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Ad Valorem Capital Tax Competition

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Abstract

Studies of tax competition have found that using a unit tax is commitment-robust for governments, while we observe ad valorem taxes on capital in practice. This study presents a model that explains the emergence of ad valorem capital tax competition, incorporating an elastic supply of capital in the standard tax competition model. Specifically, it shows that if the elasticity of capital supply is positive, governments adopt the ad valorem tax method and thereby ad valorem tax competition prevails. On the other hand, under a fixed capital supply (i.e., zero elasticity of capital supply), countries compete in unit taxes.

JEL classification: H21; H77.

Keywords: tax competition; unit tax; ad valorem tax; elastic capital supply.

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1 Introduction

Classical tax theory suggests that an ad valorem (ADV) tax is welfare superior to a unit (specific) tax in a monopoly [Suits and Musgrave (1953)]. An ADV tax reduces monopoly power in a price-setting environment and thus transfers more revenue from the monopoly to the government. Because of the deterrent effect on a firm's market power intrinsic to the ADV tax method, the equilibrium price will be lower with an ADV tax than with a unit tax, explaining the superiority of an ADV tax over a unit tax [Stiglitz (1988, p. 425)]. Since early research, many studies have reexamined and substantiated this argument, and one of them is the analyses that reconsider it in the open economy setting.[1] In most conventional studies, tax methods are compared within a single-region framework in which firms and consumers are forced to demand domestic resources irrespective of the level of prices and taxes. This is a somewhat extreme assumption since firms nowadays choose their location, consumers buy their goods, and investors shift their money across jurisdictions.

The concern over the choice of tax method in the open economy with multiple jurisdictions is inevitably linked to interregional fiscal competition analysis. The comparison of an ADV tax with a unit tax in a multiple-region framework was first presented by Lockwood (2004), where investors are able to shift the resources they own beyond the reach of the taxing authority. Based on the standard tax competition model, he studies the equilibrium properties under different tax regimes, i.e., unit taxes and ADV taxes, and finds the opposite result to the classical argument: Nash equilibrium taxes attain higher welfare when all regions employ unit taxes than when they employ ADV taxes. Akai et al. (2011) formally prove the superiority of the unit tax method in tax competition by endogenizing the tax method: selecting unit tax as a policy instrument is the dominant strategy of governments and is Pareto efficient.[2] Although the finding of the superiority of unit taxes in tax competition might be robust, this raises the issue of why governments use ADV taxes in practice.

This paper presents a model that addresses this unresolved problem. The answer to the question posed by Lockwood and Akai et al. is that unit taxes are most probably infeasible to implement. This might be acceptable by some measures, but we are not fully satisfied with this answer. In this study, we thus present a model that explains the appearance of an ADV tax in the tax competition environment, incorporating one missing key factor in the analysis, namely

[1] See Keen (1998) for a general review of the comparative analysis between these two tax methods.

[2] Departing from the symmetric tax competition model of Lockwood (2004) and Akai et al. (2011), Ogawa (2016) constructs an asymmetric model and presents the possibility that a country that imports capital chooses the ADV tax method. However, he still does not incorporate the elastic supply of capital. Oher studies have considered cross-border shopping, trade, and firm relocation in a two-country model when analyzing the effects of a unit tax and an ADV tax. See Lockwood and Wong (2000), Jorgensen and Schröder (2005), Schröder and Sørensen (2010), Aiura and Ogawa (2013), Akai et al. (2014), and Takatsuka (2014).

the elastic supply of capital. Although the theoretical literature on tax competition has drawn attention to the importance of the strategic choice of tax method before competing for mobile capital through the tax rate, it has generally done so by assuming a fixed supply of capital in the economy. This might be strange because the analysis clearly looks at inter-regional competition from a long-term perspective. The method of taxing capital is irreversible and therefore decisions on the tax method are long-term in nature, while tax rates can be changed more easily in the short-term. In this case, it is natural to assume that capital supply is inelastic in the short-term, but elastic, and thereby affected by tax policies, in the long run. However, previous studies do not account for the elastic supply of capital and its effect on the choice of tax method, which is the focus of this study.

The importance of the elastic supply of capital in tax competition was recently found by Eichner and Runkel (2012), who point to the important role that the elasticity of aggregate capital supply plays for the efficiency of decentralized policymaking.[3] By using a simplified version of the two-period approach constructed by Bucovetsky and Wilson (1991) and used by Eichner and Runkel (2012), we show the appearance of ADV tax competition in which governments employ the ADV tax method voluntarily in the standard tax competition model.

The remainder of the paper is organized as follows. In section 2, we set up the basic model. In section 3, the tax effects are analyzed and main results are derived. To make our arguments clear, we basically follow the model of Akai et al. (2011), just allowing for the elastic supply of capital. Finally, section 4 concludes.

2 The Model

We extend the symmetric two-jurisdiction model of Akai et al. (2011) so that the saving decision is endogenized, as in Bucovetsky and Wilson (1991), Keen and Kotsogiannis (2002), and Eichner and Runkel (2012).

Firms: Firms in jurisdiction i ($i = 1, 2$) are perfectly competitive and produce an identical product. Each firm uses capital and labor in production. They each have a production function $F(K_i, L_i)$, where K_i is the amount of capital and L_i is the amount of labor in jurisdiction i. We assume that F is homogeneous of degree one and normalize $L_1 = L_2 = 1$ without loss of generality. Then, we rewrite production function F as $f(k_i)$, where $k_i \equiv K_i/L_i$. Following the literature, we specify that $f(k_i)$ is quadratic with a linear marginal product of capital schedules:

[3] Eichner and Runkel (2012) study the effects of the endogenous supply of capital on the efficiency result found by Ogawa and Wildasin (2009) in which the decentralized tax competition equilibrium is efficient even if the analysis accounts for not only local pollution but also transboundary pollution.

$f(k_i) = (a - bk_i/2)k_i$, where $a, b > 0$.[4] Each unit of capital is rented on the economy-wide capital market at the net return to capital denoted by r. Each firm takes the capital tax rates and net return to capital r as given.

Governments: Governments levy taxes on the return to capital and choose between a unit tax and an ADV tax. Tax revenue g_i in each choice of tax instrument can be expressed as

$$\begin{cases} g_i = T_i k_i, & \text{if government } i \text{ uses a unit tax,} \\ g_i = t_i f_k k_i, & \text{if government } i \text{ uses an ADV tax,} \end{cases} \tag{1}$$

where T_i denotes a unit tax and t_i denotes an ADV tax in jurisdiction i. Each jurisdiction is assumed to maximize its tax revenue g_i.[5]

Investors: Investors are in the third country and live for two periods.[6] We model investors' behavior as that of the representative investor. In period 1, she has an initial endowment $\overline{k} > 0$, which can be spent on first-period consumption at rate x^1 or saved for second period consumption at rate $s = \overline{k} - x^1$. In period 2, she receives capital income $(1 + r)s$ and consumes it.

The utility of the representative investor $U(x^1, x^2)$ is given by

$$U(x^1, x^2) = x^1 + \beta x^2, \tag{2}$$

where $0 < \beta \leq 1$ is the discount factor.[7] Given the budget constraints of the first period $\overline{k} = x^1 + s$ and the second period $x^2 = (1 + r)s$, saving schedule $s(r)$ is obtained as follows:

$$\begin{cases} s(r) = 0 & \text{if } r < \theta \\ s(r) = \overline{k} & \text{if } r \geq \theta, \end{cases} \tag{3}$$

[4]The quadratic production function is widely used in the literature on capital tax competition. See, among others, Bucovetsky (1991), Itaya et al. (2008), Kempf and Rota-Graziosi (2010), Akai et al. (2011), and Janeba and Osterloh (2013).

[5]The current study considers a government that aims to maximize its tax revenue. This assumption is often made when analyzing the difference between a unit tax and an ADV tax in tax competition (e.g., Akai et al., 2011; Aiura and Ogawa, 2013), which can be justified on two grounds. First, the government may be a Leviathan, which has often been assumed in the literature on tax competition. Second, the government may face severe revenue shortfalls, meaning that raising the tax revenue becomes its primary concern.

[6]Eichner and Runkel (2012) assume that the local residents in each jurisdiction play the role of investors. Even if we alternatively assume that local residents invest, our result does not change. We assume that investors live outside the economy only for ease of exposition.

[7]Keen and Kotsogiannis (2002) and Eichner and Runkel (2012) assume $U(x^1, x^2) = u(x_1) + x_2$. We specify that $u(x_1)$ is x_1/β.

where $\theta \equiv 1/\beta - 1 > 0$. For simplicity, we assume that the investor saves all her endowment when she is indifferent between consumption and saving. The lower the investor's discount factor, the higher the net return to capital must be for saving to be positive.

Capital market: Capital is perfectly mobile and the capital market is globally integrated. Capital supply is determined by the amount of saving s, whereas capital demand follows from firms' profit maximization. Moreover, the net return to capital in each jurisdiction must be equalized because of perfect capital mobility. Therefore, the capital market equilibrium is characterized by the following equations:

$$\begin{cases} r = f_k(k_i) - T_i & \text{if government } i \text{ uses a unit tax,} \\ r = (1 - t_i) f_k(k_i) & \text{if government } i \text{ uses an ADV tax,} \end{cases} \tag{4}$$

$$k_1 + k_2 = s(r), \tag{5}$$

where $s(r)$ is given by (3).

3 Equilibrium Analysis

The timing of the game is as follows. In the first stage, each jurisdiction independently and simultaneously chooses whether it employs a unit tax or an ADV tax. In the second stage, both investors and jurisdictions act independently and simultaneously. Investors split their endowment between first-period consumption and saving given the tax rates and each jurisdiction sets its capital tax rate given the amount of saving. This timing reflects our conjecture that a government can commit to a tax method it chooses, while committing to a certain tax rate is infeasible. Shifting tax instruments takes a lot of time and cost, whereas changing tax rates seems much easier. Indeed, tax rates are subject to, for example, changes in political situations and macroeconomic conditions. Given this, it would be implausible to assume that a government can commit to future tax rates, while commitment to tax instruments is plausible.[8]

The game is solved from the second stage, following the standard backward induction process.

[8] Bucovetsky and Wilson (1991) and subsequent studies assume that a government can commit to future tax rates. We do not think, however, that this assumption is always plausible. Since the saving decision has an intertemporal nature, the game has a long-term perspective. A commitment device in such a game has to be effective for a long time. However, for various reasons, tax rates are often changed in an uncommitted way. Thus, we do not assume that a government can commit to future tax rates. We discuss this assumption further at the end of Section 4.

3.1 Second Stage

3.1.1 Tax Competition

Jurisdiction $i \neq j$ sets its capital tax rate given the tax rate of jurisdiction j and the amount of saving s. Thus, as long as $s > 0$, the equilibrium tax rates are obtained from Lemma 1 of Akai et al. (2011), in which the total amount of capital is exogenously given. When $s = 0$, there is nothing to analyze: regardless of whether each jurisdiction uses a unit tax or an ADV tax, any tax rate is optimal with zero tax revenue. We summarize the arguments so far in the following lemma.[9]

Lemma 1 (*Akai et al., 2011*). *Given the amount of saving s, the equilibrium tax rates in the second stage are obtained as follows:*

(i) *When $s > 0$ and both jurisdictions adopt unit taxes,*

$$T_i = bs, \;\; i = 1, 2. \tag{6}$$

(ii) *When $s > 0$ and both jurisdictions adopt ADV taxes,*

$$t_i = \frac{bs}{a}, \;\; i = 1, 2. \tag{7}$$

(iii) *When $s > 0$, jurisdiction i uses a unit tax and jurisdiction $j \neq i$ uses an ADV tax,*

$$T_i = \frac{bs}{2} + \frac{H}{4}, \text{and } t_j = \frac{H}{2(a - bs)} \tag{8}$$

where $H \equiv 6a - bs - \sqrt{25a^2s^2 + 36a^2 - 36abs}$.

(iv) *When $s = 0$, regardless of whether each jurisdiction uses a unit tax or an ADV tax, any tax rate constitutes an equilibrium.*

Once tax rates are determined, by using (4) and (5), we can obtain the net return to capital in each case (see Appendix A). Since Lemma 1 shows that tax rates are functions of saving s, the net return to capital can be expressed as a function of s. We denote $r_{UU}(s)$ ($r_{AA}(s)$, $r_{UA}(s)$) as the net return to capital when both jurisdictions use unit taxes (both jurisdiction uses ADV taxes, jurisdiction i uses a unit tax, while jurisdiction $j \neq i$ uses an ADV tax). Given the amount of saving s, we have the following result.

Lemma 2. *For any $s > 0$, $r_{UU}(s) < r_{UA}(s) < r_{AA}(s)$.*

[9] Following Akai et al. (2011), we impose $2a > 3b\bar{k}$ to make the solutions meaningful in the sense that capital demands and the post-tax rate of return to capital are positive in all cases with $s > 0$.

Proof. Following Lockwood (2004, p. 768), we convert a unit tax into an ADV tax by using the formula $T_i = t_i f_k(k_i)$. By applying this to the equilibrium tax rates in Lemma 1, we can show that $t_i^{AA} < t_i^{UA} < t_i^{AU} < t_i^{UU}$, where the first superscript denotes the tax instrument of jurisdiction i and the second one denotes that of jurisdiction $j \neq i$.[10]

Now, suppose one jurisdiction uses a unit tax, while the other uses an ADV tax. Then, the jurisdiction that uses an ADV tax has less than half of the capital in the market since $t_i^{UA} < t_i^{AU}$. That is, $k_i^{AU} < k_i^{UU} = s/2$. Then, we have $r_{UU} = (1 - t_i^{UU}) f_k(k_i^{UU}) < (1 - t_i^{AU}) f_k(k_i^{AU}) = r_{UA}$.

Similarly, the jurisdiction that uses a unit tax has more than half of the capital in the market since $t_i^{UA} < t_i^{AU}$. That is, $k_i^{UA} > k_i^{AA} = s/2$. Thus, we have $r_{UA} = (1 - t_i^{UA}) f_k(k_i^{UA}) < (1 - t_i^{AA}) f_k(k_i^{AA}) = r_{AA}$. Q.E.D.

Lemma 2 shows that the net return to capital increases as the number of jurisdictions that adopt an ADV tax increases. This is because committing to an ADV tax works as a means of committing to a lower future tax rate. We do not allow for commitment to certain tax rates as this seems infeasible. However, committing to a certain tax instrument works as a substitute commitment device. Thus, the equilibrium tax rate becomes lower when a jurisdiction uses an ADV tax, which increases the net return to capital.

In Lockwood (2004) and Akai et al. (2011), the extent to which the tax method influences the net return to capital is neglected because it cannot affect the tax revenue. In the current study, however, it matters because it affects the tax revenue by changing saving decisions.

3.1.2 Decision on Investment

By obtaining the equilibrium net return to capital given s, we turn to analyzing investors' behavior. A representative investor splits her endowment between consumption and saving given the tax rates and thus the net return to capital. This behavior is characterized by the optimal saving schedule $s(r)$ defined in (3). From (3), the optimal saving is either 0 or $\overline{k}$ depending on the net return to capital.

3.1.3 Equilibrium Conditions

To derive Lemma 1 and Lemma 2, we took the amount of saving s as given. However, since we endogenize the saving decision, an equilibrium in the second stage requires that the equilibrium amount of saving satisfies some equilibrium conditions. We discuss this point next.

[10] For the formal proof, see the working paper version of Akai et al. (2011) (Osaka University Working Paper no. 10-01).

Lemma 1 determines the net return to capital given the tax instruments and amount of saving s. By contrast, the amount of saving depends on the net return to capital. In the equilibrium, the amount of saving is such that the net return to capital implied by Lemma 1 and optimal investment condition (3) become consistent. More formally, by letting s^* denote the amount of saving in the equilibrium, the following conditions must be satisfied:

$$\begin{cases} s^* = s(r_{UU}(s^*)) & \text{if both jurisdictions use unit taxes,} \\ s^* = s(r_{UA}(s^*)) & \text{if one jurisdiction uses a unit tax and the other uses an ADV tax,} \\ s^* = s(r_{AA}(s^*)) & \text{if both jurisdictions use ADV taxes.} \end{cases} \tag{9}$$

If no equation holds in (9), the capital supply and amount of capital in the market are different. Such a situation is implausible. From (3) and (9), equilibrium saving s^* is either 0 or $\overline{k}$. If the net return to capital exceeds θ when saving is $\overline{k}$, then this saving is consistent with the equilibrium. If not, $s^* = \overline{k}$ cannot be supported in an equilibrium since investors do not act optimally.

Since the net return to capital depends on whether each government uses a unit tax or an ADV tax, the choice of tax instrument does affect saving. Lemma 3 formalizes this point.

Lemma 3. *The possible equilibria in the second stage are classified as follows:*

(i) *If $\theta \leq r_{UU}(\overline{k})$, the optimal investment $s^* = \overline{k}$ is consistent with an equilibrium for any tax regime.*

(ii) *If $r_{UU}(\overline{k}) < \theta \leq r_{UA}(\overline{k})$, the optimal investment $s^* = \overline{k}$ is consistent with an equilibrium if and only if at least one jurisdiction uses an ADV tax. If both jurisdictions use unit taxes, s^* must be zero.*

(iii) *If $r_{UA}(\overline{k}) < \theta \leq r_{AA}(\overline{k})$, the optimal investment $s^* = \overline{k}$ is consistent with an equilibrium if and only if both jurisdictions use ADV taxes. Otherwise, s^* must be zero.*

(iv) *If $r_{AA}(\overline{k}) < \theta$, the optimal investment s^* must be zero under any tax regime.*

Proof. These results directly follow from Lemma 2 and (9). Q.E.D.

Lemma 3 states that an equilibrium with $s^* = \overline{k}$ may exist. Whether this is the case depends on the tax regime. As the number of jurisdictions using ADV taxes increases, the net return to capital increases, as shown in Lemma 2. A high net return to capital makes investment more lucrative. Consequently, $s^* = \overline{k}$ becomes more likely to be consistent with an equilibrium as the number of jurisdictions using an ADV tax increases. In cases (i) and (iv), whether $s^* = \overline{k}$ is consistent with an equilibrium does not depend on the number of jurisdictions using an ADV tax. However, in cases (ii) and (iii), whether $s^* = \overline{k}$ is consistent with an equilibrium does depend on the number of jurisdictions using an ADV tax. Therefore, using an ADV tax has a "saving-inducing effect," while the choice of a unit tax has a "saving-reducing effect." These properties have not been noted in studies comparing a unit tax with an ADV tax; hence, Lemma 3 provides a new perspective on the comparison between a unit tax and an ADV tax.

On the contrary, no saving ($s^* = 0$) becomes consistent with an equilibrium in any case.[11] Therefore, when $s^* = \overline{k}$ is consistent with an equilibrium, there are two kinds of equilibria: one with $s^* = \overline{k}$ and one with $s^* = 0$. Here, an equilibrium selection issue arises. To highlight the fact that changing tax regime may change the saving decision, we assume that when $s^* = \overline{k}$ is consistent with an equilibrium, an equilibrium with $s^* = \overline{k}$ is selected.[12]

3.2 First Stage

In the first stage, each government determines whether it uses a unit tax or an ADV tax. Let $g_i^{UA}(s)$ denote the equilibrium tax revenue of jurisdiction i with capital supply s when jurisdiction i uses a unit tax and jurisdiction $j \neq i$ uses an ADV tax. $g_i^{AU}(s), g_i^{UU}(s)$, and $g_i^{AA}(s)$ are defined in a similar manner. The equilibrium tax revenues are formally given in Appendix A.

In Table 1, we complete the payoff matrix to find the equilibria in the four cases, where the first (second) coordinate in each pair represents the tax revenue in country 1 (2).

	Unit	ADV
Unit	$g_1^{UU}(s), g_2^{UU}(s)$	$g_1^{UA}(s), g_2^{AU}(s)$
ADV	$g_1^{AU}(s), g_2^{UA}(s)$	$g_1^{AA}(s), g_2^{AA}(s)$

Table 1: Payoff matrix

Case 1: $\theta \leq r_{UU}(\overline{k})$. In this case, the interest rate is sufficiently high or investors are sufficiently patient such that investors save all of their endowment regardless of the tax regime: $s = \overline{k}$. The payoff matrix in case 1 is therefore given as in Table 2.

In case 1, capital supply is fixed at $\overline{k}$ regardless of the choice of tax instrument, reducing our analysis to the fixed capital supply model of Akai et al. (2011). In this case, as in Proposition 1 of Akai et al. (2011), we can easily find that choosing a unit tax is the dominant strategy ($g_i^{UU} > g_i^{AU}$ and $g_i^{UA} > g_i^{AA}$); both jurisdictions choose unit taxes in the first stage. We also

[11] Suppose that when $s = 0$, both jurisdictions set extremely high tax rates. Lemma 1(iv) shows that this forms the equilibrium in tax competition. Moreover, the consistency requirement (9) is satisfied because r can become negative and hence no investment is made. Thus, an equilibrium with $s^* = 0$ always exists.

[12] Note that when $s^* = \overline{k}$ is consistent with an equilibrium, the equilibrium with $s^* = \overline{k}$ brings higher utility for investors and jurisdictions than $s^* = 0$ does. That is, the equilibrium with $s^* = \overline{k}$ Pareto dominates that with $s^* = 0$.

	Unit	ADV
Unit	$g_1^{UU}(\bar{k}), g_2^{UU}(\bar{k})$	$g_1^{UA}(\bar{k}), g_2^{AU}(\bar{k})$
ADV	$g_1^{AU}(\bar{k}), g_2^{UA}(\bar{k})$	$g_1^{AA}(\bar{k}), g_2^{AA}(\bar{k})$

Table 2: Payoff matrix: Case 1 $[\theta \leq r_{UU}(\bar{k})]$

find that the revenue of both jurisdictions is higher when they choose unit taxes than when they choose ADV taxes ($g_i^{UU} > g_i^{AA}$).

We follow Akai et al. (2011) to explain the intuition behind the result that jurisdictions use a unit tax when the total amount of capital is fixed. The fiscal externality is inherent in capital tax competition, which leads jurisdictions to choose an inefficiently low level of capital tax rate. Jurisdiction i can gain by making jurisdiction $j \neq i$ set a higher tax rate to ease the fiscal externality problem. However, choosing an ADV tax forces rival jurisdictions to face higher tax elasticity, leading them to set a lower tax rate, which worsens the situation.[13] Therefore, to avoid the escalation of the fiscal externality problem, a jurisdiction avoids choosing an ADV tax and commits itself to choosing a unit tax.

Case 2: $r_{UU}(\bar{k}) < \theta \leq r_{UA}(\bar{k})$. In this case, the amount of capital in the market becomes zero if both jurisdictions choose unit taxes. When the amount of capital is zero, the tax revenue is also zero since there is nothing to levy a tax on. In the other cases, capital supply is $\bar{k}$. Thus, we have the payoff matrix presented in Table 3.

	Unit	ADV
Unit	$0, 0$	$g_1^{UA}(\bar{k}), g_2^{AU}(\bar{k})$
ADV	$g_1^{AU}(\bar{k}), g_2^{UA}(\bar{k})$	$g_1^{AA}(\bar{k}), g_2^{AA}(\bar{k})$

Table 3: Payoff matrix. Case 2 $[r_{UU}(\bar{k}) < \theta \leq r_{UA}(\bar{k})]$

In this case, $g_1^{UA}(\bar{k}), g_2^{AU}(\bar{k}), g_1^{AU}(\bar{k}), g_2^{UA}(\bar{k}), g_1^{AA}(\bar{k})$, and $g_2^{AA}(\bar{k})$ are all positive. Moreover, Proposition 1 of Akai et al. (2011) reveals that $g_i^{UA}(\bar{k}) > g_i^{AA}(\bar{k})$. Thus, there are two pure-strategy equilibria in which one government jurisdiction adopts an ADV tax and the other employs

[13] This result follows because an ADV tax creates a side effect that makes the rival lower its tax rate. When jurisdiction i uses a unit tax, jurisdiction $j \neq i$ takes the unit tax rate T_i as given to determine its tax rate. Suppose, by contrast, that jurisdiction i uses an ADV tax. Its effective unit tax rate is written as $T_i = t_i f_k(k_i)$. To choose its tax rate, jurisdiction j takes t_i as given, but not k_i. If jurisdiction j lowers its tax rate, it increases k_i, which induces more capital inflow because the effective unit tax rate T_i lowers. Thus, the perceived elasticity of capital with respect to the tax rate of jurisdiction j is larger if jurisdiction i uses an ADV tax. This makes jurisdiction j lower its tax rate.

a unit tax.

The intuition behind this result is explained as follows. In this case, the net return to capital becomes too low to induce saving if both jurisdictions use unit taxes. If capital supply is fixed, using a unit tax is beneficial for a jurisdiction because it can mitigate the fiscal externality problem in tax competition. However, when the saving decision is endogenous, increasing the rival's tax rate has an important side effect: it decreases the net return to capital and may decrease the amount of capital in the market. In such a case, this "saving-reducing effect" of a unit tax is dominant if the rival uses a unit tax. That is, each jurisdiction wants to use an ADV tax if the rival uses a unit tax. On the contrary, if the rival uses an ADV tax, the net return to capital becomes sufficiently high that the saving-reducing effect is not significant. In this case, using a unit tax is beneficial because mitigating the fiscal externality problem becomes the primary concern. Therefore, the two jurisdictions select different tax methods.

Case 3: $r_{UA}(\overline{k}) < \theta \leq r_{AA}(\overline{k})$. The amount of capital in the market becomes zero unless both jurisdictions choose ADV taxes in this case. Thus, we have the payoff matrix shown in Table 4.

	Unit	ADV
Unit	$0, 0$	$0, 0$
ADV	$0, 0$	$g_1^{AA}(\overline{k}), g_2^{AA}(\overline{k})$

Table 4: Payoff matrix: Case 3 $[r_{UA}(\overline{k}) < \theta \leq r_{AA}(\overline{k})]$

$g_1^{AA}(\overline{k})$ and $g_2^{AA}(\overline{k})$ are positive. Thus, the pure-strategy equilibria are that both jurisdictions use the same tax method, namely either the unit tax method or the ADV tax method. However, we claim that the latter equilibrium is much more plausible because choosing an ADV tax weakly dominates choosing a unit tax for each jurisdiction.

Intuitively, in this case, the saving-reducing effect of a unit tax is severe: if one jurisdiction uses a unit tax, the net return to capital becomes so low that saving is zero. Thus, if the rival uses an ADV tax, the jurisdiction has an incentive to adopt an ADV tax because it increases saving significantly. By contrast, suppose the rival uses a unit tax. There is no benefit to use a unit tax to mitigate the "race to the bottom" in tax competition because the amount of capital in the market is zero. The saving-inducing effect of an ADV tax is also zero because the net return to capital remains too high to induce saving as long as the rival uses a unit tax. Thus, a jurisdiction is indifferent between using a unit tax and an ADV tax. As a result, an ADV tax weakly dominates a unit tax.

Case 4: $r_{AA}(\overline{k}) < \theta$. In this case, under any tax regime, investors do not save at all irrespective of the tax regime because the net return to capital is too low or investors place too much weight on current consumption. The tax revenue is always zero because there is no tax base. Consequently, the choice of tax instrument does not matter. Any strategy profile constitutes a Nash equilibrium.

We summarize the analysis so far as our main proposition.

Proposition 1. *The equilibrium choice of tax instruments in the first stage is as follows:*

(i) *If $\theta \leq r_{UU}(\overline{k})$, a unit tax strictly dominates an ADV tax for each jurisdiction. Thus, both jurisdictions use unit taxes in the equilibrium.*

(ii) *If $r_{UU}(\overline{k}) < \theta \leq r_{UA}(\overline{k})$, one jurisdiction uses a unit tax and the other uses an ADV tax.*

(iii) *If $r_{UA}(\overline{k}) < \theta \leq r_{AA}(\overline{k})$, an ADV tax weakly dominates a unit tax for each jurisdiction. Thus, both jurisdictions use ADV taxes in the plausible equilibrium.*

(iv) *If $r_{AA}(\overline{k}) < \theta$, any choice of tax instruments becomes an equilibrium because capital is never supplied to the market.*

Case (i) corresponds to the result presented by the fixed-capital supply model of Akai et al. (2011) and cases (ii)–(iv) are derived in our model as this involves the elasticity of capital supply. By generalizing the aspect of capital supply, we show that the equilibrium pattern derived by Akai et al. (2011) prevails if investors always spend their resources on saving, irrespective of the tax method adopted by the jurisdictions. Our extension further shows that if saving and thereby the capital supply depend on interest rates, which are affected by the tax method chosen by the jurisdictions, ADV tax competition prevails.

4 Concluding Remarks

In this study, we reexamined the conventional assumption on the tax method employed in the tax competition model. It has been assumed and also proven that governments compete on unit taxes, which leads us to ask why we do not observe unit tax competition in practice. In contrast to the traditional argument, the main result of our study sheds new light on the choice of tax instrument in tax competition analysis: by incorporating the elastic supply of capital into the standard model of tax competition, we show that ADV tax competition prevails as the subgame perfect Nash equilibrium when capital supply is elastic with respect to the (after-tax) price of capital. Conversely, unit tax competition prevails only when the supply of capital is inelastic, which would be acceptable from the short-term perspective, but violated in the long run. Overall, our study thus explains why countries employ the ADV tax method in a tax competition environment.

ADV taxes are widely used in practice, while the use of unit taxes is relatively limited. Motivated by today's fast pace of globalization, a growing literature is trying to explain the prevalence of ADV taxes under tax competition. While Aiura and Ogawa (2013) succeed in explaining the prevalence of ADV taxes under commodity tax competition, the prevalence of ADV taxes under capital tax competition remains a puzzle since Akai et al. (2011) find that a unit tax strictly dominates an ADV tax in the standard model of capital tax competition.[14] Our study presents a possible answer to this puzzle by introducing an important but often neglected aspect into capital tax competition, namely endogenous saving, as motivated by Bucovetsky and Wilson, (1991), Keen and Kotsogiannis (2002), and Eichner and Runkel (2012).

In closing the paper, two remarks are in order. First, we specify the functions of preferences and technologies, which considerably simplifies the analysis, to produce clear-cut results. More general formulations may quantitatively change the range of parameters that produce our results. Still, in rather general analysis, the equilibrium form of tax competition depends on the elasticity of capital supply, and ADV tax competition could emerge when the elasticity of capital supply is high. Second, we changed the assumption made in the literature by assuming that governments cannot commit to a future tax rate. Bucovetsky and Wilson, (1991), Keen and Kotsogiannis (2002), and Eichner and Runkel (2012) assume that the government sets tax rates by taking into account the effect of taxes on the amount of saving. We argued that such a commitment may be infeasible, while commitment to tax instruments may be feasible. In such a situation, commitment to an ADV tax works as a commitment to lower future tax rates. Our argument is that even when governments cannot commit to tax rates, the choice of tax instrument can be used as a commitment device; hence, we observe ADV tax competition. If a government can commit to future tax rates as assumed in the literature, our results may not be informative. Rather than committing to an ADV tax, a government could just commit to a lower tax rate. Thus, our results are valid when a government lacks a way of committing to a future tax rate, but can use the tax method as a commitment device.

[14] Recently, Hoffmann and Runkel (2016) also showed that in the context of capital tax competition, an equilibrium may be more efficient with ADV taxes than with unit taxes because an ADV tax is useful for absorbing rents from firms. They also noted that an ADV tax may be selected in the equilibrium even if the choice of tax instrument is endogenized. Although their study may explain why an ADV tax is observed in the real world, they take the saving decision as exogenous, unlike in the current study. In other words, the driving force of their result is different from that of ours.

Appendix A

For reference, we provide information about the tax revenues and net return to capital in the equilibrium. By taking the amount of saving s as given and using (1), (4), and (5), we obtain the following results.

Case 1: $s > 0$ and both jurisdictions adopt unit taxes.

In this case, tax rates are given by (6). By using (4) and (5), $k_1 = k_2 = s/2$ since both jurisdictions set the same tax rate. From (4), the net return to capital $r_{UU}(s)$ and tax revenues $g_1^{AA}(s)$ are, respectively, given as

$$r_{UU}(s) = \frac{2a - 3bs}{2}, \tag{A.1}$$

$$g_1^{UU}(s) = g_2^{UU}(s) = \frac{bs^2}{2}. \tag{A.2}$$

Case 2: $s > 0$ and both jurisdictions adopt ADV taxes.

Tax rates are given by (7). Again, the capital allocation is $k_1 = k_2 = s/2$ because of symmetry. From (4), the net return to capital $r_{AA}(s)$ and tax revenues $g_1^{AA}(s)$ are obtained as

$$r_{AA}(s) = \frac{(2a - bs)(a - bs)}{2a}, \tag{A.3}$$

$$g_1^{AA}(s) = g_2^{AA}(s) = \frac{(2a - bs)bs^2}{4a}. \tag{A.4}$$

Case 3: $s > 0$, jurisdiction i uses a unit tax and jurisdiction $j \neq i$ uses an ADV tax.

In this case, the tax rates are given by (8). Since the tax rates are not the same, the capital allocation is no longer symmetric. By using (4), (5), and (8), after tedious calculations, we obtain

$$k_i = \frac{(H + 2bs)(a - bs)}{2b[4(a - bs) - H]}, \quad k_j = \frac{(a - bs)(6bs - H) - 2bsH}{2b[4(a - bs) - H]}. \tag{A.5}$$

By using (4), (8), and (A.6), we obtain the net return to capital $r_{UA}(s)$:

$$r_{UA}(s) = \frac{[2(4a - 3bs) - H][2(a - bs) - H]}{4[4(a - bs) - H]}. \tag{A.6}$$

From (1), (8), and (A.5), the tax revenues are obtained as follows:

$$g_i^{UA}(s) = \frac{(H + 2bs)^2(a - bs)}{8b[4(a - bs) - H]}, \; g_j^{AU}(s) = \frac{H[(a - bs)(6bs - H) - 2bsH][2(4a - 3bs) - H]}{8b[4(a - bs) - H]^2}. \tag{A.7}$$

Note that under the assumption of $2a > 3b\bar{k}$, we can show that (A.1)–(A.7) are all positive (Akai et al., 2011).

Case 4: $s = 0$.

This case is trivial since capital is never supplied in the market. Tax rates are indeterminate as stated in Lemma 1 and so the net return to capital is also indeterminate. Tax revenues are always zero for both jurisdictions.

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