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Bayesian Estimation of Entry Games With Multiple Players and Multiple Equilibria

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Abstract

Entry game models are often used to study the nature of firms' profit and the nature of competition among firms in empirical studies. However, when there are multiple players in an oligopoly market, resulting multiple equilibria have made it difficult in previous studies to estimate the payoff functions of players in complete information, static and discrete games without using unreasonable assumptions. To overcome this difficulty, this paper proposes a practical estimation method for an entry game with three players using a Bayesian approach. Some mild assumptions are imposed on the payoff function, and the average competitive effect is used to capture the entry effect of the number of firms.

Our proposed methodology is applied to Japanese airline data in 2000, when there are three major airline companies, ANA, JAL and JAS. The model comparison is conducted to investigate the nature of strategic interaction among these Japanese airline companies.

1 Introduction

There has been a growing number of studies of statistical inference in discrete games the class of empirical studies of entry games originated by Bresnahan and Reiss (1991) and Berry (1992). Bresnahan and Reiss (1991) proposed estimation methods for a symmetric case where only the number of entrants in each market was considered important, ignoring who entered the market. Bajari, Hong, and Ryan (2005) took a complementary approach, imposing assumptions on how equilibrium is selected in complete information games. Estimation of the confidence region has been proposed for models with inequality restrictions that apply to entry models (Andrews, Berry and Jia (2003) and Pakes, Porter, Ho and Is (2005)). Ferrall, Houde, Imai, and Pak (2008) considered the entry game with incomplete information and proposed a methodological framework using Bayesian

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techniques. In this paper, we focus on an entry game model of oligopoly and propose a practical estimation method for the payoff function in complete information, static and discrete games. The response variables are typically binary (entry or exit) and are subject to multiple equilibria and strategic interactions, which makes the estimation difficult. An econometric framework that allows for multiple equilibria is presented in which we use only the simple condition that firms enter into a market only when they expect to operate profitably in equilibrium.

Thus, we assume that firms take a pure strategy but do not make any assumptions regarding equilibrium selection. Because a unique solution is not guaranteed to exist in such an entry game, we use the concept of a Nash equilibrium as in Bresnahan and Reiss (1991) to address this problem. Some games have multiple Nash equilibria or no Nash equilibrium; such games are known to cause an identification problem in the literature. The identification problem arises in the estimation because econometricians can observe only one realized strategy among multiple equilibria. Although game theory describes the possibility of multiple equilibria, there is no further information about how the equilibrium is selected from the observations. The presence of multiple equilibria could result in an incoherency problem (see Tamer (2003)) in that the choice probabilities implied by the model do not sum to one. To overcome these problems, in this paper, we employ a Bayesian estimation and introduce a latent random variable that determines the choice among multiple alternatives.

Some earlier studies proposed a Bayesian estimation of an entry game with two players(e.g., Sugawara and Omori (2012)). These studies classify the possible patterns of strategic interactions by four combinations of sign conditions. We extend this model to an entry game with three players. There are many possible patterns of strategic interaction when there are three players in game. To reduce the number of patterns to be examined, we make some assumptions about the payoff function.

In an entry game with two players, possible patterns of strategic interactions are classified by four combinations of sign conditions. However, in an entry game with more than two players, it becomes extremely complicated to classify many possible patterns of strategic interactions by the sign of competitive effects. To reduce the number of patterns as discussed in Berry (1992), we assume that the competitive effect measures how much the firm's payoff increases when the other firms enter into the market and that it is a function of the number of firms entered into the market. The number of other firms in the market is considered important deciding whether the firm enters, but it ignores which firms enter the market. We note that our models can capture asymmetric effects; while the entry of one player increases the payoff function of other players, the entry of others may decrease the payoff function of the player. This is more general than in previous studies, which assumed that entry effects are symmetric and that the strategic interaction is known.

This paper is organized as follows. Section 2 describes the class of entry game models with three players and derives the likelihood function. In Section 3, we describe the estimation methods using a Bayesian approach. Section 4 applies our proposed model to Japanese airline data. Section 5 concludes the paper.

2 Entry Game Model

2.1 Model

Following Berry (1992), we consider entry models where there are three players and two strategies. The structure of the game is as follows. A market is composed of three players who face two options, $A = \{0, 1\}$. Each player i bases her action $y_{im} \in A$ on a publicly observed variable $x_{im} \in X$. We assume that $y_{im} \in A = \{0, 1\}$ for $i \in I = \{1, 2, 3\}$ and $m = 1, \cdots, M$.

The strategy $y_{im} = 1$ implies the entrance of the i-th player in the m-th market, while the strategy $y_{im} = 0$ implies no entrance. Our goal is to estimate payoff functions $\pi^i_{(y_{1m},y_{2m},y_{3m})}$ where $\pi^i_{(y_{1m},y_{2m},y_{3m})}$ denotes the payoff function for the i-th player in the m-th market when the i-th player chooses strategy y_{im}.

2.2 Three-player two-strategy discrete game

We describe a parametric specification for the profit equation. Profit is assumed to depend on exogenous data (some of which are unobserved by the econometrician) and on the endogenous number of firms. The entry effect of firm j on firm i is different than that of firm j on firm k. For example, we can assume a payoff function like $\pi^i_{(y_{1m},y_{2m},y_{3m})} = x'_{im}\beta_i + \Delta^i_j y_{jm} + \Delta^i_{j'} y_{j'm} + u_{im}$. However, under this parameterization, the model becomes extremely complicated when we classify many possible patterns of strategic interactions by the sign of their competitive effects. For simplicity, we instead use the average competitive effect to capture the entry effect, depending on the number of firms. We assume that the payoff function is a linear function;

$$\pi^i_{(y_{1m},y_{2m},y_{3m})} = x'_{im}\beta_i + \delta_i f(n_m) + u_{im},$$

where u_{im} is the part of profit that is unobserved by the econometrician but observed by all players in the m-th market. β_i is a $K \times 1$ regression coefficient vector for the i-th player and payoff of the i-th player increases by $\delta_i f(n_m)$ when the n_m is the number of players in the m-th market. Extending Berry (1992), who used $\delta \log n_m$ as the average competitive effect, we choose $\delta_i \log n_m$ as $\delta_i f(n_m)$ payoff function as

$$\pi^i_{(y_{1m},y_{2m},y_{3m})} = x'_{im}\beta_i + \delta_i \log n_m + u_{im}.$$

Table 1: Payoff matrix for three players

	$y_{1m} = 1$	$y_{1m} = 0$
$y_{2m} = 1, y_{3m} = 1$	$x'_{1m}\beta_1 + \delta_1 \log n_m + u_{1m}$, $x'_{2m}\beta_2 + \delta_2 \log n_m + u_{2m}$, $x'_{3m}\beta_3 + \delta_3 \log n_m + u_{3m}$	0, $x'_{2m}\beta_2 + \delta_1 \log n_m + u_{2m}$ $x'_{3m}\beta_3 + \delta_2 \log n_m + u_{3m}$
$y_{2m} = 1, y_{3m} = 0$	$x'_{1m}\beta_1 + \delta_1 \log n_m + u_{1m}$, $x'_{1m}\beta_1 + \delta_2 \log n_m + u_{1m}$, 0	0, $x'_{2m}\beta_2 + u_{2m}$, 0
$y_{2m} = 0, y_{3m} = 1$	$x'_{1m}\beta_1 + \delta_1 \log n_m + u_{1m}$, 0, $x'_{3m}\beta_3 + \delta_3 \log n_m + u_{3m}$	0, 0, $x'_{3m}\beta_3 + u_{3m}$
$y_{2m} = 0, y_{3m} = 0$	$x'_{1m}\beta_1 + u_{1m}$, 0, 0	0, 0, 0

Furthermore, the payoff is assumed to be zero when the i-th player does not enter the market ($y_{im} = 0$).

We assume that players have complete information and only use pure strategies. The assumption of complete information implies that $(x_{im}, \beta_i, \delta_i, u_{im})$ are known to both players in the m-th market, while the econometrician can observe only x_{im}. Thus, (β_i, δ_i) are treated as parameters and u_{im} as a random error term in the estimation of the payoff function. We assume that if there is a unique equilibrium based on the iterative elimination of dominated strategies, players choose the equilibrium. Otherwise, players choose the Nash equilibrium. If, furthermore, there are multiple Nash equilibria, players choose one of them at random.

Under these assumptions, players play a two-stage game. In the first stage, they decide whether to enter the market. If the i-th player chooses not to enter the market, she receives nothing, and the payoff function is equal to zero. If the i-th player chooses to enter the market, she plays in a competition. The i-th player receives $x'_{im}\beta_i + \delta_i \log n_m + u_{im}$ when n_m players enter into market m. Otherwise, she receives $x'_{im}\beta_i + u_{im}$.

In this paper, we classify types of strategic interaction according to the signs of $(\delta_1, \delta_2, \delta_3)$ as follows. When $\delta_1 < 0, \delta_2 < 0$ and $\delta_3 < 0$, the market is "strategically substitutive", where the entry of other players will reduce the payoff of the entrant. When $\delta_1 > 0, \delta_2 > 0$ and $\delta_3 > 0$, the market is "strategically compensative", where the entry of one player is also beneficial for other players. When the signs of $(\delta_1, \delta_2, \delta_3)$ are the same, the entry of one player has a symmetric effect on the payoff functions of other players in each competition. On the other hand, when the signs of $(\delta_1, \delta_2, \delta_3)$ are different, the entry of one player may have an asymmetric effect. Thus, the signs have empirically important implications for the analysis of the market structure. Moreover, as we shall describe later, they lead to different game structures and econometric models depending on the type of strategic interaction (e.g., Kooreman (1994)).

2.3 Choice Probability and the Likelihood Function

2.3.1 The case $\delta_1 < 0, \delta_2 < 0, \delta_3 < 0$

Let $y_m = (y_{1m}, y_{2m}, y_{3m}), \beta = (\beta_1, \beta_2, \beta_3), \delta = (\delta_1, \delta_2, \delta_3)$ and F denote the distribution function of u. To show how we derive the likelihood function, we first consider the case where $\delta_1 < 0, \delta_2 < 0$ and $\delta_3 < 0$. Figure 1 shows the regions that correspond to the outcomes of the entry game based on the coordinates of the unobserved components (u_{1m}, u_{2m}, u_{3m}). In Region 1 of the top left of Figure 1, $\{(u_{1m}, u_{2m}, u_{3m}) | u_{1m} > -x'_{1m}\beta_1 - \delta_1 \log 3, u_{2m} > -x'_{2m}\beta_2 - \delta_2 \log 3, u_{3m} > -x'_{3m}\beta_3 - \delta_3 \log 3\}$, the following inequalities hold:

$$0 < x'_{im}\beta_i + \delta_i \log 3 + u_{im} < x'_{im}\beta_i + \delta_i \log 2 + u_{im} < x'_{im}\beta_i + u_{im} \quad \text{for } i = 1, 2, 3.$$

From Table 1, it is clear that all players take strategy 1 no matter what action their competitors take. Thus, all players enter the market, i.e., $(y_{1m}, y_{2m}, y_{3m}) = (1, 1, 1)$. Strategy 0 is strictly dominated by strategy 1 for all players, and outcome $(1, 1, 1)$ must be a unique equilibrium. A similar statement holds true in Region 8, where this unique equilibrium is $(0, 0, 0)$.

Assumption If there is a unique equilibrium based on iterative elimination of dominated strategies, players choose the equilibrium. Otherwise, players choose the Nash equilibrium. If, furthermore, there are multiple Nash equilibria, players choose one of them at random.

Under this assumption, we can obtain Regions 2 to 7 by iterative elimination of dominated strategies. For example, in the case where

$$\begin{aligned}
&0 < x'_{1m}\beta_1 + \delta_1 \log 3 + u_{1m} < x'_{1m}\beta_1 + \delta_1 \log 2 + u_{1m} < x'_{1m}\beta_1 + u_{1m}, \\
&x'_{2m}\beta_2 + \delta_2 \log 3 + u_{2m} < 0 < x'_{2m}\beta_2 + \delta_2 \log 2 + u_{2m} < x'_{2m}\beta_2 + u_{2m}, \\
&0 < x'_{3m}\beta_3 + \delta_3 \log 3 + u_{3m} < x'_{3m}\beta_3 + \delta_3 \log 2 + u_{3m} < x'_{3m}\beta_3 + u_{3m},
\end{aligned}$$

the first player and the third player take strategy 1 regardless of the strategy of the second player. The strategy of the second player depends on the other players and takes strategy 0 ($(y_{1m}, y_{2m}, y_{3m}) = (1, 0, 1)$). Thus, the first and third players enter the market while the second player does not, and the unique equilibrium is derived. Similarly, we obtain the unique equilibria for Regions 2 to 7.

Regions 2 to 7 are derived by the iterated elimination of dominated strategies, where equilibrium selection is an optimization problem with a unique solution as in perfect competition. The obtained equilibria are the rational solution, and it is natural for econometricians to assume that they are observed. However, such a discussion does not hold true for Region M1, $\{(u_{1m}, u_{2m}, u_{3m}) | -x'_{1m}\beta_1 - \delta_1 \log 2 < u_{1m} < -x'_{1m}\beta_1 - \delta_1 \log 3, -x'_{2m}\beta_2 - \delta_2 \log 2 < u_{2m} < -x'_{2m}\beta_2 - \delta_2 \log 3, -x'_{3m}\beta_3 - \delta_3 \log 3 < u_{3m}\}$. We have

$$x'_{im}\beta_i + \delta_i \log 3 + u_{im} < 0 < x'_{im}\beta_i + \delta_i \log 2 + u_{im} < x'_{im}\beta_i + u_{im} \quad \text{for } i = 1, 2.$$

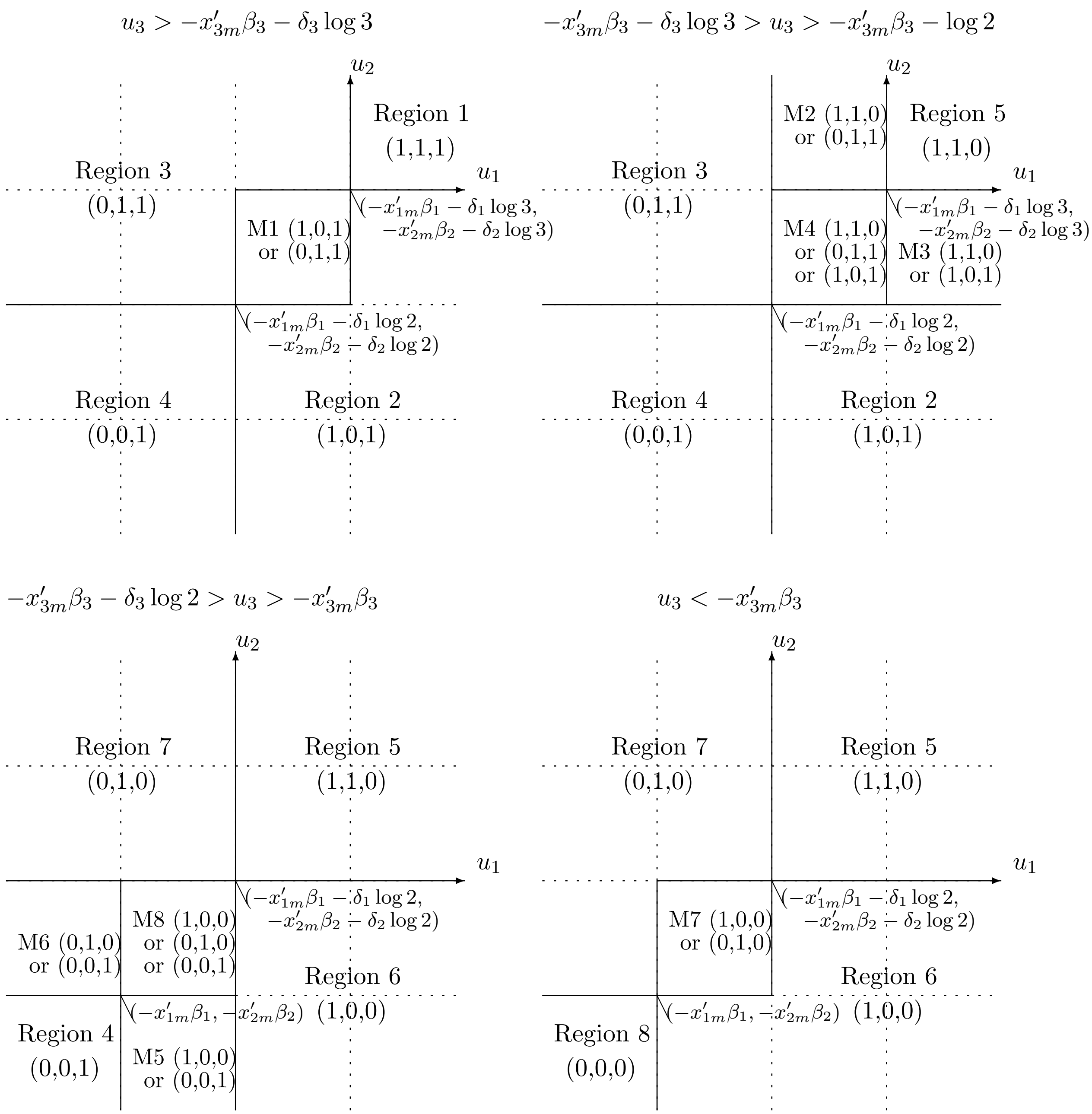
The better strategy is determined by the action of the counterpart of both players, and no equilibrium can be obtained. In this paper, we use the concept of Nash equilibrium as the selection rule to determine the equilibrium. Because the outcomes $(1, 0, 1)$ and $(0, 1, 1)$ are both Nash equilibria, we further assume that players choose one of these equilibria, based on some factors unknown to econometricians (Kooreman(1994)).

Under these assumptions, we can obtain Figure 1 and, hence, the likelihood function. We define F_i^n as the cdf (cumulative distribution function) at $-x'_{im}\beta_i - \delta_i \log n$. Let $\theta = (\beta, \delta)$ and assume that F is the normal distribution function, and that $\bar{F} \equiv 1 - F$. We define sixteen choice probabilities for the outcomes in Figure 1 as follows:

$$\begin{aligned}
P_{1m} &\equiv \bar{F}_1^3 \bar{F}_2^3 \bar{F}_3^3, & P_{M1m} &\equiv [F_1^3 - F_1^2][F_2^3 - F_2^2]\bar{F}_3^3, \\
P_{2m} &\equiv \bar{F}_1^2 F_2^2 \bar{F}_3^2 + \bar{F}_1^3 [F_2^3 - F_2^2]\bar{F}_3^3, & P_{M2m} &\equiv [F_1^3 - F_1^2]\bar{F}_2^3[F_3^3 - F_3^2], \\
P_{3m} &\equiv F_1^2 \bar{F}_2^2 \bar{F}_3^2 + [F_1^3 - F_1^2]\bar{F}_2^3 \bar{F}_3^3, & P_{M3m} &\equiv \bar{F}_1^3[F_2^3 - F_2^2][F_3^3 - F_3^2], \\
P_{4m} &\equiv F_1^2 F_2^2 \bar{F}_3^2 + F_1 F_2 [F_3^2 - F_3], & P_{M4m} &\equiv [F_1^3 - F_1^2][F_2^3 - F_2^2][F_3^3 - F_3^2], \\
P_{5m} &\equiv \bar{F}_1^2 \bar{F}_2^2 F_3^2 + \bar{F}_1^3 \bar{F}_2^3 [F_3^3 - F_3^2], & P_{M5m} &\equiv [F_1^2 - F_1]F_2[F_3^2 - F_3], \\
P_{6m} &\equiv \bar{F}_1^2 F_2^2 F_3^2 + [F_1^2 - F_1]F_2 F_3, & P_{M6m} &\equiv F_1[F_2^2 - F_2][F_3^2 - F_3], \\
P_{7m} &\equiv F_1^2 \bar{F}_2^2 F_3^2 + F_1[F_2^2 - F_2]F_3, & P_{M7m} &\equiv [F_1^2 - F_1][F_2^2 - F_2]F_3, \\
P_{8m} &\equiv F_1 F_2 F_3, & P_{M8m} &\equiv [F_1^2 - F_1][F_2^2 - F_2][F_3^2 - F_3],
\end{aligned}$$

where $\sum_{i=1}^{8} P_{im} + \sum_{i=1}^{8} P_{Mim} = 1$ and P_{im} donate the choice probability of Region i. Furthermore, we introduce the latent binomial (multinomial) random variable, $d_{im}(D_{ijm})$, which selects one

Figure 1: The case where $\delta_1 < 0, \delta_2 < 0, \delta_3 < 0$



outcome among possible outcomes of Region M_i. Then the likelihood function is given by:

$$
\begin{aligned}
f(1,1,1|\theta,\mathbf{d}_m,\mathbf{D}_m) &= P_{1m} \\
f(1,0,1|\theta,\mathbf{d}_m,\mathbf{D}_m) &= P_{2m} + d_{1m}P_{M1m} + d_{3m}P_{M3m} + D_{41m}P_{M4m} \\
f(0,1,1|\theta,\mathbf{d}_m,\mathbf{D}_m) &= P_{3m} + (1-d_{1m})P_{M1m} + d_{2m}P_{M2m} + D_{42m}P_{M4m} \\
f(0,0,1|\theta,\mathbf{d}_m,\mathbf{D}_m) &= P_{4m} + d_{5m}P_{M5m} + d_{6m}P_{M6m} + D_{81m}P_{M8m} \\
f(1,1,0|\theta,\mathbf{d}_m,\mathbf{D}_m) &= P_{5m} + (1-d_{2m})P_{M2m} + (1-d_{3m})P_{M3m} + D_{43m}P_{M4m} \\
f(1,0,0|\theta,\mathbf{d}_m,\mathbf{D}_m) &= P_{6m} + (1-d_{5m})P_{M5m} + d_{7m}P_{M7m} + D_{82m}P_{M8m} \\
f(0,1,0|\theta,\mathbf{d}_m,\mathbf{D}_m) &= P_{7m} + (1-d_{6m})P_{M6m} + (1-d_{7m})P_{M7m} + D_{83m}P_{M8m} \\
f(0,0,0|\theta,\mathbf{d}_m,\mathbf{D}_m) &= P_{8m}
\end{aligned}
$$

where $\mathbf{d}_m = (d_{1m}, d_{2m}, d_{3m}, d_{5m}, d_{6m}, d_{7m})'$ with $d_{im} = 0, 1 (j = 1, 2, 3, 5, 6, 7)$, and $\mathbf{D}_m = (D_{41m}, D_{42m}, D_{43m}, D_{81m}, D_{82m}, D_{83m})'$ with $D_{ijm} = 0, 1$ and $\sum_j D_{ijm} = 1$.

2.3.2 The case $\delta_1 > 0, \delta_2 < 0, \delta_3 < 0$

In this subsection, we will introduce the case in which we include no Nash equilibrium. In the case where $\delta_1 > 0, \delta_2 < 0, \delta_3 < 0$, some regions have no Nash equilibrium. Figure 2 shows the regions that correspond to the outcomes of the entry game of the unobserved components for the case $\delta_1 > 0, \delta_2 < 0, \delta_3 < 0$. In Region NE1 at the top left of Figure 2, $\{(u_{1m}, u_{2m}, u_{3m})| -x'_{1m}\beta_1 - \delta_1 \log 3 < u_{1m} < -x'_{1m}\beta_1 - \delta_1 \log 2, -x'_{2m}\beta_2 - \delta_2 \log 2 < u_{2m} < -x'_{2m}\beta_2 - \delta_2 \log 3, -x'_{3m}\beta_3 - \delta_3 \log 3 < u_{3m}\}$; there is no Nash equilibrium. Because we obtain the following inequalities,

$$
\begin{aligned}
x'_{1m}\beta_1 + u_{1m} < x'_{1m}\beta_1 + \delta_1 \log 2 + u_{1m} < 0 < x'_{1m}\beta_1 + \delta_1 \log 3 + u_{1m}, \\
x'_{2m}\beta_2 + \delta_2 \log 3 + u_{2m} < 0 < x'_{2m}\beta_2 + \delta_2 \log 2 + u_{2m} < x'_{2m}\beta_2 + u_{2m}, \\
0 < x'_{3m}\beta_3 + \delta_3 \log 3 + u_{3m} < x'_{3m}\beta_3 + \delta_3 \log 2 + u_{3m} < x'_{3m}\beta_3 + u_{3m},
\end{aligned}
$$

the third player always enters the market, but the entrances of the first player and the second player depend on each other's entrance. If all players enter the market, the players have the following payoff functions:

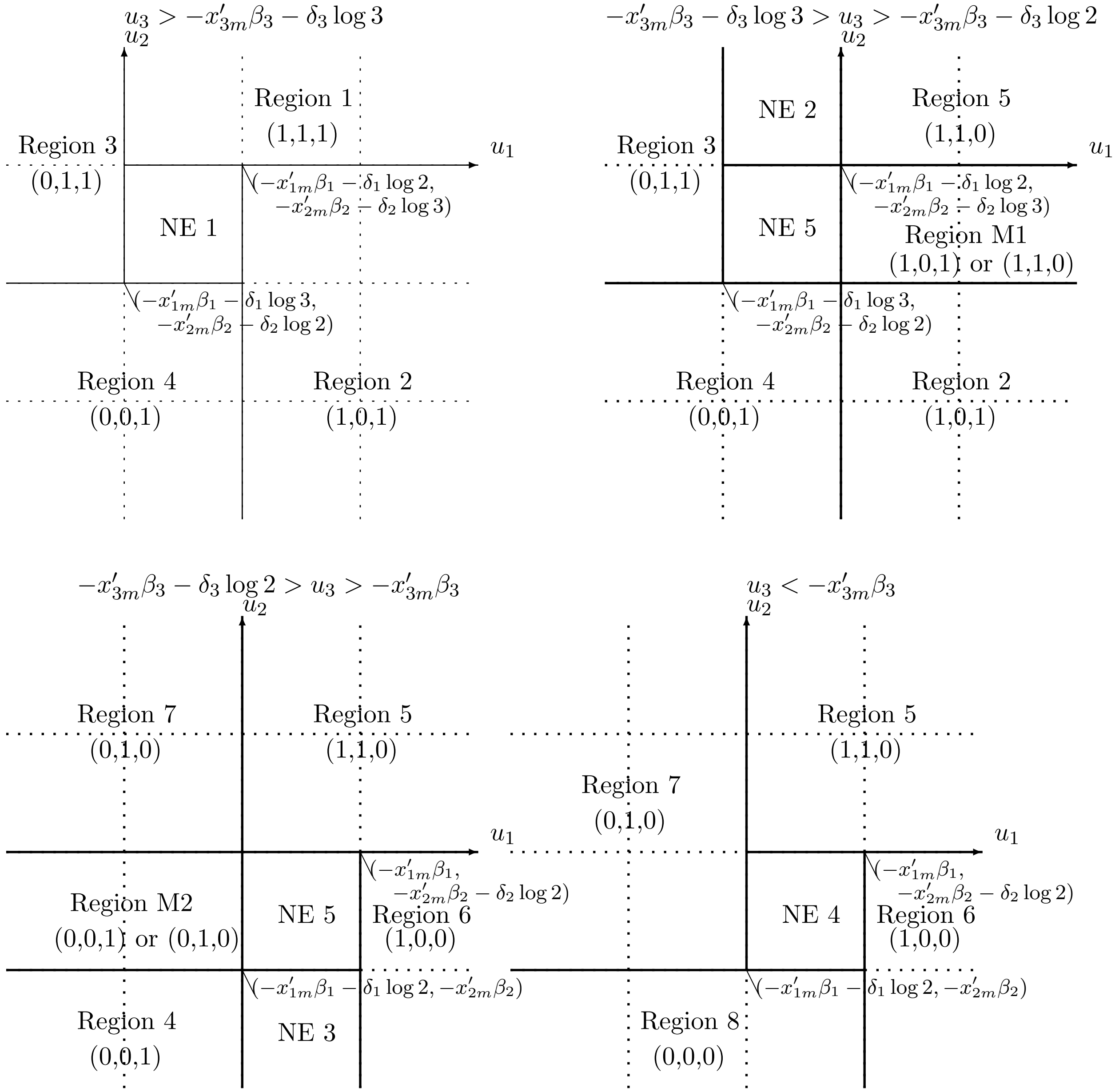
$$
\begin{aligned}
0 < x'_{1m}\beta_1 + \delta_1 \log 3 + u_{1m}, \\
x'_{2m}\beta_2 + \delta_2 \log 3 + u_{2m} < 0, \\
0 < x'_{3m}\beta_3 + \delta_3 \log 3 + u_{3m}.
\end{aligned}
$$

In this situation, the second player has the negative payoff function and would not enter the market if he expected the first player to enter. In the other situation, i.e., only the first and third players enter the market, (the second player does not enter), the players' payoff functions are

$$
\begin{aligned}
&x'_{1m}\beta_1 + \delta_1 \log 2 + u_{1m} < 0,\\
&0 < x'_{2m}\beta_2 + \delta_2 \log 2 + u_{2m},\\
&0 < x'_{3m}\beta_3 + \delta_3 \log 2 + u_{3m}.
\end{aligned}
$$

Then, the first player has the negative payoff function and would not enter the market if he expected that the second player would not enter the market. The second player, however, has the positive payoff function if he enters the market and the first player does not enter the market. Therefore, we know that the third players always enter the market and have no information regarding the entrance of the first and second players in this region. Thus, we will observe one of outcomes $(0, 1, 1), (0, 0, 1), (1, 1, 1)$ or $(1, 0, 1)$.

Figure 2: The case where $\delta_1 > 0, \delta_2 < 0, \delta_3 < 0$



To drive the likelihood function, we let

$$
\begin{aligned}
P_{1m} &\equiv \bar{F}_1^3\bar{F}_2^3\bar{F}_3^3, & P_{M1m} &\equiv \bar{F}_1^2[F_2^3-F_2^2][F_3^3-F_3^2],\\
P_{2m} &\equiv \bar{F}_1^2F_2^3\bar{F}_3^3+\bar{F}_1^2F_2^2[F_3^3-F_3^2], & P_{M2m} &\equiv F_1^2[F_2^2-F_2][F_3^2-F_3],\\
P_{3m} &\equiv F_1^3\bar{F}_2^2\bar{F}_3^2, & P_{NE1m} &\equiv [F_1^2-F_1^3][F_2^3-F_2^2]\bar{F}_3^3,\\
P_{4m} &\equiv F_1^2F_2^2\bar{F}_3^2+F_1^2F_2[F_3^2-F_3], & P_{NE2m} &\equiv [F_1^2-F_1^3]\bar{F}_2^3[F_3^3-F_3^2],\\
P_{5m} &\equiv \bar{F}_1^2\bar{F}_2^3[F_3^3-F_3^2]+\bar{F}_1^2\bar{F}_2^2F_3^2, & P_{NE3m} &\equiv [F_1-F_1^2]F_2[F_3^2-F_3],\\
P_{6m} &\equiv \bar{F}_1F_2^2F_3^2, & P_{NE4m} &\equiv [F_1-F_1^2][F_2^3-F_2^2]F_3,\\
P_{7m} &\equiv F_1^2\bar{F}_2^2[F_3^2-F_3]+F_1^2\bar{F}_2F_3, & P_{NE5m} &\equiv [F_1^2-F_1^3][F_2^3-F_2^2][F_3^3-F_3^2]\\
P_{8m} &\equiv F_1F_2F_3, & &\quad +[F_1-F_1^2][F_2^2-F_2][F_3^2-F_3].
\end{aligned}
$$

We introduce the latent binomial (multinomial) random variables $d_{im}(D_{ijm})$, which select one outcome from possible outcomes Region $\mathrm{M}_i(\mathrm{NE}_i)$. Then, we obtain the likelihood function by

$$
\begin{aligned}
f(1,1,1|\theta,\mathbf{d}_m,\mathbf{D}_m) &= P_{1m}+D_{11m}P_{NE1m}+D_{21m}P_{NE2m}+D_{51m}P_{NE5m},\\
f(1,0,1|\theta,\mathbf{d}_m,\mathbf{D}_m) &= P_{2m}+d_{1m}P_{M1m}+D_{12m}P_{NE1m}+D_{31m}P_{NE3m}+D_{52m}P_{NE5m},\\
f(0,1,1|\theta,\mathbf{d}_m,\mathbf{D}_m) &= P_{3m}+D_{22m}P_{NE2m}+D_{13m}P_{NE1m}+D_{53m}P_{NE5m},\\
f(0,0,1|\theta,\mathbf{d}_m,\mathbf{D}_m) &= P_{4m}+d_{2m}P_{M2m}+D_{14m}P_{NE1m}+D_{32m}P_{NE3m}+D_{54m}P_{NE5m},\\
f(1,1,0|\theta,\mathbf{d}_m,\mathbf{D}_m) &= P_{5m}+(1-d_{1m})P_{M1m}+D_{23m}P_{NE2m}+D_{41m}P_{NE4m}+D_{55m}P_{NE5m},\\
f(1,0,0|\theta,\mathbf{d}_m,\mathbf{D}_m) &= P_{6m}+D_{33m}P_{NE3m}+D_{42m}P_{NE4m}+D_{56m}P_{NE5m},\\
f(0,1,0|\theta,\mathbf{d}_m,\mathbf{D}_m) &= P_{7m}+(1-d_{2m})P_{M2m}+D_{24m}P_{NE2m}+D_{43m}P_{NE4m}+D_{57m}P_{NE5m},\\
f(0,0,0|\theta,\mathbf{d}_m,\mathbf{D}_m) &= P_{8m}+D_{34m}P_{NE3m}+D_{44m}P_{NE4m}+D_{58m}P_{NE5m},
\end{aligned}
$$

when $\mathbf{d_m}=(\mathbf{d_{1m}},\mathbf{d_{2m}})'$, $\mathbf{D}_m = (D_{11m},...,D_{14m},D_{21m},...,D_{24m},D_{31m},...,D_{34m},D_{41m},...,D_{44m})$. Likelihood functions for other cases where $(\delta_1>0,\delta_2<0,\delta_3<0)$, $(\delta_1>0,\delta_2>0,\delta_3<0)$ and $(\delta_1>0,\delta_2>0,\delta_3>0)$ are described in Appendix A.

3 Bayesian Estimation

In this section, we describe how to estimate the model parameters using a hierarchical Bayesian model. Consider, for example, the case where $\delta_1<0,\delta_2<0,\delta_3<0$.

For the likelihood function, we define $\mathbf{d}_m=(d_{1m},d_{2m},d_{3m},d_{5m},d_{6m},d_{7m})'$ and $\mathbf{D}_m=(D_{41m},D_{42m},D_{43m},D_{81m},D_{82m},D_{83m})'$. For parameter $\mathbf{d}_m$ and $\mathbf{D}_m$, we assume the binomial and multinomial distribution, respectively. Thus, d_{im} (D_{ijm}) takes 1 with probability p_{im} (p_{ijm}),

$$d_{im}|p_{im} \sim Bernoulli(p_{im}), \qquad p_{im} \sim \mathcal{B}eta(a_{i1m}, a_{i2m}),$$

$$\mathbf{D}_{im}|\mathbf{p}_{im} \sim Multi(\mathbf{p}_{im}), \qquad \mathbf{p}_{im} \sim \mathcal{D}irichlet(\mathbf{a}_{im})$$

where a_{i1m}, a_{i2m} are the parameters of Beta distribution and $\mathbf{a}_{im} = (a_{i1m}, a_{i2m}, a_{i3m})'$ is the parameter vector of the Dirichlet distribution. The likelihood function $f(\mathbf{y}|\theta, \mathbf{d}, \mathbf{D})$ is given by

$$\begin{aligned} f(\mathbf{y}|\theta, \mathbf{d}, \mathbf{D}) &= \prod_{m=1}^{M} f(y_m|\theta, \mathbf{d}_m, \mathbf{D}_m) \\ &= \prod_{m=1}^{M} f(1,1,1|\theta, \mathbf{d}_m, \mathbf{D}_m)^{I[y=(1,1,1)]} f(1,0,1|\theta, d_m\mathbf{d}_m, \mathbf{D}_m)^{I[y=(1,0,1)]} \dots \\ &\quad f(0,0,0|\theta, \mathbf{d}_m, \mathbf{D}_m)^{I[y=(0,0,0)]} \end{aligned}$$

where $\mathbf{y} = (y_1, \cdots, y_m)'$, $\mathbf{d} = (\mathbf{d}_1, \cdots, \mathbf{d}_M)$ and $\mathbf{D} = (\mathbf{D}_1, ..., \mathbf{D}_M)$.

For the prior distributions, we assume a multivariate normal distribution for β and a truncated normal distribution for δ,

$$\beta \sim \mathcal{N}(\beta_0, \Sigma_0),$$

$$\delta_i \sim \mathcal{TN}_{R_i}(\delta_{i0}, \Sigma_{\delta_i 0}).$$

The prior distribution of δ_i is truncated on region R_i, for example, the region $R_i = (-\infty, 0)$, for the case where $\delta_1 < 0, \delta_2 < 0, \delta_3 < 0$. Then, the joint posterior probability density is

$$\pi(\theta, \mathbf{d}, \mathbf{D}, p|\mathbf{y}) \propto f(y|\theta, d)\pi(\theta) \prod_{m=1}^{M} \left[\prod_{i=1}^{8} \left[p_{im}^{(d_{im}+a_{i1m})-1} (1-p_{im})^{(1-d_{im}+a_{i2m})-1} \right] \right] \left[\prod_{i} p_{ijm}^{(D_{ijm}+a_{ijm}-1)} \right],$$

where $\pi(\theta)$ denotes a probability density function of the multivariate normal distribution $\mathcal{N}(\beta_0, \Sigma_0)$ and the truncated normal distribution $\mathcal{TN}_{R_i}(\delta_{i0}, \Sigma_{\delta_i 0})$. The conditional posterior probability distributions of d_m and p_m are: for $i = 1, \ldots, 8$,

$$d_{im}|\theta, p_{im}, \mathbf{y} \sim Bernoulli(q_{im}),$$

$$D_{im}|\theta, p_m, \mathbf{y} \sim Multi(\mathbf{q}_{im}),$$

where $\mathbf{q}_{im} = (q_{i1m}, q_{i2m}, q_{i3m})'$,

$$q_{im} = \frac{p_{im} f(y_m|\theta, \mathbf{D}_m, d_{im}=1, d_{-im})}{p_{im} f(y_m|\theta, \mathbf{D}_m, d_{im}=1, d_{-im}) + (1-p_{im}) f(y_m|\theta, \mathbf{D}_m d_{im}=0, d_{-im})},$$

$$q_{ijm} = \frac{p_{ijm} f(y_m|\theta, D_{ijm} = 1, D_{ikm} = 0, D_{ilm} = 0, D_{-im}, \mathbf{d}_m)}{\sum_{k=1}^{3} p_{ikm} f(y_m|\theta, D_{ikm} = 1, D_{-ikm} = 0, \mathbf{D}_{-im}, \mathbf{d}_m)},$$

where $d_{-im}(\mathbf{D}_{-im})$ implies that all elements of $\mathbf{d}_m(\mathbf{D}_m)$ except $d_{im}(\mathbf{D}_{im})$, $D_{-ikm} = 0$ imply that $D_{ilm} = 0(for\ l \neq k)$. Finally, conditional posterior distributions of p_{im} and $\mathbf{p}_{im}$ are

$$p_{im}|\theta, d_{im}, \mathbf{y} \sim \mathcal{B}eta(a_{i1m} + d_{im}, a_{i2m} + 1 - d_{im}),$$

$$\mathbf{p}_{im}|\theta, d_{im}, \mathbf{y} \sim \mathcal{D}irichlet(\mathbf{a}_i + \mathbf{D}_{im}).$$

Markov Chain Monte Carlo (MCMC) algorithm. We implement a MCMC algorithm in six blocks:

1. Initialize $\mathbf{p}, \mathbf{d}, \boldsymbol{\beta}, \boldsymbol{\delta}$
2. Generate $\mathbf{p}|\boldsymbol{\beta}, \boldsymbol{\delta}, \mathbf{d}, \mathbf{D}, \mathbf{y}$
3. Generate $\mathbf{d}, \mathbf{D}|\boldsymbol{\beta}, \boldsymbol{\delta}, \mathbf{p}, \mathbf{y}$
4. Generate $\boldsymbol{\beta}|\mathbf{p}, \mathbf{d}, \mathbf{D}, \boldsymbol{\delta}, \mathbf{y}$
5. Generate $\boldsymbol{\delta}|\boldsymbol{\beta}, \mathbf{d}, \mathbf{D}, \mathbf{p}, \mathbf{y}$
6. Go to Step 2

To sample in steps 4 and 5 from $\pi(\boldsymbol{\beta}|\mathbf{p}, \mathbf{d}, \mathbf{D}, \boldsymbol{\delta}, \mathbf{y})$ and $\pi(\boldsymbol{\delta}|\boldsymbol{\beta}, \mathbf{p}, \mathbf{d}, \mathbf{D}, \mathbf{y})$, we employ the Metropolis-Hastings algorithm using the normal or truncated normal proposal densities, which approximate the conditional posterior densities. The approximation is based on a Taylor expansion of the conditional posterior density around the conditional mode, where it can be found numerically by the Newton-Raphson method. See Appendix B for illustrative examples.

4 Application to Japanese airline competition

4.1 Brief history of the Japanese airline industry

In Japan, there used to be three major airline companies: Japan Airlines (JAL), Air Nippon Airways (ANA) and Japan Air System (JAS, formerly called Toa Domestic Airlines). These three companies were permitted to operate in the Japanese airline market. Japanese airline markets are strictly regulated by the government. Airline companies need government approvals and licenses to determine airfares and to operate new routes. This system is also called the 45/47 system (where 45 and 47 imply the years 1970 and 1972 of the Showa era in Japan). Under the 45/47 system, JAL, as the Japanese flag carrier, was assigned to operate international flights and main domestic routes. ANA operated both main domestic flights and local routes, and JAS operated local flights only. Consequently, ANA operated the most domestic routes and JAL has the fewest domestic routes. Until JAS was merged with JAL in 2001, the market share of ANA, JAL and JAS exceeded 90% and the market was almost an oligopoly; airfares for the three airlines were almost the same. Therefore, for the most part, there were only three players, ANA, JAL and JAS, in the Japanese airline industry at that time.

4.2 Data description

In this paper, we focus on Japanese airline competition as an entry game where the three players are ANA, JAL and JAS, including their affiliated companies. We define a market as a trip between two airports. If a firm has at least one flight between the airports, we say the i-th player enters the m-th market and let $y_{im} = 1$. Our data come from the timetable (Japan Railway Company, 2000) for January, 2000. Japan had 94 airports in 2000, but some of them were located in islands and primarily served the island residents. Such airports handled few flights and operated by government policy. To provide transportation for those who lived in the islands, the government subsidized an airline to operate flights. These airports are not part of market competition, so we removed these from our sample. With one exception, we also eliminated airports with fewer than two routes or that were located in isolated islands. The exception was Naha, which is the largest city in Okinawa prefecture and plays an important role in both business and tourism. Thus, we have 38 airports and 703 markets.

We describe explanatory variables in the rest of this subsection. In this study, there are four types of explanatory variables in the payoff function: firm-market specific variables, a dummy variable for a strong competitor of an airline and variables of important factors for demand and for cost.

As a firm-market specific explanatory variable, we added sums of the number of flights from the two airports for each firm (e.g., Flight ANA for ANA). The firm needs little additional cost to operate a new route if it already has another route from the airport, because it does not need to install a variety of facilities such as a new check-in counter or ticket office. Thus, the sum of the number of fights represents the marginal cost of operations, and we expect these variables have positive coefficients.

In Japan, railways are a popular transportation service; they have a large share of domestic transportation. In particular, the bullet train called the Shinkansen has become a great competitor of the airlines in domestic long-distance transportation. In this study, we include a dummy variable for the Shinkansen in their payoff function (Train). The Shinkansen dummy will take the value of one if one can go to the destination with no transfer by Shinkansen, and the station is located within an hour of the airport.

Table 2: Summary Statistics

Variable	Mean	Stdev
Distance	0.000	0.490
Population	0.664	1.201
$\sqrt{\text{Population}}$	2.041	1.574
Train	0.825	2.751
Flight ANA	3.167	2.849
Flight JAL	1.941	2.139
Flight JAS	1.298	1.741

We focus on two factors as explanatory variables following previous studies such as Sugawara and Omori (2011), Berry (1992) and Cilberlto and Tamer (2009). First, we consider travel cost, which was measured by the direct distance between airports (Distance). The greater is the distance between the airports, the more cost we will incur to travel. Direct distances are calculated by the geodesic distance formula (Banerjee et al. (2004, pp.17-18)). Second, we used the population of the prefecture where an airport was located as proxy variables for demand and market size factor. We made two explanatory variables from the population, (1) the product of the city population and (2) the square root of the product of the population of the prefecture where the airports were located. Including the squared root of the population is intended to capture the non-linear effect. We do not adopt the squared population for the non-linear effect because it did not improve the model comparison criterion, DIC (see Section 4.4). We used the population based on the census data conducted in 2000. Table 2 shows summary statistics for the independent variable.

The numbers of observed entries in the markets for ANA, JAS and JAL are 107, 72 and 44, respectively; Table 3 shows the details of entrance status. The number of markets where only JAL is present is few because JAL had been regulated to operate only main routes under the 45/47 system. In Japanese airline markets, we can see that ANA and JAL establish a presence in the domestic market in 2000.

4.3 Estimation results

Let δ_{ANA}, δ_{JAL}, and δ_{JAS} denote the coefficient of the logarithm of n_m for ANA, JAL and JAS. $\delta_{ANA} > 0, \delta_{JAL} > 0, \delta_{JAS} > 0$ among eight competing models because it is selected as the best model based on DIC as we shall see in Section 4.4.

Table 4 shows posterior means, posterior standard deviations, 95% credible intervals and inefficiency factors. The 3,000 MCMC samples were generated after discarding 300 initial samples as the burn-in period. The inefficiency factors (IF) are 1-12, implying that the chain mixes very well (IF is described in Appendix B). The acceptance rates of the independence Metropolis-Hastings algo-

Table 3: Entrance status

	Number of markets
ANA, JAL and JAS	27
ANA and JAL	13
JAS and JAL	1
ANA and JAS	13
ANA	54
JAS	31
JAL	3
No entrant	561

rithm are sufficiently high and the proposal distribution seems to approximate well the conditional posterior distribution.

The restriction, $\delta_{ANA} > 0, \delta_{JAL} > 0, \delta_{JAS} > 0$, means that the market is strategically compensative, where the entry of one airline is also beneficial for other airlines. Estimation results indicate that the posterior mean of $\delta_{JAL}(1.862)$ is much greater than are those of $\delta_{ANA}(0.312)$ and $\delta_{JAS}(0.205)$ (Table 4). The entry decision of JAL is to follow the market leaders JAS and ANA. Because ANA and JAS are allowed to operate in more domestic routes than is JAL, they were able to accumulate more knowledge of the domestic market than JAL, which could contribute only to main routes. This is because the segregation policy, the 45/47 system, has been historically applied to the Japanese airline industry. The 45/47 system allows JAL to operate only limited domestic or local flights, while ANA and JAS operate most local routes making them the market leaders.

Estimation results of the model with $\delta_{ANA} > 0, \delta_{JAL} > 0, \delta_{JAS} > 0$ provide several implications. First, posterior means of the train dummy in all payoff function are negative. This implies that all airlines' profit would decrease when there was a bullet train, Shinkansen. The bullet train is the greatest competitor for airlines in the Japanese transportation market. In fact, price competition is very severe between railway companies and airlines in certain markets. A discounted plane ticket fare between Tokyo and Osaka, for example, is almost same cost as a train ticket. Second, all signs of the sum of the number of flights, Flight ANA, Flight JAS and Flight JAL, are positive. As the marginal cost of flight operations decreases, the payoff would add as expected. If ANA has already entered into the airport, even if it add a new flight, there is little additional cost for operation. This result is consistent with the reality. Third, the posterior means of Population and $\sqrt{\text{Population}}$ are negative and positive, respectively. Some population size is needed to operate an airplane profitably, but an excess of population does not increase the profit.

Table 4: Estimation results for model with $\delta_{ANA} > 0, \delta_{JAL} > 0, \delta_{JAS} > 0$. Posterior means, standard deviations, 95% credible intervals and inefficiency factors.

Parameter	Mean	Std	95% Interval	IF
ANA				
Constant**	-2.712	0.220	(-3.150,-2.271)	2
Distance	-0.195	0.170	(-0.523,0.139)	2
Population**	-0.389	0.147	(-0.664,-0.097)	1
$\sqrt{\text{Population}}$*	0.256	0.128	(0.003,0.499)	2
Train**	-0.102	0.036	(-0.175,-0.031)	2
Flight ANA**	0.340	0.036	(0.270,0.412)	2
δ_{ANA}**	0.312	0.214	(0.017,0.806)	3
JAL				
Constant**	-4.988	0.643	(-6.316,-3.837)	1
Distance**	1.072	0.330	(0.409,1.723)	1
Population*	-0.589	0.256	(-1.100,-0.111)	2
$\sqrt{\text{Population}}$**	0.892	0.270	(0.376,1.427)	2
Train	-0.041	0.047	(-0.140,0.051)	2
Flight JAL**	0.208	0.070	(0.070,0.348)	4
δ_{JAL}**	1.862	0.380	(1.130,2.637)	12
JAS				
Constant**	-3.406	0.300	(-3.994,-2.844)	2
Distance	0.030	0.207	(-0.369,0.426)	2
Population**	-0.488	0.167	(-0.823,-0.173)	1
$\sqrt{\text{Population}}$**	0.646	0.155	(0.344,0.953)	1
Train**	-0.179	0.041	(-0.262,-0.099)	1
Flight JAS**	0.339	0.041	(0.258,0.422)	2
δ_{JAS}**	0.205	0.158	(0.007,0.592)	4

4.4 Model comparison

The Japanese airline market has been gradually deregulated since the 1980s, but some features remain from the former regulation regime. Therefore, the type of strategic interaction between the three major companies is unclear, and the signs of δ_i's are unknown a priori. Thus, we first estimate the eight models we considered in Section 2 and determine the type of strategic interaction in the Japanese airline market by conducting model selection based on the DIC.

To conduct model comparisons, we use DIC (deviance information criterion; see e.g., Spiegelhalter et al. (2002)) which is defined as

$$DIC = E_{\pi(\theta|x)}\left\{D(\theta)\right\} + p_D = D(\theta^*) + 2p_D,$$

where $\theta^* = E_{\pi(\theta|x)}(\theta)$, $D(\theta) = -\log f(x|\theta)$ and,

$$\begin{aligned} p_D &= E_{\pi(\theta|x)}\{D(\theta)\} - D(\theta^*) \\ &= E_{\pi(\theta|x)}\{-2\log f(x|\theta)\} + 2\log f(x|\theta^*), \end{aligned}$$

p_D represents the effective number of parameters and is used as a measure of the model complexity of a Bayesian model. Allowing for both goodness-of-fit and complexity, we select the model with the smallest DIC.

In the analysis of Japanese airline data, we use the following prior distributions:

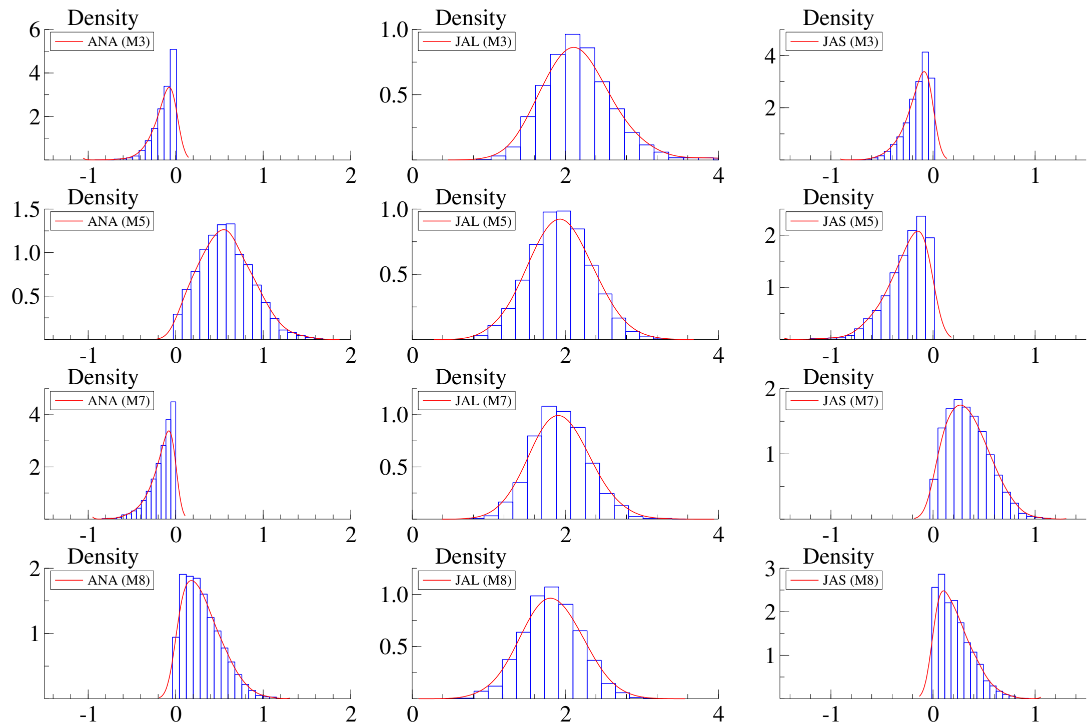
$$\begin{aligned} \beta &\sim \mathcal{N}(0, 100 I_6), \\ \delta_i &\sim \mathcal{TN}_{(-\infty,0)}(-1, 10) \; for \; \delta_i < 0, \\ \delta_i &\sim \mathcal{TN}_{(0,\infty)}(1, 10) \; for \; \delta_i > 0, \\ p_{im} &\sim i.i.d.Beta(1,1) \text{ for } i = 1,2,3,5,6,7, m = 1, \ldots M, \\ \mathbf{p}_{im} &\sim i.i.d.Dirichlet(\mathbf{1}) \text{ for } i = 4,8, m = 1, \ldots M, \end{aligned}$$

The DICs for eight competing models are shown in Table 5.

Table 5: DIC for eight competing models

Model	Sign of δ			Ranking	DIC	Std. Err.
M1	$\delta_{ANA} < 0$	$\delta_{JAL} < 0$	$\delta_{JAS} < 0$	8	838.9	(0.07)
M2	$\delta_{ANA} > 0$	$\delta_{JAL} < 0$	$\delta_{JAS} < 0$	6	828.5	(0.08)
M3	$\delta_{ANA} < 0$	$\delta_{JAL} > 0$	$\delta_{JAS} < 0$	4	803.4	(0.37)
M4	$\delta_{ANA} < 0$	$\delta_{JAL} < 0$	$\delta_{JAS} > 0$	7	833.9	(0.47)
M5	$\delta_{ANA} > 0$	$\delta_{JAL} > 0$	$\delta_{JAS} < 0$	2	801.1	(0.40)
M6	$\delta_{ANA} > 0$	$\delta_{JAL} < 0$	$\delta_{JAS} > 0$	5	826.8	(0.22)
M7	$\delta_{ANA} < 0$	$\delta_{JAL} > 0$	$\delta_{JAS} > 0$	3	802.5	(0.24)
M8	$\delta_{ANA} > 0$	$\delta_{JAL} > 0$	$\delta_{JAS} > 0$	1	800.9	(0.30)

Figure 3: Estimated posterior densities of $\delta_{ANA}, \delta_{JAL}$ and δ_{JAS} for models M3, M5, M7 and M8



First, when we compare two models in which the signs of δ_{ANA} and δ_{JAS} are the same, the model with the positive δ_{JAL} is always preferred. For example, comparing model M1 ($\delta_{JAL} < 0, \delta_{ANA} < 0, \delta_{JAS} < 0$) and model M3 ($\delta_{JAL} > 0, \delta_{ANA} < 0, \delta_{JAS} < 0$), model M3 has a smaller DIC (803.2) than that of model M1 (838.9) and is preferred. We obtain similar results when we compare models M2 and M5, models M4 and M7 and models M6 and M8. This implies that positive δ_{JAL} is always supported by the dataset.

Therefore, we focus on four models with positive δ_{JAL} (models M3, M5, M7 and M8), below. The DICs suggest that model M8 is the best ($\delta_{ANA} > 0, \delta_{JAL} > 0, \delta_{JAS} > 0$) among the four models. The payoffs of all players increase when other players enter the market and the market is a strategically compensative competition. Furthermore, Figure 3 shows estimated posterior densities of δ_i's for these models. The posterior densities of δ_{JAL} are very similar for these models, while those of δ_{ANA} and δ_{JAS} appear different depending on the model. This result is reasonable because, under the 45/47 system, ANA and JAS were allowed to operate more routes than JAL; this superiority is advantageous to JAL, allowing it to construct more networks that are beneficial after deregulation. JAL could enter only main routes where ANA and JAS already entered. This compensative property of the Japanese airline market is a clear difference from that of the US market, where airlines engage in severe substitution competition (where all δ_i's are negative).

We note that the DIC of model M5 is close to that of model M8. As shown in Table 7 (Appendix

C), the estimation results of model M5 are very similar to those of model M8 except for δ_{JAS}. Moreover, as shown in Figure 3, posterior densities of δ_{JAS} have high peaks close to zero in both models suggesting that the entry decision for JAS is not affected by the other airline companies.

5 Conclusions

This paper proposes a Bayesian estimation of an entry game model without making equilibrium selection assumptions when there are multiple equilibria. In an entry game model with three players, asymmetric entry effects are allowed in the payoff function for each player through the number of players in the market. The proposed estimation method was applied to an empirical study of Japanese airline competition. We compared models with all possible combinations of signs of competitive effects and found that the competition had a substitutive structure in the Japanese airline industry in the year 2000.

Based on estimates of the competitive effect, the entry decision of JAL is found to follow market leaders JAS and ANA, which is, considering the history of the Japanese airline industry. This paper also reveals that airlines and trains are in rivals. In fact, there is little of a difference between the costs of flying and train travel.

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Appendix

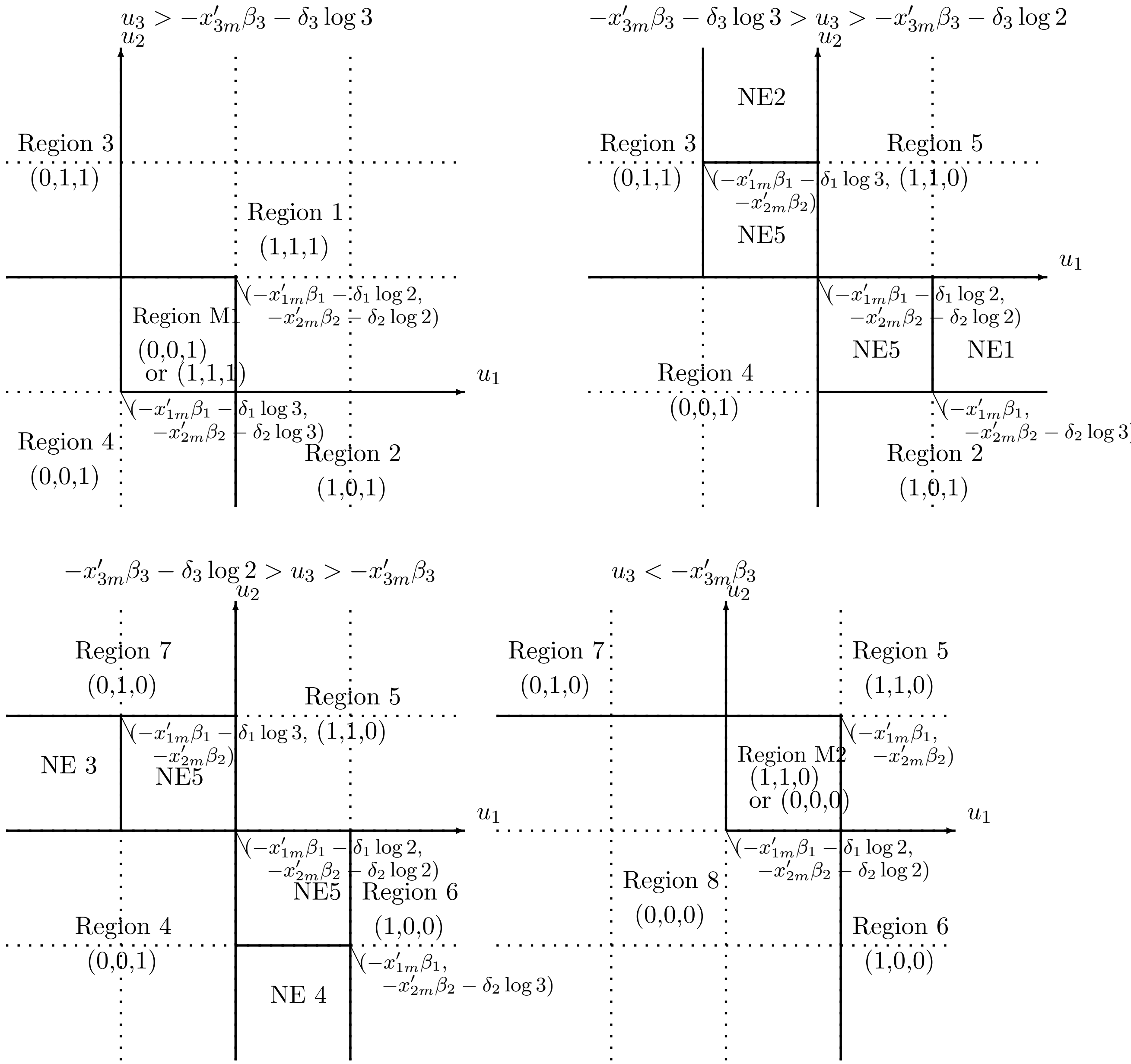
A. Other Cases

There are some other cases of combinations of signs of δ_t’s. In this section, we will introduce likelihood functions for other cases. We can obtain the likelihood under the same assumptions and with the same logic of Section 2.

A.1 The case $\delta_1 > 0, \delta_2 > 0, \delta_3 < 0$

Figure 4 shows the regions that correspond to the outcomes of the entry game of the unobserved components in the case where $\delta_1 > 0, \delta_2 > 0, \delta_3 < 0$.

Figure 4: The case where $\delta_1 > 0, \delta_2 > 0, \delta_3 < 0$



Let us denote

$$
\begin{aligned}
P_{1m} &\equiv \bar{F}_1^2\bar{F}_2^3\bar{F}_3^3 + [F_1^2 - F_1^3]\bar{F}_2^2\bar{F}_3^3, & P_{M1m} &\equiv [F_1^2 - F_1^3][F_2^2 - F_2^3]\bar{F}_3^3,\\
P_{2m} &\equiv \bar{F}_1^2 F_2^3\bar{F}_3^2, & P_{M2m} &\equiv [F_1 - F_1^2][F_2 - F_2^2]F_3,\\
P_{3m} &\equiv F_1^3\bar{F}_2^2\bar{F}_3^2, & P_{NE1m} &\equiv \bar{F}_1[F_2^2 - F_2^3][F_3^3 - F_3^2],\\
P_{4m} &\equiv F_1^2F_2^3\bar{F}_3^3 + F_1^3[F_2^2 - F_2^3]\bar{F}_3^3 + F_1^2F_2^2[F_3^3 - F_3], & P_{NE2m} &\equiv [F_1^2 - F_1^3]\bar{F}_2[F_3^3 - F_3^2],\\
P_{5m} &\equiv \bar{F}_1^2\bar{F}_2^2[F_3^3 - F_3] + \bar{F}_1\bar{F}_2^2F_3 + [F_1 - F_1^2]\bar{F}_2F_3, & P_{NE3m} &\equiv F_1^3[F_2 - F_2^2][F_3^2 - F_3],\\
P_{6m} &\equiv \bar{F}_1F_2^2F_3^2, & P_{NE4m} &\equiv [F_1 - F_1^2]F_2^3[F_3^2 - F_3],\\
P_{7m} &\equiv F_1^2\bar{F}_2F_3^2, & P_{NE5m} &\equiv [F_1 - F_1^2][F_2^2 - F_2^3][F_3^3 - F_3],\\
P_{8m} &\equiv F_1F_2^2F_3 + F_1^2[F_2 - F_2^2]F_3, & &\quad + [F_1^2 - F_1^3][F_2 - F_2^2][F_3^3 - F_3],
\end{aligned}
$$

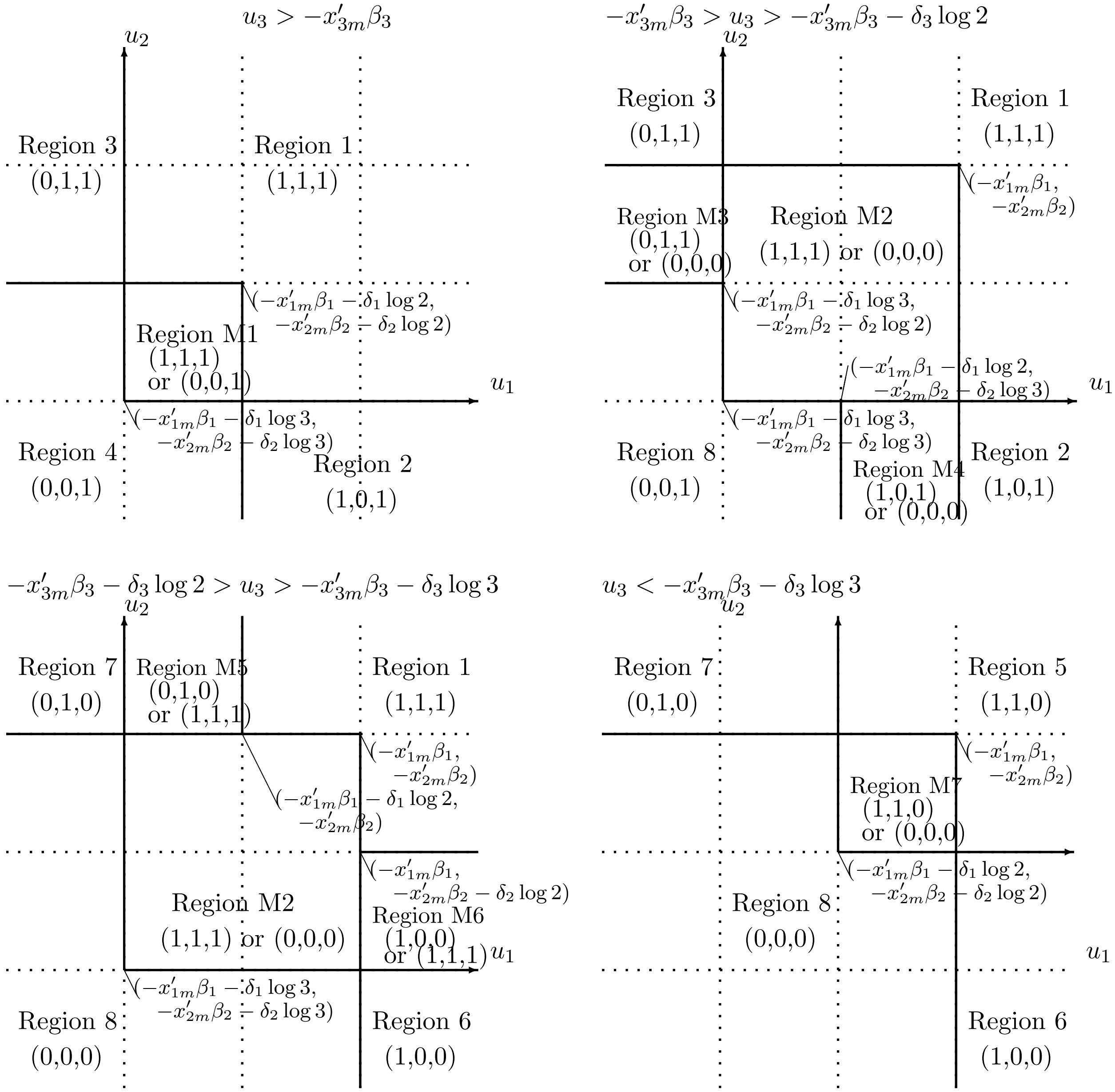
where $\sum_i P_{im}(\theta)+\sum_i P_{Mim}(\theta)+\sum_i P_{NEim}(\theta) = 1$ and P_{im} denote the choice probability of Region i. We introduce the latent binomial (multinomial) random variables $d_{im}(D_{ijm})$, which select one outcome from possible outcomes of Region M_i (NE_i). Then, we obtain the likelihood function by

$$
\begin{aligned}
f(1,1,1|\theta,\mathbf{d}_m,\mathbf{D}_m) &= P_{1m} + d_{1m}P_{M1m} + D_{11m}P_{NE1m} + D_{21m}P_{NE2m} + D_{51m}P_{NE5m},\\
f(1,0,1|\theta,\mathbf{d}_m,\mathbf{D}_m) &= P_{2m} + D_{12m}P_{NE1m} + D_{41m}P_{NE4m} + D_{52m}P_{NE5m},\\
f(0,1,1|\theta,\mathbf{d}_m,\mathbf{D}_m) &= P_{3m} + D_{22m}P_{NE2m} + D_{31m}P_{NE3m} + D_{53m}P_{NE5m},\\
f(0,0,1|\theta,\mathbf{d}_m,\mathbf{D}_m) &= P_{4m} + (1-d_{1m})P_{M1m} + D_{32m}P_{NE3m} + D_{42m}P_{NE4m} + D_{54m}P_{NE5m},\\
f(1,1,0|\theta,\mathbf{d}_m,\mathbf{D}_m) &= P_{5m} + d_{2m}P_{M2m} + D_{13m}P_{NE1m} + D_{23m}P_{NE2m} + D_{55m}P_{NE5m},\\
f(1,0,0|\theta,\mathbf{d}_m,\mathbf{D}_m) &= P_{6m} + D_{14m}P_{NE1m} + D_{43m}P_{NE4m} + D_{56m}P_{NE5m},\\
f(0,1,0|\theta,\mathbf{d}_m,\mathbf{D}_m) &= P_{7m} + D_{24m}P_{NE2m} + D_{33m}P_{NE3m} + D_{57m}P_{NE5m},\\
f(0,0,0|\theta,\mathbf{d}_m,\mathbf{D}_m) &= P_{8m} + (1-d_{2m})P_{M2m} + D_{34m}P_{NE3m} + D_{44m}P_{NE4m} + D_{58m}P_{NE5m}.
\end{aligned}
$$

A.2 The case $\delta_1 > 0, \delta_2 > 0, \delta_3 > 0$

Figure 5 shows the regions that correspond to the outcomes of the entry game of the unobserved components in the case where $\delta_1 > 0, \delta_2 > 0, \delta_3 > 0$.

Figure 5: The case where $\delta_1 > 0, \delta_2 > 0, \delta_3 > 0$



We define

$$
\begin{aligned}
P_{1m} \equiv& \bar{F}_1^3 \bar{F}_2^2 \bar{F}_3 + \bar{F}_1^2 [F_2^2 - F_2^3] \bar{F}_3 \\
&+ \bar{F}_1 \bar{F}_2^3 [F_3 - F_3^2] + [F_1 - F_1^3] \bar{F}_2 [F_3 - F_3^2] \\
&+ \bar{F}_1 \bar{F}_2^2 [F_3^2 - F_3^3] + [F_1 - F_1^2] \bar{F}_2 [F_3^2 - F_3^3], \\
P_{2m} \equiv& \bar{F}_1 F_2^3 [F_3 - F_3^2] + \bar{F}_1^2 F_2^3 \bar{F}_3, \\
P_{3m} \equiv& F_1^3 \bar{F}_2 [F_3 - F_3^2] + F_1^3 \bar{F}_2^2 \bar{F}_3, \\
P_{4m} \equiv& F_1^2 F_2^3 \bar{F}_3 + F_1^3 [F_2^2 - F_2^3] \bar{F}_3, \\
P_{5m} \equiv& \bar{F}_1 \bar{F}_2^2 F_3^3 + [F_1 - F_1^2] \bar{F}_2 F_3^3, \\
P_{6m} \equiv& \bar{F}_1 F_2^3 [F_3^2 - F_3^3] + \bar{F}_1 F_2^2 F_3^3, \\
P_{7m} \equiv& F_1^3 \bar{F}_2 [F_3^2 - F_3^3] + F_1^2 \bar{F}_2 F_3^3, \\
P_{8m} \equiv& F_1^2 F_2^3 [F_3 - F_3^2] + F_1^3 [F_2^2 - F_2^3][F_3 - F_3^2] \\
&+ F_1 F_2^3 [F_3^2 - F_3^3] + F_1^3 [F_2 - F_2^3][F_3^2 - F_3^3] \\
&+ F_1 F_2^2 F_3^3 + F_1^2 [F_2 - F_2^2] F_3^3,
\end{aligned}
$$

$$
\begin{aligned}
P_{M1m} \equiv& [F_1^2 - F_1^3][F_2^2 - F_2^3] \bar{F}_3, \\
P_{M2m} \equiv& [F_1 - F_1^3][F_2 - F_2^3][F_3 - F_3^3], \\
P_{M3m} \equiv& F_1^3 [F_2 - F_2^2][F_3 - F_3^2], \\
P_{M4m} \equiv& [F_1 - F_1^2] F_2^3 [F_3 - F_3^2], \\
P_{M5m} \equiv& [F_1^2 - F_1^3] \bar{F}_2 [F_3^2 - F_3^3], \\
P_{M6m} \equiv& \bar{F}_1 [F_2^2 - F_2^3][F_3^2 - F_3^3], \\
P_{M7m} \equiv& [F_1 - F_1^2][F_2 - F_2^2] F_3^3,
\end{aligned}
$$

where $\sum_i P_{im}(\theta) + \sum_i P_{Mim}(\theta) = 1$ and P_{im} denote the choice probability of Region i. We introduce the latent binomial random variables d_{im}, which select one outcome from possible outcomes of Region M_i. Then, we obtain the likelihood function by

$$
\begin{aligned}
f(1,1,1|\theta, \mathbf{d}_m) &= P_{1m} + d_{1m} P_{M1m} + d_{2m} P_{M2m} + d_{5m} P_{M5m} + d_{6m} P_{M6m}, \\
f(1,0,1|\theta, \mathbf{d}_m) &= P_{2m} + d_{4m} P_{M4m}, \\
f(0,1,1|\theta, \mathbf{d}_m) &= P_{3m} + d_{3m} P_{M3m}, \\
f(0,0,1|\theta, \mathbf{d}_m) &= P_{4m} + (1 - d_{1m}) P_{M1m}, \\
f(1,1,0|\theta, \mathbf{d}_m) &= P_{5m} + d_{7m} P_{M7m}, \\
f(1,0,0|\theta, \mathbf{d}_m) &= P_{6m} + (1 - d_{6m}) P_{M6m}, \\
f(0,1,0|\theta, \mathbf{d}_m) &= P_{7m} + (1 - d_{5m}) P_{M5m}, \\
f(0,0,0|\theta, \mathbf{d}_m) &= P_{8m} + (1 - d_{2m}) P_{M2m} + (1 - d_{3m}) P_{M3m} + (1 - d_{4m}) P_{M4m} + (1 - d_{7m}) P_{7m}.
\end{aligned}
$$

B. Illustrative Example Using Simulated Data

We illustrate our proposed estimation procedure using simulated data. We generated 1000 observations. Prior distributions of parameters of the choice probability models are assumed to be:

$$\begin{aligned} \beta &\sim \mathcal{N}(0, 10I_6) \\ \delta_i &\sim \mathcal{TN}_{(-\infty,0)}(-1, 10), \\ p_{im} &\sim i.i.d.Beta(1,1) \text{ for } i = 1, \ldots, 8, m = 1, \ldots M, \\ p_{im} &\sim i.i.d.Dirichlet(\mathbf{1}) \end{aligned}$$

where $\mathbf{1} = (1,1,1)'$. There are 2600 MCMC iterations after discarding 300 initial samples as the burn-in period. Table 6 shows the true value, posterior means, posterior standard deviations, 95% credible intervals, and inefficiency factors (IF). The true values of all coefficients are fairly close to the posterior mean estimates and are included in the 95% credible intervals.

Table 6: Simulation results
True value, posterior means, posterior standard deviations, 95% credible intervals, inefficiency factors (IF)

Prm.	True	Mean	Std	95% Interval	IF
β_{11}	0	0.041	0.141	(-0.235,0.332)	22
β_{12}	-1	-0.977	0.060	(-1.100,-0.864)	2
β_{21}	1	0.981	0.122	(0.747,1.243)	10
β_{22}	-1	-0.938	0.060	(-1.057,-0.824)	2
β_{31}	1.5	1.406	0.195	(1.027,1.782)	19
β_{32}	2.5	2.613	0.161	(2.310,2.930)	2
δ_1	-0.5	-0.656	0.174	(-1.011,-0.319)	23
δ_2	-0.5	-0.474	0.165	(-0.817,-0.150)	11
δ_3	-1	-1.006	0.246	(-1.502,-0.524)	19

The inefficiency factor is defined as $1 + 2\sum_{s=0}^{\infty} \rho_s$, where ρ_s is the sample autocorrelation at lag s calculated from the sampled values and is used to measure how well the chain mixes(see, e.g., Chib (2001)). Because it is the ratio of the numerical variance of the posterior sample mean to the variance of the posterior sample mean from the hypothetical uncorrelated draws, it suggests the relative number of correlated draws necessary to attain the same variance of the posterior sample mean from uncorrelated draws. The obtained inefficient factors are 1-22, indicating that the chain mixes very well. The acceptance rates for the independent Metropolis-Hastings algorithm are as high as 85%, for all parameters.

C. Estimation Result of Model M5

In Table 5, model M5 has the second smallest DIC. In this section, we will introduce the result of Model M5. Table 7 shows estimation results for model M5, which are true where $\delta_{ANA} > 0, \delta_{JAL} > 0$, and $\delta_{JAS} < 0$. Model M5 assumes that ANA and JAL have a positive effect when the other airline enters the market but that only JAS has a negative effect from the entrance of other airlines. Table 7 shows posterior means, posterior standard deviations, 95% credible intervals, and inefficiency factors.

Table 7: Estimation results for model M5
Posterior means, posterior standard deviations, 95% credible intervals, inefficiency factors.

Parameter	Mean	Std	95% Interval	IF
ANA				
Constant**	-2.668	0.224	(-3.122, -2.221)	2
Distance	-0.203	0.161	(-0.520, 0.109)	1
Population*	-0.378	0.149	(-0.659, -0.079)	1
$\sqrt{\text{Population}}$	0.234	0.132	(-0.024, 0.485)	1
Train**	-0.098	0.036	(-0.173, -0.028)	2
Flight ANA**	0.329	0.037	(0.257, 0.404)	2
δ_{ANA}**	0.581	0.299	(0.073, 1.222)	4
JAL				
Constant**	-4.999	0.654	(-6.364, -3.828)	3
Distance**	1.082	0.338	(0.437, 1.761)	2
Population*	-0.593	0.260	(-1.109, -0.065)	2
$\sqrt{\text{Population}}$**	0.897	0.273	(0.371, 1.448)	1
Train	-0.043	0.046	(-0.133, 0.046)	1
Flight JAL**	0.211	0.069	(0.068, 0.345)	3
δ_{JAL}**	1.874	0.354	(1.201, 2.565)	13
JAS				
Constant**	-3.502	0.316	(-4,145,-2.917)	2
Distance	0.069	0.212	(-0.357,0.479)	1
Population**	-0.533	0.163	(-0.858,-0.212)	2
$\sqrt{\text{Population}}$**	0.707	0.158	(0.407,1.031)	2
Train**	-0.186	0.040	(-0.265,-0.109)	2
Flight JAS**	0.366	0.040	(0.286,0.449)	3
δ_{JAS}**	-0.248	0.183	(-0.681,-0.011)	6

From Table 7, the estimation results of model M5 are very similar to those of model M8. The signs of all parameters' posterior means except δ_{JAS} in model M5 are the same as those of model M8 and the posterior means in model M8 are included in the 95% credible interval in model M5. δ_{JAS} in model M5 and model M8 are restricted in their signs to negative and positive, respectively. As shown in Table 4, the 95% credible interval and posterior mean of δ_{JAS} in model M8 are very close to zero. In Table 7, those of δ_{JAS} in model M5 are also very close to zero. These results suggest that the entry decision of JAS is not affected by the other airlines.