38E-01	1.22E-01	1.92E-01	2.64E-01	3.19E-01	4.99E-01

4. Asymptotic Properties of the SIML Estimation

It is important to investigate the asymptotic properties of the SIML estimator when the instantaneous volatility function $\sigma_x^2(s)$ is not constant over time. When the integrated variance-covariance matrix is a positive (deterministic) constant a.s. (i.e. Σ_x is not stochastic) while the instantaneous covariance function is time varying, we have the consistency and the asymptotic normality of the SIML estimator as $n \to \infty$. For the deterministic time varying case with the non-stochastic sampling, the asymptotic properties of the SIML estimator can be summarized as the next proposition and the proof of Theorem 4.1 is quite similar to the one given by Kunitomo and Sato (2008, 2011), and Misaki and Kunitomo (2013). Thus we omit its details.

Theorem 4.1 : We assume that $X(t)$ and $u_s(t_i^s), u_f(t_j^f)$ $(i, j \geq 1)$ in (2.3), (2.5) and (2.6) are independent, $\mathbf{\Sigma}_x = \int_0^1 \mathbf{\Sigma}_x(s)ds$ is a positive definite and constant (or deterministic), $\mathbf{\Sigma}_x(s)$ is bounded, and $\mathcal{E}[u_s(t_i^s)^4] < \infty$, $\mathcal{E}[u_f(t_j^f)^4] < \infty$ under Assumptions 2.1 and 2.2 with $c_s = c_f = 1$. Define the SIML estimator $\hat{\sigma}_{ss}^{(x)}$ of $\sigma_{ss}^{(x)}$ by (2.14) and the SIML estimator $\hat{\sigma}_{sf}^{(x)}$ of $\sigma_{sf}^{(x)}$ by (2.17).

(i) For $m_{n_s} = n_s^{\alpha}, m_{n_f} = n_f^{\alpha}$, and $0 < \alpha < 0.5$, as $n \longrightarrow \infty$ (and $m_{n_s^*}/m_{n_s} \xrightarrow{p} 1, m_{n_f^*}/m_{n_f} \xrightarrow{p} 1$),

$$\hat{\sigma}_{ss}^{(x)} - \sigma_{ss}^{(x)} \xrightarrow{p} 0 \ , \ \hat{\sigma}_{ff}^{(x)} - \sigma_{ff}^{(x)} \xrightarrow{p} 0 \tag{4.1}$$

and

$$\hat{\sigma}_{sf}^{(x)} - \sigma_{sf}^{(x)} \xrightarrow{p} 0 \ . \tag{4.2}$$

(ii) For $m_{n_s} = n_s^{\alpha}, m_{n_f} = n_f^{\alpha}$, and $0 < \alpha < 0.4$, as $n \longrightarrow \infty$

$$\sqrt{m_{n_s}}\left[\hat{\sigma}_{ss} - \sigma_{ss}\right] \xrightarrow{d} N\left[0, V_{ss}\right] \ , \ \sqrt{m_{n_f}}\left[\hat{\sigma}_{ff} - \sigma_{ff}\right] \xrightarrow{d} N\left[0, V_{ff}\right] \ , \tag{4.3}$$

and for $m_n = \min\{m_{n_s}, m_{n_f}\}$

$$\sqrt{m_n}\left[\hat{\sigma}_{sf} - \sigma_{sf}\right] \xrightarrow{d} N\left[0, V_{sf}\right] \ , \tag{4.4}$$

where

$$V_{ss} = 2\int_0^1 \left[\sigma_{ss}^{(x)}(s)\right]^2 ds \ , \ V_{ff} = 2\int_0^1 \left[\sigma_{ff}^{(x)}(s)\right]^2 ds \tag{4.5}$$

and

$$V_{sf} = \int_0^1 \left[\sigma_{ss}^{(x)}(s)\sigma_{ff}^{(x)}(s) + (\sigma_{sf}^{(x)}(s))^2\right] ds \ . \tag{4.6}$$

When the instantaneous covariance matrix is constant, then

$$V_{ss} = 2[\sigma_{ss}^{(x)}]^2 \ , \ V_{sf} = \sigma_{ss}^{(x)}\sigma_{ff}^{(x)} + [\sigma_{sf}^{(x)}]^2 \ . \tag{4.7}$$

When Σ_x is a random matrix, we need the concept of stable convergence. Some discussions on stable convergence along Jacod and Shryaev (2003) and Jacod (2007) have been given in Kunitomo and Sato (2013). The next result includes Theorem 4.1 as a special case in a sense, but in order to make the developments clearer, we state it separately. The proof of Theorem 4.2 has been similar to the one given by Kunitomo and Sato (2008a, 2011), and Misaki and Kunitomo (2013).

Theorem 4.2 : We assume that $X(t)$ and $u_s(t_i^s), u_f(t_j^f)$ $(i, j \geq 1)$ in (2.3), (2.5) and (2.6) are independent, $\mathbf{\Sigma}_x = \int_0^1 \mathbf{\Sigma}_x(s)ds$ is a positive definite and stochastic, $\mathbf{\Sigma}_x(s)$ is bounded, and $\mathcal{E}[u_s(t_i^s)^4] < \infty \ , \mathcal{E}[u_f(t_j^f)^4] < \infty$ under Assumptions 2.1 and 2.2 with $c_s = c_f = 1$. Define the SIML estimator $\hat{\sigma}_{ss}^{(x)}$ of $\sigma_{ss}^{(x)}$ by (2.14) and the SIML estimator $\hat{\sigma}_{sf}^{(x)}$ of $\sigma_{sf}^{(x)}$ by (2.17).

(i) For $m_{n_s} = n_s^{\alpha}, m_{n_f} = n_f^{\alpha}$, and $0 < \alpha < 0.5$, as $n \longrightarrow \infty$ (and $m_{n_s^*}/m_{n_s} \xrightarrow{p} 1, m_{n_f^*}/m_{n_f} \xrightarrow{p} 1$),

$$\hat{\sigma}_{ss}^{(x)} - \sigma_{ss}^{(x)} \xrightarrow{p} 0 \ , \ \hat{\sigma}_{ff}^{(x)} - \sigma_{ff}^{(x)} \xrightarrow{p} 0 \ , \tag{4.8}$$

and

$$\hat{\sigma}_{sf}^{(x)} - \sigma_{sf}^{(x)} \xrightarrow{p} 0 \ . \tag{4.9}$$

(ii) For $m_{n_s} = n_s^{\alpha}, m_{n_f} = n_f^{\alpha}$, and $0 < \alpha < 0.4$, as $n \longrightarrow \infty$ we have the weak convergence

$$Z_{ss} = \sqrt{m_{n_s}}\left[\hat{\sigma}_{ss}^{(x)} - \sigma_{ss}^{(x)}\right] \xrightarrow{w} Z_{ss}^* \ , \tag{4.10}$$

where the characteristic function $g_n(t) = \mathcal{E}[\exp(itZ_{ss})]$ of Z_{ss} (or Z_{ff}) converges to the characteristic function of Z_{ss}^* (or Z_{ff}^*), which is written as

$$g(t) = \mathcal{E}[e^{-\frac{V_{ss}t^2}{2}}] \tag{4.11}$$

(or V_{ss}) and

(4.12) $$V_{ss} = 2\int_0^1 \left[\sigma_{ss}^{(x)}(s)\right]^2 ds\ ,\ V_{ff} = 2\int_0^1 \left[\sigma_{ff}^{(x)}(s)\right]^2 ds\ .$$

As $n \longrightarrow \infty$ for $m_n = \min\{m_{n_s}, m_{n_f}\}$

(4.13) $$Z_{sf} = \sqrt{m_n}\left[\hat{\sigma}_{sf}^{(x)} - \sigma_{sf}^{(x)}\right] \xrightarrow{w} Z_{sf}^*\ ,$$

where the characteristic function $g_n(t) = \mathcal{E}[\exp(itZ_{sf})]$ of Z_{sf} converges to the characteristic function of Z_{sf}^*, which is written as

(4.14) $$g(t) = \mathcal{E}[e^{-\frac{V_{sf}t^2}{2}}]$$

and

(4.15) $$V_{sf} = \int_0^1 \left[\sigma_{ss}^{(x)}(s)\sigma_{ff}^{(x)}(s) + (\sigma_{sf}^{(x)}(s))^2\right] ds\ .$$

By using the estimators of the integrated volatility and the integrated covariance, the SIML estimator of the hedging ratio $H = \sigma_{sf}^{(x)}/\sigma_{ff}^{(x)}$ has been defined by (2.18). From Theorem 4.1, the asymptotic variance can be expressed as

(4.16) $$AV[\sqrt{m_n}\left[\hat{H} - H\right]] = [\frac{1}{\sigma_{ff}^{(x)}}]^2 V_{sf} + [\frac{\sigma_{sf}^{(x)}}{\sigma_{ff}^{(x)2}}]^2 V_{ff} - 4\frac{\sigma_{sf}^{(x)}}{\sigma_{ff}^{(x)3}}\int_0^1 \sigma_{sf}^{(x)}(s)\sigma_{ff}^{(x)} ds\ .$$

Although the above formula looks complicated, when the volatility and covariance functions are constant we have the next result.

Corollary 4.1 : Assume that the instantaneous volatility matrix $\mathbf{\Sigma}_x$ is constant. Then the asymptotic variance of the limiting distribution of $\sqrt{m_n}\left[\hat{H} - H\right]$ is given by

(4.17) $$\omega_H = \frac{\sigma_{ss}^{(x)}}{\sigma_{ff}^{(x)}}\left[1 - \frac{\sigma_{sf}^{(x)2}}{\sigma_{ss}^{(x)}\sigma_{ff}^{(x)}}\right]\ .$$

It would be straightforward to derive the corresponding results on the asymptotic distribution of the SIML estimator when we have the non-linear transformations of (2.5) and (2.6). Some of the results have been reported in Sato and Kunitomo (2012) for the case of fixed observation intervals. We shall report the extensions of Sato and Kunitomo (2012) subsequently.

5. Further Simulations

By extending the basic simulation framework reported in Section 3, we have conducted a large number of simulations. We have adopted the micro-market adjustment model and the round-off error models investigated by Sato and Kunitomo (2011). Among many Monte-Carlo simulations, we summarize our main results as Tables 5.1-5.10.

As the fourth case (Case 4) we use the EACD(1,1) model, which was proposed by Engle and Russel (1997). Financial econometricians have been interested in the autoregressive conditional duration (ACD) models because the assumption of Poisson random sampling leads to the sequence of i.i.d. random variables for durations while we may have the dependent structure on the observed durations. Although there can be many types of duration dependence models, we shall use the EACD(1,1) model as a representative one. Let $\tau_i^a = t_i^a - t_{i-1}^a$ and $\tau_i^a = \psi_i^a \epsilon_i^a$, where

$$\psi_i^a = \omega + \delta \epsilon_i^a + \gamma \psi_{i-1}^a \quad (5.1)$$

and ϵ_i^a are a sequence of i.i.d. exponential random variables with $\delta > 0, \gamma > 0, \omega > 0$ $\mathcal{E}[\epsilon_i^a] = 0$ and $V[\epsilon_i^a] = 1$ for $a = s$ or f. In order to make a fair comparison with the Poisson sampling and other possible cases, we have set $\mathcal{E}[\tau_i^a] = 1/n$.

In our simulations we use several non-linear transformation models in the form of (2.5) and (2.6). (Sato and Kunitomo (2012) have discussed the economic meaning of models in the details.) We set $\mathbf{X}(t_i^s) = X_s(t_i^s)$ and $\mathbf{X}(t_j^f) = X_f(t_j^f)$ for the sake of the resulting simplicity in our simulations. Then each model corresponds to

Case 5 $\quad h_1(x,y,u) = y + g(x-y) + u \ (g : \text{a constant})$,

Case 6 $\quad h_2(x,y,u) = y + g_\eta(x-y+u) \ \ (g_\eta(\cdot) \text{ is } (2.6))$,

Case 7 $\quad h_3(x,y,u) = y + g_\eta(x-y) + u \ \ (g_\eta(\cdot) \text{ is } (2.6))$,

Case 8 $\quad h_4(x,y,u) = y + u + \begin{cases} g_1(x-y) & \text{if } y \geq 0 \ (g_1 : \text{a constant}) \\ g_2(x-y) & \text{if } y < 0 \ (g_2 : \text{a constant}) \end{cases}$, respectively.

Among them Case 5 is the basic additive model when $g = 1$. When $0 < g < 2$, Case 5 corresponds to the linear model with the micro-market noise. Case 6 and Case 7 are the micro-market models with the round-off errors. Case 6 is the basic round-off model and Case 7 has a more complicated nonlinearity. Case 8 is the SSAR model, which has been known as a nonlinear (discrete) time series model.

When there are micro-market noise components with the martingale signal part, the value of RCV often differs substantially from the true integrated covariance of the signal part. However, we have found that it is possible to estimate the integrated variances, the integrated covariance and the noise variances when we have the signal-noise ratio as $10^{-2} \sim 10^{-6}$ by the SIML estimation method. Although we have omitted the details of the second example, the estimation results are similar in the stochastic volatility model.

For the estimation of the hedging coefficients, We have confirmed that by using the H-Y method we can improve the historical covariance method as was pointed out by Hayashi and Yoshida (2005). However, when we have micro-market noise terms, the SIML-SIML combination of the integrated variance and the integrated covariance. This point is vivid and important on the hedging coefficient estimation as Tables 5.1-5.8 have shown.

By examining these results of our simulations we can conclude that we can estimate the integrated volatility of the hidden martingale part reasonably by the SIML estimation method. It may be surprising to find that the SIML method gives reasonable estimates even when we have nonlinear transformations of the original unobservable security (intrinsic) values. We have conducted a number of further simulations, but the results are quite similar as we have reported in this section.

Table 5.1 : Estimation of covariance and hedging coefficient :

Case 4 (ACD; $\lambda = 1800$)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ^2_{x1}	2.00E-04	2.03E-04	2.23E-04	2.04E-04	2.03E-04	2.03E-04	2.02E-04
		6.61E-05	4.59E-05	6.40E-05	8.36E-05	9.54E-05	1.30E-04
σ^2_{v1}	2.00E-06	2.04E-06	1.05E-07	9.78E-07	1.89E-06	2.16E-06	2.98E-06
		1.51E-07	8.16E-09	9.92E-08	2.20E-07	3.06E-07	8.48E-07
RV1	2.00E-04	7.46E-03	7.08E-03	4.72E-03	2.46E-03	1.39E-03	4.38E-04
		5.26E-04	4.89E-04	2.96E-04	1.77E-04	1.30E-04	8.87E-05
σ^2_{x2}	2.00E-04	2.08E-04	2.24E-04	2.10E-04	2.08E-04	2.08E-04	2.12E-04
		6.98E-05	4.74E-05	6.74E-05	8.76E-05	1.02E-04	1.37E-04
σ^2_{v2}	2.00E-06	2.03E-06	1.04E-07	9.72E-07	1.89E-06	2.17E-06	3.00E-06
		1.42E-07	8.37E-09	9.51E-08	2.10E-07	3.06E-07	8.34E-07
RV2	2.00E-04	7.38E-03	7.02E-03	4.68E-03	2.45E-03	1.39E-03	4.41E-04
		5.21E-04	4.85E-04	2.92E-04	1.70E-04	1.30E-04	8.35E-05
σ^2_{x12}	1.00E-04	1.02E-04	1.00E-04	1.01E-04	1.02E-04	1.04E-04	1.02E-04
		5.47E-05	3.42E-05	4.84E-05	6.43E-05	7.55E-05	1.06E-04
σ^2_{v12}	0.00E+00	1.01E-08	-8.89E-11	1.97E-09	2.43E-08	6.93E-08	4.71E-07
		1.22E-07	2.10E-09	4.90E-08	1.52E-07	2.26E-07	5.94E-07
RCV	1.00E-04	6.82E-05	3.41E-06	3.80E-05	7.20E-05	8.57E-05	9.75E-05
		1.72E-04	5.30E-05	1.25E-04	1.24E-04	9.39E-05	6.17E-05
HY	1.00E-04	1.05E-04					
		1.23E-04					
RCV-RV	5.00E-01	9.13E-03	4.78E-04	8.08E-03	2.94E-02	6.17E-02	2.24E-01
		2.32E-02	7.47E-03	2.64E-02	5.03E-02	6.74E-02	1.38E-01
HY-RV	5.00E-01	1.42E-02	1.49E-02	2.24E-02	4.32E-02	7.68E-02	2.51E-01
		1.66E-02	1.75E-02	2.62E-02	5.06E-02	8.99E-02	3.00E-01
HY-SIML	5.00E-01	5.90E-01	4.92E-01	5.78E-01	6.48E-01	6.87E-01	8.83E-01
		7.53E-01	5.98E-01	7.33E-01	9.13E-01	1.05E+00	1.74E+00
SIML-SIML	5.00E-01	5.07E-01	4.50E-01	4.96E-01	5.06E-01	5.16E-01	4.94E-01
		2.46E-01	1.28E-01	2.08E-01	2.73E-01	3.23E-01	4.92E-01
1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ^2_{x1}	2.00E-04	1.97E-04	2.00E-04	1.97E-04	1.97E-04	1.97E-04	1.96E-04
		6.41E-05	4.11E-05	6.13E-05	8.07E-05	9.14E-05	1.27E-04
σ^2_{v1}	2.00E-08	4.92E-08	3.85E-09	3.84E-08	1.07E-07	2.01E-07	1.03E-06
		4.05E-09	1.92E-10	2.81E-09	1.17E-08	2.99E-08	2.99E-07
RV1	2.00E-04	2.73E-04	2.69E-04	2.45E-04	2.23E-04	2.12E-04	2.03E-04
		1.23E-05	1.22E-05	1.21E-05	1.40E-05	1.81E-05	3.82E-05
σ^2_{x2}	2.00E-04	2.02E-04	2.00E-04	2.03E-04	2.02E-04	2.02E-04	2.05E-04
		6.82E-05	4.14E-05	6.60E-05	8.48E-05	9.87E-05	1.31E-04
σ^2_{v2}	2.00E-08	4.92E-08	3.83E-09	3.83E-08	1.07E-07	2.02E-07	1.02E-06
		4.09E-09	1.90E-10	2.92E-09	1.24E-08	2.97E-08	2.85E-07
RV2	2.00E-04	2.71E-04	2.68E-04	2.44E-04	2.22E-04	2.12E-04	2.03E-04
		1.22E-05	1.22E-05	1.24E-05	1.45E-05	1.81E-05	3.74E-05
σ^2_{x12}	1.00E-04	1.01E-04	1.00E-04	1.01E-04	1.02E-04	1.04E-04	1.03E-04
		5.33E-05	3.04E-05	4.65E-05	6.27E-05	7.36E-05	1.04E-04
σ^2_{v12}	0.00E+00	9.08E-09	-1.49E-12	1.22E-09	1.87E-08	6.26E-08	4.77E-07
		4.17E-09	7.96E-11	1.97E-09	8.59E-09	2.18E-08	2.30E-07
RCV	1.00E-04	6.86E-05	4.69E-06	3.60E-05	6.72E-05	8.27E-05	9.80E-05
		9.08E-06	2.15E-06	6.40E-06	9.82E-06	1.30E-05	2.82E-05
HY	1.00E-04	1.01E-04					
		1.12E-05					
RCV-RV	5.00E-01	2.51E-01	1.74E-02	1.47E-01	3.02E-01	3.91E-01	4.83E-01
		3.18E-02	7.98E-03	2.51E-02	4.04E-02	5.32E-02	1.11E-01
HY-RV	5.00E-01	3.68E-01	3.74E-01	4.10E-01	4.52E-01	4.77E-01	5.10E-01
		3.82E-02	3.86E-02	4.14E-02	4.69E-02	5.47E-02	1.04E-01
HY-SIML	5.00E-01	5.69E-01	5.22E-01	5.65E-01	6.14E-01	6.59E-01	8.82E-01
		2.09E-01	1.17E-01	2.04E-01	3.10E-01	4.18E-01	1.07E+00
SIML-SIML	5.00E-01	5.19E-01	5.02E-01	5.16E-01	5.24E-01	5.34E-01	5.19E-01
		2.45E-01	1.21E-01	2.05E-01	2.77E-01	3.25E-01	5.13E-01

Table 5.2 : Estimation of covariance and hedging coefficient :

Case 4 (ACD ; $\lambda = 18000$)

18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ^2_{x1}	2.00E-04	2.05E-04	2.06E-04	2.05E-04	2.04E-04	2.02E-04	2.03E-04
		4.30E-05	4.27E-05	6.64E-05	8.52E-05	9.33E-05	1.27E-04
σ^2_{v1}	2.00E-06	2.00E-06	9.32E-07	2.03E-06	2.08E-06	2.18E-06	2.98E-06
		5.55E-08	3.36E-08	1.41E-07	2.28E-07	3.16E-07	8.14E-07
RV1	2.00E-04	7.22E-02	4.51E-02	7.40E-03	2.60E-03	1.40E-03	4.38E-04
		1.62E-03	9.34E-04	3.01E-04	1.79E-04	1.37E-04	8.63E-05
σ^2_{x2}	2.00E-04	2.05E-04	2.06E-04	2.07E-04	2.10E-04	2.12E-04	2.12E-04
		4.25E-05	4.27E-05	6.85E-05	8.83E-05	1.03E-04	1.42E-04
σ^2_{v2}	2.00E-06	2.01E-06	9.31E-07	2.03E-06	2.10E-06	2.20E-06	3.04E-06
		5.58E-08	3.34E-08	1.46E-07	2.28E-07	3.07E-07	8.41E-07
RV2	2.00E-04	7.22E-02	4.52E-02	7.41E-03	2.61E-03	1.41E-03	4.45E-04
		1.78E-03	9.57E-04	3.02E-04	1.79E-04	1.32E-04	8.89E-05
σ^2_{x12}	1.00E-04	1.01E-04	1.00E-04	1.02E-04	1.03E-04	1.04E-04	1.03E-04
		3.70E-05	3.37E-05	5.13E-05	6.49E-05	7.66E-05	1.03E-04
σ^2_{v12}	0.00E+00	-9.58E-10	-3.15E-10	6.07E-09	3.90E-08	1.05E-07	4.83E-07
		4.56E-08	1.83E-08	1.03E-07	1.67E-07	2.17E-07	5.84E-07
RCV	1.00E-04	7.51E-05	3.09E-05	8.39E-05	9.73E-05	1.04E-04	1.00E-04
		5.01E-04	3.78E-04	2.16E-04	1.26E-04	9.30E-05	6.06E-05
HY	1.00E-04	1.31E-04					
		3.80E-04					
RCV-RV	5.00E-01	1.04E-03	6.88E-04	1.13E-02	3.75E-02	7.38E-02	2.32E-01
		6.94E-03	8.36E-03	2.91E-02	4.86E-02	6.60E-02	1.39E-01
HY-RV	5.00E-01	1.81E-03	2.90E-03	1.78E-02	5.08E-02	9.60E-02	3.10E-01
		5.26E-03	8.41E-03	5.14E-02	1.48E-01	2.72E-01	9.22E-01
HY-SIML	5.00E-01	6.77E-01	6.76E-01	7.16E-01	8.01E-01	8.07E-01	1.03E+00
		2.00E+00	1.98E+00	2.14E+00	2.47E+00	2.68E+00	4.58E+00
SIML-SIML	5.00E-01	4.90E-01	4.85E-01	4.98E-01	5.09E-01	5.21E-01	5.24E-01
		1.46E-01	1.28E-01	2.06E-01	2.75E-01	3.46E-01	5.14E-01
18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ^2_{x1}	2.00E-04	2.00E-04	2.00E-04	1.99E-04	1.98E-04	1.96E-04	1.96E-04
		4.15E-05	4.13E-05	6.40E-05	8.17E-05	9.08E-05	1.24E-04
σ^2_{v1}	2.00E-08	2.28E-08	1.21E-08	4.89E-08	1.09E-07	2.04E-07	1.02E-06
		6.36E-10	4.01E-10	3.47E-09	1.24E-08	3.01E-08	2.83E-07
RV1	2.00E-04	9.20E-04	6.49E-04	2.72E-04	2.25E-04	2.13E-04	2.01E-04
		1.73E-05	1.09E-05	9.35E-06	1.34E-05	1.79E-05	3.72E-05
σ^2_{x2}	2.00E-04	2.00E-04	2.00E-04	2.02E-04	2.05E-04	2.06E-04	2.06E-04
		4.10E-05	4.11E-05	6.63E-05	8.60E-05	1.02E-04	1.38E-04
σ^2_{v2}	2.00E-08	2.29E-08	1.21E-08	4.91E-08	1.09E-07	2.03E-07	1.02E-06
		6.27E-10	4.11E-10	3.38E-09	1.24E-08	3.09E-08	2.87E-07
RV2	2.00E-04	9.20E-04	6.50E-04	2.72E-04	2.24E-04	2.12E-04	2.04E-04
		1.90E-05	1.17E-05	9.09E-06	1.33E-05	1.73E-05	3.78E-05
σ^2_{x12}	1.00E-04	1.01E-04	1.00E-04	1.02E-04	1.03E-04	1.03E-04	1.03E-04
		3.54E-05	3.22E-05	4.98E-05	6.30E-05	7.47E-05	9.97E-05
σ^2_{v12}	0.00E+00	8.14E-10	1.11E-10	1.14E-08	4.19E-08	9.04E-08	5.00E-07
		5.59E-10	2.40E-10	2.53E-09	9.52E-09	2.37E-08	2.23E-07
RCV	1.00E-04	6.84E-05	3.59E-05	8.92E-05	9.69E-05	9.93E-05	1.01E-04
		6.91E-06	5.18E-06	6.92E-06	1.02E-05	1.34E-05	2.88E-05
HY	1.00E-04	1.00E-04					
		6.34E-06					
RCV-RV	5.00E-01	7.44E-02	5.53E-02	3.28E-01	4.31E-01	4.67E-01	5.01E-01
		7.53E-03	7.97E-03	2.32E-02	3.73E-02	5.14E-02	1.14E-01
HY-RV	5.00E-01	1.09E-01	1.55E-01	3.70E-01	4.49E-01	4.76E-01	5.16E-01
		7.11E-03	9.88E-03	2.53E-02	3.80E-02	4.97E-02	1.02E-01
HY-SIML	5.00E-01	5.25E-01	5.26E-01	5.60E-01	6.15E-01	6.58E-01	8.42E-01
		1.16E-01	1.16E-01	1.93E-01	3.17E-01	4.24E-01	8.77E-01
SIML-SIML	5.00E-01	5.03E-01	5.02E-01	5.13E-01	5.25E-01	5.30E-01	5.35E-01
		1.42E-01	1.25E-01	1.99E-01	2.68E-01	3.29E-01	4.94E-01

Table 5.3 : Estimation of covariance and hedging coefficient :

Case 5 ($g = 0.2; \lambda = 1800$)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ^2_{x1}	2.00E-04	2.30E-04	4.20E-04	2.37E-04	2.11E-04	2.05E-04	2.08E-04
		7.45E-05	9.49E-05	7.63E-05	8.78E-05	9.84E-05	1.33E-04
σ^2_{v1}	2.00E-06	6.45E-07	5.68E-08	5.81E-07	1.72E-06	3.22E-06	6.33E-06
		4.76E-08	2.94E-09	4.88E-08	1.96E-07	4.76E-07	1.78E-06
RV1	2.00E-04	4.02E-03	3.98E-03	3.66E-03	3.06E-03	2.42E-03	8.33E-04
		1.69E-04	1.73E-04	1.87E-04	2.14E-04	2.20E-04	1.73E-04
σ^2_{x2}	2.00E-04	2.39E-04	4.19E-04	2.44E-04	2.17E-04	2.13E-04	2.10E-04
		7.90E-05	8.79E-05	7.97E-05	8.99E-05	1.02E-04	1.35E-04
σ^2_{v2}	2.00E-06	6.44E-07	5.69E-08	5.80E-07	1.74E-06	3.21E-06	6.29E-06
		4.55E-08	2.79E-09	4.80E-08	1.98E-07	4.81E-07	1.69E-06
RV2	2.00E-04	4.03E-03	3.99E-03	3.67E-03	3.08E-03	2.43E-03	8.34E-04
		1.65E-04	1.67E-04	1.84E-04	2.06E-04	2.30E-04	1.66E-04
σ^2_{x12}	1.00E-04	9.76E-05	9.42E-05	9.76E-05	9.90E-05	1.00E-04	1.01E-04
		6.16E-05	6.58E-05	5.93E-05	6.80E-05	7.78E-05	1.04E-04
σ^2_{v12}	0.00E+00	-1.63E-09	8.77E-12	-6.23E-11	1.24E-09	1.49E-08	3.57E-07
		5.16E-08	1.17E-09	2.96E-08	1.37E-07	3.32E-07	1.23E-06
RCV	1.00E-04	9.97E-06	2.88E-07	7.30E-06	2.12E-05	4.27E-05	8.36E-05
		1.03E-04	2.96E-05	8.65E-05	1.34E-04	1.55E-04	1.21E-04
HY	1.00E-04	1.74E-05					
		1.16E-04					
RCV-RV	5.00E-01	2.46E-03	9.30E-05	1.98E-03	6.85E-03	1.76E-02	1.01E-01
		2.57E-02	7.48E-03	2.37E-02	4.37E-02	6.41E-02	1.48E-01
HY-RV	5.00E-01	4.32E-03	4.35E-03	4.74E-03	5.56E-03	6.97E-03	2.12E-02
		2.89E-02	2.91E-02	3.17E-02	3.78E-02	4.83E-02	1.47E-01
HY-SIML	5.00E-01	7.18E-02	4.00E-02	6.80E-02	7.64E-02	1.08E-01	1.45E-01
		6.02E-01	3.01E-01	5.87E-01	7.59E-01	9.28E-01	1.54E+00
SIML-SIML	5.00E-01	4.25E-01	2.26E-01	4.11E-01	4.72E-01	4.98E-01	4.92E-01
		2.45E-01	1.54E-01	2.23E-01	2.83E-01	3.34E-01	5.17E-01
1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ^2_{x1}	2.00E-04	1.94E-04	1.91E-04	1.95E-04	1.94E-04	1.93E-04	1.97E-04
		6.38E-05	3.97E-05	6.27E-05	7.95E-05	9.18E-05	1.26E-04
σ^2_{v1}	2.00E-08	6.83E-09	8.78E-10	9.10E-09	2.99E-08	7.27E-08	8.08E-07
		4.97E-10	4.68E-11	7.62E-10	3.58E-09	1.11E-08	2.29E-07
RV1	2.00E-04	6.22E-05	6.36E-05	7.50E-05	9.62E-05	1.19E-04	1.76E-04
		2.74E-06	2.96E-06	4.64E-06	7.85E-06	1.20E-05	3.39E-05
σ^2_{x2}	2.00E-04	2.00E-04	1.91E-04	2.00E-04	1.99E-04	2.01E-04	2.01E-04
		6.43E-05	3.91E-05	6.35E-05	8.29E-05	9.67E-05	1.32E-04
σ^2_{v2}	2.00E-08	6.81E-09	8.80E-10	9.10E-09	2.99E-08	7.29E-08	8.13E-07
		4.82E-10	4.58E-11	7.75E-10	3.54E-09	1.13E-08	2.30E-07
RV2	2.00E-04	6.23E-05	6.37E-05	7.53E-05	9.65E-05	1.20E-04	1.78E-04
		2.65E-06	2.88E-06	4.45E-06	7.56E-06	1.18E-05	3.26E-05
σ^2_{x12}	1.00E-04	9.83E-05	9.39E-05	9.88E-05	9.94E-05	1.01E-04	1.02E-04
		5.40E-05	2.98E-05	4.83E-05	6.37E-05	7.45E-05	1.01E-04
σ^2_{v12}	0.00E+00	1.99E-11	-2.10E-13	5.63E-11	1.47E-09	1.20E-08	3.61E-07
		7.07E-10	1.83E-11	4.66E-10	2.39E-09	7.73E-09	1.81E-07
RCV	1.00E-04	1.34E-05	9.91E-07	9.43E-06	2.49E-05	4.19E-05	8.38E-05
		2.48E-06	4.92E-07	1.99E-06	4.52E-06	7.85E-06	2.59E-05
HY	1.00E-04	2.01E-05					
		3.54E-06					
RCV-RV	5.00E-01	2.15E-01	1.56E-02	1.26E-01	2.58E-01	3.51E-01	4.75E-01
		3.73E-02	7.73E-03	2.48E-02	4.16E-02	5.69E-02	1.16E-01
HY-RV	5.00E-01	3.22E-01	3.16E-01	2.67E-01	2.09E-01	1.68E-01	1.17E-01
		5.28E-02	5.14E-02	4.27E-02	3.28E-02	2.63E-02	2.44E-02
HY-SIML	5.00E-01	1.14E-01	1.09E-01	1.13E-01	1.22E-01	1.33E-01	1.80E-01
		4.21E-02	2.48E-02	4.01E-02	5.97E-02	8.67E-02	2.41E-01
SIML-SIML	5.00E-01	5.10E-01	4.93E-01	5.09E-01	5.13E-01	5.29E-01	5.23E-01
		2.39E-01	1.24E-01	2.04E-01	2.71E-01	3.34E-01	4.97E-01

Table 5.4 : Estimation of covariance and hedging coefficient :

Case 5 ($g = 0.2; \lambda = 18000$)

18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ^2_{x1}	2.00E-04	2.25E-04	2.28E-04	2.09E-04	2.09E-04	2.10E-04	2.15E-04
		4.53E-05	4.53E-05	6.83E-05	8.68E-05	1.02E-04	1.38E-04
σ^2_{v1}	2.00E-06	6.27E-07	5.63E-07	4.31E-06	5.63E-06	5.73E-06	6.53E-06
		1.78E-08	1.67E-08	3.12E-07	6.10E-07	8.19E-07	1.80E-06
RV1	2.00E-04	4.00E-02	3.63E-02	1.74E-02	6.83E-03	3.52E-03	8.59E-04
		5.26E-04	5.97E-04	7.05E-04	4.78E-04	3.38E-04	1.81E-04
σ^2_{x2}	2.00E-04	2.27E-04	2.29E-04	2.10E-04	2.11E-04	2.12E-04	2.13E-04
		4.74E-05	4.78E-05	6.83E-05	8.79E-05	1.02E-04	1.34E-04
σ^2_{v2}	2.00E-06	6.27E-07	5.63E-07	4.30E-06	5.59E-06	5.72E-06	6.45E-06
		1.73E-08	1.71E-08	2.99E-07	6.19E-07	8.15E-07	1.81E-06
RV2	2.00E-04	4.00E-02	3.63E-02	1.74E-02	6.80E-03	3.51E-03	8.55E-04
		5.37E-04	5.98E-04	6.80E-04	4.74E-04	3.36E-04	1.74E-04
σ^2_{x12}	1.00E-04	9.98E-05	9.96E-05	1.02E-04	1.04E-04	1.06E-04	1.06E-04
		3.80E-05	3.66E-05	5.16E-05	6.67E-05	7.95E-05	1.07E-04
σ^2_{v12}	0.00E+00	5.23E-11	-3.47E-10	5.64E-09	3.10E-08	8.30E-08	5.67E-07
		2.06E-08	1.14E-08	2.12E-07	4.39E-07	5.76E-07	1.30E-06
RCV	1.00E-04	2.40E-05	1.16E-05	6.91E-05	8.26E-05	9.35E-05	1.05E-04
		3.42E-04	2.87E-04	4.83E-04	3.41E-04	2.31E-04	1.30E-04
HY	1.00E-04	2.74E-05					
		3.75E-04					
RCV-RV	5.00E-01	5.94E-04	3.18E-04	3.96E-03	1.20E-02	2.70E-02	1.23E-01
		8.55E-03	7.91E-03	2.77E-02	5.01E-02	6.58E-02	1.50E-01
HY-RV	5.00E-01	6.83E-04	7.46E-04	1.57E-03	3.82E-03	7.87E-03	2.98E-02
		9.39E-03	1.04E-02	2.16E-02	5.51E-02	1.08E-01	4.63E-01
HY-SIML	5.00E-01	1.23E-01	1.23E-01	8.24E-02	1.05E-01	6.97E-02	1.51E-01
		1.77E+00	1.75E+00	2.13E+00	2.44E+00	2.80E+00	4.26E+00
SIML-SIML	5.00E-01	4.43E-01	4.36E-01	4.89E-01	5.04E-01	5.11E-01	5.08E-01
		1.45E-01	1.36E-01	2.05E-01	2.68E-01	3.20E-01	4.93E-01

18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ^2_{x1}	2.00E-04	1.99E-04	1.99E-04	2.00E-04	2.02E-04	2.02E-04	2.04E-04
		4.07E-05	4.05E-05	6.57E-05	8.36E-05	9.89E-05	1.30E-04
σ^2_{v1}	2.00E-08	6.31E-09	5.95E-09	5.29E-08	1.21E-07	2.14E-07	1.02E-06
		1.79E-10	1.77E-10	3.87E-09	1.31E-08	3.08E-08	2.95E-07
RV1	2.00E-04	4.22E-04	4.01E-04	2.96E-04	2.37E-04	2.19E-04	2.03E-04
		5.35E-06	6.44E-06	1.05E-05	1.35E-05	1.78E-05	3.82E-05
σ^2_{x2}	2.00E-04	2.00E-04	2.00E-04	2.01E-04	2.04E-04	2.03E-04	2.03E-04
		4.14E-05	4.13E-05	6.55E-05	8.47E-05	9.74E-05	1.25E-04
σ^2_{v2}	2.00E-08	6.31E-09	5.95E-09	5.29E-08	1.19E-07	2.13E-07	1.03E-06
		1.74E-10	1.78E-10	3.69E-09	1.37E-08	3.10E-08	2.94E-07
RV2	2.00E-04	4.22E-04	4.01E-04	2.96E-04	2.37E-04	2.18E-04	2.04E-04
		5.50E-06	6.37E-06	1.01E-05	1.33E-05	1.78E-05	3.82E-05
σ^2_{x12}	1.00E-04	1.01E-04	1.00E-04	1.03E-04	1.06E-04	1.07E-04	1.07E-04
		3.49E-05	3.21E-05	5.00E-05	6.48E-05	7.72E-05	1.03E-04
σ^2_{v12}	0.00E+00	-6.54E-13	1.29E-12	3.78E-09	3.09E-08	7.83E-08	4.82E-07
		2.13E-10	1.20E-10	2.65E-09	9.79E-09	2.34E-08	2.24E-07
RCV	1.00E-04	1.34E-05	9.35E-06	5.67E-05	8.35E-05	9.20E-05	9.88E-05
		3.58E-06	3.14E-06	7.35E-06	1.03E-05	1.37E-05	2.97E-05
HY	1.00E-04	2.00E-05					
		4.21E-06					
RCV-RV	5.00E-01	3.17E-02	2.33E-02	1.92E-01	3.52E-01	4.20E-01	4.87E-01
		8.45E-03	7.83E-03	2.40E-02	3.77E-02	5.27E-02	1.17E-01
HY-RV	5.00E-01	4.74E-02	4.98E-02	6.76E-02	8.44E-02	9.18E-02	1.02E-01
		9.96E-03	1.05E-02	1.42E-02	1.80E-02	2.03E-02	2.81E-02
HY-SIML	5.00E-01	1.04E-01	1.04E-01	1.11E-01	1.18E-01	1.27E-01	1.61E-01
		3.01E-02	3.01E-02	4.34E-02	6.35E-02	8.28E-02	1.80E-01
SIML-SIML	5.00E-01	5.04E-01	5.01E-01	5.15E-01	5.29E-01	5.33E-01	5.40E-01
		1.40E-01	1.25E-01	2.00E-01	2.62E-01	3.12E-01	4.84E-01

Table 5.5 : Estimation of covariance and hedging coefficient :

Case 6 ($\eta = 0.001; \lambda = 1800$)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ^2_{x1}	2.00E-04	2.01E-04	2.22E-04	2.04E-04	2.01E-04	2.00E-04	2.06E-04
		6.61E-05	4.60E-05	6.52E-05	8.25E-05	9.54E-05	1.31E-04
σ^2_{v1}	2.00E-06	2.12E-06	1.09E-07	1.03E-06	1.99E-06	2.25E-06	3.10E-06
		1.54E-07	6.31E-09	9.02E-08	2.22E-07	3.30E-07	8.53E-07
RV1	2.00E-04	7.71E-03	7.35E-03	4.95E-03	2.57E-03	1.44E-03	4.49E-04
		3.68E-04	3.54E-04	2.66E-04	1.84E-04	1.42E-04	8.84E-05
σ^2_{x2}	2.00E-04	2.07E-04	2.23E-04	2.09E-04	2.06E-04	2.08E-04	2.07E-04
		6.75E-05	4.54E-05	6.65E-05	8.68E-05	1.01E-04	1.37E-04
σ^2_{v2}	2.00E-06	2.12E-06	1.09E-07	1.02E-06	2.00E-06	2.25E-06	3.08E-06
		1.51E-07	6.21E-09	8.86E-08	2.29E-07	3.21E-07	8.27E-07
RV2	2.00E-04	7.72E-03	7.35E-03	4.94E-03	2.58E-03	1.44E-03	4.51E-04
		3.52E-04	3.38E-04	2.66E-04	1.87E-04	1.40E-04	8.54E-05
σ^2_{x12}	1.00E-04	9.92E-05	9.99E-05	9.98E-05	9.98E-05	1.01E-04	1.01E-04
		5.60E-05	3.45E-05	5.05E-05	6.58E-05	7.66E-05	1.04E-04
σ^2_{v12}	0.00E+00	4.56E-09	-2.12E-11	1.74E-09	2.01E-08	5.90E-08	4.53E-07
		1.25E-07	2.20E-09	5.14E-08	1.54E-07	2.27E-07	5.94E-07
RCV	1.00E-04	6.59E-05	6.90E-06	4.29E-05	6.47E-05	8.02E-05	9.54E-05
		1.85E-04	5.58E-05	1.29E-04	1.24E-04	9.63E-05	6.29E-05
HY	1.00E-04	1.05E-04					
		1.31E-04					
RCV-RV	5.00E-01	8.53E-03	9.49E-04	8.62E-03	2.50E-02	5.56E-02	2.15E-01
		2.41E-02	7.60E-03	2.62E-02	4.85E-02	6.66E-02	1.38E-01
HY-RV	5.00E-01	1.36E-02	1.42E-02	2.11E-02	4.06E-02	7.28E-02	2.43E-01
		1.70E-02	1.78E-02	2.64E-02	5.08E-02	9.21E-02	3.11E-01
HY-SIML	5.00E-01	5.77E-01	4.82E-01	5.67E-01	6.29E-01	6.88E-01	8.93E-01
		7.88E-01	6.21E-01	7.74E-01	9.11E-01	1.13E+00	1.88E+00
SIML-SIML	5.00E-01	4.96E-01	4.49E-01	4.90E-01	5.00E-01	5.16E-01	5.03E-01
		2.43E-01	1.30E-01	2.06E-01	2.73E-01	3.30E-01	5.13E-01
1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ^2_{x1}	2.00E-04	1.97E-04	2.01E-04	1.97E-04	1.96E-04	1.95E-04	1.99E-04
		6.46E-05	4.17E-05	6.36E-05	8.03E-05	9.27E-05	1.27E-04
σ^2_{v1}	2.00E-08	1.22E-07	7.72E-09	7.62E-08	1.84E-07	2.84E-07	1.10E-06
		9.67E-09	4.25E-10	6.14E-09	2.01E-08	4.05E-08	3.05E-07
RV1	2.00E-04	5.47E-04	5.33E-04	4.30E-04	3.18E-04	2.62E-04	2.12E-04
		2.55E-05	2.50E-05	2.08E-05	1.93E-05	2.16E-05	4.06E-05
σ^2_{x2}	2.00E-04	2.02E-04	2.01E-04	2.03E-04	2.02E-04	2.02E-04	2.02E-04
		6.51E-05	4.05E-05	6.40E-05	8.40E-05	9.82E-05	1.34E-04
σ^2_{v2}	2.00E-08	1.22E-07	7.73E-09	7.67E-08	1.82E-07	2.85E-07	1.11E-06
		9.80E-09	4.21E-10	5.90E-09	1.98E-08	4.18E-08	3.01E-07
RV2	2.00E-04	5.47E-04	5.33E-04	4.30E-04	3.17E-04	2.62E-04	2.14E-04
		2.53E-05	2.48E-05	2.13E-05	1.93E-05	2.20E-05	3.80E-05
σ^2_{x12}	1.00E-04	9.94E-05	9.98E-05	1.00E-04	1.00E-04	1.02E-04	1.03E-04
		5.46E-05	3.15E-05	4.88E-05	6.44E-05	7.54E-05	1.02E-04
σ^2_{v12}	0.00E+00	7.82E-09	1.09E-11	1.40E-09	1.89E-08	6.34E-08	4.70E-07
		8.36E-09	1.53E-10	3.93E-09	1.44E-08	3.10E-08	2.42E-07
RCV	1.00E-04	6.64E-05	4.99E-06	3.73E-05	6.80E-05	8.36E-05	9.76E-05
		1.42E-05	4.10E-06	1.08E-05	1.41E-05	1.70E-05	3.11E-05
HY	1.00E-04	9.98E-05					
		1.50E-05					
RCV-RV	5.00E-01	1.22E-01	9.35E-03	8.68E-02	2.14E-01	3.19E-01	4.60E-01
		2.59E-02	7.69E-03	2.46E-02	4.25E-02	5.80E-02	1.16E-01
HY-RV	5.00E-01	1.83E-01	1.88E-01	2.32E-01	3.15E-01	3.83E-01	4.86E-01
		2.72E-02	2.78E-02	3.39E-02	4.65E-02	5.71E-02	1.14E-01
HY-SIML	5.00E-01	5.67E-01	5.17E-01	5.62E-01	6.09E-01	6.62E-01	9.11E-01
		2.22E-01	1.28E-01	2.12E-01	3.10E-01	4.37E-01	1.39E+00
SIML-SIML	5.00E-01	5.09E-01	4.97E-01	5.10E-01	5.12E-01	5.28E-01	5.22E-01
		2.39E-01	1.24E-01	2.04E-01	2.72E-01	3.36E-01	4.99E-01

Table 5.6 : Estimation of covariance and hedging coefficient :
Case 6 ($\eta = 0.001; \lambda = 18000$)

18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ^2_{x1}	2.00E-04	2.04E-04	2.05E-04	2.06E-04	2.07E-04	2.08E-04	2.11E-04
		4.20E-05	4.19E-05	6.76E-05	8.60E-05	1.01E-04	1.34E-04
σ^2_{v1}	2.00E-06	2.09E-06	9.76E-07	2.11E-06	2.18E-06	2.27E-06	3.09E-06
		5.82E-08	3.12E-08	1.45E-07	2.34E-07	3.25E-07	8.68E-07
RV1	2.00E-04	7.52E-02	4.76E-02	7.69E-03	2.70E-03	1.46E-03	4.51E-04
		1.12E-03	8.45E-04	2.90E-04	1.84E-04	1.35E-04	8.90E-05
σ^2_{x2}	2.00E-04	2.05E-04	2.06E-04	2.06E-04	2.08E-04	2.09E-04	2.10E-04
		4.26E-05	4.26E-05	6.68E-05	8.73E-05	1.01E-04	1.31E-04
σ^2_{v2}	2.00E-06	2.09E-06	9.79E-07	2.12E-06	2.17E-06	2.28E-06	3.07E-06
		5.75E-08	3.22E-08	1.56E-07	2.44E-07	3.25E-07	8.75E-07
RV2	2.00E-04	7.52E-02	4.76E-02	7.71E-03	2.70E-03	1.46E-03	4.51E-04
		1.13E-03	8.21E-04	3.05E-04	1.91E-04	1.39E-04	8.98E-05
σ^2_{x12}	1.00E-04	1.00E-04	9.99E-05	1.02E-04	1.05E-04	1.07E-04	1.07E-04
		3.59E-05	3.35E-05	5.11E-05	6.60E-05	7.90E-05	1.05E-04
σ^2_{v12}	0.00E+00	-3.00E-10	-1.33E-09	1.39E-08	4.25E-08	9.67E-08	5.15E-07
		4.74E-08	1.97E-08	1.04E-07	1.63E-07	2.31E-07	6.03E-07
RCV	1.00E-04	7.50E-05	4.12E-05	1.02E-04	1.02E-04	1.04E-04	1.02E-04
		5.60E-04	3.98E-04	2.17E-04	1.31E-04	9.66E-05	6.47E-05
HY	1.00E-04	9.48E-05					
		3.99E-04					
RCV-RV	5.00E-01	9.95E-04	8.70E-04	1.32E-02	3.76E-02	7.16E-02	2.27E-01
		7.45E-03	8.37E-03	2.82E-02	4.86E-02	6.62E-02	1.38E-01
HY-RV	5.00E-01	1.26E-03	2.00E-03	1.23E-02	3.47E-02	6.51E-02	2.12E-01
		5.30E-03	8.38E-03	5.19E-02	1.48E-01	2.76E-01	9.31E-01
HY-SIML	5.00E-01	4.84E-01	4.81E-01	4.97E-01	5.14E-01	5.49E-01	8.87E-01
		2.08E+00	2.06E+00	2.33E+00	2.67E+00	3.10E+00	5.53E+00
SIML-SIML	5.00E-01	4.91E-01	4.86E-01	4.99E-01	5.12E-01	5.18E-01	5.19E-01
		1.44E-01	1.30E-01	2.02E-01	2.69E-01	3.19E-01	5.17E-01
18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ^2_{x1}	2.00E-04	2.00E-04	2.00E-04	2.00E-04	2.02E-04	2.03E-04	2.05E-04
		4.10E-05	4.08E-05	6.61E-05	8.39E-05	9.92E-05	1.30E-04
σ^2_{v1}	2.00E-08	8.51E-08	4.24E-08	1.27E-07	1.92E-07	2.84E-07	1.10E-06
		3.63E-09	1.94E-09	9.69E-09	2.03E-08	4.06E-08	3.08E-07
RV1	2.00E-04	3.23E-03	2.17E-03	5.61E-04	3.24E-04	2.62E-04	2.13E-04
		9.51E-05	6.61E-05	2.26E-05	1.88E-05	2.18E-05	3.93E-05
σ^2_{x2}	2.00E-04	2.01E-04	2.01E-04	2.01E-04	2.04E-04	2.03E-04	2.03E-04
		4.15E-05	4.14E-05	6.56E-05	8.48E-05	9.75E-05	1.26E-04
σ^2_{v2}	2.00E-08	8.50E-08	4.25E-08	1.28E-07	1.92E-07	2.84E-07	1.11E-06
		3.78E-09	2.00E-09	9.50E-09	2.03E-08	4.06E-08	3.07E-07
RV2	2.00E-04	3.23E-03	2.17E-03	5.63E-04	3.24E-04	2.62E-04	2.14E-04
		9.94E-05	6.91E-05	2.27E-05	1.83E-05	2.14E-05	3.97E-05
σ^2_{x12}	1.00E-04	1.01E-04	1.00E-04	1.02E-04	1.05E-04	1.06E-04	1.08E-04
		3.50E-05	3.22E-05	5.02E-05	6.48E-05	7.73E-05	1.03E-04
σ^2_{v12}	0.00E+00	6.72E-10	1.12E-10	1.17E-08	4.15E-08	8.75E-08	4.96E-07
		2.03E-09	8.42E-10	6.40E-09	1.53E-08	3.07E-08	2.34E-07
RCV	1.00E-04	6.67E-05	3.68E-05	8.95E-05	9.67E-05	9.85E-05	1.01E-04
		2.34E-05	1.75E-05	1.48E-05	1.45E-05	1.64E-05	3.10E-05
HY	1.00E-04	9.94E-05					
		1.97E-05					
RCV-RV	5.00E-01	2.07E-02	1.70E-02	1.60E-01	2.98E-01	3.75E-01	4.73E-01
		7.24E-03	8.02E-03	2.56E-02	4.05E-02	5.47E-02	1.20E-01
HY-RV	5.00E-01	3.08E-02	4.59E-02	1.77E-01	3.08E-01	3.81E-01	4.83E-01
		6.11E-03	9.13E-03	3.57E-02	6.29E-02	8.03E-02	1.35E-01
HY-SIML	5.00E-01	5.18E-01	5.18E-01	5.52E-01	5.88E-01	6.32E-01	8.05E-01
		1.50E-01	1.50E-01	2.24E-01	3.23E-01	4.11E-01	9.52E-01
SIML-SIML	5.00E-01	5.03E-01	5.00E-01	5.13E-01	5.26E-01	5.30E-01	5.39E-01
		1.40E-01	1.25E-01	2.00E-01	2.62E-01	3.13E-01	4.83E-01

Table 5.7 : Estimation of covariance and hedging coefficient :

Case 7 ($\eta = 0.001; \lambda = 1800$)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ^2_{x1}	2.00E-04	2.01E-04	2.23E-04	2.04E-04	2.01E-04	2.01E-04	2.06E-04
		6.63E-05	4.63E-05	6.53E-05	8.22E-05	9.54E-05	1.31E-04
σ^2_{v1}	2.00E-06	2.12E-06	1.09E-07	1.03E-06	2.00E-06	2.25E-06	3.11E-06
		1.54E-07	6.26E-09	9.11E-08	2.26E-07	3.33E-07	8.65E-07
RV1	2.00E-04	7.70E-03	7.34E-03	4.95E-03	2.57E-03	1.44E-03	4.50E-04
		3.66E-04	3.53E-04	2.67E-04	1.85E-04	1.43E-04	8.96E-05
σ^2_{x2}	2.00E-04	2.08E-04	2.23E-04	2.09E-04	2.07E-04	2.08E-04	2.08E-04
		6.75E-05	4.51E-05	6.65E-05	8.70E-05	1.01E-04	1.36E-04
σ^2_{v2}	2.00E-06	2.11E-06	1.09E-07	1.02E-06	2.00E-06	2.25E-06	3.07E-06
		1.48E-07	6.18E-09	8.82E-08	2.30E-07	3.14E-07	8.34E-07
RV2	2.00E-04	7.71E-03	7.35E-03	4.94E-03	2.57E-03	1.44E-03	4.51E-04
		3.52E-04	3.39E-04	2.67E-04	1.87E-04	1.37E-04	8.60E-05
σ^2_{x12}	1.00E-04	1.00E-04	1.00E-04	1.01E-04	1.01E-04	1.02E-04	1.02E-04
		5.60E-05	3.46E-05	5.04E-05	6.56E-05	7.64E-05	1.04E-04
σ^2_{v12}	0.00E+00	4.84E-09	-2.39E-11	2.40E-09	1.91E-08	5.86E-08	4.48E-07
		1.26E-07	2.18E-09	5.19E-08	1.54E-07	2.32E-07	5.97E-07
RCV	1.00E-04	6.49E-05	7.41E-06	4.27E-05	6.39E-05	8.07E-05	9.51E-05
		1.85E-04	5.53E-05	1.30E-04	1.25E-04	9.81E-05	6.30E-05
HY	1.00E-04	1.04E-04					
		1.31E-04					
RCV-RV	5.00E-01	8.41E-03	1.01E-03	8.57E-03	2.48E-02	5.59E-02	2.15E-01
		2.40E-02	7.53E-03	2.63E-02	4.88E-02	6.77E-02	1.40E-01
HY-RV	5.00E-01	1.35E-02	1.41E-02	2.09E-02	4.02E-02	7.19E-02	2.39E-01
		1.71E-02	1.80E-02	2.66E-02	5.10E-02	9.17E-02	3.10E-01
HY-SIML	5.00E-01	5.71E-01	4.79E-01	5.62E-01	6.22E-01	6.71E-01	8.53E-01
		7.92E-01	6.25E-01	7.75E-01	9.22E-01	1.11E+00	1.84E+00
SIML-SIML	5.00E-01	4.99E-01	4.51E-01	4.93E-01	5.03E-01	5.19E-01	5.05E-01
		2.40E-01	1.29E-01	2.05E-01	2.72E-01	3.31E-01	5.16E-01
1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ^2_{x1}	2.00E-04	1.97E-04	2.01E-04	1.98E-04	1.96E-04	1.95E-04	1.99E-04
		6.48E-05	4.18E-05	6.38E-05	8.02E-05	9.27E-05	1.27E-04
σ^2_{v1}	2.00E-08	1.16E-07	7.55E-09	7.57E-08	1.83E-07	2.84E-07	1.10E-06
		9.28E-09	4.16E-10	6.12E-09	2.04E-08	4.14E-08	3.02E-07
RV1	2.00E-04	5.34E-04	5.22E-04	4.27E-04	3.17E-04	2.61E-04	2.12E-04
		2.52E-05	2.46E-05	2.10E-05	1.99E-05	2.25E-05	3.99E-05
σ^2_{x2}	2.00E-04	2.03E-04	2.01E-04	2.03E-04	2.02E-04	2.03E-04	2.03E-04
		6.49E-05	4.04E-05	6.40E-05	8.43E-05	9.79E-05	1.33E-04
σ^2_{v2}	2.00E-08	1.15E-07	7.54E-09	7.60E-08	1.84E-07	2.85E-07	1.11E-06
		8.95E-09	4.16E-10	6.22E-09	2.08E-08	4.20E-08	2.93E-07
RV2	2.00E-04	5.34E-04	5.23E-04	4.30E-04	3.18E-04	2.62E-04	2.15E-04
		2.55E-05	2.49E-05	2.13E-05	1.95E-05	2.23E-05	3.76E-05
σ^2_{x12}	1.00E-04	1.00E-04	1.00E-04	1.01E-04	1.01E-04	1.03E-04	1.03E-04
		5.48E-05	3.17E-05	4.88E-05	6.44E-05	7.51E-05	1.02E-04
σ^2_{v12}	0.00E+00	7.66E-09	3.45E-12	1.36E-09	1.98E-08	6.40E-08	4.66E-07
		8.48E-09	1.53E-10	3.81E-09	1.36E-08	3.01E-08	2.41E-07
RCV	1.00E-04	6.66E-05	5.02E-06	3.79E-05	6.90E-05	8.40E-05	9.78E-05
		1.49E-05	4.04E-06	1.08E-05	1.34E-05	1.63E-05	3.11E-05
HY	1.00E-04	1.01E-04					
		1.51E-05					
RCV-RV	5.00E-01	1.25E-01	9.61E-03	8.86E-02	2.18E-01	3.22E-01	4.61E-01
		2.81E-02	7.73E-03	2.52E-02	4.06E-02	5.76E-02	1.17E-01
HY-RV	5.00E-01	1.89E-01	1.93E-01	2.36E-01	3.18E-01	3.87E-01	4.90E-01
		2.84E-02	2.90E-02	3.53E-02	4.68E-02	5.93E-02	1.14E-01
HY-SIML	5.00E-01	5.70E-01	5.21E-01	5.65E-01	6.11E-01	6.65E-01	8.97E-01
		2.19E-01	1.29E-01	2.11E-01	3.08E-01	4.35E-01	1.20E+00
SIML-SIML	5.00E-01	5.11E-01	5.00E-01	5.12E-01	5.15E-01	5.32E-01	5.24E-01
		2.38E-01	1.23E-01	2.03E-01	2.72E-01	3.37E-01	5.02E-01

Table 5.8 : Estimation of covariance and hedging coefficient :

Case 7 ($g = 0.001; \lambda = 18000$)

18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ^2_{x1}	2.00E-04	2.04E-04	2.06E-04	2.06E-04	2.07E-04	2.08E-04	2.11E-04
		4.20E-05	4.20E-05	6.77E-05	8.54E-05	1.00E-04	1.34E-04
σ^2_{v1}	2.00E-06	2.09E-06	9.76E-07	2.11E-06	2.17E-06	2.27E-06	3.09E-06
		5.89E-08	3.18E-08	1.45E-07	2.33E-07	3.25E-07	8.57E-07
RV1	2.00E-04	7.52E-02	4.76E-02	7.70E-03	2.70E-03	1.46E-03	4.51E-04
		1.12E-03	8.46E-04	2.91E-04	1.84E-04	1.34E-04	8.82E-05
σ^2_{x2}	2.00E-04	2.06E-04	2.07E-04	2.07E-04	2.09E-04	2.10E-04	2.11E-04
		4.27E-05	4.27E-05	6.71E-05	8.75E-05	1.01E-04	1.32E-04
σ^2_{v2}	2.00E-06	2.09E-06	9.79E-07	2.12E-06	2.17E-06	2.28E-06	3.08E-06
		5.74E-08	3.16E-08	1.50E-07	2.38E-07	3.31E-07	8.68E-07
RV2	2.00E-04	7.52E-02	4.76E-02	7.72E-03	2.70E-03	1.46E-03	4.51E-04
		1.13E-03	8.02E-04	3.02E-04	1.89E-04	1.40E-04	8.91E-05
σ^2_{x12}	1.00E-04	1.01E-04	1.00E-04	1.03E-04	1.05E-04	1.07E-04	1.08E-04
		3.60E-05	3.36E-05	5.13E-05	6.60E-05	7.89E-05	1.06E-04
σ^2_{v12}	0.00E+00	-2.79E-10	-1.04E-09	1.41E-08	4.46E-08	9.49E-08	5.12E-07
		4.82E-08	1.97E-08	1.05E-07	1.64E-07	2.30E-07	6.01E-07
RCV	1.00E-04	8.49E-05	3.92E-05	1.02E-04	1.02E-04	1.03E-04	1.02E-04
		5.70E-04	3.98E-04	2.16E-04	1.31E-04	9.63E-05	6.48E-05
HY	1.00E-04	1.03E-04					
		4.01E-04					
RCV-RV	5.00E-01	1.13E-03	8.25E-04	1.32E-02	3.79E-02	7.08E-02	2.26E-01
		7.58E-03	8.37E-03	2.80E-02	4.86E-02	6.60E-02	1.37E-01
HY-RV	5.00E-01	1.37E-03	2.17E-03	1.34E-02	3.80E-02	7.11E-02	2.41E-01
		5.34E-03	8.43E-03	5.22E-02	1.49E-01	2.79E-01	9.39E-01
HY-SIML	5.00E-01	5.26E-01	5.23E-01	5.36E-01	5.46E-01	5.74E-01	8.80E-01
		2.09E+00	2.07E+00	2.36E+00	2.63E+00	2.97E+00	4.96E+00
SIML-SIML	5.00E-01	4.93E-01	4.87E-01	5.01E-01	5.14E-01	5.20E-01	5.18E-01
		1.44E-01	1.30E-01	2.02E-01	2.68E-01	3.17E-01	5.14E-01
18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ^2_{x1}	2.00E-04	2.00E-04	2.00E-04	2.01E-04	2.02E-04	2.03E-04	2.05E-04
		4.09E-05	4.06E-05	6.60E-05	8.35E-05	9.91E-05	1.31E-04
σ^2_{v1}	2.00E-08	6.77E-08	3.85E-08	1.32E-07	1.92E-07	2.85E-07	1.10E-06
		2.74E-09	1.62E-09	9.24E-09	2.03E-08	4.18E-08	3.16E-07
RV1	2.00E-04	2.86E-03	2.07E-03	5.71E-04	3.24E-04	2.62E-04	2.13E-04
		6.90E-05	5.09E-05	1.96E-05	1.84E-05	2.18E-05	3.94E-05
σ^2_{x2}	2.00E-04	2.01E-04	2.01E-04	2.02E-04	2.04E-04	2.04E-04	2.04E-04
		4.15E-05	4.13E-05	6.58E-05	8.48E-05	9.76E-05	1.26E-04
σ^2_{v2}	2.00E-08	6.79E-08	3.86E-08	1.32E-07	1.92E-07	2.84E-07	1.11E-06
		2.78E-09	1.60E-09	9.02E-09	2.10E-08	4.02E-08	3.09E-07
RV2	2.00E-04	2.86E-03	2.07E-03	5.71E-04	3.24E-04	2.62E-04	2.14E-04
		6.81E-05	4.99E-05	2.03E-05	1.91E-05	2.15E-05	3.95E-05
σ^2_{x12}	1.00E-04	1.01E-04	1.01E-04	1.03E-04	1.06E-04	1.07E-04	1.08E-04
		3.50E-05	3.21E-05	5.02E-05	6.49E-05	7.74E-05	1.03E-04
σ^2_{v12}	0.00E+00	7.58E-10	1.56E-10	1.19E-08	4.15E-08	8.67E-08	4.93E-07
		1.74E-09	7.47E-10	6.63E-09	1.53E-08	3.21E-08	2.35E-07
RCV	1.00E-04	6.76E-05	3.77E-05	8.97E-05	9.60E-05	9.78E-05	1.01E-04
		2.26E-05	1.63E-05	1.46E-05	1.44E-05	1.66E-05	3.05E-05
HY	1.00E-04	1.00E-04					
		1.91E-05					
RCV-RV	5.00E-01	2.36E-02	1.83E-02	1.57E-01	2.97E-01	3.73E-01	4.74E-01
		7.92E-03	7.86E-03	2.51E-02	4.15E-02	5.52E-02	1.18E-01
HY-RV	5.00E-01	3.50E-02	4.85E-02	1.76E-01	3.11E-01	3.85E-01	4.88E-01
		6.64E-03	9.20E-03	3.37E-02	6.17E-02	8.09E-02	1.33E-01
HY-SIML	5.00E-01	5.23E-01	5.24E-01	5.59E-01	5.95E-01	6.41E-01	8.17E-01
		1.48E-01	1.48E-01	2.31E-01	3.24E-01	4.28E-01	1.01E+00
SIML-SIML	5.00E-01	5.06E-01	5.03E-01	5.16E-01	5.30E-01	5.34E-01	5.39E-01
		1.40E-01	1.24E-01	2.00E-01	2.63E-01	3.13E-01	4.87E-01

Table 5.9 : Estimation of covariance and hedging coefficient :

Case 8 ($g_1 = 0.2, g_2 = 5; \lambda = 1800$)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ^2_{x1}	2.00E-04	2.15E-04	3.20E-04	2.20E-04	2.06E-04	2.03E-04	2.08E-04
		7.00E-05	7.04E-05	7.05E-05	8.47E-05	9.66E-05	1.33E-04
σ^2_{v1}	2.00E-06	7.43E-07	6.11E-08	6.22E-07	1.76E-06	3.00E-06	4.88E-06
		5.55E-08	3.20E-09	5.30E-08	2.03E-07	4.35E-07	1.37E-06
RV1	2.00E-04	4.33E-03	4.26E-03	3.74E-03	2.88E-03	2.09E-03	6.61E-04
		1.84E-04	1.86E-04	1.91E-04	2.03E-04	1.96E-04	1.36E-04
σ^2_{x2}	2.00E-04	2.23E-04	3.19E-04	2.26E-04	2.12E-04	2.11E-04	2.10E-04
		7.31E-05	6.59E-05	7.33E-05	8.85E-05	1.02E-04	1.35E-04
σ^2_{v2}	2.00E-06	7.43E-07	6.13E-08	6.21E-07	1.77E-06	2.98E-06	4.82E-06
		5.29E-08	3.07E-09	5.16E-08	2.04E-07	4.36E-07	1.30E-06
RV2	2.00E-04	4.34E-03	4.28E-03	3.75E-03	2.89E-03	2.09E-03	6.58E-04
		1.81E-04	1.82E-04	1.88E-04	1.98E-04	2.04E-04	1.30E-04
σ^2_{x12}	1.00E-04	9.95E-05	9.81E-05	9.98E-05	1.00E-04	1.02E-04	1.02E-04
		5.85E-05	4.90E-05	5.46E-05	6.64E-05	7.67E-05	1.04E-04
σ^2_{v12}	0.00E+00	-1.82E-09	-7.44E-12	4.13E-11	9.95E-10	2.59E-08	3.94E-07
		5.67E-08	1.26E-09	3.17E-08	1.38E-07	3.07E-07	9.41E-07
RCV	1.00E-04	1.70E-05	1.01E-06	1.27E-05	3.00E-05	5.44E-05	8.93E-05
		1.09E-04	3.18E-05	8.95E-05	1.27E-04	1.39E-04	9.43E-05
HY	1.00E-04	2.90E-05	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
		1.14E-04	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
RCV-RV	5.00E-01	3.92E-03	2.56E-04	3.40E-03	1.04E-02	2.59E-02	1.37E-01
		2.51E-02	7.48E-03	2.40E-02	4.42E-02	6.64E-02	1.47E-01
HY-RV	5.00E-01	6.71E-03	6.80E-03	7.77E-03	9.95E-03	1.37E-02	4.53E-02
		2.64E-02	2.68E-02	3.05E-02	3.96E-02	5.51E-02	1.83E-01
HY-SIML	5.00E-01	1.45E-01	9.16E-02	1.40E-01	1.57E-01	1.91E-01	2.43E-01
		6.35E-01	3.88E-01	6.17E-01	7.59E-01	9.30E-01	1.56E+00
SIML-SIML	5.00E-01	4.64E-01	3.09E-01	4.53E-01	4.91E-01	5.12E-01	4.97E-01
		2.42E-01	1.45E-01	2.13E-01	2.78E-01	3.32E-01	5.22E-01
1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ^2_{x1}	2.00E-04	1.95E-04	1.95E-04	1.96E-04	1.95E-04	1.94E-04	1.98E-04
		6.42E-05	4.08E-05	6.31E-05	7.97E-05	9.21E-05	1.26E-04
σ^2_{v1}	2.00E-08	8.85E-09	1.15E-09	1.20E-08	4.02E-08	9.87E-08	8.85E-07
		6.73E-10	6.29E-11	9.88E-10	4.73E-09	1.51E-08	2.47E-07
RV1	2.00E-04	8.17E-05	8.34E-05	9.71E-05	1.21E-04	1.43E-04	1.86E-04
		3.79E-06	4.01E-06	5.78E-06	9.24E-06	1.36E-05	3.55E-05
σ^2_{x2}	2.00E-04	2.01E-04	1.96E-04	2.01E-04	2.00E-04	2.02E-04	2.01E-04
		6.46E-05	3.99E-05	6.36E-05	8.33E-05	9.71E-05	1.32E-04
σ^2_{v2}	2.00E-08	8.84E-09	1.16E-09	1.20E-08	4.00E-08	9.92E-08	8.89E-07
		6.49E-10	6.27E-11	1.00E-09	4.76E-09	1.51E-08	2.48E-07
RV2	2.00E-04	8.20E-05	8.37E-05	9.74E-05	1.21E-04	1.44E-04	1.87E-04
		3.71E-06	3.96E-06	5.59E-06	8.93E-06	1.35E-05	3.40E-05
σ^2_{x12}	1.00E-04	9.90E-05	9.69E-05	9.95E-05	9.99E-05	1.01E-04	1.02E-04
		5.42E-05	3.07E-05	4.84E-05	6.39E-05	7.47E-05	1.01E-04
σ^2_{v12}	0.00E+00	4.57E-10	-9.06E-13	1.44E-10	3.38E-09	2.23E-08	4.03E-07
		9.13E-10	2.51E-11	6.24E-10	3.17E-09	1.08E-08	1.98E-07
RCV	1.00E-04	2.06E-05	1.51E-06	1.38E-05	3.41E-05	5.34E-05	8.92E-05
		3.36E-06	6.73E-07	2.53E-06	5.45E-06	9.13E-06	2.74E-05
HY	1.00E-04	3.13E-05	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
		4.84E-06	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
RCV-RV	5.00E-01	2.52E-01	1.82E-02	1.43E-01	2.82E-01	3.73E-01	4.80E-01
		3.78E-02	8.06E-03	2.45E-02	3.98E-02	5.46E-02	1.16E-01
HY-RV	5.00E-01	3.83E-01	3.75E-01	3.22E-01	2.59E-01	2.19E-01	1.73E-01
		5.39E-02	5.25E-02	4.44E-02	3.52E-02	3.01E-02	3.64E-02
HY-SIML	5.00E-01	1.78E-01	1.66E-01	1.76E-01	1.90E-01	2.07E-01	2.79E-01
		6.56E-02	3.81E-02	6.28E-02	9.23E-02	1.34E-01	3.55E-01
SIML-SIML	5.00E-01	5.10E-01	4.96E-01	5.10E-01	5.13E-01	5.30E-01	5.23E-01
		2.39E-01	1.24E-01	2.03E-01	2.71E-01	3.35E-01	4.98E-01

Table 5.10 : Estimation of covariance and hedging coefficient :

Case 8 ($g_1 = 0.2, g_2 = 5; \lambda = 18000$)

18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ^2_{x1}	2.00E-04	2.15E-04	2.17E-04	2.08E-04	2.09E-04	2.10E-04	2.15E-04
		4.36E-05	4.35E-05	6.83E-05	8.62E-05	1.01E-04	1.36E-04
σ^2_{v1}	2.00E-06	7.24E-07	6.00E-07	3.59E-06	4.09E-06	4.17E-06	4.98E-06
		2.07E-08	1.81E-08	2.59E-07	4.50E-07	5.99E-07	1.37E-06
RV1	2.00E-04	4.29E-02	3.69E-02	1.37E-02	4.98E-03	2.59E-03	6.75E-04
		5.75E-04	6.15E-04	5.55E-04	3.50E-04	2.44E-04	1.40E-04
σ^2_{x2}	2.00E-04	2.16E-04	2.18E-04	2.09E-04	2.11E-04	2.11E-04	2.13E-04
		4.51E-05	4.53E-05	6.79E-05	8.80E-05	1.02E-04	1.33E-04
σ^2_{v2}	2.00E-06	7.23E-07	6.01E-07	3.59E-06	4.06E-06	4.17E-06	4.94E-06
		2.00E-08	1.86E-08	2.47E-07	4.52E-07	5.98E-07	1.39E-06
RV2	2.00E-04	4.29E-02	3.69E-02	1.37E-02	4.96E-03	2.59E-03	6.74E-04
		5.91E-04	6.10E-04	5.44E-04	3.52E-04	2.50E-04	1.37E-04
σ^2_{x12}	1.00E-04	1.01E-04	1.01E-04	1.03E-04	1.06E-04	1.07E-04	1.08E-04
		3.71E-05	3.51E-05	5.17E-05	6.65E-05	7.92E-05	1.07E-04
σ^2_{v12}	0.00E+00	6.82E-11	-4.05E-10	9.73E-09	3.80E-08	9.09E-08	5.53E-07
		2.26E-08	1.22E-08	1.81E-07	3.15E-07	4.22E-07	9.86E-07
RCV	1.00E-04	3.27E-05	1.98E-05	8.14E-05	9.15E-05	9.79E-05	1.05E-04
		3.54E-04	2.95E-04	3.88E-04	2.47E-04	1.72E-04	1.01E-04
HY	1.00E-04	3.81E-05	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
		3.70E-04	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
RCV-RV	5.00E-01	7.58E-04	5.39E-04	5.92E-03	1.81E-02	3.83E-02	1.56E-01
		8.26E-03	8.00E-03	2.84E-02	4.99E-02	6.65E-02	1.48E-01
HY-RV	5.00E-01	8.89E-04	1.03E-03	2.80E-03	7.46E-03	1.48E-02	5.56E-02
		8.63E-03	1.01E-02	2.71E-02	7.46E-02	1.44E-01	5.80E-01
HY-SIML	5.00E-01	1.79E-01	1.79E-01	1.52E-01	1.76E-01	1.50E-01	2.65E-01
		1.83E+00	1.81E+00	2.11E+00	2.38E+00	2.74E+00	4.27E+00
SIML-SIML	5.00E-01	4.69E-01	4.63E-01	4.96E-01	5.11E-01	5.18E-01	5.14E-01
		1.44E-01	1.33E-01	2.04E-01	2.67E-01	3.18E-01	4.95E-01
18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
σ^2_{x1}	2.00E-04	1.99E-04	1.99E-04	2.00E-04	2.02E-04	2.02E-04	2.04E-04
		4.07E-05	4.05E-05	6.58E-05	8.35E-05	9.89E-05	1.30E-04
σ^2_{v1}	2.00E-08	7.37E-09	6.56E-09	5.10E-08	1.14E-07	2.07E-07	1.02E-06
		2.12E-10	1.95E-10	3.69E-09	1.25E-08	2.99E-08	2.93E-07
RV1	2.00E-04	4.67E-04	4.29E-04	2.83E-04	2.30E-04	2.15E-04	2.03E-04
		6.05E-06	6.90E-06	9.72E-06	1.30E-05	1.75E-05	3.80E-05
σ^2_{x2}	2.00E-04	2.00E-04	2.00E-04	2.01E-04	2.04E-04	2.03E-04	2.03E-04
		4.14E-05	4.13E-05	6.55E-05	8.47E-05	9.74E-05	1.25E-04
σ^2_{v2}	2.00E-08	7.37E-09	6.56E-09	5.10E-08	1.13E-07	2.07E-07	1.02E-06
		2.04E-10	1.98E-10	3.56E-09	1.28E-08	3.02E-08	2.91E-07
RV2	2.00E-04	4.68E-04	4.29E-04	2.83E-04	2.29E-04	2.14E-04	2.03E-04
		6.20E-06	6.79E-06	9.66E-06	1.28E-05	1.77E-05	3.80E-05
σ^2_{x12}	1.00E-04	1.01E-04	1.00E-04	1.03E-04	1.06E-04	1.07E-04	1.08E-04
		3.49E-05	3.21E-05	5.00E-05	6.48E-05	7.72E-05	1.03E-04
σ^2_{v12}	0.00E+00	3.63E-11	7.22E-12	6.01E-09	3.52E-08	8.22E-08	4.87E-07
		2.39E-10	1.33E-10	2.65E-09	9.41E-09	2.30E-08	2.23E-07
RCV	1.00E-04	2.03E-05	1.37E-05	6.76E-05	8.86E-05	9.44E-05	9.94E-05
		3.84E-06	3.38E-06	7.13E-06	1.00E-05	1.36E-05	2.97E-05
HY	1.00E-04	3.06E-05	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
		4.43E-06	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
RCV-RV	5.00E-01	4.35E-02	3.19E-02	2.39E-01	3.86E-01	4.39E-01	4.91E-01
		8.18E-03	7.90E-03	2.40E-02	3.72E-02	5.23E-02	1.17E-01
HY-RV	5.00E-01	6.54E-02	7.14E-02	1.08E-01	1.34E-01	1.43E-01	1.56E-01
		9.46E-03	1.03E-02	1.56E-02	2.01E-02	2.30E-02	3.62E-02
HY-SIML	5.00E-01	1.60E-01	1.60E-01	1.70E-01	1.81E-01	1.95E-01	2.48E-01
		3.90E-02	3.90E-02	6.22E-02	9.38E-02	1.26E-01	2.84E-01
SIML-SIML	5.00E-01	5.05E-01	5.01E-01	5.15E-01	5.29E-01	5.33E-01	5.39E-01
		1.40E-01	1.25E-01	2.00E-01	2.62E-01	3.12E-01	4.85E-01

6. Conclusions

In this paper, we have shown that the Separating Information Maximum Likelihood (SIML) estimator is useful and it has the asymptotic robustness in the sense that it is consistent and it has the asymptotic normality under a fairly general conditions when the high frequency financial data are randomly sampled. They include not only the cases when we have the micro-market noises but also the cases when the micro-market structure has the nonlinear adjustments and the round-off errors under a set of reasonable assumptions. This paper has focused on the estimation of the integrated covariance and the hedging coefficients and we have shown that the SIML estimator has reasonable asymptotic as well as finite sample properties. Micro-market factors in financial markets are common in the sense that we have the minimum price change and the minimum order size rules and also we often observe the bid-ask differences in stock markets. Therefore the robustness of the estimation methods of the integrated covariance and the integrated hedging coefficient is quite important.

As a concluding remark, we should stress on the fact that the SIML estimator is very simple and it can be practically used not only for the integrated volatility but also the integrated covariance and the hedging coefficients from the multivariate high frequency financial series. Some applications on the analysis of stock-index futures market shall be reported in another occasion.

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