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Online Supplement for "Stochastic Differential Game in High Frequency Market"

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Online Supplement for "Stochastic Differential Game in High Frequency Market"

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Abstract

This is an online supplement for "Stochastic Differential Game in High Frequency Market" which is submitted to Automatica.

A Comparative statics

This subsection presents comparative statics of the Nash equilibrium for open loop admissible strategies of the three types of players and the corresponding mid-price process by changing the parameters from the base case in Sections 4.1, where $c_1 = c_2 = c_3 = 1000$, $\eta = 1000$ and $\mu = 0$.

A.1 The case where $c_1 = 0$

Firstly, if we change c_1 from 1000 to 0, which means that the algorithmic traders do not have to close their position by T, the algorithmic traders buy the risky asset in accordance with the selling from the general traders (Figures A.1 & A.2). Then, the market makers show tight spreads to buy against the small total selling amount from the general traders and algorithmic traders (Figures A.3 & A.4). As a result, there is almost no market impact on the mid-price process (Figure A.5).

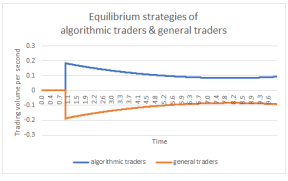

Fig. A.1. Equilibrium strategies of the algorithmic traders and the general traders, $c_1 = 0$, $c_2 = c_3 = 1000$, $\eta = 1000$, and $\mu = 0$

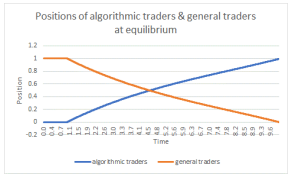

Fig. A.2. Positions of the algorithmic traders, general traders, and market makers, $c_1 = 0$, $c_2 = c_3 = 1000$, $\eta = 1000$, and $\mu = 0$

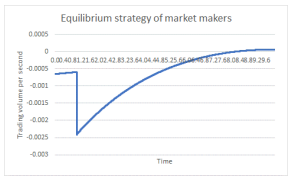

Fig. A.3. Equilibrium strategy of the market makers, $c_1 = 0$, $c_2 = c_3 = 1000$, $\eta = 1000$, and $\mu = 0$

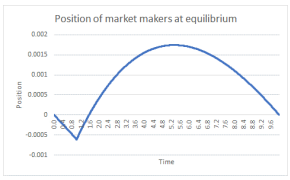

Fig. A.4. Positions of the market makers, $c_1 = 0$, $c_2 = c_3 = 1000$, $\eta = 1000$, and $\mu = 0$

Fig. A.5. Transition of the mid price of the risky asset, $c_1 = 0$, $c_2 = c_3 = 1000$, $\eta = 1000$, and $\mu = 0$

Algorithmic traders	General traders	Market makers
2.10E-03	-0.00302	2.38E-06

Table A.1
Profits and losses of the algorithmic traders, general traders, and market makers, $c_1 = 0$, $c_2 = c_3 = 1000$, $\eta = 1000$, and $\mu = 0$

A.2 The case where $c_2 = 0, \eta = 0$

Secondly, if we shift both c_2 and η from 1000 to 0, which implies that the general traders do not have to reduce their long risky asset position, neither the general traders nor the algorithmic traders trade largely (Figure A.6)

⋆ The paper entitled "Stochastic Differential Game in High Frequency Market" is submitted to Automatica.

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and the market makers set tight spreads. Consequently, the mid price is almost unchanged (Figure A.7).

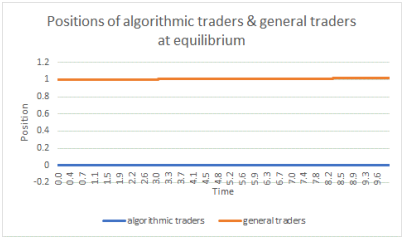

Fig. A.6. Positions of the algorithmic traders, general traders, and market makers, $c_2 = 0$, $c_1 = c_3 = 1000$, $\eta = 0$, and $\mu = 0$

Fig. A.7. Transition of the mid price of the risky asset, $c_2 = 0$, $c_1 = c_3 = 1000$, $\eta = 0$, and $\mu = 0$

Algorithmic traders	General traders	Market makers
2.64E-06	0.00459	2.40E-08

Table A.2
Profits and losses of the algorithmic traders, general traders, and market makers, $c_2 = 0$, $c_1 = c_3 = 1000$, $\eta = 0$, and $\mu = 0$

A.3 The case where σ_t is downward sloping

Thirdly, if we change σ_t from $\sigma_t \equiv 0.001$ to $\sigma_t = 0.002 - 0.0001t$, which means that the term structure of the volatility process is changed from flat to downward sloping, since the high volatility is high in the beginning of the period, the general traders, who are risk averse, reduce more long positions in the beginning of the trading period.

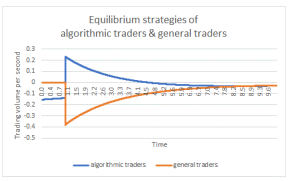

Fig. A.8. Equilibrium strategies of the algorithmic traders and the general traders, $\sigma_t : 0.002 - 0.0001t$, $c_1 = c_2 = c_3 = 1000$, $\eta = 1000$, and $\mu = 0$

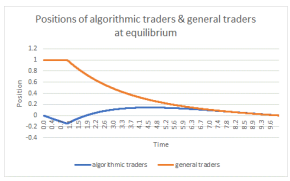

Fig. A.9. Positions of the algorithmic traders, general traders, and market makers, $\sigma_t : 0.002 - 0.0001t$, $c_1 = c_2 = c_3 = 1000$, $\eta = 1000$, and $\mu = 0$

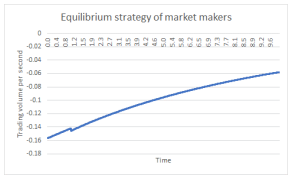

Fig. A.10. Equilibrium strategy of the market makers, $\sigma_t : 0.002 - 0.0001t$, $c_1 = c_2 = c_3 = 1000$, $\eta = 1000$, and $\mu = 0$

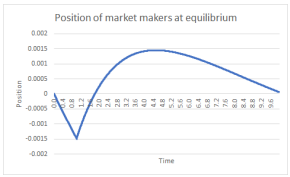

Fig. A.11. Positions of the market makers, $\sigma_t : 0.002 - 0.0001t$, $c_1 = c_2 = c_3 = 1000$, $\eta = 1000$, and $\mu = 0$

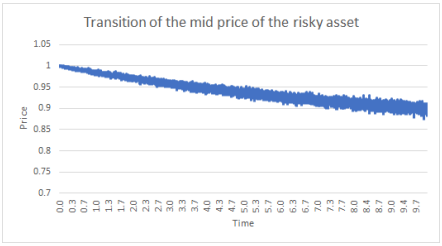

Fig. A.12. Transition of the mid price of the risky asset, $\sigma_t : 0.002 - 0.0001t$, $c_1 = c_2 = c_3 = 1000$, $\eta = 1000$, and $\mu = 0$

Algorithmic traders	General traders	Market makers
5.23E-04	-0.162	7.03E-06

Table A.3
Profits and losses of the algorithmic traders, general traders, and market makers, $\sigma_t : 0.002 - 0.0001t$, $c_1 = c_2 = c_3 = 1000$, $\eta = 1000$, and $\mu = 0$

A.4 The case where $\mu = -0.01$

Furthermore, if we change μ from 0 to -0.01, which implies that there is a negative global market shock and a price fall is observed, the patterns of the trading strategies of the three types of players are unchanged (Figures A.13-A.16). The mid price of the risky asset falls more by 10% than in the case of $\mu = 0$ (Figure A.17).

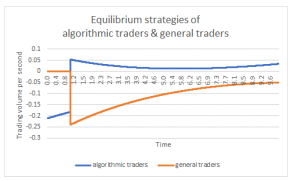

Fig. A.13. Equilibrium strategies of the algorithmic traders and the general traders, $\mu = -0.01$, $c_1 = c_2 = c_3 = 1000$, and $\eta = 1000$

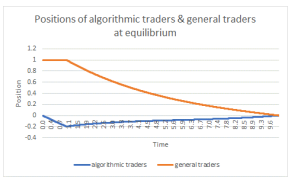

Fig. A.14. Positions of the algorithmic traders, general traders, and market makers, $\mu = -0.01$, $c_1 = c_2 = c_3 = 1000$, and $\eta = 1000$

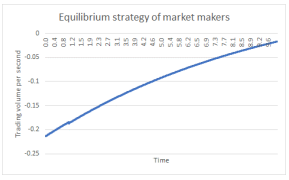

Fig. A.15. Equilibrium strategy of the market makers, $\mu = -0.01$, $c_1 = c_2 = c_3 = 1000$, and $\eta = 1000$

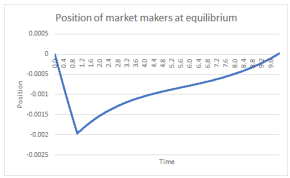

Fig. A.16. Positions of the market makers, $\mu = -0.01$, $c_1 = c_2 = c_3 = 1000$, and $\eta = 1000$

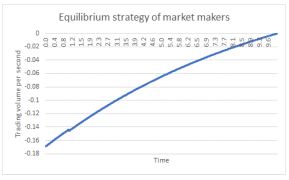

Fig. A.20. Equilibrium strategy of the market makers, $\mu = -0.01$, $c_1 = 0, c_2 = c_3 = 1000$, and $\eta = 1000$

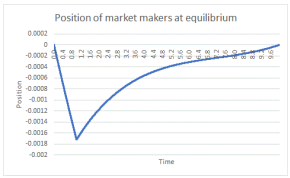

Fig. A.21. Positions of the market makers, $\mu = -0.01$, $c_1 = 0, c_2 = c_3 = 1000$, and $\eta = 1000$

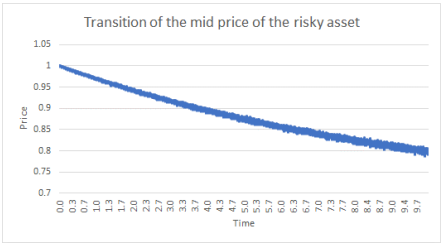

Fig. A.17. Transition of the mid price of the risky asset, $\mu = -0.01$, $c_1 = c_2 = c_3 = 1000$, and $\eta = 1000$

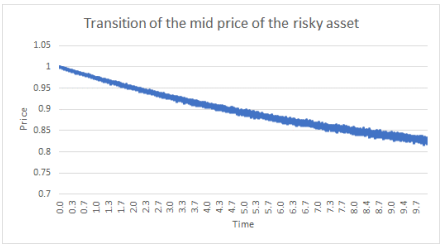

Fig. A.22. Transition of the mid price of the risky asset, $\mu = -0.01$, $c_1 = 0, c_2 = c_3 = 1000$, and $\eta = 1000$

Algorithmic traders	General traders	Market makers
3.03E-04	-0.220	3.19E-06

Table A.4
Profits and losses of the algorithmic traders, general traders, and market makers, $\mu = -0.01$, $c_1 = c_2 = c_3 = 1000$, and $\eta = 1000$

Algorithmic traders	General traders	Market makers
+6.62E-04	-0.174	+2.20E-06

Table A.5
Profits and losses of the algorithmic traders, general traders, and market makers, $\mu = -0.01$, $c_1 = 0, c_2 = c_3 = 1000$, and $\eta = 1000$

A.5 The case where $\mu = -0.01$ and $c_1 = 0$

Finally, if we change μ and c_1 from $\mu = 0$ and $c_1 = 1000$ to $\mu = -0.01$ and $c_1 = 0$, the algorithmic traders buy along with the selling of the general traders (Figures A.18 & A.19). This implies that when the speed of the price fall is not excessively fast, compared to the worst case scenario of $\mu = -0.03$ and $c_1 = 0$ in Section 4.3, where there is a large negative global market shock and both the algorithmic traders and the general traders sell in the rapid price fall. It is more profitable for the algorithmic traders to buy with the large negative spreads (Figure A.20) even when the mid price is decreasing (Figure A.22).

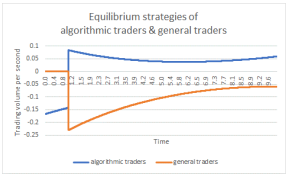

Fig. A.18. Equilibrium strategies of the algorithmic traders and the general traders, $\mu = -0.01$, $c_1 = 0, c_2 = c_3 = 1000$, and $\eta = 1000$

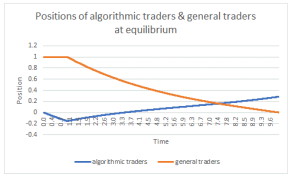

Fig. A.19. Positions of the algorithmic traders, general traders, and market makers, $\mu = -0.01$, $c_1 = 0, c_2 = c_3 = 1000$, and $\eta = 1000$