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EPIC FAIL: HOW BELOW-BID PRICING BACKFIRES IN MULTIUNIT AUCTIONS

DANIEL MARSZALEC, ALEXANDER TEYTELBOYM, AND SANNA LAKSÁ

Abstract. Auctions with below-bid pricing (e.g., uniform-price, and ascending auctions) have remarkable theoretical properties, but practitioners are skeptical about their implementation. We present a dynamic model of collusion in multiunit auctions that explains this gap between theory and practice. To sustain collusion at the reserve price, bidders submit *crank-handle bids*. The cost of sustaining crank-handle collusion depends on the degree of below-bid pricing in the auction. Our model predicts that crank-handle collusion is easier to sustain in auctions with more below-bid pricing and when bidders are more symmetric. Evidence from auctions of fishing quota in the Faroe Islands supports our predictions.

Keywords: ascending auction, uniform-price auction, pay-as-bid auction, collusion, predation, crank-handle bidding.

JEL Classification: D44, D47, L41, Q22.

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1. Introduction

One of the key insights of auction theory is that the auctioneer might find it desirable to compensate bidders to reveal their private information (Vickrey, 1961). If winning bidders must pay their bids, they will bid below their value and will not reveal their true willingness-to-pay. Auction rules with *below-bid pricing*, such as the ascending auction or the uniform-price auction, decouple winners' payments from their bids to incentivize them to bid close to their value.[1] By revealing more information, these auction rules can improve the efficiency of allocations and, under certain conditions, increase the auctioneer's revenue (Milgrom and Weber, 1982; Ausubel et al., 2014). For example, Ausubel et al. (2014) conclude that:

> Uniform pricing has several desirable properties, including: (i) it is easily understood in both static and dynamic forms; (ii) it is fair in the sense that the same price is paid by everyone; (iii) absent market power it is efficient and strategically simple ("you just bid what you think it's worth"); and (iv) the exercise of market power under uniform pricing favours smaller bidders.

However, the theoretical advantages of auctions with below-bid pricing rely crucially on two assumptions that rarely hold in practice: competitive bidding and the absence of significant asymmetries among bidders. As Klemperer (2002b, p. 169–170) argued,

> What really matters in auction design [is]: discouraging collusive, entry-deterring and predatory behavior.

Indeed, some have argued that it is precisely the uniform-price auctions design that best mitigates collusive incentives. Friedman (1960) argued that the uniform-price auction "in any of its variants, will make the price the same for all purchasers, reduce the incentive for collusion, and greatly widen the market." Chari and Weber (1992) argued that: "Uniform-price auctions are also likely to be less susceptible to market manipulation." In this paper, we argue that the desirability of uniform-price auctions for practical market design in the presence of limited competition should be reassessed.[2]

Before going on to analyze multiunit auctions, let us pause to recall insights from three classic papers on single-unit auctions. First, Robinson (1985) pointed out that if colluding bidders have perfect information about each others' payoffs, then even static collusion in which the auctioneer obtains zero revenue is easily enforceable in second-price and ascending auctions, but not in first-price auctions. Second, Klemperer (1998) and Maskin and Riley (2000) showed bidder asymmetry can have a dramatic effect on auction

[1] In a single-unit setting, second-price and ascending auctions are strategyproof and therefore EPIC (Ex-Post Incentive Compatible). In the linear-bid setting with decreasing marginal values that we consider, uniform-price auctions are also EPIC (Ausubel et al., 2014, Proposition 6). Ascending auctions are, in fact, *obviously* strategyproof (Li, 2017). In a multiunit setting, Vickrey auctions are strategyproof.

[2] Both the uniform-price and pay-as-bid auctions are popular formats for selling government debt (Armantier and Sbai, 2006; Kang and Puller, 2008; Castellanos and Oviedo, 2008; Hortaçsu and McAdams, 2010; Marszalec, 2017) and electricity supply (Fabra et al., 2002; Hurlbut et al., 2004).

outcomes. In particular, second-price and ascending auctions make it much easier for bidders to exercise their value advantage in independent value (Maskin and Riley, 2000) and (almost) common-value (Klemperer, 1998) auctions. We show that the insights of Robinson (1985), Maskin and Riley (2000), and Klemperer (1998) generalize in theory and in practice to multiunit auctions.

The contribution of this paper is two-fold. Our first contribution is a simple, dynamic model of collusive bidding in multiunit auctions with perfect information and no transfers. In the stage game of our model, bidders attempt to collude at the reserve price using (what we call) *crank-handle bids*. Such crank-handle bids drop off sharply to the reserve price at the point where the colluding bidders split the market (Figure 1). As we show, such bidding strategies are easy to achieve and are widely used in practice. In procurement auctions, similar bidding strategies are often called "hockey-sticks" (Hurlbut et al., 2004), however, as Figure 1 shows, in our setting they more closely resemble crank-handles.

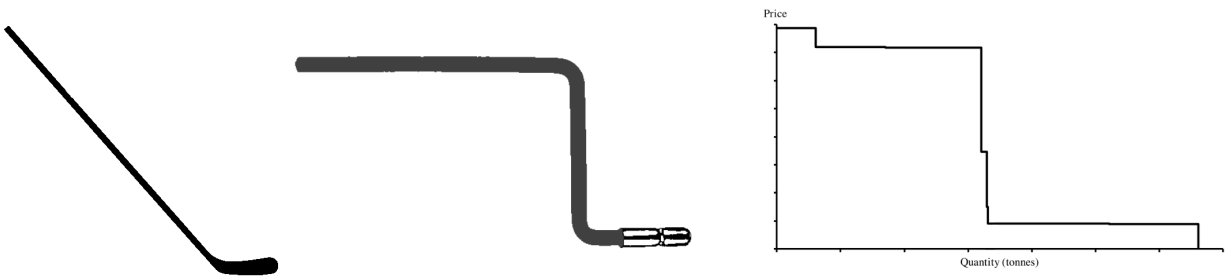


FIGURE 1. A hockey stick, a crank-handle, and a typical crank-handle bid.

If one of the bidders deviates from collusive crank-handle bidding, the bidders revert forever to a static, competitive bidding Nash equilibrium. Along the collusive bidding path, bidders end up paying a *collusion-enforcement premium*, i.e., the profit foregone by the colluding bidders even if no deviation from the collusive equilibrium occurs. Bidders are tempted to deviate either by trying to buy the same quantity at a lower price (what we call a *height-deviation*) or a larger quantity at slightly above the market-clearing price (what we call a *width-deviation*). The two most widely-used multiunit homogeneous-good auction rules—the pay-as-bid and the uniform-price auction—illustrate the two extremes of collusion incentives: the collusion-enforcement premium is zero in the uniform-price auction and the height-deviation incentive is completely absent; the collusion-enforcement premium is maximal in the pay-as-bid auction and the height-deviation incentive is strongest.

Beyond these two popular auctions, we analyze incentives in a wide variety of formats including the Vickrey auction and *mixed-price auctions* where winners pay a combination of their bids and the market-clearing price.[3] Our first result (Prediction 1 and Theorem 1) shows that bidders need to be less patient to collude in mixed-price auctions with more below-bid pricing. In other words, collusion is easier and deviation is less tempting in auctions with more below-bid pricing.

[3]Plum (1992) and Tse (2004) analyzed such mixed-price auctions in a single-unit setting.

Our model also allows us to analyze the consequences of bidder asymmetry and the possibility of predation. Collusion is easier when bidders are more symmetric in any mixed-price auction because both the incentives to deviate and the willingness to enforce collusive play are similar across bidders. Moreover, for any degree of asymmetry, collusion is easier in auctions with more below-bid pricing (Theorem 2). In particular, in the uniform-price auction, collusion is easy even when bidders are asymmetric because enforcing a collusive equilibrium is costless (Prediction 2). But even if bidders find it hard to collude, auctions with below-bid pricing are likely to perform poorly when bidders are asymmetric. In auctions with below-bid pricing strong predatory bidders find it easier to exercise their value advantage to deter entry since they can commit to aggressive crank-handle bidding at a low cost (Prediction 3). Overall, our model highlights how auctions with below-bid pricing are highly susceptible to both collusion and predation.

Our second contribution is empirical. We present a setting in which inexperienced bidders took advantage of auctions with below-bid pricing. We analyze bidding data from fishing quota auctions in the Faroe Islands which took place over four years between 2016 and 2019 and involved overlapping sets of bidders. As a fraction of GDP, these are some of the largest auctions ever held.[4] The observations from these auctions support our theory in four different ways. First, we find rampant crank-handle bidding indicating attempts to collude. Second, in (sequential) ascending auctions, we observe a transparent and successful bidding ring among incumbents. Third, we document how a strong bidder used his value advantage to deter entry and win most of the quota at just above the reserve price in (sequential) ascending auctions. Finally, we illustrate the success of a brief switch to pay-as-bid auctions in which the same bidders who colluded in uniform-price auctions found that the same collusive strategy failed in the pay-as-bid auction.

1.1. **Related work.** Our paper is related to four strands of the auctions literature.

First, an extensive theoretical literature, starting with Wilson (1979), focused on the analysis of pure-strategy Bayes-Nash equilibria in multi-unit auctions (Klemperer and Meyer, 1989; Green and Newbery, 1992; Back and Zender, 2001; Wang and Zender, 2002; Pycia and Woodward, 2020). The backbone of our model is the linear bidding equilibrium derived for uniform-price and pay-as-bid auctions in the presence of uncertain supply by Ausubel et al. (2014). Our mixed-price auction is closely related to the equilibrium analysis of "hybrid" payment rules (Viswanathan and Wang, 2002; Wang and Zender, 2002; Armantier and Sbaï, 2009; Ruddell et al., 2017). The closest paper to ours is Woodward (2019) who analyzes symmetric linear and non-linear equilibria in mixed-price auctions with uncertainty. Our linear bidding equilibrium in mixed-price auctions is a special case of his general result although our proof follows familiar methods from Ausubel et al. (2014) which we also apply to the asymmetric setting.

[4]Since the sale of the Roman Empire, the "largest ever" auction is often considered to be the 3G auction in the UK held in 2000; it raised 2.5 percent of GNP by selling *twenty*-year spectrum licenses (Binmore and Klemperer, 2002). By contrast, the Faroese *one*-year quota auctions held in 2017 raised 0.8 percent of GDP.

Second, many papers have pointed out the possibility of low-revenue equilibria in uniform-price auctions (Wilson, 1979; Klemperer and Meyer, 1989; Back and Zender, 1993; Noussair, 1995; Engelbrecht-Wiggans and Kahn, 1998; Kremer and Nyborg, 2003, 2004; LiCalzi and Pavan, 2005; McAdams, 2007). Indeed, Milgrom (2004, p. 264) says that these "extreme price equilibria [are]... of great practical importance." Such sophisticated strategies have been observed experimentally when bidders have been given access to pre-play communication (Goswami et al., 1996). When these tactics are observed in procurement or supply-function settings they are referred to as "hockey-stick" bidding (since the price offered for the first units is low and price for the final units is very high, see Figure 1)— such bidding patterns are often found in repeated electricity auctions (Hurlbut et al., 2004; Harbord and Pagnozzi, 2014; Holmberg and Newbery, 2010).[5] We point out that low-revenue equilibria can be sustained (in a dynamic setting) in a general class of mixed-price auctions using crank-handle bids. Moreover, our empirical evidence suggests that collusive crank-handle bidding is natural and prevalent.

Third, our paper is broadly related to a theoretical literature on the effect of auction design on collusion, starting with seminal analyses of Graham and Marshall (1987) and McAfee and McMillan (1992) in static settings. Collusion in auctions is generally easier in sequential (rather than one-shot) auctions, in open (rather than sealed-bid) formats and in second-price (rather than first-price) auctions (Robinson, 1985; Marshall and Marx, 2007, 2009, 2012). More recently, Pavlov (2008) and Che and Kim (2009) offered elegant designs of collusion-proof auctions. What we present is, however, not conclusive evidence of an explicitly agreed bidding ring: repeated auctions, like any repeated strategic situations, naturally offer opportunities for patient players to tacitly coordinate on outcomes that are better for participants than static, one-shot equilibria (Athey and Bagwell, 2001; Aoyagi, 2003; Skrzypacz and Hopenhayn, 2004; Athey et al., 2004; Blume and Heidhues, 2008).

Finally, our paper touches on the empirical work on auctions in the presence of collusion. Bidding rings have been studied extensively and their identification is an active area of research (Baldwin et al., 1997; Pesendorfer, 2000; Bajari and Ye, 2003; Ishii, 2009; Asker, 2010; Kawai and Nakabayashi, 2018; Conley and Decarolis, 2016; Chassang and Ortner, 2019; Chassang et al., 2020). Our data, while compelling, is insufficient for a structural econometric analysis.

This paper is organized as follows. In Section 2, we illustrate crank-handle bidding and the key incentives for collusion in static uniform-price and pay-as-bid auctions with two bidders. In Section 3, we introduce the formal model of dynamic collusion in mixed-price auctions with multiple bidders and give a rigorous analysis of the intuitive predictions made in Section 2. Section 4 describes the setting of fishing quota auctions in the Faroes. Section 5 presents evidence of successful crank-handle collusion in uniform-price auctions. Section 6 provides evidence of collusion and predation in (sequential) ascending auctions. Section 7 considers possible improvements to auction design in the Faroese context.

[5] However, Cramton (2003) argues that "such bids are entirely reasonable given reasonable assumptions about demand and supply uncertainty, forward contracts, and marginal cost curves."

Section 8 describes the consequences of a brief switch to pay-as-bid auctions. Section 9 is a conclusion. Appendix A contains proofs of the formal results. Online Appendix B presents further evidence on successful and unsuccessful crank-handle collusion.

2. Collusion and predation in uniform-price and pay-as-bid auctions

To fix ideas, let us consider a simple static model of multiunit auctions. There are two bidders E (the "Enforcer") and D (the "Deviator") and an auctioneer selling one perfectly divisible good. Without loss of generality, the seller's value for the good is zero. Bidders have full information about each others' valuations and can reach an agreement on their possible collusive strategy (Robinson, 1985). We illustrate different auctions in Figure 2.

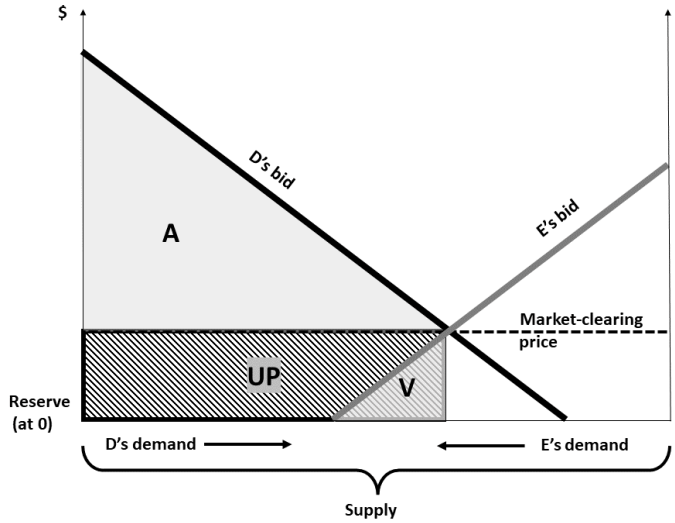
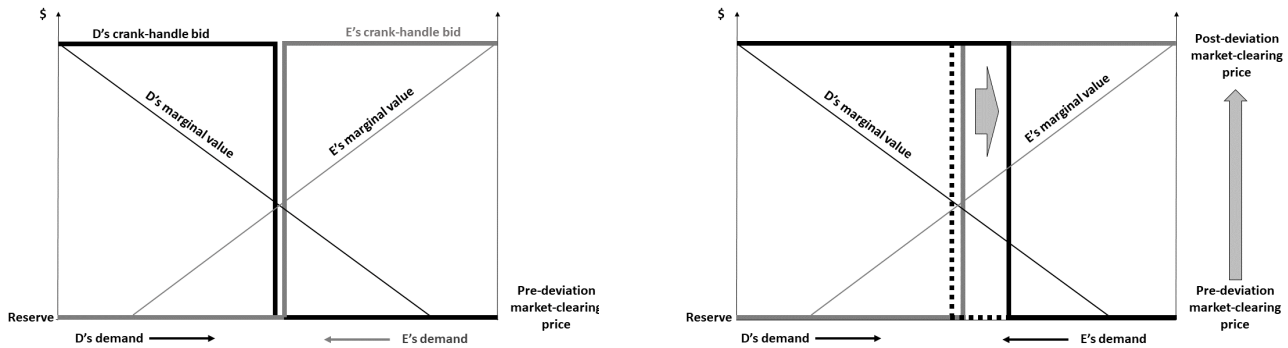


Figure 2. Auction formats. Rectangle UP = payment in uniform-price auction. Triangle A + rectangle UP = payment in the pay-as-bid auction. Triangle V = payment in the Vickrey auction.

Three of these formats are familiar: the pay-as-bid auction, in which each winner pays their bids in full; the uniform-price auction in which winners pay the market-clearing price for all units won, and the Vickrey auction in which the winner pays the value of externality on the other bidder. We now analyze whether the market can be split between bidders at a price of zero as a one-shot Nash equilibrium.

2.1. **Crank-handle collusion in uniform-price auctions.** Let us now consider how collusion can be achieved in uniform-price auctions by crank-handle bidding (see also Example 1 in Ausubel et al. (2014)).



(A) Collusion in uniform-price auctions, before deviation: market split at reserve

(B) Collusion in uniform-price auctions, after deviation: large price rise on all units

FIGURE 3. Crank-handle equilibrium in a uniform-price auction.

Suppose that both bidders submit crank-handle bids, as illustrated in Figure 3a. In this collusive equilibrium, both bidders' payments are zero. To see why these strategies constitute a Nash equilibrium, suppose that the Deviator increases his demand at the margin, as in Figure 3b. The market-clearing price jumps dramatically and now the Deviator has to pay a high price for *every* unit won. If the top part of the Enforcer's crank-handle bid is high enough, the increase in the market-clearing price can wipe out the Deviator's profit on the marginal unit, making the deviation unprofitable.[6] Moreover, it is costless for the Enforcer to submit a high crank-handle bid and enforce the collusive equilibrium making this a natural candidate for equilibria in a setting with possible bidder coordination.[7]

The possibility of such simple collusive equilibria urges caution over the arguments for uniform-price auctions. While in a competitive-bidding equilibrium uniform-price auctions may have desirable properties, once we consider collusive equilibria, the results may be reversed. Rather than bidding truthfully, it can be optimal to submit crank-handle bids, which convey inaccurate market-price signals. We therefore cannot conclude anything about the efficiency of the auction. Moreover, as we show later, the profitability of crank-handle bidding favors strong bidders over weaker ones.

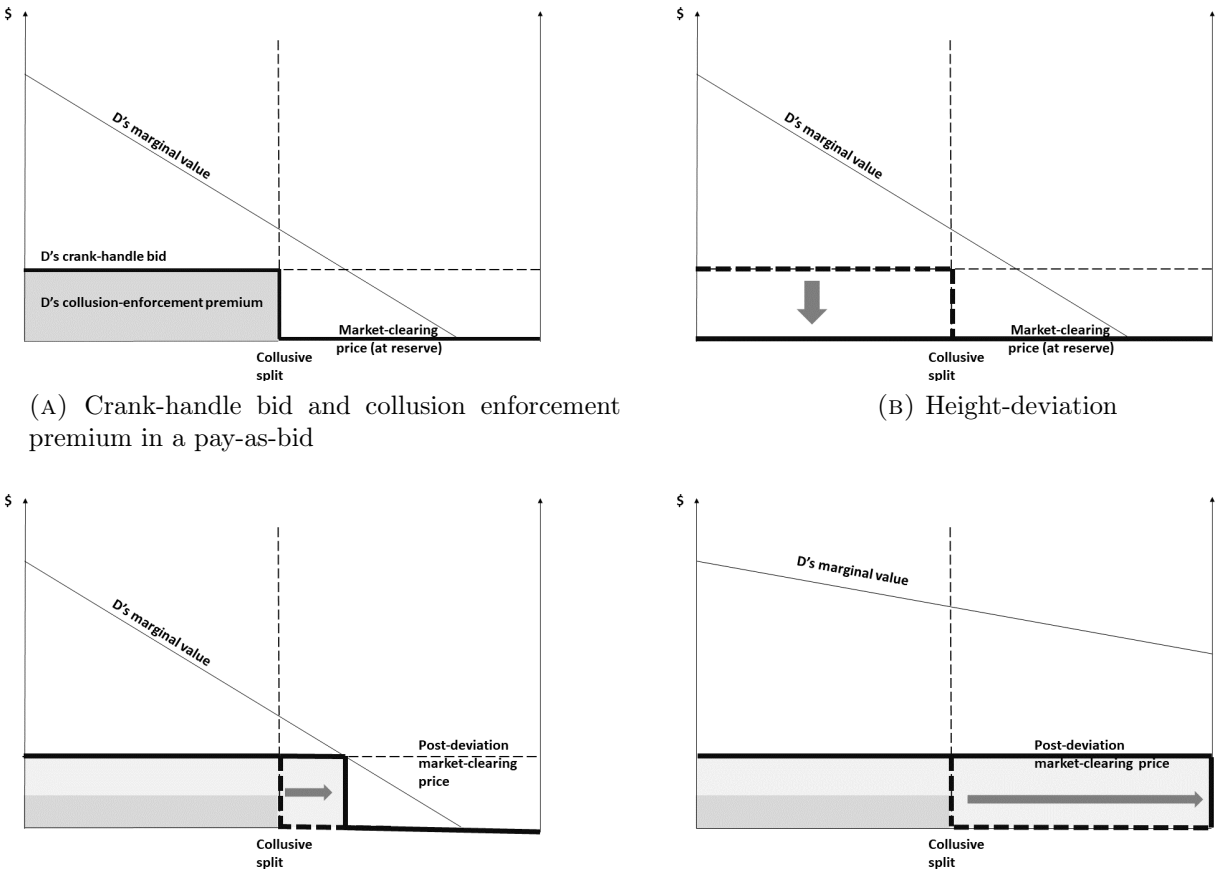
2.2. **Collusive equilibria in pay-as-bid vs uniform-price auctions.** Next, we compare the collusive incentives in pay-as-bid auctions to uniform-price auctions.[8] Consider the Deviator's attempt to submit a crank-handle bid in a pay-as-bid auction (Figure 4a). Unlike in the case of the uniform-price auction, submitting this crank-handle bid is costly for the Deviator (and, by symmetry, the Enforcer) in the pay-as-bid auction. As a result,

[6]The Enforcer's top crank of the crank-handle bid only needs to be above the Deviator's marginal value at the collusive split to prevent a deviation.

[7]The same logic of crank-handle bids, explained above, applies to the Vickrey auction. Here also the Enforcer can ensure that any marginal deviation will be unprofitable by submitting a crank-handle bid where the top part is above the Deviator's marginal value at the collusive split quantity.

[8]We restrict our attention to crank-handle bids in the collusive equilibria as these are the most efficient bids to split the market at the reserve price.

the Deviator has an additional incentive—absent in the uniform-price auction— to reduce the "height" of his crank-handle bid to slightly above zero, thereby winning his share of the collusive split for a near zero price (Figure 4b). This immediately suggests that in the pay-as-bid auction collusive equilibria will need to involve crank-handle bids whose height is below the marginal value at the collusive split quantity. However, such a "low" crank-handle bid creates other deviation opportunities. For example, the Deviator might find it profitable to deviate by increasing his bid to get all the profitable units (Figure 4c). In fact, if the crank-handle bid of the Enforcer is "low" enough, the Deviator might find it profitable to sweep the whole market (Figure 4d).



(A) Crank-handle bid and collusion enforcement premium in a pay-as-bid

(B) Height-deviation

(C) Partial width-deviation

(D) Full width-deviation

FIGURE 4. Deviations from crank-handle bidding in mixed-price auctions.

For crank-handle collusion to be sustainable in any auction, we require that the Deviator has no incentive to perform either a height- or a width-deviation. First, consider the Deviator's incentive to perform a width-deviation and increase his demand. In any format, if the Enforcer submits a high enough crank-handle bid, he can ensure that the Deviator makes a loss on the additionally won units. Any crank-handle bid whose height is above the Deviator's marginal value at the collusive split quantity will deter the Deviator from wanting to win additional units.

Second, consider the Enforcer's incentive to maintain the collusive equilibrium. In uniform-price (and Vickrey) auctions, it is costless for the Enforcer to bid with an arbitrarily

high crank-handle. If no deviation occurs, the high portion of the crank-handle is only used as a deterrent and does not affect the Enforcer's payment in the collusive equilibrium.

In the pay-as-bid auction every winning bid is paid in full. Thus, in a collusive equilibrium, all winning bids are costly to the Enforcer: the Enforcer must pay a *collusion-enforcement premium* even if no deviation occurs. To collude at a minimal cost, the Enforcer must submit a crank-handle bid where the upper portion is "low". However, such flat crank-handle bids make deviation from collusion attractive: by bidding more, the Deviator can win many more units at a marginally higher price, and therefore increase his overall profit (recall Figures 4c and 4d). Consequently, in the pay-as-bid auction, the two conditions for successful collusion are in direct conflict even in a static game. Indeed, in a static pay-as-bid auction, the Enforcer has no way of preventing the Deviator from deviating to a flat bid marginally above zero. As a result, a repeated game setting will be required to fully assess the collusive incentives in these auctions.

In summary, to disincentivize deviations, the market-clearing price must jump considerably after a deviation; but inducing such a jump is costlier when prices are more closely tied to the Enforcer's bids on inframarginal units. Since in pay-as-bid auctions prices coincide with winning bids, the cost of bidding in a crank-handle equilibrium that would make collusion sustainable must be higher. Conversely, uniform-price auctions make collusion-enforcing crank-handle bids costless to sustain. Hence:

Prediction 1. Collusive crank-handle bidding is more likely in uniform-price auctions than in pay-as-bid auctions.

2.3. **Collusion and bidder asymmetry.** In practice, bidders' values are often asymmetric: strong bidders have high valuations and weak bidders have low valuations. In uniform-price auctions, the crank-handle collusive equilibrium is sustainable for any degree of asymmetry since the collusion-enforcement premium is zero. However, in pay-as-bid auctions, the degree of asymmetry clearly matters. If bidders are asymmetric, the payoff derived from deviating on marginal collusive units is high for the strong bidder (i.e., a strong incentive for a width-deviation), while the weak bidder's ability to pay for a high enough collusion-enforcement premium that sustains crank-handle collusion is limited. On the other hand, if bidders' valuations are similar then collusion is easier: bidders can simultaneously keep each others' incentives to deviate in check while being able to absorb the collusion-enforcement premium. Hence:

Prediction 2. In uniform-price auctions, bidder asymmetry does not affect the possibility of collusion. In pay-as-bid auctions collusion is easier when bidders are symmetric.

2.4. **Predation in uniform-price vs pay-as-bid auctions.** Although bidders are less likely to collude when they have asymmetric valuations, auctions with below-bid pricing can perform poorly *even when bidders are not colluding*. As emphasized by Klemperer (2002b), competitive bidding is essential for good performance of auctions, but is unlikely if the auction features a bidder (or a group of bidders) who can predate on weaker

competitors. Mirroring the definition of predation in the context of market entry by Ordover and Willig (1981), we define a predatory bidding strategy as one that is profitable only if it discourages competitive bidding by the predator's rivals.

In practice, as Klemperer (2002b) pointed out:

> A strong bidder also has an incentive to create a reputation for aggressiveness that reinforces its advantage. For example, when Glaxo was bidding for Wellcome, it made it clear that it "would almost certainly top a rival bid."

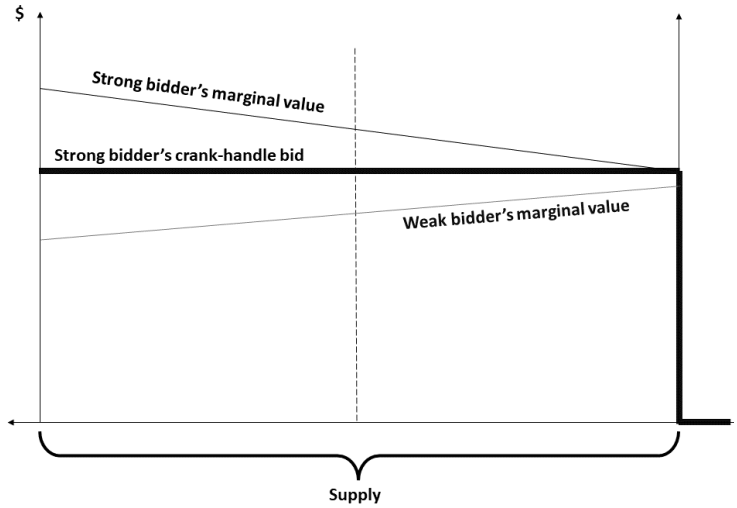


FIGURE 5. Predation with value advantage.

Suppose that the predator has value advantage (as in Figure 5). In the uniform-price auction, the predator can credibly and costlessly commit to extremely aggressive bidding by submitting a crank-handle for the entire supply. If the predator's commitment to aggressive crank-handle bidding discourages competition, ex post the cost of the predatory strategy is zero. Conversely, in a pay-as-bid auction, the predator is unable to costlessly commit to "top a rival bid," as the predator would make far less profit even in the case of successful predation. In our model, the incentives of a value-advantaged bidder to predate perfectly mirror the incentives for crank-handle collusion. Uniform-price auctions therefore exacerbate incentives for collusion, predation, and predatory collusion (among several bidders). Hence:

Prediction 3. Predation is less costly in uniform-price auctions than in pay-as-bid auctions.

Combining our three predictions, we can see that non-coordinating bidders would prefer pay-as-bid auctions to uniform-price auctions because they are less likely to be excluded from winning by crank-handle collusion.

3. A model of collusion in mixed-price auctions

To investigate the influence of below-bid pricing on collusion incentives, we introduce *mixed-price* auctions. In a mixed-price auction with a price-discrimination parameter α each, winner's payment is an α-weighted average of their bid, and the market-clearing price. More precisely, the price charged on each winning unit is

$$\alpha \times \text{bid} + (1 - \alpha) \times \text{market-clearing price}.$$

Therefore, $\alpha = 1$ recovers the pay-as-bid auction, and $\alpha = 0$ recovers the uniform-price auction.

The examples of Section 2 illustrated the corner cases of $\alpha = 0$ and $\alpha = 1$. In this section we show that those intuitions carry over to intermediate degrees of below-bid pricing; lower α indicates *more* below-bid pricing. We will use a repeated-games framework, with a mixed-price auction constituting the stage game, and look at the sets of parameters that can sustain a collusive equilibrium, at different values of α. We assume that after each iteration of the auction, the auctioneer publicly announces the market-clearing, and average winning prices, for that round. This is standard practice in auctions that feature a pay-as-bid component, including most Treasury Bill auctions. The market-clearing price is set where aggregate demand is equal to supply; in the case of ties at the market-clearing price, each bidder is rationed equally.

3.1. Competitive bidding environment.

Before moving to the collusive equilibrium, we must first characterize a competitive bidding equilibrium benchmark for our stage game. We do so by extending the model of Ausubel et al. (2014) to mixed-price auctions. Their model features linear diminishing marginal values and aggregate supply uncertainty that follows a generalized Pareto distribution, resulting in a linear competitive-bidding equilibrium. In our competitive benchmark, we also focus on linear equilibria.[9]

We assume there are $n \geq 2$ bidders, and a market supply $Q > 0$ that is uncertain, with a mean of $\bar{Q}$. We denote the average per-capita supply by $r = \frac{\bar{Q}}{n}$. Each bidder i has a linear marginal value function:

$$\text{MV}(q_i) = v - \rho_i q_i.$$

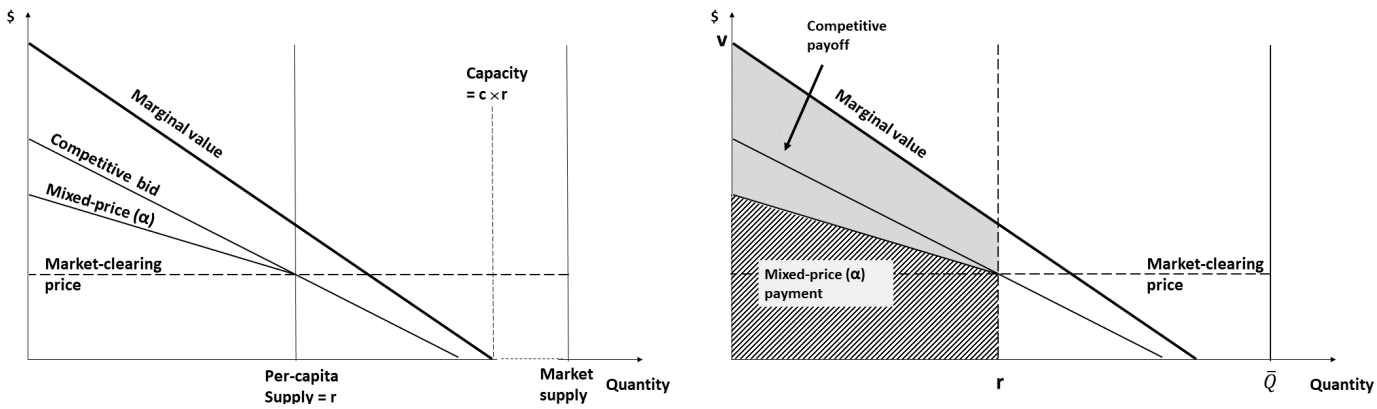
As the marginal value is linearly decreasing, it will eventually fall below zero, and the bidder will become satiated. Since it is not straightforward to see at which quantity this happens, we will frequently use the following slope-capacity substitution:

$$\rho_i = \frac{v}{c_i r}. \tag{1}$$

Here, c_i is bidder i's "capacity"—a multiple of the expected per-capita supply r at which satiation is reached. To obtain a non-trivial equilibrium, we require $\sum_i c_i > n$; otherwise the aggregate demand will be below market supply at any price, and the auction will clear

[9] Independent of our work, Woodward (2019) has also adapted the same base model of Ausubel et al. (2014) to mixed-price auctions, for a more general family of polynomial value functions.

at zero price. As $c_i \to \infty$, the marginal value function becomes flat, and we approach the constant marginal value case. Figure 6a illustrates our model and its primitives for a strictly positive α, and $1 < c < n$.



(A) Values, bids, and pricing in mixed-price.

(B) Values, payments and payoffs in mixed-price.

FIGURE 6. Pricing and payoffs in mixed-price.

Market supply Q is randomly distributed over the support $[0, r(1+\lambda)]$. Per-capita market supply has the following distribution for $\lambda > 0$:

$$F\left(\frac{Q}{n} \mid v\right) = 1 - \left(1 - \frac{Q}{r(1+\lambda)n}\right)^{\lambda},$$

which is a special case of the generalized Pareto distribution with a location of 0, shape of $-\frac{1}{\lambda}$, and scale of $r(1+\lambda)$.[10] As $\lambda \to 0$, uncertainty goes to zero and F tends to a degenerate distribution with all the mass at $\bar{Q} = rn$.

A bidder's strategy in the stage game is a bid-function, $b_i(q_i; v)$, which specifies a price for each quantity. Since we are dealing with linear equilibria, the bid-functions will be linear in each bidder's own value and quantity. Furthermore, bidders' payoffs are quasilinear in payments t_i:

$$u_i(q_i, t_i; b_i) = \int_0^{q_i} (v - \rho_i q) dq - t_i(q_i; b_i).$$

In a mixed-price auction, each winner pays:

$$t_i(q_i; b_i) = \alpha \int_0^{q_i} b_i(q) dq + (1-\alpha) p^*$$

where p^* is a market-clearing price such that $q_i + \sum_{j \neq i} b_j^{-1}(p, v) = Q$. Since in a mixed-price auction the payment is a convex combination of the bid and the market-clearing price, as illustrated in Figure 6b, each bidder's payment is conditional on the shape of the entire bidding function - not only at the margin. With these elements defined, we can

[10] We choose this parametrization because it allows us to isolate uncertainty from scale effects. If the "scale" parameter of the generalized Pareto distribution does not adjust with the shape parameter, reducing uncertainty will also reduce the mean of the distribution. To prevent the mean from collapsing to zero as uncertainty disappears, it is necessary to scale the "scale parameter". We use the same notation as Woodward (2019); in the notation of Ausubel et al. (2014) we set $\xi = -\frac{1}{\lambda}$.

solve for the symmetric equilibrium (in which $b_i^*(q_i) = b^*(q)$ for all i) in the competitive stage game benchmark.

Lemma 1. Assume $n > 2$ and $\rho_i = \rho$ for all i. In the symmetric equilibrium,

$$b_i^*(q_i) = v - \frac{\alpha r(1+\lambda)n\rho}{(n-2)\lambda + (n+\lambda)\alpha} - \frac{(n-1)\lambda\rho}{(n-2)\lambda + (n+\lambda)\alpha} q_i.$$

Proof. See Appendix A.1. □

3.2. **Collusive bidding environment.** In this section, we consider the case where supply becomes certain, i.e., $\lambda \to 0$, and the support of per-capita supply becomes $[0, r]$. Certain supply is in line with our empirical setting since the entire announced supply was allocated in every auction. We investigate the conditions under which collusive stage game strategies can be sustained in an infinitely repeated auction stage game.

Assumption 1 (Symmetric environment with at least three bidders without uncertainty)**.**

- $n > 2$,
- $c_i = c > 1$ (which implies that $\rho_i = \rho$) for all bidders i,
- consider payoffs and bidding functions in the limit as $\lambda \to 0$.

As uncertainty vanishes, the competitive bidding functions in the symmetric stage game equilibrium become:

$$b_i^*(q_i) = \begin{cases} v - \rho r & \text{if } \alpha > 0 \text{ and } q_i \leq \bar{Q} \\ v - \frac{n-1}{n-2}\rho q_i & \text{if } \alpha = 0 \end{cases}. \tag{2}$$

When $\alpha > 0$, bidders submit a flat bid that equals the bidders' marginal value at $q_i = r$; due to the assumption of equal rationing in case of ties, each bidder is allocated a quantity $q_i = r$ in this symmetric equilibrium. In the case of a uniform-price auction, bidders still submit a downward sloping bid. Using these strategies results in the following competitive stage game payoffs:

$$u_i^*(q_i) = vr - \frac{\rho r^2}{2} - \begin{cases} q_i(v - \rho r) & \text{if } \alpha > 0 \text{ and } q_i \leq \bar{Q} \\ q_i(v - \frac{n-1}{n-2}\rho q_i) & \text{if } \alpha = 0 \end{cases}.$$

To model collusion in the stage game, we focus on crank-handle bidding. A crank-handle bid is a step function given by:

$$b_i^C(h, w) = \begin{cases} h(v - \rho r) & \text{if } q_i < rw \\ 0 & \text{if } q_i \geq rw \end{cases}.$$

with $h, w \in [0, 1]$. Here, w represents the "width" of the crank-handle (as a proportion of per-capita supply) and h represents the "height" of the crank handle. In a symmetric collusive stage game, for any height of the crank, every bidder will set $w = 1$ as his

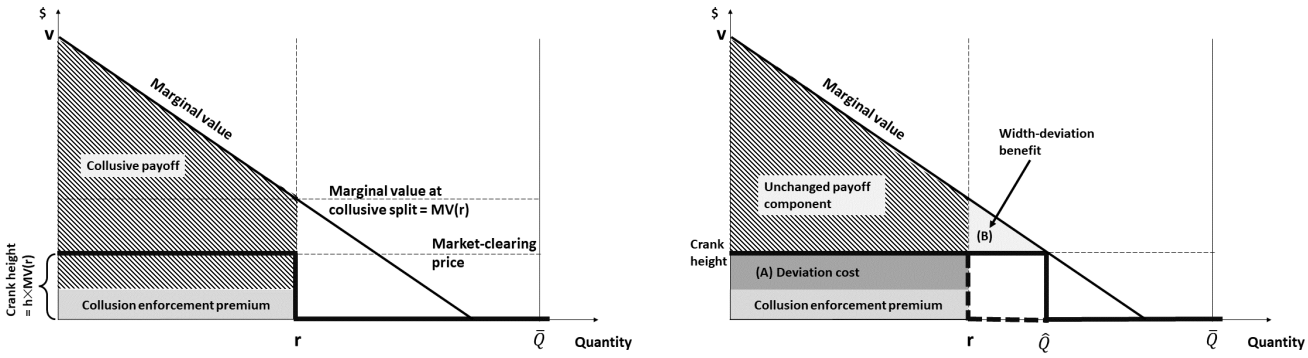
payment is less than his value on each unit he could possibly demand.[11] Therefore, to find the optimal collusive bidding in a one-stage game, it suffices to find the optimal h. Using these parameters, we can express the collusive equilibrium payoffs as:

$$u_i^C(h) = vr - \rho\frac{r^2}{2} - \alpha r h(v - \rho r).$$

Consequently, the collusion-enforcement premium is given by:

$$\text{CEP}(\alpha, h) = \alpha h(v - r\rho)r. \tag{3}$$

Figure 7a depicts the collusive stage game equilibrium from the viewpoint of an individual bidder. To ensure that any kind of deviations – including those that leave the market-clearing price unaltered – are detectable, we assume that the auctioneer reports average prices in addition to the market-clearing price, after each iteration of the stage game.[12]



(A) Crank-handle bid, payoffs, and the collusion-enforcement premium.

(B) Deviations from crank-handle bidding in mixed-price auctions. Net effect of a width-deviation is (B-A).

FIGURE 7. Crank-handle bids and deviation incentives in mixed-price auctions.

As in Section 2, bidders face two deviations: a *height-deviation* (bidding the collusive split at zero price), and a *width-deviation* (bidding slightly above the crank-price, to win more units). The least-cost of performing both deviations is, again, to bid a specific shape of a crank-handle. For the height-deviation, this will be:

$$b_i^{DH}(q_i) = \begin{cases} \epsilon & \text{if } q_i < r \\ 0 & \text{if } q_i \geq r \end{cases}.$$

The benefit from a height-deviation is that the bidder no longer needs to pay the collusion-enforcement premium, which is indicated by a rectangle in both Figures 7a and 7b.

[11]Given that all other bidders follow collusive crank-handle bidding, the residual supply for the remaining bidder is r. Since in the collusive equilibrium the market-clearing price is zero, and the remaining bidder's values are at least $v - \rho r$ for all quantities up to r, it is optimal for the remaining bidder to pick the widest feasible collusive crank-handle; that is, to set $w = 1$.

[12]This is a plausible assumption, which is satisfied in most practical applications of pay-as-bid auctions for commodities such as Treasury Bills.

As Eq. (3) shows, the situations when $h = 0$ and $\alpha = 0$ are equivalent: in a uniform-price auction, the collusion-enforcement premium is always zero, so the height-deviation is never more profitable than sticking to any other collusive crank handle, in equilibrium.

For the width-deviation, we first find the quantity to which a bidder would find it most profitable to deviate:

$$\widehat{Q}_i = \min\left\{\bar{Q}, \frac{v - h(v - \rho_i r)}{\rho_i}\right\}.$$

Using the capacity-slope substitution, and the fact that $\bar{Q} = rn$, we get:

$$\widehat{Q}_i = r \times \min\{n, c(1-h) - h\}. \tag{4}$$

The intuition here is that when a bidder performs a width-deviation, they will deviate to a quantity where their marginal value equals the height of the crank (which is identical for all bidders). If the deviator's demand is flat (c is high), he is more likely to want to "sweep the entire market", and demand the entire market supply, $\bar{Q} = rn$; this is a corner solution. In the interior case, the deviator will demand a quantity that is strictly less than the full market supply. We now define the width-deviation bidding function as:

$$b_i^{DW}(q_i) = \begin{cases} h(v - \rho r) + \epsilon & \text{if } q_i < \widehat{Q}_i \\ 0 & \text{if } q_i \geq \widehat{Q}_i \end{cases}.$$

Next, to pin down the optimal h^* we need to calculate the one-stage deviation payoffs for both height- and width-deviations (ϵ is suppressed as it goes to zero):

$$u_i^{DW}(q_i) = (v - h(v - \rho r)) \times \widehat{Q}_i - \frac{\rho \widehat{Q}_i^2}{2},$$
$$u_i^{DH}(q_i) = vr - \frac{\rho r^2}{2}.$$

Let $u_i^D(h) = \max\{u_i^{DW}(q_i), u_i^{DH}(q_i)\}$. The optimal h equates payoffs from both the width- and height-deviations, so $u_i^D(h^*) = u_i^{DW}(q_i) = u_i^{DH}(q_i)$. Using the substitution that $\rho = \frac{v}{cr}$, we obtain the following result.

Lemma 2. Suppose Assumption 1 holds. Then,

$$h^* = \begin{cases} \frac{1-2c+2cn-n^2}{2n(c-1)} & \text{if } c \geq \frac{n^2+1}{2} \\ \frac{c-\sqrt{2c-1}}{c-1} & \text{otherwise} \end{cases}.$$

Proof. See Appendix A.2. □

With the crank height set optimally at h^*, we find the stage game payoffs from competition, collusion, and the most profitable deviation. Using the slope-capacity substitution from Eq. (1), we obtain:

Lemma 3. Suppose Assumption 1 holds. Then the competitive bidding payoffs are:

$$u_i^*(q_i) = \frac{vr}{2c} + \begin{cases} 0 & \text{if } \alpha > 0 \\ \frac{vr}{c(n-2)} & \text{if } \alpha = 0 \end{cases}.$$

Moreover, the collusive bidding and deviation payoffs are given by:

$$u_i^C(h^*) = \frac{vr}{2c} + \frac{c-1}{c}(1-\alpha h^*)vr,$$
$$u_i^D(h^*) = \frac{vr}{2c} + \frac{c-1}{c}vr.$$

Lemma 3, combined with Eq. (3) and (4) are sufficient to explain why in the uniform-price auction it is straightforward to construct a collusive equilibrium where even a single-period deviation is not (strictly) profitable. When $\alpha = 0$, the collusive payoff u_i^C is independent of h: for any crank height, the collusion-enforcement premium is zero. In particular, it is costless to set $h = 1$. But then, Eq. (4) shows that $\widehat{Q}_i = r$: the optimal width-deviation is to the same quantity as the collusive split. In other words: there is no strictly profitable width-deviation, even in the stage game.[13]

We now consider a repeated game, where the payoffs from each consecutive auction stage game is discounted by the discount factor δ. A deviation from crank-handle collusion with h^* is detected by observing average prices and is punished by reversion to competitive bidding, given by Eq. (2), forever. We look for the lowest collusion-sustaining discount factor, δ^* which is given by:

$$\delta^* = \inf\left\{ \delta \in [0,1] \;\middle|\; \frac{u_i^C(h^*)}{1-\delta} \geq u_i^D(h^*) + \frac{\delta}{1-\delta} u_i^*(q_i) \right\}.$$

Using Lemma 3 to solve for δ^* at every $\alpha \in [0,1]$, we obtain:

Theorem 1. Suppose Assumption 1 holds. Then,

$$\delta^* = \alpha h^*.$$

Proof. See Appendix A.2. □

Since h^* does not depend on α, Theorem 1 indicates that below-bid pricing makes collusion easier. When α decreases, so does the collusion-sustaining discount factor.

Combining Theorem 1 and Lemma 2, we immediately obtain the following result for constant marginal values.

Corollary 1. Suppose Assumption 1 holds. Then, as $c \to \infty$,

$$\delta^* \to \frac{n-1}{n}\alpha.$$

With constant marginal values, a higher α and a larger number of bidders both make collusion harder.

[13] This result also extends to other auctions where inframarginal units are charged at, or below, the market-clearing price, including the Vickrey auction. Even though the Vickrey auction resides outside of the mixed-price(α) parameter space, when there is no uncertainty, the result for $\alpha = 0$ also shows how single-period deviations from the collusive equilibrium can be blocked in the Vickrey auction.

3.3. **Collusion with two asymmetric bidders.** We now consider the case when bidders are asymmetric and restrict to $n = 2$. There are two bidders: weak and strong $i \in \{W, S\}$. First, note that when $\alpha = 0$, just as in the symmetric case we can enforce any collusive equilibrium because the collusion-enforcement premium in the uniform-price auction is zero. In what follows, we therefore assume that $\alpha > 0$. As in Section 3.2, we look at the limit case when uncertainty tends to zero.

Assumption 2 (Asymmetric environment with two bidders without uncertainty)**.**

- $n = 2$,
- $\alpha > 0$,
- consider payoffs and bidding functions in the limit as $\lambda \to 0$.

We model asymmetry as difference in the slope on marginal values of the two bidders: the strong bidder's marginal values are "flatter". We denote the weak bidder's slope as ρ as before, and strong bidder's slope as $\rho_S = (1-\sigma)\rho$ where $\sigma \in [0, 1)$. Therefore, σ is the *asymmetry* parameter. The corresponding slope-capacity substitutions become $\rho = \frac{v}{rc}$, and $\rho = \frac{v}{(1-\sigma)rc}$. Since we have only two bidders, $\bar{Q} = 2r$. As before, we assume that $c > 1$, which results in a positive market-clearing price.

In a competitive bidding equilibrium with a market-clearing price that equates the marginal values of both bidders,[14] the proportion of supply won by the weak bidder is:

$$\phi = \frac{1-\sigma}{2-\sigma},$$

and the proportion of supply won by the strong bidder is:

$$1 - \phi = \frac{1}{2-\sigma}.$$

We investigate a collusive equilibrium at the same (efficient) market split at a zero price. The weak bidder has a "narrow" crank:

$$b_W^C(h;\sigma) = \begin{cases} h(v - \rho\phi\bar{Q}) & \text{if } q_W < \phi\bar{Q} \\ 0 & \text{if } q_W \geq \phi\bar{Q} \end{cases}.$$

However, the strong bidder has a "wide" crank:

$$b_S^C(h;\sigma) = \begin{cases} h(v - \rho\phi\bar{Q}) & \text{if } q_S < (1-\phi)\bar{Q} \\ 0 & \text{if } q_S \geq (1-\phi)\bar{Q} \end{cases}.$$

Since the market split is unequal, both the competitive and collusive payoffs for the two bidder types will differ. Therefore, the optimal crank heights (h_S^{**} and h_W^{**}) and collusion-sustaining discount factors (δ_S^{**} and δ_W^{**}) will also differ. For collusion to be sustained at the market level, we need the more stringent constraint to bind, that is the discount factor for each bidder must be at least $\delta^{**} = \max\{\delta_W^{**}, \delta_S^{**}\}$. The optimal crank height now depends on the bidders' capacities, and the degree of asymmetry.

[14] We discuss strategies that sustain such an equilibrium in Appendix A.4

Lemma 4. Suppose Assumption 2 holds. Then:

$$h^{**} = \begin{cases} \frac{c(2-\sigma)+2\sigma-3}{(2-\sigma)(c(2-\sigma)-2(1-\sigma))} & \text{if } c \geq 2 + \frac{1}{1-\sigma} - \frac{1}{2-\sigma} \\ \\ \frac{c(2-\sigma)-2\sqrt{(1-\sigma)(c(2-\sigma)-(1-\sigma))}}{c(2-\sigma)-2(1-\sigma)} & \text{otherwise} \end{cases}.$$

Proof. See Appendix A.3. □

Analogously to the symmetric case, we also obtain a result on the difficulty of collusion under different degrees of below-bid pricing.

Theorem 2. Suppose Assumption 2 holds. Then,

$$\delta^{**} = \alpha h^{**}.$$

Also: (1) for any σ, δ^{**} is strictly increasing in α; (2) for any $\alpha > 0$, δ^{**} is increasing in σ.

Proof. See Appendix A.3. □

Theorem 2 gives two predictions. First, for any mixed-price auction and any degree of asymmetry, collusion is harder in auctions with less below-bid pricing. Second, in any mixed-price auction collusion is easier when there is less asymmetry.[15]

4. Fishing industry in the Faroes

The Faroe Islands is a small country in the North Atlantic which is heavily dependent on the fishing industry. In 2017, over 20 percent of GDP and 93 percent of the country's exports came from the fishing and aquaculture sector which employed 15 percent of the workforce (Statistics Faroe Islands, 2020). There are three main types of fisheries in the Faroes: demersal fisheries (e.g., cod, haddock) in Faroese waters; pelagic fisheries (e.g., mackerel, herring, blue whiting); and demersal fisheries in the Barents Sea. Only the latter two are commercially lucrative.

Just three companies operate in the Barents sea (P/F Enniberg, Sp/f Framherji and P/F JFK Trol). The pelagic fisheries industry is also concentrated: four companies (P/F Christian í Grótinum, Sp/f Framherji, P/F JFK Trol and P/F Varðin) with ten vessels fish approximately 90 percent of the pelagic catch (Faroese Fisheries Inspectorate, 2017).

Since the introduction of fishing licenses in 1987, quota in the Faroe Islands had been almost exclusively allocated based on historical fishing rights (i.e., "grandfathered"). Starting from 2016, as part of the ongoing fisheries reform the government decided to run trial auctions of 10 percent of the TAC ("total allowable catch") for demersal fish in the Barents Sea, blue whiting, herring, and mackerel. Between then and 2019, the proportion of TAC allocated through auction gradually increased (Faroe Islands Government, 2018).

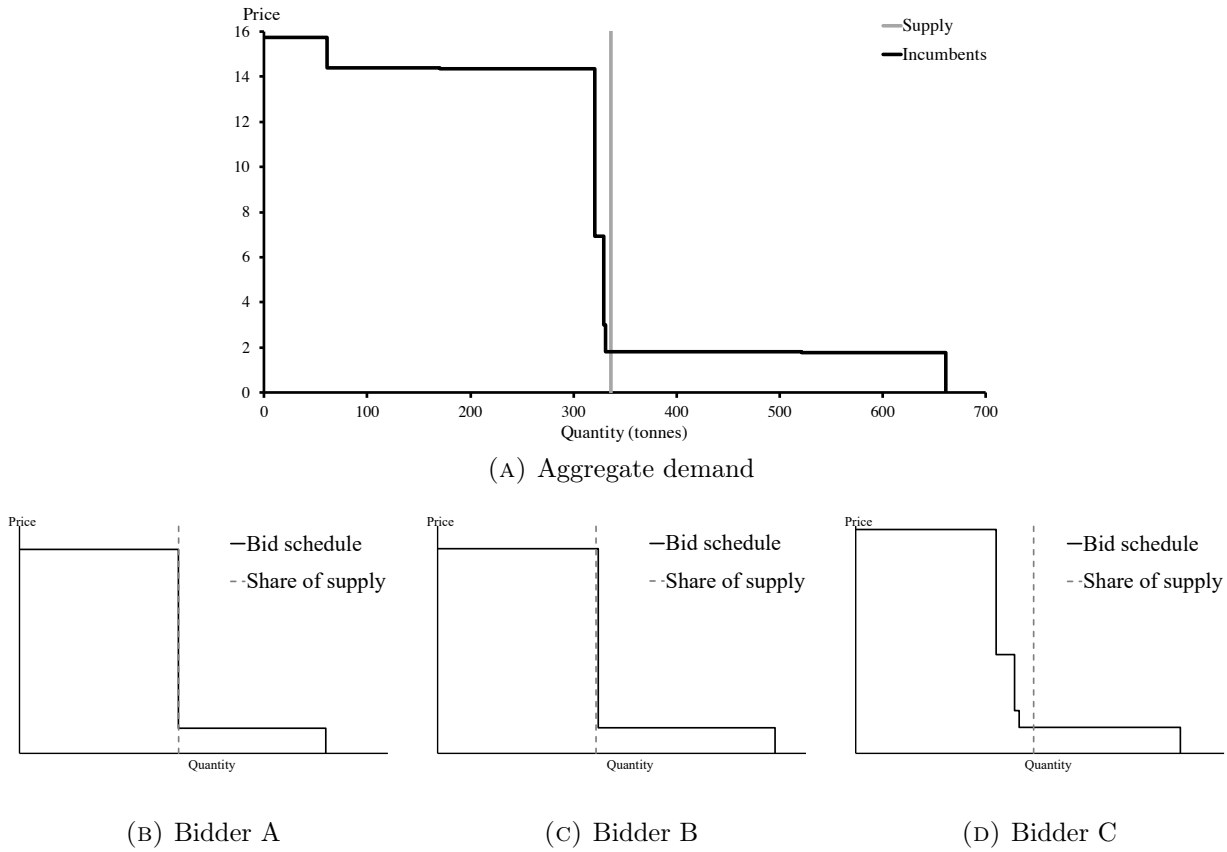
[15] Our parameterization modelled asymmetry via a difference in slopes, while keeping the same intercept for both bidders. An alternative parameterization would allow for varying intercepts v instead of, or in addition to, slopes.

Globally, auctioning of fishing rights is still comparatively rare. Fishing quota auctions have been implemented in Chile, Estonia, New Zealand, Russia and Washington State in the U.S. (see, Lynham (2014) and State of Washington (2018)).

Many economists argue that auction-based quota allocation can improve the efficiency of operations, give long-run incentives for innovation investment, and provide opportunities for revenue-recycling (Kominers et al., 2017; Marszalec, 2018a; Teytelboym, 2019). We do not consider broader questions of the desirability of quota auctions in the Faroes and focus on the performance of the auctions instead.

5. Evidence of crank-handle bidding in uniform-price auctions

In 2018, quota for demersal fish in Svalbard was sold in a uniform-price auction. In these auctions, bidders were allowed to bid entire demand curves (at most five price-quantity bids that comprise an individual demand). Supply of quota was pre-announced and fixed.[16] There were three incumbents whom we anonymize as Bidders A, B, and C.



(A) Aggregate demand

(B) Bidder A (C) Bidder B (D) Bidder C

Figure 8. Aggregate and individual bids of the incumbents and the supply of quota in the 2018 auction of quota for demersal fish in Svalbard. Individual demands are not to scale to preserve anonymity.

Although the industry's break-even cost is below 8 kr./kg, most of the supply was allocated to bids over 14 kr./kg. Figure 8a shows the final aggregate demand and the

[16] Last-accepted-bid pricing rule was used (Burkett and Woodward, 2020). If the marginal bid were rationed, bidders had the option of refusing to purchase the associated quota.

supply of quota. The aggregate demand has a pronounced crank-handle shape: the sharply declining portion of the aggregate demand takes place at quantities between 321 and 331 tonnes. The final step-decrease in aggregate demand occurs less than 1.5 percent short of the supply of 336 tonnes. Individual bids presented in Figures 8b–8d also have a crank-handle shape, in line with Prediction 1. Indeed, the evidence for Prediction 1 is robust: in Online Appendix C, we show that crank-handle bidding was pervasive in most uniform-price quota auctions of demersal fish between 2016 and 2019.

Without knowing exactly the bidders' motivation for submitting crank-handle bids, we cannot be certain that they are colluding; it is possible that crank-handles reflect their true demand functions. However, the point at which incumbents' bids drop off sharply coincide precisely with the incumbents' grandfathered quota share held prior to the auction. In Figures 8b–8d, we illustrate the grandfathered quota shares of the supply by dotted lines. It would be a remarkable coincidence that all three bidders' valuations of the quota would change so dramatically at the grandfathered quota shares, in particular since the industry as a whole had considerable spare capacity (Leo, 2018). The pattern of using grandfathered shares to split the market was pervasive in all six uniform-price auctions in 2018 (see Online Appendix C).

6. Evidence of collusion and predation in ascending auctions

In this section, we illustrate how a series of ascending auctions for fishing quota also performed poorly. The logic for their failure is analogous to the reasons for why uniform-price auction failed. As Klemperer (2002b, p. 171) argued:

> Since, with many units, the lowest winning bid in a uniform-price auction is typically not importantly different from the highest losing bid, [the uniform-price] auction is analogous to an ascending auction (in which every winner pays the runner-up's willingness-to-pay). The "threats" that support collusion in a uniform-price auction are likewise analogous to the implicit threats supporting collusion in an ascending auction.

6.1. **Bidding rings.** To evaluate Prediction 2, we analyze how ascending auctions facilitated bidding ring formation among incumbents. In 2017, 5,447 tonnes of quota for mackerel were originally split into 24 lots, and sold in a sequence of ascending auctions. Seven bidders (anonymized as A–G) participated in the auction and all, except Bidder G, won some quota.

Figure 9 shows the price paid by the winning bidder on each lot. The stability of the prices is striking: after three rounds, the prices settle at around 3.10 kr./kg. From Lot 3 onward, all but two lots were sold at prices between 3.09 and 3.12 kr./kg (the reserve price was 1.25 kr./kg).

The prices for all lots, except Lots 13.2 and 21, were set by Bidder A who exited last in 23 out of 25 rounds (see Online Appendix B.1). The identities of the winning bidders rotated across rounds, with five bidders (i.e., Bidders B–F) taking turns to outbid Bidder

A. Moreover, in every round, except Rounds 14 and 21, four out of five rotating bidders either did not participate or dropped out at prices far below 3.00 kr./kg.

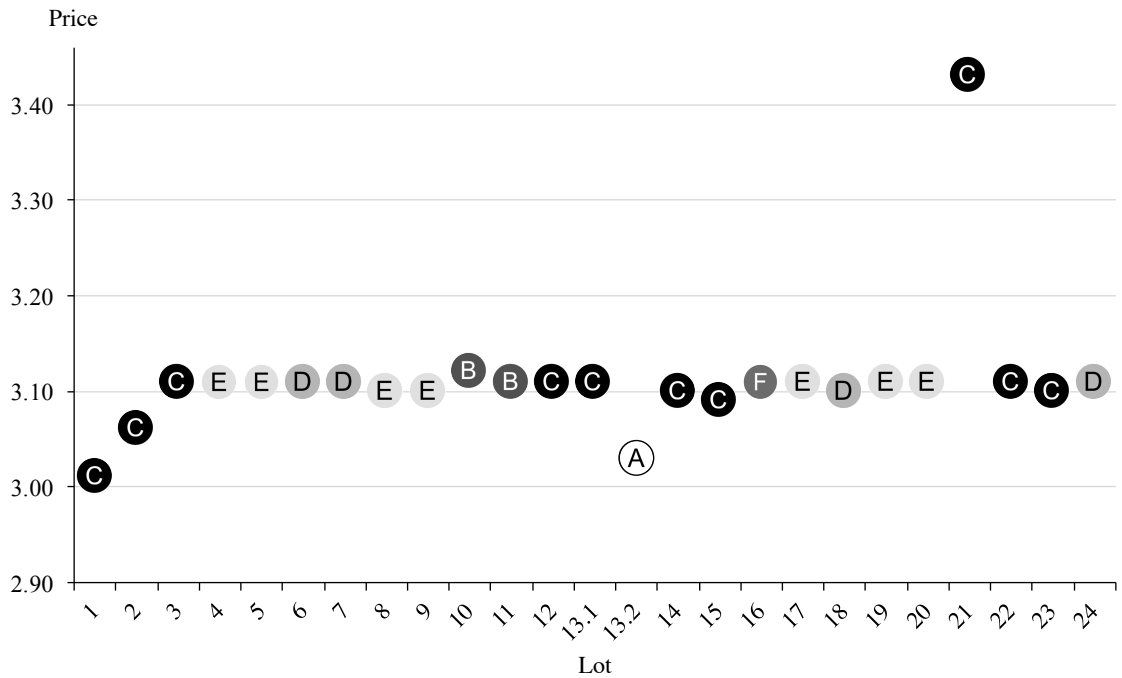


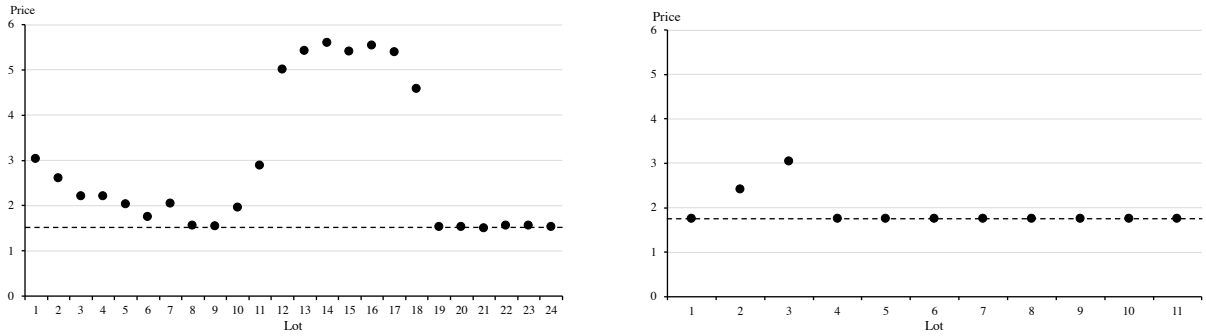
FIGURE 9. Final prices in the 2017 ascending auctions of quota for mackerel (24 lots: Lot 13 was split into two during the course of the auction; see Online Appendix B.1). Labels denote the identities of winners.

The bidding pattern in here is consistent with the presence of a bidding ring among Bidders B–F. If that were the case, there are at least two explanations for the bidding pattern: either Bidder A was trying to inflate the price paid by all members of the bidding ring, or Bidder A served to commit members of the bidding ring to pay similar prices.[17]

Why was collusion so easy in these auctions? Prediction 2 suggested that collusion in uniform-price auctions is costless (irrespective of bidder asymmetry) because the collusion-enforcement premium is zero. A special case of our Prediction 2 is the single-unit, ascending auction setting: conditional on the leader bidding aggressively in the ascending auction, other ring-members cannot make a profit, and can simply drop out at the reserve (Robinson, 1985). In sequential auctions, collusion is just as easy: if bidders can agree on a winner rotation pattern, there is no incentive to deviate in an individual auction.

6.2. **Value advantage and predation.** The 2016 and 2017 auctions of quota for demersal fish in the Russian part of the Barents Sea were also run as a series of ascending auctions (with similar amounts of quota being auctioned in each round). In these auctions, there was a strong incumbent and a weak entrant, allowing us to evaluate Prediction 3.

[17]See Online Appendix B.1 for further evidence on the former explanation.



(A) 2016 auction (24 lots). Prices in the auctions of quota for demersal fish in the Russian part of the Barents Sea. Dots: final prices. Dashed line: reserve price.

(B) 2017 auction (11 lots). Prices in the auctions of quota for demersal fish in the Russian part of the Barents Sea. Dots: final prices. Dashed line reserve price.

FIGURE 10. Sequential ascending auctions in the Faroe Islands.

In 2016, two weeks after the auction design had been announced—and just one day before the auction was run—the dominant incumbent made an announcement in an interview on Faroese national radio that he was committed to winning all the quota at whatever price necessary. As we showed in Section 2.4, such announcements of predatory bidding are credible in ascending auctions. Consequently, all the 24 auctions had only two bidders: the interviewed incumbent and one entrant. Two other incumbents, who held quota in the Barents Sea, did not participate.

The outcome of the auctions was that the incumbent won all 24 lots and the entrant was the price-setter on each lot. Figure 10a shows the final price in each auction round. The entrant competed most keenly for Lots 10 to 18, and the last six auction rounds finished marginally above the reserve price. Overall, the average price for Lots 1-18 was approximately 3.4 kr./kg, but only 1.5 kr./kg in the last six rounds. It appears that in Rounds 10-18, the entrant was either testing the incumbent's resolve, or checking whether the incumbent had bought enough quota to be willing to exit the auction. With only six lots left, the entrant stopped competing altogether: one plausible reason for such behavior is that the remaining amount of quota would not have been sufficient for the entrant to run a profitable operation. Without meaningful competition on the last six lots, the incumbent bought them marginally above the reserve price.

As Figure 10b illustrates, the same auction run in 2017 continued from where the previous year's auction left off: with the incumbent showing a strong commitment to bid whatever necessary to win everything. The entrant only bid in Rounds 2 and 3 to test the incumbent's commitment, but dropped out thereafter. All lots other than Lots 2 and 3 were sold at the reserve price to the incumbent.

Clearly, the standard competitive-bidding and symmetry assumptions did not hold in these auctions. In the potential bidder pool there were three incumbents (one of whom was stronger than the others) and one entrant (who was value-disadvantaged). Moreover,

bidding was not competitive. Instead, bidding was consistent with Prediction 3: the value-advantaged bidder was able to commit to aggressive bidding in the ascending auctions and to drive other bidders out.

6.3. **Consequences of the sequential design.** The poor performance of ascending auctions in the Faroes could be partly attributed to their sequential design. As observed by Marshall and Marx (2007, 2009, 2012), sequential designs in any auction facilitate bidding rings because bidders can send each other more signals, making both detection and punishment easier.

In Section 2.4 we explained why predation is easy in auctions with below-bid pricing. In fact, Klemperer (2002b) emphasized that:

> Predation may be particularly easy in repeated ascending auctions, such as in a series of spectrum auctions. A bidder who buys assets that are complementary to assets for sale in a future auction or who simply bids very aggressively in early auctions can develop a reputation for aggressiveness (Bikhchandani, 1988). Potential rivals in future auctions will be less willing to participate and will bid less aggressively if they do participate (Klemperer, 2002a).

Klemperer's argument about "complementary assets" also applies to minimum feasible scale or to "essential facilities" in production (Motta, 2004, p. 66). In the Faroese case, the incumbent could block entry by aggressive bidding in earlier rounds and thus denying the entrant the chance to win enough quota to operate at a minimum feasible scale.

7. Improvements to auction design

The main reasons for the poor performance of uniform-price and (sequential) ascending auctions of fishing quota in the Faroes were collusion—via crank-handle bidding and bidding rings—and predation.

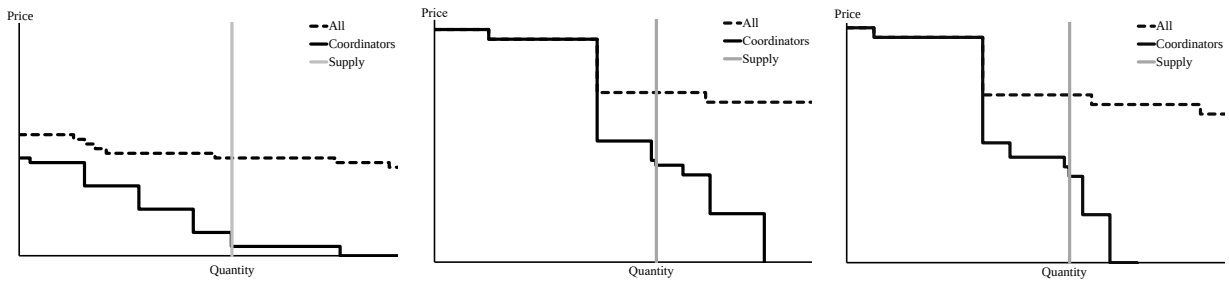
One format that could have mitigated both collusion and predation is the (multiunit) pay-as-bid auction. In this design, bidders submit their demand curves in a sealed-bid manner and the winners pay their bids on units won. The sealed-bid nature of pay-as-bid auctions means that little information is transmitted to and between bidders making the formation of bidding rings harder (Marshall and Marx, 2007, 2009, 2012). As our model shows, pay-as-bid auctions have several other advantages. First, collusion and crank-handle bidding are difficult to sustain in an equilibrium of a pay-as-bid auction (Predictions 1 and 2). Second, pay-as-bid auctions mitigate the ability of strong bidders to exercise their value advantage to predatory effect compared to auctions with below-bid pricing (Prediction 3).

There are two possible variations on the pay-as-bid auction. First, one could run a single auction of all the species at the same time, allowing bidders to express package bids across different species. Package bidding could also allow bidders to express preferences over operational scale which makes it harder to exclude entrants. Although pay-as-bid

package auctions have complex Bayes-Nash equilibria, they can be easier to explain to bidders than an auction with more subtle core-selecting pricing rules.[18] Second, one could also run an auction with uncertain supply of quota, which is most easily achieved by giving the auctioneer discretion to reduce supply if the market-clearing price is too low. Uncertain supply mitigates the ease with which bidders could enforce crank-handle collusion and is used in practice in some Treasury Bill auctions.[19] However, uncertain supply might not be feasible in the fishing quota context since the total allowable catch is set seasonally by a political and scientific consensus.

8. Recent evidence from uniform-price and pay-as-bid auctions

In October 2018, we publicly shared a version of this paper in which we recommended the pay-as-bid package auction. Following discussions with officials, industry experts, and civil servants, the Faroese government decided to trial standard pay-as-bid auctions to sell one-year quota for blue whiting in March 2019. Uniform-price auctions continued to be used for three-year and eight-year quota. Results from the auctions are summarized in Figure 11. Scaling on the vertical axis is the same for all auctions, and the quantity axes have been scaled proportionately. Price and quantity numbers have been removed to preserve confidentiality. For each auction, we show the aggregate demand, and a separate demand function for a subset of bidders who attempted to coordinate on a low market-clearing price.



(A) One-year quota: pay-as-bid auction.

(B) Three-year quota: uniform-price auction.

(C) Eight-year quota: uniform-price auction.

Figure 11. Aggregate demands of the coordinating and of all bidders in the March 2019 auctions of quota for blue whiting.

Overall, the bidding data are consistent with our predictions. First, coordinating bidders submitted a substantially lower crank-handle aggregate demand in the pay-as-bid auction than in the uniform-price auction (Prediction 1). The low crank-handle suggests that bidders were responding to presence of the collusion-enforcement premium

[18] See, for example, Prendergast (2017, footnote 4) and Marszalec (2018b).

[19] Mariño and Marszalec (2020) observe an extreme example of such strategic supply adjustment in the Philippines, where the Treasury frequently restricts supply. Consequently, the market-clearing price is never unexpectedly low. Earlier discussions of strategic supply choice include Back and Zender (2001), LiCalzi and Pavan (2005), and McAdams (2007).

in the pay-as-bid auction. However, the aggregate demands of coordinating bidders in the uniform-price auctions more closely resemble the high crank-handle bids observed in previous years (Prediction 1).

Second, aggregate demand in the pay-as-bid auction intersects supply at a price which is strictly above the highest bid of the coordinating bidders; in this auction, a non-coordinating bidder would have won the entire supply, save for a technical error (see Online Appendix B.2). As outlined in Section 3, bidders attempting to coordinate in a pay-as-bid auction have an incentive to submit a flat bid. However, this strategy is vulnerable to deviation by a non-coordinating bidder who can win the entire supply. By comparison, in the uniform-price auctions, non-coordinating bidders would have won only around a quarter of the three-year quota or 40 percent of the eight-year quota. While these are not *ceteris paribus* comparisons, the evidence from these auctions supports our Prediction 3 that non-coordinating bidders have a higher likelihood of winning in pay-as-bid auctions.

This is where the story on the Faroese quota auctions ends. The use of fishing quota auctions became a major campaign issue in the August 2019 elections. The 2015–2019 coalition government was voted out of office. As of October 2020, the new government had no plans to run further fishing quota auctions (Føroya Løgting, 2019).

9. Conclusion

Economic theory based on realistic assumptions can be very informative for auction design (Milgrom, 2000; Klemperer, 2002b). However, models that do not take bidder asymmetry or the possibility of collusion and predation into account can offer misleading predictions of market behavior and result in unfavourable auction outcomes.

In this paper, we construct a model which shows that auctions with below-bid pricing facilitate collusion, and undo many of the desirable properties that such auctions exhibit in symmetric, competitive-bidding equilibria. Our model covers the Vickrey, uniform-price auctions, pay-as-bid, as well as the entire family of *mixed-price auctions*. Our results cast doubt on the conventional theoretical wisdom that decoupling winners' payments from their own bids is a recipe for robust auction design. To the contrary, pay-as-bid auctions are likely to be less vulnerable to collusion and predation which are pervasive in practice.

Using recent evidence from the Faroe Islands, we show that ascending and uniform-price auctions performed poorly in a manner consistent with our model. We document an unusual case of auctions with many interesting bidding strategies developed by a group of inexperienced bidders. Our examples of crank-handle bidding are probably the most exciting as there appears to be no other clear example from the field (except in electricity markets where bidders are much more experienced). While fishing companies participating in Faroese auctions neither had much prior auction experience nor (as far as we know) hired auction consultants, their experience of running profitable businesses in a tightly-knit industry allowed them to easily implement profitable bidding strategies that surprised auction designers.

We therefore caution any auction designer who considers running (sequential) ascending or uniform-price auctions without flexible supply wherever there is serious bidder asymmetry and opportunities for industry coordination. Our results suggest that in practice, by mitigating collusion and predation, pay-as-bid auctions may be more efficient and raise higher revenue than auctions with below-bid pricing. Meanwhile, the majesty of ascending, uniform-price, and Vickrey auctions continues to walk on theoretical stilts.

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Appendix A. Proofs

A.1. Symmetric competitive bidding equilibrium.

Proof of Lemma 1. We solve for a linear bidding function $b_i(q_i, v) = A(v) - B(v)q_i$ in a mixed-price auction. Consider bidder i in a mixed-price auction, whose residual supply has slope μ_i and intercept x, i.e,

$$p = x + \mu_i q_i.$$

Intercept x has c.d.f. $G_i(\cdot)$ and a corresponding density $g_i(\cdot)$. Following, the method developed in the proofs of Proposition 6 and 7 in Ausubel et al. (2014), we first discretize the distribution of intercept x by partitioning its support into a countable number of intervals of length Δx. Let π_s denote the probability that the realization of the intercept is in interval s. Consider a small deviation from the equilibrium bid so the market-clearing price changes by $dp = \mu_i dq$. The first-order condition (FOC) is then:

(5)
$$dq \times \pi^s \times (v - \rho q_i) = dq \times \pi^s \times (b_i + \alpha\mu_i \times \Delta q + (1-\alpha)\mu_i q_i) + \alpha dq \times (\Delta p + \mu_i \Delta q) \times \sum_{k>s} \pi^k.$$

By rearranging and making the necessary substitution, we obtain the Euler equation for a mixed-price auction with any $\alpha \in [0, 1]$:

$$\frac{\pi^s}{\Delta x}\frac{\Delta x}{\Delta q} \times (v - \rho q_i - b_i - \alpha\mu_i \times \Delta q - (1-\alpha)\mu_i q_i) = \alpha\left(\frac{\Delta p}{\Delta q} + \mu_i\right) \times \sum_{k>s} \pi^k. \tag{6}$$

Now let $\Delta x \to 0$. In the limit, the Euler equation (6) becomes:

$$g_i(x)\left(-\frac{\partial x}{\partial q}\right) \times (v - \rho q_i - b - (1-\alpha)\mu_i q_i) = \alpha\left[\mu - \frac{\partial b_i(v)}{\partial q_i}\right] \times G_i(x). \tag{7}$$

Next, we rewrite the Euler equation (7) as a function of equilibrium quantity q_i. Using the p.d.f $g(q_i) = g_i(x)\left(-\frac{\partial x}{\partial q}\right)$, the c.d.f. $G_i(q) = 1 - G_i(x(q_i))$, and the inverse hazard rate $h_i(q) = \frac{(1-G_i(q))}{g_i(q)}$, of q_i, we obtain:

$$v - \rho q_i - b_i - (1-\alpha)\mu_i q_i = \alpha\left[\mu - \frac{\partial b_i(v)}{\partial q_i}\right] \times h_i(q). \tag{8}$$

Substitute in the hazard rate $h_i(q) = \sigma - \frac{1}{\lambda}q_i$ and the (negative of) the slope of the bidding function $\psi_i = -\frac{\partial b_i(v)}{\partial q_i}$ into Eq. (8) to obtain a new expression for the FOC:

$$v - [\rho + (1-\alpha)\mu_i]\, q_i - b_i = \alpha\,[\mu_i + \psi_i]\left[\sigma - \frac{1}{\lambda}q_i\right]. \tag{9}$$

Collecting the q terms only from Eq. (9) and solving for ψ_i, we obtain:

$$\psi_i = \frac{\rho + (1-\alpha)\mu_i - \alpha\mu_i\frac{1}{\lambda}}{1 + \frac{1}{\lambda}\alpha}. \tag{10}$$

The slope of bidder i's residual supply (known as "price impact" in Ausubel et al. (2014)) is generally given by:

$$\mu_i = \frac{1}{\sum_{j\neq i}\frac{1}{\psi_j}}. \tag{11}$$

Substitute Eq. (11) into Eq. (10) and, assuming symmetry, $\psi_i = \psi$, solve for ψ to obtain:

$$\psi = \frac{\rho(n-1)}{n - 2 + \alpha(1 + \frac{1}{\lambda}n)}. \tag{12}$$

Under symmetry, the price impact becomes:

$$\mu = \frac{\psi}{(n-1)} = \frac{\rho}{n - 2 + \alpha(1 + \frac{1}{\lambda}n)}. \tag{13}$$

Once again, collecting the q terms from Eq. (9), assuming symmetry, and using Eqs. (12) and (13), we can simplify the expression for the slope of the bidding function as follows:

$$-[\rho + (1-\alpha)\mu] + \alpha[\mu + \psi]\frac{1}{\lambda} = -\left[\rho + (1-\alpha)\mu_i - \alpha[\mu+\psi]\frac{1}{\lambda}\right] \tag{14}$$

$$= -\left[\rho + \frac{(1-\alpha)\rho}{n - 2 + \alpha(1 + \frac{1}{\lambda}n)} - \frac{\alpha\rho n\frac{1}{\lambda}}{I - 2 + \alpha(1 + \frac{1}{\lambda}n)}\right] \tag{15}$$

$$= -\left[\frac{\rho(n-1)}{I - 2 + \alpha(1 + \frac{1}{\lambda}n)}\right] \tag{16}$$

$$= -\left[(\mu + \psi)\frac{n-1}{n}\right]. \tag{17}$$

Substituting Eq. (17) into the FOC (9), the optimal bid can be written as:

$$b_i = v - \frac{\rho(n-1)}{n-2+\alpha(1+\frac{1}{\lambda}n)}\left[q_i + \frac{n}{n-1}\alpha\sigma\right].$$

Finally, substituting in $\sigma = r(1+\frac{1}{\lambda})$, and slightly rewriting the above equation gives:

$$b_i = v - \frac{\alpha r(1+\lambda)n\rho}{(n-2)\lambda+(n+\lambda)\alpha} - \frac{(n-1)\lambda\rho}{(n-2)\lambda+(n+\lambda)\alpha}q_i, \tag{18}$$

as required. □

A.2. **Collusion in the symmetric model.**

Proof of Theorem 1. In our application, supply is fixed, which corresponds to the case where $\lambda \to 0$ so, using Eq. (18), we obtain the following symmetric equilibrium bidding function:

$$b_i^*(q_i) = \begin{cases} v - \rho r & \text{if } \alpha > 0 \text{ and } q_i \leq \bar{Q} \\ v - \frac{n-1}{n-2}\rho q & \text{if } \alpha = 0 \end{cases}.$$

The competitive per-capita equilibrium payoffs then are:

$$u_i^*(q_i) = \begin{cases} (v-(v-\rho))r - \frac{\rho r^2}{2} & \text{if } \alpha > 0 \text{ and } q_i \leq \bar{Q} \\ (v-(v-\frac{n-1}{n-2}\rho))r - \frac{\rho r^2}{2} & \text{if } \alpha = 0 \end{cases}.$$

Using the slope-capacity substitution from Eq. (1), $\rho = \frac{v}{cr}$, yields:

$$u_i^*(q_i) = \begin{cases} \frac{rv}{2c} & \text{if } \alpha > 0 \\ \frac{rv}{2c} + \frac{rv}{c(n-2)} & \text{if } \alpha = 0 \end{cases}.$$

This payoff can be rewritten more succinctly, using the indicator function $I[\alpha = 0]$, which takes the value 1 when $\alpha = 0$, and 0 otherwise:

$$u_i^*(q_i) = \frac{rv}{2c} + \frac{rv}{c(n-2)}I[\alpha = 0].$$

The payoff from bidding according to a collusive crank-handle with height h, in a mixed-price auction is:

$$u_i^C(h) = vr - \frac{\rho r^2}{2} - (v-\rho r)\alpha h r.$$

Using the slope-capacity substitution, as above, this expression simplifies to:

$$u_i^C(h) = \frac{rv}{2c} + \frac{(c-1)}{c}(1-\alpha h)rv. \tag{19}$$

The payoff from a (downward) height-deviation is equivalent to the payoff from bidding a crank-handle with $h = 0$, i.e.:

$$u_i^{DH}(q_i) = \frac{rv}{2c} + \frac{(c-1)}{c}rv. \tag{20}$$

Note that the situations when $h = 0$ and $\alpha = 0$ are equivalent: in a uniform-price auction, the collusion-enforcement premium is always zero, so the height-deviation is never more profitable than sticking to any other collusive crank handle, in equilibrium.

To evaluate the payoffs from a width-deviation, we must first find the quantity (i.e. width) to which a bidder would find it most profitable to deviate. It is given by the following equation:

$$\widehat{Q}_i = \min\left\{\bar{Q}, \frac{v - h(v - \rho_i r)}{\rho_i}\right\}.$$

Using the slope-capacity substitution from Eq. (1), and the fact that $\bar{Q} = rn$, we get:

$$\widehat{Q}_i = r \times \min\{n, c(1-h) - h\} = rZ,$$

where $Z = \min\{n, c(1-h) - h\}$. The intuition here is that when a bidder performs a width-deviation, they will deviate to a quantity where their marginal value equals the height of the crank (which, recall, is identical for all bidders); if the deviator's demand is flat (c is high), he is more likely to want to "sweep the entire market", and demand the entire market supply, $\bar{Q} = rn$; this is a corner solution. In the interior case, the deviator will demand a quantity that is strictly less than the full market supply. The payoff from a width-deviation is therefore:

$$u_i^{DW}(q_i) = vr - \frac{\rho(rZ)^2}{2} - (v - \rho r)h(rZ).$$

Applying the slope-capacity substitution from Eq. (1), we obtain:

$$u_i^{DW}(q_i) = \frac{rv}{2c} + \frac{rv}{2c}\left(2cZ - Z^2 - 2(c-1)hZ - 1\right). \tag{21}$$

The optimal crank height will equalize the expected payoff from height- and width-deviations. To see why this must be the case, note that collusive payoffs are decreasing in crank height (Eq. (19)) – so the bidders want to set the lowest feasible crank that deters deviation. However, from Eq. (21) above, the width-deviation payoffs are also higher when h is low; in particular, at $h = 0$, every bidder would like to deviate. Meanwhile, the payoff from a height-deviation is fixed (i.e. invariant in h; see Eq. (20)). Therefore, to maximize payoffs along the equilibrium path, while simultaneously minimizing deviation incentives, the bidders must pick the lowest h, subject to width-deviation incentives not exceeding incentives for a height-deviation. At this height, h^*, the expected payoffs from both deviations are equal. Solving for h^* gives:

$$h^*(c,n) = \begin{cases} \frac{1-2c+2cn-n^2}{2n(c-1)} & \text{if } c \geq \frac{n^2+1}{2} \\ \frac{c-\sqrt{2c-1}}{c-1} & \text{otherwise} \end{cases}.$$

Note also that, as expected:

$$u_i^{DW}(q_i) = u_i^{DH}(q_i) = \frac{rv}{2c} + \frac{(c-1)}{c}rv = u^D(h^*).$$

The minimum collusion-sustaining discount factor is given by:

$$\delta^* = \inf\left\{ \delta \in [0,1) \;\middle|\; \frac{u_i^C(h^*)}{1-\delta} \geq u^D(h^*) + \frac{\delta}{1-\delta}u_i^*(q_i) \right\}.$$

Rearranging this for δ^* gives us:

$$\delta^* = \frac{u^D(h^*) - u_i^C(h^*)}{u^D(h^*) - u_i^*(q_i)}.$$

Substituting in for all three payoffs gives:

$$\delta^* = \frac{\frac{rv}{2c} + \frac{(c-1)}{c}rv - (\frac{rv}{2c} + \frac{(c-1)}{c}(1-\alpha h^*)rv)}{\frac{rv}{2c} + \frac{(c-1)}{c}rv - (\frac{rv}{2c} + \frac{rv}{c(n-2)}I[\alpha = 0])}.$$

Noting that $v, r > 0$ and $c > 1$, after appropriate cancellations we obtain:

$$\delta^* = \frac{c-1}{(c-1) + \frac{I[\alpha=0]}{n-2}}\alpha h^*.$$

The indicator function in the denominator takes the value 1 only when $\alpha = 0$, but in that case $\delta^* = 0$ anyway. Without loss of generality, we can therefore conclude that:

$$\delta^* = \alpha h^*.$$

□

A.3. **Collusion with two asymmetric bidders.** The derivation of the minimal collusion-sustaining discount factor in the asymmetric model is largely similar to the symmetric case. Once we have established what the "collusive market split" is, both the weak and the strong bidder are facing the same options as before: stick to the collusive split or perform a width- or height-deviation, followed by Nash reversion to the competitive stage game equilibrium. Previously, in equilibrium, each bidder would win $q_i = \bar{Q}/2$, and that would scale competitive payoffs, crank-handle payoffs, as well as payoffs from the height-deviation. Now, the scaling factor for these three amounts will be some market split, with the strong bidder obtaining more than $\bar{Q}/2$, and the weak bidder obtaining less. In both the symmetric and asymmetric cases, the incentives for a width-deviation will be driven by the slope of demand and the crank height, in equilibrium.

Since the demand slopes and original market shares differ between the two bidders, their incentives for a width-deviation will diverge. It is still the case that the optimal crank height equates incentives from width- and height-deviations for one of the bidders (the weak bidder).

Proof of Theorem 2 and Lemma 4. The weak bidder's marginal value slope is ρ and strong bidder's slope is $(1-\sigma)\rho$ where $\sigma \in [0,1)$. The marginal values for the weak bidder are

given by:

$$\mathrm{MV}_W(q_W) = v - \rho q_W,$$

and for the strong bidder, by:

$$\mathrm{MV}_S(q_S) = v - (1-\sigma)\rho q_S.$$

As before, we can re-parametrize the slope in terms of bidder capacities, so that $c = \frac{v}{r\rho}$ for the weak bidder, and $\frac{c}{1-\sigma}$ for the strong bidder. Therefore, σ is the *asymmetry* parameter. Let ϕ denote the weak bidder's share. As we show in Appendix A.4 below, it can be pinned down by the condition that equates the two bidders' marginal values:

$$v - \rho\phi\bar{Q} = v - \rho(1-\sigma)(1-\phi)\bar{Q} \implies \phi = \frac{1-\sigma}{2-\sigma}.$$

Using the slope-capacity substitution from Eq. (1), we find the market-clearing price:

$$p^* = \frac{v}{c}\left(c - \frac{2(1-\sigma)}{2-\sigma}\right).$$

Next, analogously to the symmetric case, we derive the competitive bidding and collusive payoffs, as well as height- and width-deviation incentives:

$$\begin{aligned} u_S^*(q_S) &= (v-p^*)(1-\phi)\bar{Q} - \frac{\rho(1-\sigma)((1-\phi)\bar{Q})^2}{2} \\ u_W^*(q_W) &= (v-p^*)\phi\bar{Q} - \frac{\rho(\phi\bar{Q})^2}{2}. \end{aligned}$$

Substituting in for the market-clearing price, and using the slope-capacity substitution:

$$\begin{aligned} u_S^*(q_S) =& \frac{2rv(1-\sigma)}{c(2-\sigma)^2} \\ u_W^*(q_W) =& \frac{2rv(1-\sigma)^2}{c(2-\sigma)^2} = (1-\sigma)u_S(q_S). \end{aligned}$$

The collusive payoffs are:

$$\begin{aligned} u_S^C(h) &= (v-\alpha h p^*)(1-\phi)\bar{Q} - \frac{\rho(1-\sigma)((1-\phi)\bar{Q})^2}{2} \\ u_W^C(h) &= (v-\alpha h p^*)\phi\bar{Q} - \frac{\rho(\phi\bar{Q})^2}{2}. \end{aligned}$$

After applying the slope-capacity substitution from Eq. (1), we get:

$$\begin{aligned} u_S^C(h) =& \frac{2rv(c(2-\sigma)(1-\alpha h) - 2ah\sigma + 2ah + \sigma - 1)}{c(2-\sigma)^2} \\ u_W^C(h) =& \frac{2rv(c(2-\sigma)(1-\alpha h) - 2\alpha h\sigma + 2\alpha h + \sigma - 1)}{c(2-\sigma)^2}(1-\sigma) = (1-\sigma)u_S^C(r,v). \end{aligned}$$

Since the payoff from a height-deviation is equivalent to crank-handle payoffs with $h = 0$, we immediately obtain:

$$u_S^{DH}(q_S) = \frac{2rv(c(2-\sigma)+\sigma-1)}{c(2-\sigma)^2}$$
$$u_W^{DH}(q_W) = (1-\sigma)u_S^{DH}(q_S)$$

As before, the crank height that minimizes the collusion-sustaining discount factor equates the incentives from height- and width-deviations. Therefore, above equations are sufficient to derive δ^{**} as a function of h^{**}. Note that each bidder will have a different discount factor. Analogously to the symmetric case:

$$\delta_S^{**} = \frac{u_S^D(h_S^{**}) - u_S^C(h_S^{**})}{u_S^D(h_S^{**}) - u_S^*(q_S)}, and$$

$$\delta_W^{**} = \frac{u_W^D(h_W^{**}) - u_W^C(h_W^{**})}{u_W^D(h_W^{**}) - u_W^*(q_W)}.$$

Substituting in for payoffs and simplifying, as in the symmetric case, we obtain:

$$\delta_S^{**} = \alpha h_S^{**} \text{ and } \delta_W^{**} = \alpha h_W^{**}.$$

To pin down h_S^{**} and h_W^{**}, we consider width-deviations for both bidder types. The optimal deviation quantity for each bidder is given by:

$$\widehat{Q}_S = \min\left\{\bar{Q}, \frac{v - h_S(v-(1-\sigma)\rho r)}{\rho(1-\sigma)}\right\}.$$
$$\widehat{Q}_W = \min\left\{\bar{Q}, \frac{v - h_W(v-\rho r)}{\rho}\right\}.$$

Using the slope-capacity substitution from Eq. (1):

$$\widehat{Q}_S = \min\left\{2, \frac{c(1-h_S)}{1-\sigma} + \frac{2h_S}{2-z}\right\} \cdot r.$$
$$\widehat{Q}_W = \min\left\{2, c(1-h_W) + \frac{2h_W(1-\sigma)}{2-\sigma}\right\} \cdot r.$$

Using these quantities to define the optimal width-deviations, we get:

$$u_S^{DW}(h_S) = (v-hp^*)\widehat{Q}_S - \frac{\rho(1-\sigma)(\widehat{Q}_S)^2}{2}$$
$$u_W^{DW}(h_W) = (v-hp^*)\widehat{Q}_W - \frac{\rho(\widehat{Q}_W)^2}{2}.$$

Using the slope-capacity substitution, the expression for market-clearing price, and the market split, we then solve for h^{**} analogously to the symmetric case. This yields:

$$h_W^{**} = \begin{cases} \frac{c(2-\sigma)+2\sigma-3}{(2-\sigma)(c(2-\sigma)-2(1-\sigma))} & \text{if } c \geq 2 + \frac{1}{1-\sigma} - \frac{1}{2-\sigma} \\ \\ \frac{c(2-\sigma)-2\sqrt{(1-\sigma)(c(2-\sigma)-(1-\sigma))}}{c(2-\sigma)-2(1-\sigma)} & \text{otherwise} \end{cases}$$

$$h_S^{**} = \begin{cases} \frac{(1-\sigma)(c(2-\sigma)-3(1-\sigma)+\sigma(1-\sigma))}{(2-\sigma)(c(2-\sigma)-2(1-\sigma))} & \text{if } c \geq 3(1-\sigma) + \sigma^2 - \frac{1}{2-\sigma} \\ \\ \frac{c(2-\sigma)-2\sqrt{(1-\sigma)(c(2-\sigma)-(1-\sigma))}}{c(2-\sigma)-2(1-\sigma)} & \text{otherwise} \end{cases}.$$

For collusion to be successfully sustained, the discount factor must be higher than the maximum of δ_S^{**} and δ_W^{**}. Since $c > 1$, and $0 \leq \sigma < 1$, in our model $\delta_S^{**} \leq \delta_W^{**}$: it is the weak bidder's constraint that is stricter. To show that asymmetry makes collusion harder, we need to show that δ_W^{**} is increasing in σ. Looking at its derivative, we get:

$$\frac{\partial h_W^{**}}{\partial \sigma} = \begin{cases} \frac{1}{2}\left(\frac{(c-2)^2}{(c(2-\sigma)-2(1-\sigma))^2} + \frac{1}{(2-\sigma)^2}\right) & \text{if } c \geq 2 + \frac{1}{1-\sigma} - \frac{1}{2-\sigma} \\ \\ \frac{c\left(c(2-\sigma)-2\sqrt{(1-\sigma)(c(2-\sigma)-(1-\sigma))}\right)}{(c(2-\sigma)-2(1-\sigma))^2\sqrt{(1-\sigma)(c(2-\sigma)-(1-\sigma))}} & \text{otherwise} \end{cases}.$$

Since both terms are positive, that shows that h_W^{**} is increasing in σ, whereby $\delta_W^* = \alpha h_W^{**}$ is also increasing in σ. This concludes the proof. □

A.4. **Equilibrium existence under asymmetry.**

A.4.1. *Non-existence of asymmetric linear equilibria without uncertainty.* The linear optimal bidding equilibrium derived in Lemma 1 does not always exist when bidders are asymmetric and when the level of uncertainty is low. We discuss this result for the case of two asymmetric bidders.

To see why the non-existence may occur, look at Eq. (10) which gives each bidder's optimal slope (ψ_i), and Eq. (11) which expresses their price-impact (μ_i):

$$\psi_i = \frac{\rho + (1-\alpha)\mu_i - \alpha\mu_i\frac{1}{\lambda}}{1 + \frac{1}{\lambda}\alpha} \text{ and, } \mu_i = \frac{1}{\sum_j \frac{1}{\psi_j}}.$$

These equations pin down the slopes of equilibrium bidding functions for all bidders. In the symmetric case, this simplifies to same condition for each bidder. With asymmetry, we get a pair of simultaneous equations, indexed by S for the strong bidder, and W for the weak bidder:

$$\begin{cases} \psi_S = \frac{1-\alpha-\frac{1}{\lambda}\alpha}{1+\frac{1}{\lambda}\alpha}\psi_W + \frac{\rho_S}{1+\frac{1}{\lambda}\alpha} \\ \psi_W = \frac{1-\alpha-\frac{1}{\lambda}\alpha}{1+\frac{1}{\lambda}\alpha}\psi_S + \frac{\rho_W}{1+\frac{1}{\lambda}\alpha} \end{cases} \implies \begin{cases} \psi_S = \frac{(\alpha+\lambda)\rho_S-(\alpha-\lambda-\alpha\lambda)\rho_W}{(2-\alpha)(2+\lambda)\alpha} \\ \psi_W = \frac{(\alpha+\lambda)\rho_W-(\alpha-\lambda-\alpha\lambda)\rho_S}{(2-\alpha)(2+\lambda)\alpha}. \end{cases}$$

To ensure that both bidder's optimal bidding functions are non-decreasing, both ψ_S and ψ_W must be non-negative. However, as uncertainty vanishes, and $\lambda \to 0$:

$$\begin{cases} \lim_{\lambda \to 0} \psi_S = \frac{\rho_S - \rho_W}{2(2-\alpha)} \\ \lim_{\lambda \to 0} \psi_W = \frac{\rho_W - \rho_S}{2(2-\alpha)} \end{cases}$$

Both parameters can only be non-negative if $\rho_S = \rho_W$, that is – when we have full symmetry. For a linear equilibrium to exist with asymmetric bidders, some residual uncertainty must remain. If, as in Section A.3, we let set $\rho_s = (1-\sigma)\rho_W$, then the equilibrium existence condition reduces to:

$$\lambda \geq \frac{\sigma}{1-\sigma}.$$

As a result, to ensure that both bidders' optimal linear bidding functions are downward-sloping, the market must entail a considerable amount of residual supply uncertainty. Since in our application the supply is fixed, the pre-requisites for the existence of an asymmetric linear equilibrium are not satisfied.

A.4.2. *Construction of an asymmetric Nash equilibrium without uncertainty.* To obtain a Nash equilibrium benchmark to be used in the punishment phase of our Theorem 2, we construct an equilibrium that is intuitively similar to that in the competitive symmetric case. We retain the requirement that bids be non-increasing, as is standard in these types of auctions. In the symmetric case, for any $\alpha > 0$, it was equilibrium for each bidder to submit a flat bid, equal to their marginal valuation at the expected level of the per-capita supply.

There are multiple equilibria in this setting, but we will show that there is a natural equilibrium at the efficient allocation, i.e., marginal values are equal (as shown in Figure 12a). Since the slopes of the two bidders' marginal values differ, this occurs at an uneven split, with the strong bidder winning more than half of the aggregate supply. When the weak bidder's marginal value is given by $v - \rho q_i$ and the strong bidder's marginal value is $v - (1-\sigma)\rho q_i$, in equilibrium the weak bidder's share, ϕ, of the aggregate supply of $\bar{Q}$ will be:

$$v - \rho\phi\bar{Q} = v - \rho(1-\sigma)(1-\phi)\bar{Q} \implies \phi = \frac{1-\sigma}{2-\sigma}.$$

The marginal value at this point (which will also be the market-clearing price) is then:

$$p^* = \frac{v}{c}\left(c - \frac{2(1-\sigma)}{2-\sigma}\right).$$

As in Ausubel et al. (2014), we proceed to construct the equilibrium by considering a discretized version of the bidding space, and assume that the grid on quantity is coarser than it is on price; given the setting of our application, this is a plausible assumption. We can, furthermore scale the grid so that the intersection of the marginal values occurs at

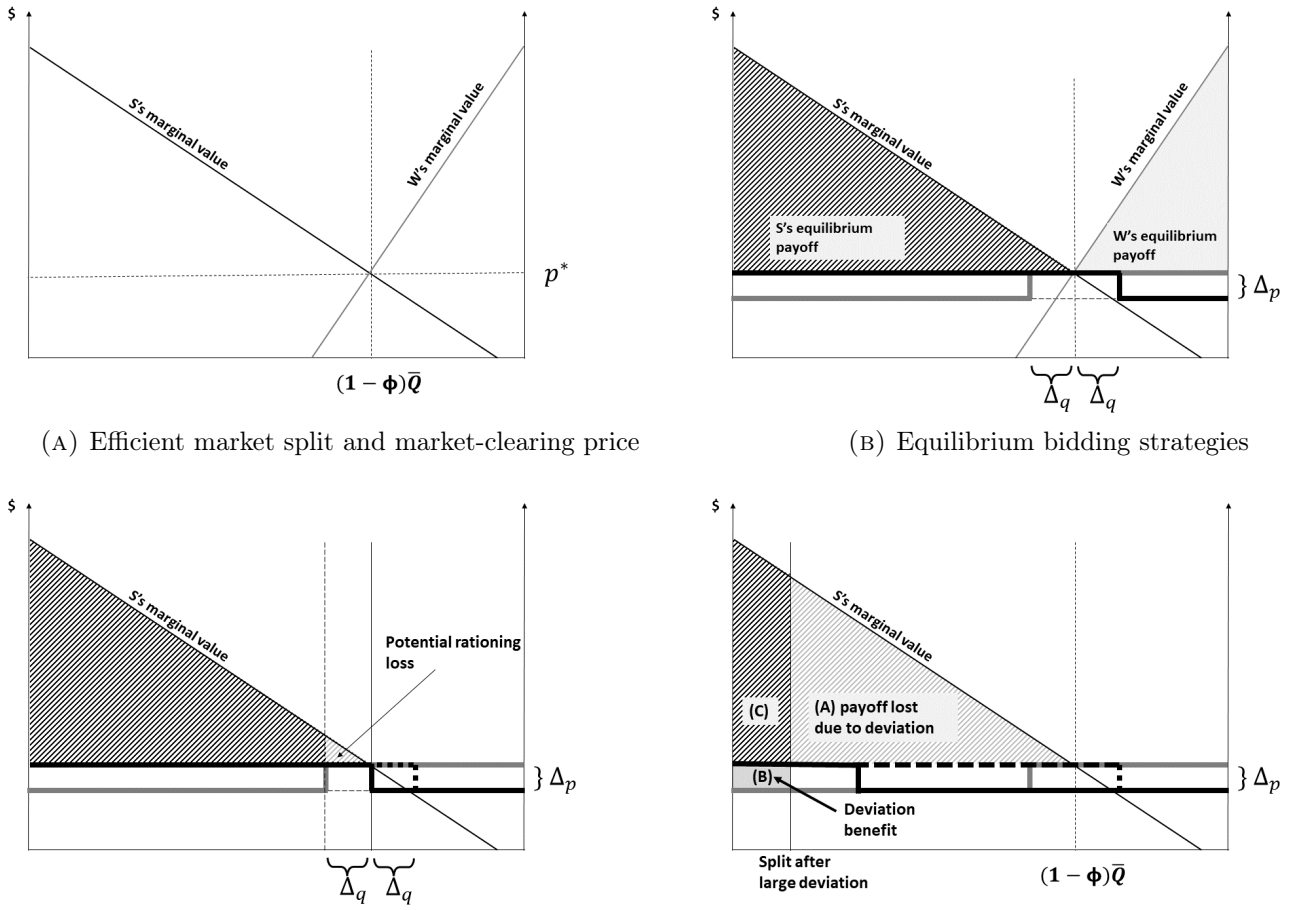
an intersection of the grid mesh. Let Δ_q denote the quantity-increment (horizontal grid resolution), and Δ_p the increment on the price grid. Figure 12b illustrates this setup.

For market clearing, we assume that the auctioneer sets the market-clearing price at the price based on the lowest at least partially-accepted bid. If there is excess demand at this price (i.e. the market does not clear precisely), the auctioneer rations both bidders equally at the margin;[20] if this does not lead to a quantity split that coincides with the quantity grid, the marginal unit is allocated to each bidder with equal probability.

Consider the following pair of bidding strategies:

$$b_W^{\Delta}(q_W) = \begin{cases} p^* & \text{if } q_W \leq \phi\bar{Q} + \Delta_q \\ p^* - \Delta_p & \text{otherwise} \end{cases}$$

$$b_S^{\Delta}(q_S) = \begin{cases} p^* & \text{if } q_S \leq (1-\phi)\bar{Q} + \Delta_q \\ p^* - \Delta_p & \text{otherwise} \end{cases}.$$



(A) Efficient market split and market-clearing price

(B) Equilibrium bidding strategies

(C) Deviation on the last unit is unprofitable

(D) A larger deviation is also unprofitable

FIGURE 12. Deviations from crank-handle bidding in mixed-price auctions.

[20]That is, both bidders have their bids reduced by the same amount, which is half the excess demand at the market-clearing price. For example, if there are 2 two units of excess demand, each bidder's allocation is rationed by one unit.

These are piecewise-linear strategies, defined by a step function where the bid of each bidder drops down by one price-increment on the second quantity-increment beyond the efficient market split quantity. Given the rationing and price-setting rules outlined above, this pair of strategies results in a market-clearing price of p^*, with the weak bidder winning a share of ϕ.

To see that this is a Nash equilibrium, consider the strong bidder's deviation incentives on the marginal unit, where $q = (1 - \phi)\bar{Q} + \Delta_q$ (the argument for the weak bidder is exactly analogous). Deviating from p^* to a higher price would require the bidder to increase his bid to $p^* + \Delta_p$ on all inframarginal units also; this would lead to an outright loss on the marginal unit (since marginal value at this point is already lower than p^*), and an increased payment on all inframarginal units. Therefore, an upward deviation is unprofitable.

Next, consider a downward deviation by one increment, on the marginal unit (illustrated in Figure 12c). Despite this deviation, there remains one unit (Δ_q) of excess demand at p^*, so the market-clearing price does not change. The auctioneer will then randomize which of the two bidders gets rationed by Δ_q. Therefore, with a 50% probability the allocation does not change, and with 50% probability the deviating bidder loses their last previously-won unit; neither outcome results in increased payoffs.

Note also that deviating to bidding $p^* - \Delta_p$ on inframarginal units is also unprofitable, as illustrated on Figure 12d. Since rationing happens at the margin, bidding lower on inframarginal units leads to a slightly lower price, but a much lower quantity won by the deviating bidder – whereas previously he was making strictly positive surplus on each of these units, even at the market clearing price of p^*. Therefore a price-reducing deviation on a larger quantity is strictly unprofitable. Since neither an upward nor downward deviation (marginal or otherwise) is profitable, and since the same argument applies to both bidders, the strategies b_S^Δ and b_S^Δ constitute a Nash equilibrium. Finally, to bring our step-function Nash equilibrium arbitrarily close to a linear one, we can take the limit by letting both Δ_p and Δ_q collapse to zero at the same rate.

Appendix B. For Online Publication.
Additional information about auctions

B.1. **Sequential ascending mackerel auctions (August, 2017).** Lot 13, originally sized at 100 tonnes, was split because Bidder C chose to buy only half of the lot: auction rules permitted buying a fraction of a lot after winning. Therefore, Lot 13 was split into two lots of 50 tonnes each. As a result, the entire auction ended up with 25 lots in total instead of the pre-announced 24 lots.

Since Bidder C won another five lots after winning Lot 13, buying half of Lot 13 appears to have been a deliberate strategy to identify the price-setter in the bidding ring (whether it was a deviating ring-member or an outsider). The strategy appears to have worked: Bidder A seems to have unexpectedly won Lot 13.2. In Round 14, two non-winning ring bidders dropped out at 3.08 kr./kg and 3.09 kr./kg respectively, possibly due to confusion caused by the splitting of Lot 13 into two lots.

In Round 21, it appears that there was further confusion about whether Bidder E or Bidder C was the designated winner. As a result, Bidder E, who won in Round 20, continued bidding until a price of 3.42 kr./kg in Round 21. Bidder E did not participate in subsequent rounds.

B.2. **Pay-as-bid blue whiting auctions (March, 2019).** The non-coordinating bidder did not, in fact, win the entire supply: due to a technical mistake, this bidder submitted their bids using incorrect formatting and ended up winning 1,000 times less than their intended amount. This mistake was caused by an inconsistency in the bidding form which confused several of the bidders.

The paper bidding forms for this auction presented the supply denominated in tonnes, but bids were elicited in kilograms. Some bidders did not notice this difference, and proceeded to bid as if they were bidding on tonnes, rather than kilograms, of quota (thereby resulting in bids for quantities that were 1,000 times smaller).

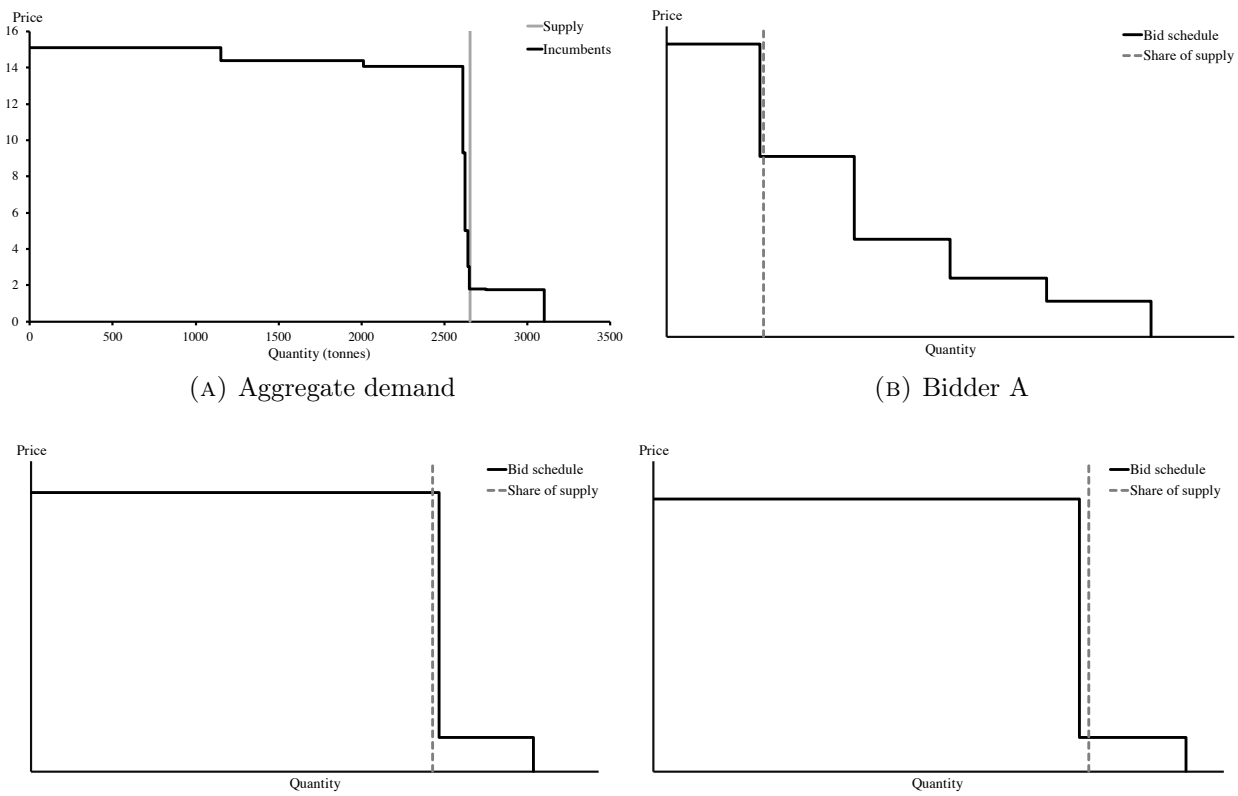
Since the non-coordinating bidder in our example was not alone in making precisely the same mistake, we interpret their *actual* bid as an error, and use the bidder's *intended* bid for our analysis.

Noticing that multiple bidders had made the same mistake, the auctioneer delayed the announcement of the results by a day, likely because they were considering rejecting all bids and re-running the auction. We take the auctioneer's actions to support our interpretation of what happened: submission of bids for negligible quantities was a result of confusion rather than a deliberate strategic choice.

Appendix C. For Online Publication.
Additional examples of crank-handle bidding

In this Appendix, we present additional examples of crank-handle bidding in uniform-price auctions of fishing quota in the Faroe Islands, strengthening the support for Predictions 1 and 3 of our model.

C.1. **Successful crank-handle collusion without exact crank-handle bidding.** In this auction of one-year quota for demersal fish in the Russian part of the Barents Sea in 2018, the incumbents submitted crank-handle bids (Figure 13). However, the drop-off in Bidder A's individual bid was not exactly in line with his grandfathered quota shares. Nevertheless, the aggregate demand still fell off dramatically just before the total quota supply (Figure 13a). This example indicates that not all bidders need to submit exact crank-handle bids to coordinate on a low-price equilibrium.



(A) Aggregate demand (B) Bidder A

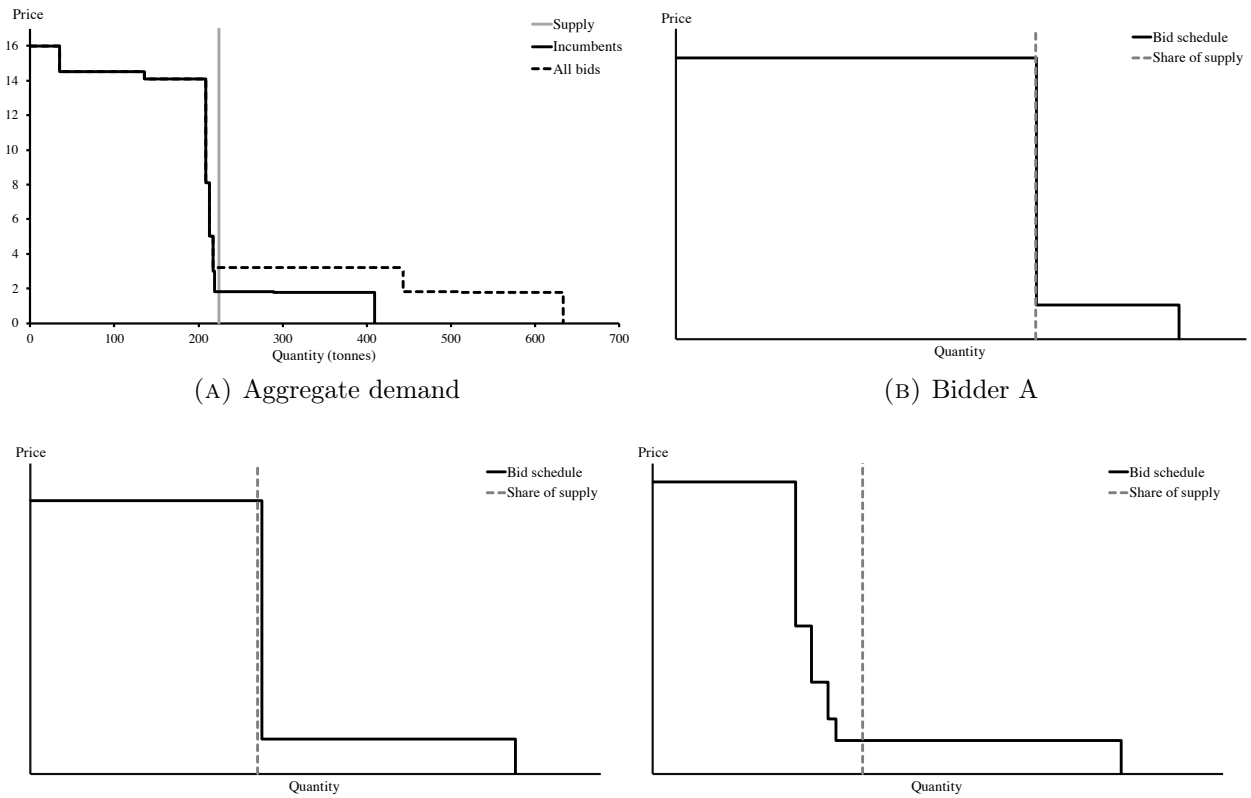
(C) Bidder B (D) Bidder C

Figure 13. Aggregate demand and individual bids in the 2018 auction of one-year quota for demersal fish in Russian part of the Barents Sea and grandfathered quota share of supply (prior to the auction). Individual demands are not to scale to preserve anonymity.

C.2. **Crank-handle collusion, predation, and entry .** In the next four uniform-price auctions of fishing quota in 2018, we observe attempts at low-price equilibria by three incumbent bidders. However, all these attempts were partially frustrated because entrants unexpectedly set a price higher than the one that the incumbents were attempting to coordinate on. However, in each case, incumbents successfully ensured that the entrants won almost no quota.

C.2.1. *Eight-year quota for demersal fish in the Norwegian part of the Barents Sea.*

- Without the entrant, the incumbents would have won the entire supply at 1.83 kr./kg.
- The entrant's bid pushed the price up to 3.20 kr./kg (i.e., a 75 percent increase).
- The entrant only won 7 tonnes out of 224 tonnes that he demanded at the market-clearing price.



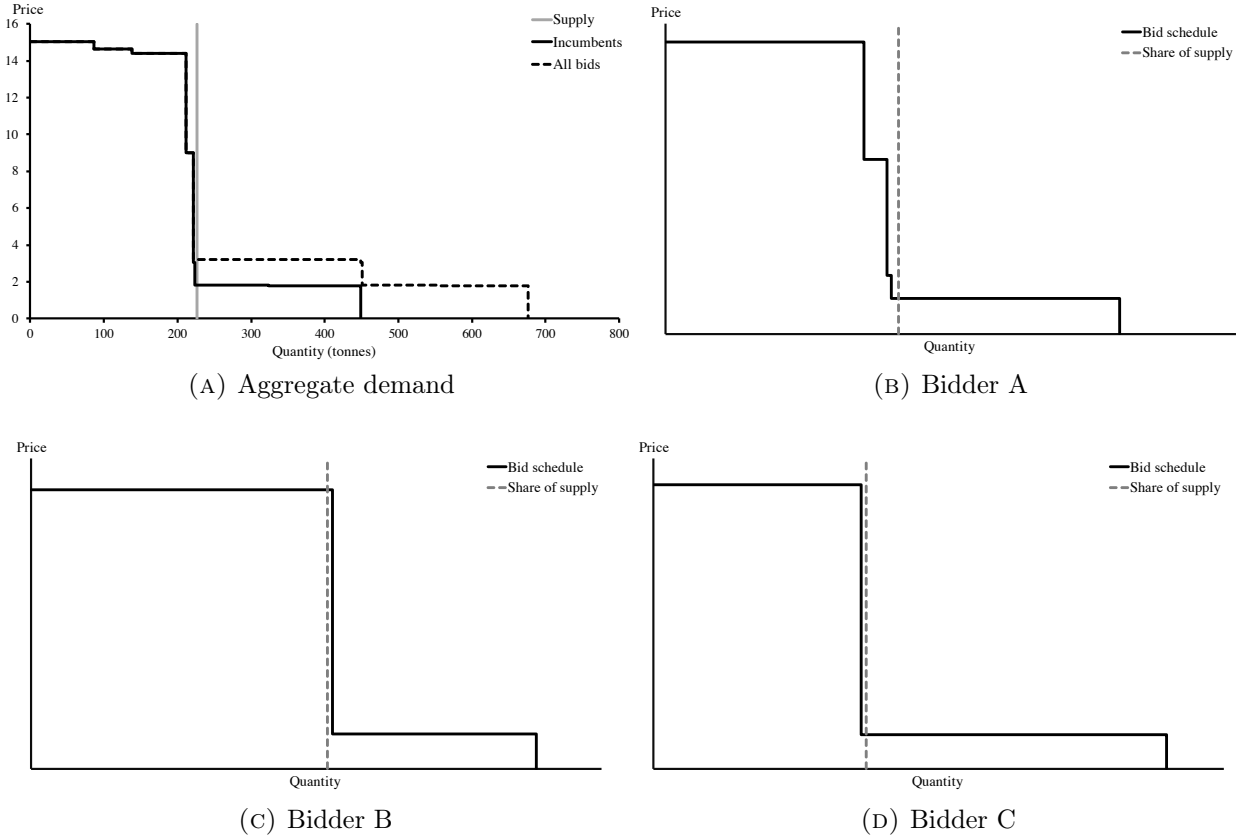
(A) Aggregate demand (B) Bidder A

(C) Bidder B (D) Bidder C

FIGURE 14. Aggregate demand and individual bids in the 2018 auction of eight-year quota for demersal fish in the Norwegian part of the Barents sea and grandfathered quota share of supply (prior to the auction). Individual demands are not to scale to preserve anonymity.

C.2.2. *Three-year quota for demersal fish in the Russian part of the Barents Sea.*

- Without the entrant, the incumbents would have won the entire supply at 1.82 kr./kg.
- The entrant's bid pushed the price up to 3.20 kr./kg (i.e., a 76 percent increase).
- The entrant only won 5 tonnes out of 227 tonnes that he demanded at the market-clearing price.



(A) Aggregate demand

(B) Bidder A

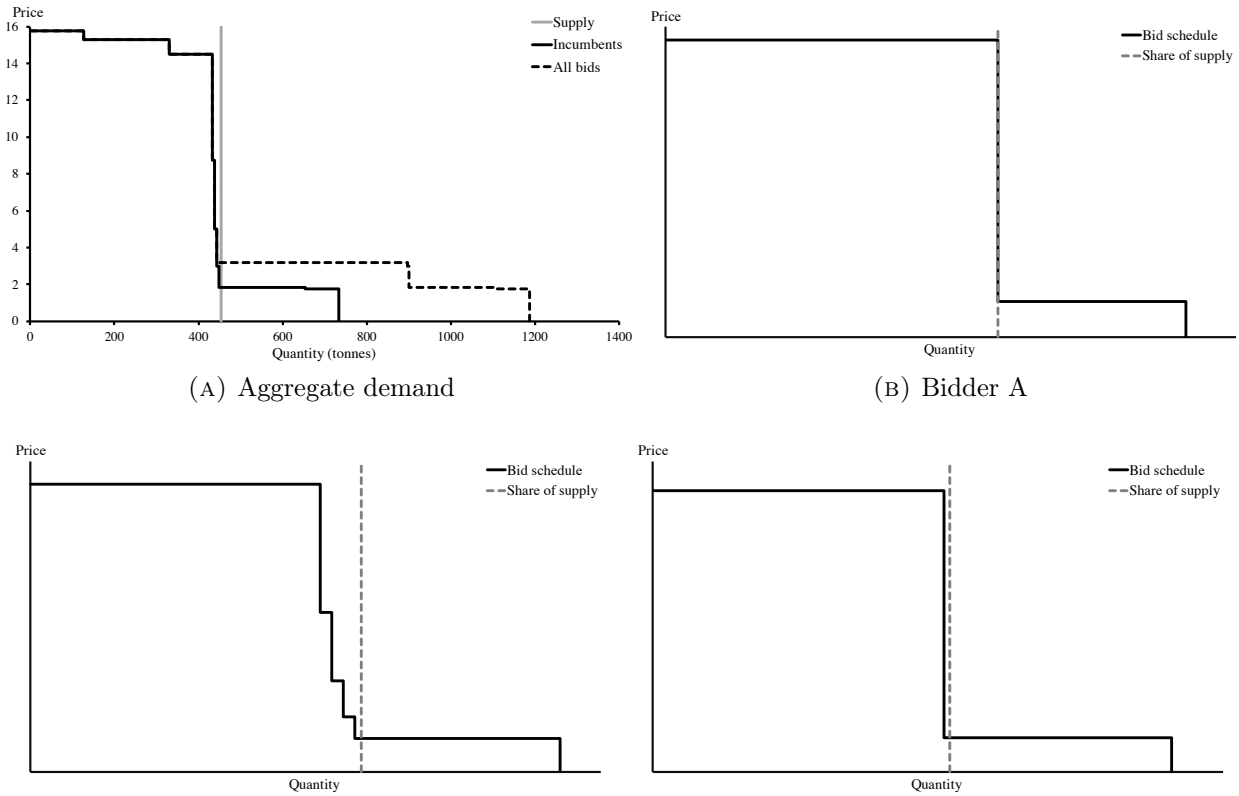
(C) Bidder B

(D) Bidder C

FIGURE 15. Aggregate demand and individual bids in the 2018 auction of three-year quota for demersal fish in Russian part of the Barents Sea and grandfathered quota share of supply (prior to the auction). Individual demands are not to scale to preserve anonymity.

C.2.3. *Eight-year quota for demersal fish in the Russian part of the Barents Sea, 2018.*

- Without the entrant, the incumbents would have won the entire supply at 1.85 kr./kg.
- The entrant's bid pushed the price up to 3.20 kr./kg (i.e., a 73 percent increase).
- The entrant only won 10 tonnes out of 453 tonnes that he demanded at the market-clearing price.



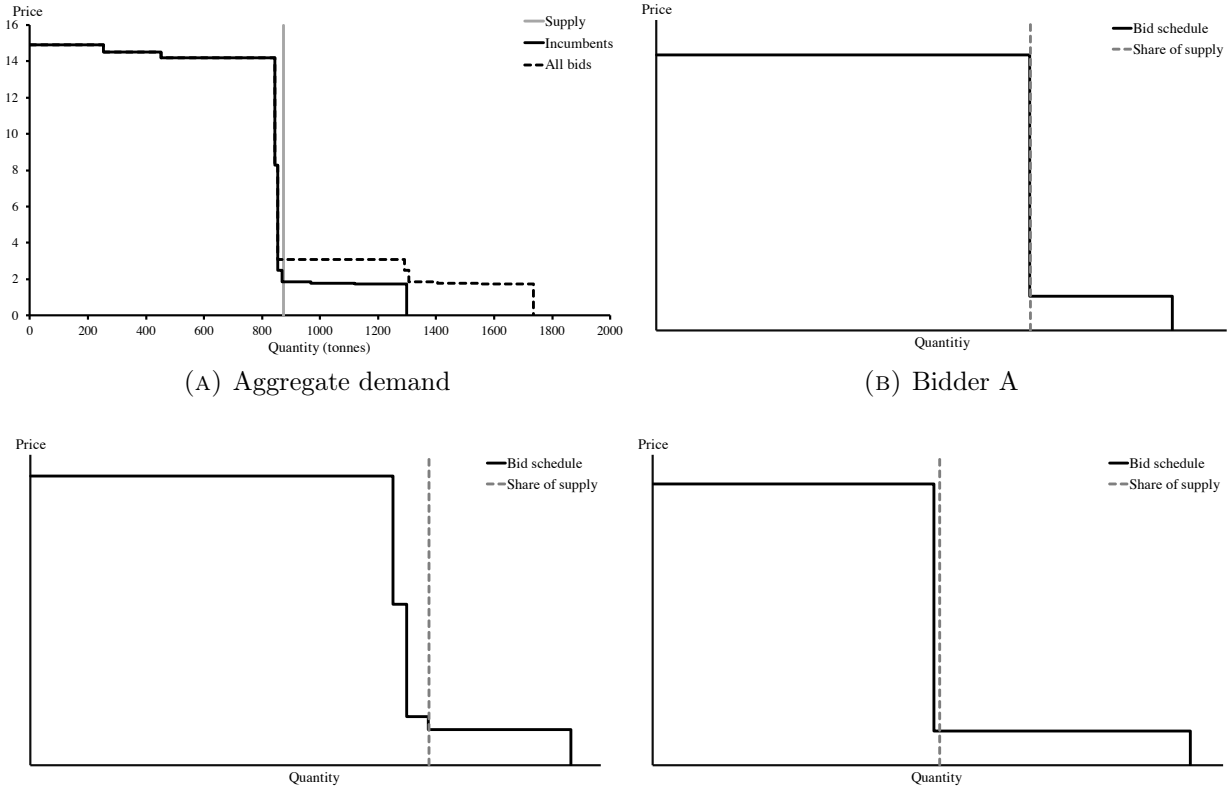
(A) Aggregate demand (B) Bidder A

(C) Bidder B (D) Bidder C

FIGURE 16. Aggregate demand and individual bids in the 2018 auction of eight-year quota for demersal fish in Russian part of the Barents Sea and grandfathered quota share of supply (prior to the auction). Individual demands are not to scale to preserve anonymity.

C.2.4. *One-year quota for demersal fish in the Norwegian part of the Barents Sea.*

- Without the entrant, the incumbents would have won the entire supply at 1.85 kr./kg.
- The entrant's bid pushed the price up to 3.10 kr./kg (i.e., a 68 percent increase).
- The entrant only won 20 tonnes out of 437 tonnes that he demanded at the market-clearing price.



(A) Aggregate demand (B) Bidder A

(C) Bidder B (D) Bidder C

FIGURE 17. Aggregate demand and individual bids in the 2018 auction of one-year quota for demersal fish in the Norwegian part of the Barents Sea and grandfathered quota share of supply (prior to the auction). Individual demands are not to scale to preserve anonymity.

C.3. **Crank-handle collusion with coarse bidding languages.** In this Appendix, we show that crank-handle bidding was present from the very start of the Faroese quota auctions. We analyze the 2016 and 2017 uniform-price auctions of one-year quota for demersal fish in the Russian part of the Barents Sea. Aggregate demand of incumbents form stark crank-handles despite their being allowed to only submit one (in 2016) or three (in 2017) bids per vessel. Therefore, the 2016 auction shows that aggregate crank-handles are possible and effective even when individual bidders cannot submit crank-handles themselves.

In both auctions, an entrant set the market-clearing price, frustrating the incumbents' attempts at low-price equilibria. However, in both cases the price-setting entrant declined to buy any quota—the auction rules allowed bidders to decline any marginal winning bid that was not accepted in full. Although this pushed up the market-clearing price paid by the incumbents, they successfully managed to prevent entry.

C.3.1. *One-year quota for demersal fish in the Russian part of the Barents Sea in 2016 .*

- Each bidder was allowed to submit one bid per vessel. Any bidder could submit multiple bids if they owned multiple vessels.
- Five bids were submitted by three incumbents and one entrant. The bids were public, and reproduced in full as Figure 19.
- Without the entrant, the incumbents would have won the entire supply at 2.03 kr./kg.
- The entrant's bid pushed the price up to 3.06 kr./kg (i.e., a 51 percent increase).
- The entrant only won 50 tonnes out of 385 tonnes that he demanded at the market-clearing price.

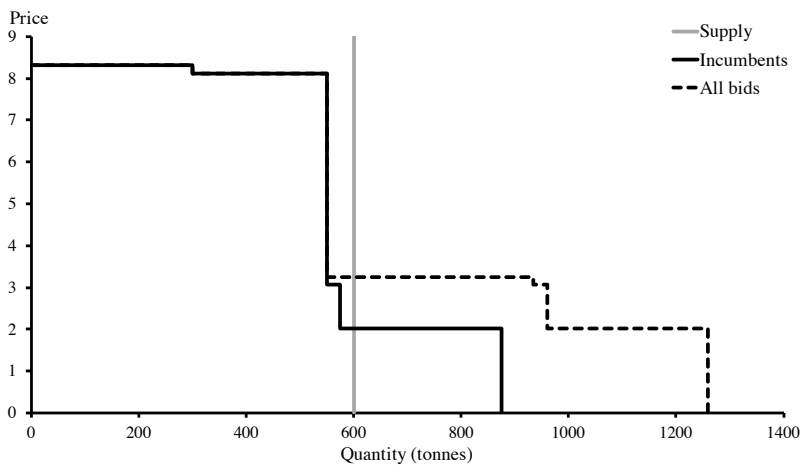


FIGURE 18. Aggregate demand in the 2016 auction of one-year quota for demersal fish in the Russian part of the Barents Sea.

Vessel	Company	Price kr./kg	Amount kg	Aggregate demand	Leftover
Enniberg	P/F Enniberg	8.31	300,000	300,000	
Gadus	P/F JFK Trol	8.12	250,000	550,000	
Sjagaklettur	P/F Jókin	3.25	385,000	935,000	50,000
Sjúrðarberg	P/F JFK Trol	3.06	25,000		
Akraberg	Sp/f Framherji	2.03	300,000		

FIGURE 19. All bids in the 2016 auction for one-year quota for demersal fish in the Russian part of the Barents Sea.

C.3.2. *One-year quota for demersal fish in the Russian part of the Barents Sea in 2017.*

- Each bidder was allowed to submit three bids per vessel.
- Eight bids were submitted by three incumbents and one entrant.
- Without the entrant, the incumbents would have won the entire supply at 1.75 kr./kg.
- The entrant's bid pushed the price up to 3.01 kr./kg (i.e., a 72 percent increase).
- The entrant only won 52 tonnes out of 450 tonnes that he demanded at the market-clearing price.

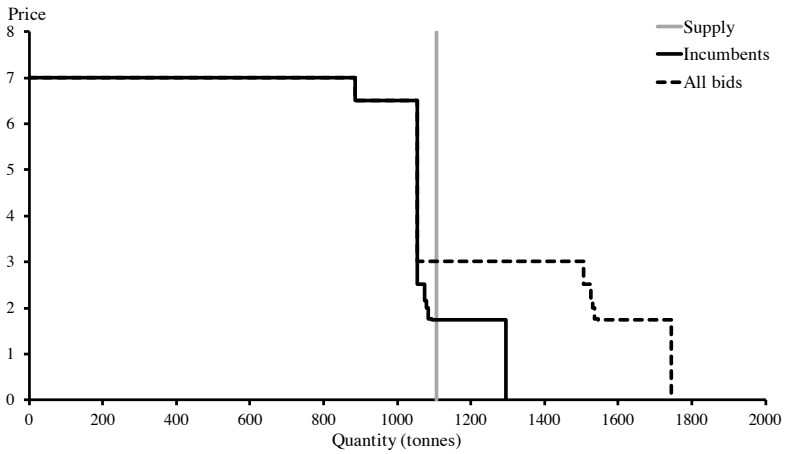


FIGURE 20. Aggregate demand in the 2017 auction of one-year quota for demersal fish in the Russian part of the Barents Sea.