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Optimal Mechanism Design: Type-Independent Preference Orderings

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Abstract

We investigate revenue maximization in general allocation problems with incomplete information, where we assume quasi-linearity, private values, independent type distributions, and single-dimensionality of type spaces. We require a mechanism to satisfy strategy-proofness and ex-post individual rationality. We assume that each player has a type-independent preference ordering over deterministic allocations. We show that the Myerson's technique to solve the incentive-constrained revenue maximization problem in single-unit auctions can be applied to general allocation problems, where the incentive-constrained revenue maximization problem can be reduced to the simple maximization problem of the sum of players' marginal revenues without imposing any incentive constraint.

Keywords: Revenue Maximization, Strategy-Proofness, Ex-Post Individual Rationality, Type-Independent Preference Orderings, Marginal Revenue. Myerson's Reduction Technique.
JEL Classification Numbers: D44, D61, D82.

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1. Introduction

This paper investigates revenue maximization in general allocation problems with incomplete information under the assumptions of expected utility, quasi-linearity, private values, independent type distributions, and the single-dimensionality of type spaces. The seminal work by Myerson (1981) considered single-unit auctions and demonstrated a tractable reduction technique to solve the incentive-constrained revenue maximization problem; Myerson showed a sufficient condition under which the revenue maximization problem under the constraints of Bayesian incentive compatibility and interim individual rationality can be reduced to the maximization problem of the sum of players' marginal revenues (virtual valuations). The latter problem is much simpler to be solved than the original problem, because it imposes no explicit incentive constraints. Without any substantial modification, this technique can be applied to the revenue maximization problem in single-unit auctions that imposes strategy-proofness (SP) and ex-post individual rationality (EPIR), instead of Bayesian incentive compatibility and interim individual rationality, respectively.

With imposing SP and EPIR, this paper clarifies the possibility that the Myerson's reduction technique is extended to more general allocation problems. The main contribution of this paper is to demonstrate the following informational condition that guarantees this reduction technique to be available even in general allocation problems. Suppose that players have their respective preference orderings over (deterministic) allocations that are independent of their types. In this case, the central planner knows their preference orderings in advance, but does not know their valuations in the absolute and relative terms. With this supposition, we can generalize the definition of marginal revenues and the sufficient condition addressed by Myerson (1981), and then show that the revenue maximization problem under the constraints of SP and EPIR can be reduced to the maximization problem of the sum of players' marginal revenues without imposing any explicit incentive constraint.

There are a number of previous works that applied the Myerson's reduction technique to various private good allocation problems. For instance, Myerson and Satterthwaite (1983) investigated bilateral bargaining with single-unit commodity, where the intermediator maximized his (or her) expected revenue under the constraints of Bayesian incentive compatibility and interim individual rationality[3]. Maskin and Riley (1989) investigated the seller's revenue maximization in multi-unit auctions. We can also apply this technique to position auctions, or sponsored search auctions, in which the search engine maximizes his (or her) expected revenue by allocating heterogeneous positions across advertisers[4].

These works commonly assumed that each player has a type-independent preference ordering over allocations assigned to him (or her). This implies that we can regard these works as the special cases of this paper's framework.

These works, however, commonly assumed that players' preference orderings are the same with each other. In contrast, this paper allows for the heterogeneity across players in term of preference ordering. For instance, while the previous study of position auctions generally assumed that advertisers have the same preference orderings over positions with each other, this paper does not need to make such assumptions. Because of this allowance for heterogeneity, this paper's framework includes general combinatorial allocation problems, in which players have heterogeneous preferences concerning substitutes and complements.

Moreover, the above-mentioned previous works did not allow for the presence externality. In contrast, this paper allows for the presence of externality in the manner that each player's welfare is influenced by not only the allocation assigned to himself but also the allocations to the other players.

Figueroa and Skreta (2012) and Ulku (2013) investigated general combinatorial allocations that allow for substitutes, complements, and externality. These papers implied that it is generally impossible to apply the Myerson's reduction technique to such general problems in a tractable manner. In contrast, this paper clarifies the informational condition under which the Myerson's reduction technique can be directly applied to such general problems. This informational condition implies that the central planner knows player's preference orderings over deterministic allocations in advance,

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[3] See Matsushima (2012) for the multiunit case.

[4] The relevant works are Edelman, Ostrovsky, and Schwarz (2007), Edelman and Schwarz (2010), and others.

but does not know their valuations, productivities, or profitabilities, in the absolute and relative terms.

The organization of this paper is as follows. Section 2 explains the model and the type-independence of preference ordering. Section 3 defines the revenue maximization problem under the constraints of strategy-proofness and ex-post individual rationality, defines the concept of marginal revenue, and shows the main theorem.

2. The Model

We investigate the following allocation problem with incomplete information, where we assume expected utility, quasi-linearity, private values, independent type distributions, and single-dimensionality of type spaces. A denotes the non-empty and finite set of all (deterministic) allocations. $N = \{1,...,n\}$ denotes the non-empty and finite set of all players, where $n \geq 2$. Each player $i \in N$ has a single-dimensional type space $\Omega_i \equiv [0,1]$. His (or her) type $\omega_i \in \Omega_i$ is randomly and independently drawn according to a probability density function $p_i(\omega_i) > 0$. Each player $i's$ payoff is given by $v_i(a,\omega_i) - t_i$, where $a \in A$ denotes the selected allocation, $t_i \in R$ denotes the monetary payment from player i to the central planner, and $v_i : A \times \Omega_i \to R$ denotes his type-dependent valuation function. We assume that v_i is differentiable in $\omega_i \in \Omega_i$. Let $v_{i2}(a,\omega_i) \equiv \dfrac{\partial v_i(a,\omega_i)}{\partial \omega_i}$.

Assumption 1: For every $i \in N$, $\omega_i \in (0,1]$, $\omega_i' \in (0,1]$, $a \in A$, and $a' \in A$,

(1) $$[v_i(a,\omega_i) = v_i(a',\omega_i)] \Rightarrow [v_i(a,\omega_i') = v_i(a',\omega_i')],$$

and

(2) $$[v_i(a,\omega_i) > v_i(a',\omega_i)] \Rightarrow [v_i(a,\omega_i') > v_i(a',\omega_i')].$$

Assumption 1 implies that each player $i \in N$ has a type-independent preference ordering $\succsim_i$ over deterministic allocations, where for every $\omega_i \in (0,1]$, $a \in A$, and $a' \in A$,

$$[v_i(a,\omega_i) > v_i(a',\omega_i)] \Leftrightarrow [a \succ_i a'],$$

and

$$[\, v_i(a,\omega_i) = v_i(a',\omega_i)\,] \Leftrightarrow [\, a \sim_i a'\,].$$ [5][6]

Since each player's preference ordering $\succsim_i$ over deterministic allocations is independent of his (or her) type ω_i, the central planner knows the profile of players' preference orderings $(\succsim_i)_{i\in N}$ in advance. However, the central planner does not know their valuations in the absolute and relative terms.

Assumption 2: For every $i \in N$, $\omega_i \in \Omega_i$, and $a \in A$,

(3) $$v_{i2}(a,\omega_i) \geq 0,$$

and for every $a' \in A$ such that $a \succsim_i a'$,

(4) $$v_{i2}(a,\omega_i) \geq v_{i2}(a',\omega_i).$$

The inequalities (3) imply that each player $i's$ valuation $v_i(a,\omega_i)$ is non-decreasing in his type ω_i. The inequalities (4) imply that the difference in valuation $v_i(a,\omega_i) - v_i(a',\omega_i)$ is non-decreasing in his type ω_i, where $a \succsim_i a'$.

A *direct mechanism*, in short, a *mechanism*, is defined by (g, x), where $g : \Omega \to A$ denotes an *allocation rule*, $x = (x_i)_{i\in N} : \Omega \to R^n$ denotes a *payment rule*, and $x_i : \Omega \to R$. We denote by G the set of all allocation rules. We denote by X the set of all payment rules[7].

Strategy-Proofness (SP): For every $i \in N$, $\omega \in \Omega$, and $\omega_i' \in \Omega_i$,

$$v_i(g(\omega),\omega_i) - x_i(\omega) \geq v_i(g(\omega_i',\omega_{-i}),\omega_i) - x_i(\omega_i',\omega_{-i}).$$

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[5] We denote $a \succ_i a'$ if and only if $a \succsim_i a'$ and $a' \not\succsim_i a$. We denote $a \sim_i a'$ if and only if $a \succsim_i a'$ and $a' \succsim_i a$.

[6] Assumption 1 allows for the case that $v_i(a,0) = v_i(a',0)$, but $v_i(a,\omega_i) \neq v_i(a',\omega_i)$ for all $\omega_i \in (0,1]$.

[7] We denote $\Omega \equiv \times_{i\in N} \Omega_i$, $\omega \equiv (\omega_i)_{i\in N} \in \Omega$, $\Omega_{-i} \equiv \times_{j\in N\setminus\{i\}} \Omega_j$, and $\omega_{-i} \equiv (\omega_j)_{j\in N\setminus\{i\}} \in \Omega_{-i}$.

An allocation rule g is said to be *monotonic* if for every $i \in N$, $\omega \in \Omega$, and $\omega_i' \in \Omega_i$,

$$g(\omega_i', \omega_{-i}) \succsim_i g(\omega) \quad \text{if} \quad \omega_i' > \omega_i .$$

Monotonicity implies that the greater player $i's$ type ω_i is, the more preferable to this player the selected allocation is. Let us denote by $\hat{G} \subset G$ the set of all monotonic allocation rules.

Proposition 1: *A mechanism (g, x) satisfies SP if g is monotonic and there exists $d_i : \Omega_{-i} \to R$ for each $i \in N$ such that*

(5) $$x_i(\omega) = v_i(g_i(\omega), \omega_i) - \int_{s_i=0}^{\omega_i} v_{i2}(g_i(s_i, \omega_{-i}), s_i) ds_i + d_i(\omega_{-i}) \quad \textit{for all} \quad \omega \in \Omega .$$

If (g, x) satisfies SP, there exists $d_i : \Omega_{-i} \to R$ for each $i \in N$ that satisfies (5).

Proof: Suppose that (g, x) satisfies monotonicity and (5). Then, from (4), it follows that for every $i \in N$, $\omega \in \Omega$, and $\omega_i' \in \Omega_i$, if $\omega_i > \omega_i'$, then

$$v_i(g(\omega), \omega_i) - x_i(\omega) - \{v_i(g(\omega_i', \omega_{-i}), \omega_i') - x_i(\omega_i', \omega_{-i})\}$$

$$= \int_{s_i=\omega_i'}^{\omega_i} v_{i2}(g(s_i, \omega_{-i}), s_i) ds_i \leq v_i(g(\omega), \omega_i) - v_i(g(\omega), \omega_i') ,$$

which implies that

$$v_i(g(\omega), \omega_i') - x_i(\omega) \leq v_i(g(\omega_i', \omega_{-i}), \omega_i') - x_i(\omega_i', \omega_{-i}) .$$

In the same manner,

$$v_i(g(\omega), \omega_i) - x_i(\omega) - \{v_i(g(\omega_i', \omega_{-i}), \omega_i') - x_i(\omega_i', \omega_{-i})\}$$

$$= \int_{s_i=\omega_i'}^{\omega_i} v_{i2}(g(s_i, \omega_{-i}), s_i) ds_i \geq v_i(g(\omega_i', \omega_{-i}), \omega_i) - v_i(g(\omega_i', \omega_{-i}), \omega_i') ,$$

which implies that

$$v_i(g(\omega), \omega_i) - x_i(\omega) \geq v_i(g(\omega_i', \omega_{-i}), \omega_i) - x_i(\omega_i', \omega_{-i}) .$$

Hence, we have proved that (g, x) satisfies SP.

We define

$$y_i(\omega_i', \omega) \equiv v_i(g(\omega_i', \omega_{-i}), \omega_i) - x_i(\omega_i', \omega_{-i})$$

and

$$y_i^*(\omega) \equiv v_i(g(\omega), \omega_i) - x_i(\omega).$$

Suppose that (g, x) satisfies SP. Then, $y_i(\omega)$ is absolutely continuous in ω_i, implying that there exist $\rho_i : \Omega \to R$ and $d_i : \Omega_{-i} \to R$ such that for every $\omega \in \Omega$,

$$y_i^*(\omega) = \int_{s_i=0}^{\omega_i} \rho_i(s_i, \omega_{-i}) ds_i - d_i(\omega_{-i}).$$

The envelope theorem[8] implies

$$\rho_i(\omega) = v_{i2}(g(\omega), \omega_i),$$

and therefore,

$$y_i^*(\omega) = v_i(g(\omega), \omega_i) - x_i(\omega) = \int_{s_i=0}^{\omega_i} v_{i2}(g(\omega), \omega_i) ds_i - d_i(\omega_{-i}),$$

implying (5).

Q.E.D.

We assume that each player has the outside opportunity $U_i(\omega_i) \in R$ that is contingent on his type $\omega_i \in \Omega_i$. We assume that there exists a 'status quo' allocation $e \in A$ such that for every $i \in N$ and every $\omega_i \in \Omega_i$,

$$U_i(\omega_i) = v_i(e, \omega_i).$$

Ex-Post Individual Rationality (EPIR): For every $i \in N$ and $\omega \in \Omega$,

$$v_i(g_i(\omega), \omega_i) - x_i(\omega) \geq v_i(e, \omega_i).$$

[8] See Milgrom and Segal (2002).

3. Revenue Maximization

We define the *marginal revenue* (virtual valuation) for player $i \in N$ associated with $(a,\omega_i) \in A \times \Omega_i$, denoted by $MR_i(a,\omega_i)$, as follows:

$$MR_i(a,\omega_i) \equiv v_i(a,\omega_i) - \frac{1-P_i(\omega_i)}{p_i(\omega_i)} v_{i2}(a,\omega_i) \qquad \text{if } a \underset{\sim i}{\succ} e,$$

and

$$MR_i(a,\omega_i) \equiv v_i(a,\omega_i) - \frac{v_{i2}(e,\omega_i)}{p_i(\omega_i)} + \frac{P_i(\omega_i)}{p_i(\omega_i)} v_{i2}(a,\omega_i) \qquad \text{if } e \underset{\sim i}{\succ} a.$$

The marginal revenue $MR_i(a,\omega_i)$ implies the valuation $v_i(a,\omega_i)$ minus the informational rent given by

$$\frac{1-P_i(\omega_i)}{p_i(\omega_i)} v_{i2}(a,\omega_i)$$

and the bargaining rent given by

$$\frac{\max[v_{i2}(e,\omega_i) - v_{i2}(a,\omega_i), 0]}{p_i(\omega_i)}.$$

Note that the above-defined bargaining rent is positive only if player i prefers e to a. We assume that $MR_i(a,\omega_i)$ is differentiable in ω_i. Let $MR_{i2}(a,\omega_i) \equiv \frac{\partial MR_i(a,\omega_i)}{\partial \omega_i}$.

The following proposition holds straightforwardly from Proposition 1 and the definition of EPIR.

Proposition 2: *Suppose that a mechanism* (g,x) *satisfies SP. Then, it satisfies EPIR if and only if for every* $i \in N$ *, the equalities (5) hold and*

(6) $$d_i(\omega_{-i}) \geq -\max_{\omega_i}\{v_i(e,\omega_i) - \int_{s_i=0}^{\omega_i} v_{i2}(g(s_i,\omega_{-i}),s_i)ds_i\} \quad \textit{for all} \quad \omega_{-i} \in \Omega_{-i}.$$

We define the revenue maximization problem as

(7) $$\max_{(g,x)\in G\times X} E[\sum_{i\in N} x_i(\omega)] \quad \text{subject to SP and EPIR.}$$

The following proposition states that the expected revenue induced by the revenue maximization problem (7) is equivalent to the expected value of the sum of players' marginal revenues.

Proposition 3: *If a mechanism* (g, x) *is a solution to the revenue maximization problem (7) and* g *is monotonic, then*

(8) $$x_i(\omega) = v_i(g(\omega), \omega_i) - \int_{s_i=0}^{\omega_i} v_{i2}(g(s_i, \omega_{-i}), s_i) ds_i$$

$$-\max_{\omega_i}\{v_i(e, \omega_i) - \int_{s_i=0}^{\omega_i} v_{i2}(g(s_i, \omega_{-i}), s_i) ds_i\} \text{ for all } i \in N \textit{ and all } \omega \in \Omega,$$

and

(9) $$E[\sum_{i \in N} x_i(\omega)] = E[\sum_{i \in N} MR_i(g(\omega), \omega_i)].$$

Proof: It is clear from Proposition 2 that the solution to the revenue maximization problem (7) satisfies (6) with equality. This along with (5) implies (8).

For each $i \in N$ and $\omega_{-i} \in \Omega_{-i}$, let us define $\omega_i(\omega_{-i}) \in \Omega_i$ as maximizing the value of $v_i(e_i, \omega_i) - \int_{s_i=0}^{\omega_i} v_{i2}(g(s_i, \omega_{-i}), s_i) ds_i$ in terms of ω_i. From the monotonicity and Assumption 2,

$$\omega_i(\omega_{-i}) = 1 \quad \text{if } e \succ_i g(\omega) \text{ for all } \omega_i \in [0,1],$$

$$\omega_i(\omega_{-i}) = 0 \quad \text{if } g(\omega) \succ_i e \text{ for all } \omega_i \in [0,1],$$

and

$$g(\omega_i(\omega_{-i}), \omega_{-i}) = e \quad \text{otherwise},$$

where

$$e \succ_i g(\omega) \text{ for all } \omega_i \in [0, \omega_i(\omega_{-i})),$$

and

$$g(\omega) \succsim_i e \text{ for all } \omega_i \in (\omega_i(\omega_{-i}), 1].$$

Hence, we can write

$$\max_{\omega_i}\{v_i(e,\omega_i)-\int_{s_i=0}^{\omega_i} v_{i2}(g(s_i,\omega_{-i}),s_i)ds_i\}$$

$$=\int_{s_i=0}^{\omega_i(\omega_{-i})}\{v_{i2}(e,s_i)-v_{i2}(g(s_i,\omega_{-i}),s_i)\}ds_i\ .$$

Since the type distributions are independent with each other, we can write

$$E[x_i(\omega)]=E[v_i(g(\omega),\omega_i)-\int_{s_i=0}^{\omega_i} v_{i2}(g(s_i,\omega_{-i}),s_i)ds_i$$

$$-\int_{s_i=0}^{\omega_i(\omega_{-i})}\{v_{i2}(e,s_i)-v_{i2}(g(s_i,\omega_{-i}),s_i)\}ds_i]\ .$$

Let us specify $z_i : A\times[0,1]\rightarrow R$ by

$$z_i(a,\omega_i)=0 \qquad\qquad \text{if } a\underset{\sim i}{\succ} e,$$

and

$$z_i(a_i,\omega_i)=v_{i2}(e,\omega_i)-v_{i2}(a,\omega_i) \qquad \text{if } e\underset{\sim i}{\succ} a\ .$$

Hence,

$$E[x_i(\omega)]=E[v_i(g(\omega),\omega_i)-\int_{s_i=0}^{\omega_i} v_{i2}(g(s_i,\omega_{-i}),s_i)ds_i$$

$$-\int_{s_i=0}^{1} z_i(g(s_i,\omega_{-i}),s_i)ds_i]$$

$$=E[v_i(g(\omega),\omega_i)-v_{i2}(g(\omega),\omega_i)\frac{\{1-P_i(\omega_i)\}}{p_i(\omega_i)}-\frac{z_i(g(\omega),\omega_i)}{p_i(\omega_i)}]\ .$$

Since

$$MR_i(a,\omega_i)=v_i(a,\omega_i)-v_{i2}(a,\omega_i)\frac{\{1-P_i(\omega_i)\}}{p_i(\omega_i)}-\frac{z_i(a,\omega_i)}{p_i(\omega_i)}\ ,$$

it follows that

$$E[x_i(\omega)]=E[MR_i(g(\omega),\omega_i)] \quad \text{for all } \ i\in N\ ,$$

which implies (9).

Q.E.D.

We assume that players' marginal revenues satisfy the following monotonic properties.

Assumption 3: For every $i \in N$ and $a \in A$,

$$MR_i(a, \omega_i) \text{ is non-decreasing in } \omega_i .$$

For every $i \in N$, $a \in A$, and $a' \in A$ such that $a' \succ_i a$,

$$MR_i(a', \omega_i) > MR_i(a, \omega_i) \text{ for all } \omega_i \in [0,1],$$

and

$$MR_i(a', \omega_i) - MR_i(a, \omega_i) \text{ is increasing in } \omega_i .$$

Assumption 3 imply that the marginal revenue $MR_i(a, \omega_i)$ and the difference in marginal revenue $MR_i(a', \omega_i) - MR_i(a, \omega_i)$ are monotonic in terms of ω_i, and the marginal revenue $MR_i(a, \omega_i)$ is monotonic in terms of a in the order of $\succ_i$.

With Assumptions 1, 2, and 3, we can show as the main theorem of this paper that the Myerson's reduction technique is available in general allocation problems; the incentive-constrained revenue maximization problem (7) can be reduced to the simple maximization problem of the sum of players' marginal revenues without imposing any explicit incentive constraint.

Theorem 4: *A mechanism* (g, x) *is a solution to the revenue maximization problem (7) if and only if*

$$(10) \qquad g(\omega) \in \arg\max_{a \in A} \sum_{i \in N} MR_i(a, \omega_i) \text{ for all } \omega \in \Omega ,$$

and x *is specified according to (8).*

Proof: From Proposition 3, it is enough to show that the allocation rule g specified by (10) is monotonic. Suppose that g is not monotonic. Then, there exist $i \in N$, $\omega \in \Omega$, and $\omega'_i > \omega_i$ such that

$$g(\omega) \succ_i g(\omega'_i, \omega_{-i}) .$$

From (10),

(11) $$MR_i(g(\omega_i', \omega_{-i}), \omega_i) - MR_i(g(\omega), \omega_i)$$
$$+\sum_{j \neq i} \{MR_j(g(\omega_i', \omega_{-i}), \omega_j) - MR_j(g(\omega), \omega_j)\} \leq 0 \text{ ,}$$

while

(12) $$MR_i(g(\omega_i', \omega_{-i}), \omega_i') - MR_i(g(\omega), \omega_i')$$
$$+\sum_{j \neq i} \{MR_j(g(\omega_i', \omega_{-i}), \omega_j) - MR_j(g(\omega), \omega_j)\} \geq 0 \text{ .}$$

From Assumption 3 and $g(\omega) \succ_i g(\omega_i', \omega_{-i})$,

$$MR_i(g(\omega_i', \omega_{-i}), \omega_i) - MR_i(g(\omega), \omega_i)$$
$$> MR_i(g(\omega_i', \omega_{-i}), \omega_i') - MR_i(g(\omega), \omega_i') \text{ ,}$$

which along with (11) implies

$$MR_i(g(\omega_i', \omega_{-i}), \omega_i') - MR_i(g(\omega), \omega_i')$$
$$+\sum_{j \neq i} \{MR_j(g(\omega_i', \omega_{-i}), \omega_j) - MR_j(g(\omega), \omega_j)\} < 0 \text{ .}$$

This contradicts (12).

Q.E.D.

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