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Optimal Taxation of Couples' Incomes with Endogenous Bargaining Power

Mizuki Komura
Musashi University

Hikaru Ogawa
The University of Tokyo

Yoshitomo Ogawa
Kindai University

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Mizuki Komura
Musashi University
m.komura@cc.musashi.ac.jp

Hikaru Ogawa
University of Tokyo
ogawa@e.u-tokyo.ac.jp

Yoshitomo Ogawa
Kindai University
yogawa@kindai.ac.jp

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Abstract

This paper studies the optimal income taxation for a two-earner household where a couple bargains over private goods consumption and time allocation between market work and leisure. In the model, their bargaining power is endogenously determined by the income gap between male and female earners in the economy. The optimal tax expression obtained in this model shows that the optimal tax rule is characterized by two components: the price distortion consideration (Ramsey tax consideration) and the endogenous bargaining power consideration. Taking account of two household members with different productivity levels in the labor market, our numerical analysis demonstrates that the optimal tax rate for the household member with higher productivity, typically, the member with smaller wage elasticity, is lower if the required tax revenue is relatively small and the influence of gender income gap on the power balance of the couple is moderate. This result contrasts with the Ramsey tax rule.

JEL Classification: H21; J16; J22
Keywords: Optimal income taxation; Endogenous bargaining power

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1 Introduction

Many studies have applied Ramsey's (1927) monumental work in optimal commodity tax theory under a revenue constraint to analyze two-earner household income taxation. The growth in female labor participation has also contributed to the invention of an optimal income taxation policy for households. Rosen (1977), while analyzing two-earner household income taxation, proposes that differential taxation of men and women, who are assumed to have different wage elasticities of labor supply, leads to efficiency gains. This is based on the view that taxation should consider the properties and characteristics of individuals. Boskin and Sheshinski (1983) show that the Ramsey tax rule (the inverse elasticity rule) for labor incomes of two household members holds; an optimal tax policy imposes a higher tax on a household member whose wage elasticity of labor supply in the labor market is less elastic (typically, the first earner in the household) than her/his partner's. Their results are derived in a unitary model where a household behaves like a single entity and does not allow distribution of resources between household members.

Recent studies focusing on individual members within a household have challenged this conventional theory of optimal taxation.[1] Meier and Rainer (2015) demonstrated, using a model of couples' non-cooperative public provision game, that the Ramsey rule does not hold. In the model, domestic public good production is underprovided so that a tax on female workers, who typically have comparative advantage in domestic production, can work as a Pigovian tax to improve efficiency. Alesina et al. (2011) examined the gender-based taxation system constructing a model where each family member endogenizes his or her bargaining position by premarital investment. They showed that a higher tax rate for male labor can be justified when the distribution effect is resolved through the alternative policy tool of lump-sum subsidies for each gender group. This result suggests the possibility that the Ramsey rule does not necessarily hold when lump-sum transfers are not available. In contrast, Cremer et al. (2016) consider the bargaining couple in the Mirrleesian tradition, which allows individuals to adjust their labor decisions under asymmetric information (Mirrlees 1971). Using the model, they showed that a higher tax rate for women is optimal, which contradicts the Ramsey rule. The purpose of our paper is to reexamine the Ramsey taxation principle in relation to a couple, following an approach analogous to their methods. A special feature of our study is that it questions the validity of the Ramsey rule by taking into account the interaction of different households' decisions with the social norm.

For the purpose of this study, our model follows Basu (2006), wherein a couple makes a decision in a collective model framework, and the power balance of the couple is endogenous so that it depends on the resource allocation of other couples in the household. The key feature of the model is that when two individuals (i.e., a couple) make decisions within their household, the couple considers the bargaining power between the two individuals as given. However, the bargaining power itself depends on the income gap between the income of men and women in society, and it is endogenously determined at the social level. With this specification, he succeeded in exploring the interaction of household decisions through the social norm through peer pressure. This approach is

[1] Apps and Rees (2018) also investigate the optimality of the Ramsey rule, considering the distribution problem among households. They showed that taxing female labor is optimal when differential earnings of female workers cause large income gaps among households.

plausible since the literature has found that households' interaction with the social norm plays a significant role in determining the couple's time allocation (Fernandez et al. 2004; Feyrer et al. 2008; Gay 2018).[2] In addition, this social norm is largely influenced by public policies. Using a data set of 15 EU countries, Cipollone et al. (2014) found that public policies, including the taxation system, influence female labor participation more significantly than economic conditions characterized by business cycles. Jaumotte (2003) observed, with data of 17 OECD countries, that the treatment of married individuals and tax incentives to share market work between spouses have a substantial impact on women's labor supply choice, which also reflects the cultural attitude toward gender roles. If each couple interdependently influences the social norm, which is also shaped by some public policies, it is important to understand how this interaction affects the optimality of the taxation system itself.

Specifically, our model proposes that each household member perceives utility from good consumption and her/his own leisure. The couple negotiates resource allocation based on a collective model.[3] To enjoy more consumption, the couple needs to work to earn money. A conflict then arises—both partners want to enjoy more consumption, but neither wishes to work to earn money; that is, each partner wants the other to work for goods consumption. The autonomy within a couple depends on the income gap of typical couples in the economy. In our model, we assume that each couple is interdependent through peer pressure by reflecting on this power balance. A couple's time allocation decision forces an impression about the relationship between typical wives and husbands onto other households that are about to make resource allocation decisions. If a typical husband is expected to earn a certain amount of money, households anticipate a power balance based on the expected income gap in the economy. The expected income gap, as the determinant, is influenced by not only the wage rates but other couples' decisions on labor supply. Moreover, the tax system also affects the bargaining power by changing the after-tax income of typical spouses. In this setting, the government collects revenue by taxing labor incomes of household members, knowing their interaction.

Our main results show that the optimal individual taxation is characterized not only by the Ramsey taxation rule, which represses tax-induced price distortions (deadweight loss), but also by the element stemmed from the endogenous bargaining, which induces individual with higher productivity works more and hence increases the consumption of the good by the household. The former component is expected to result in higher tax rates for an individual with less-elastic labor supply—typically, the first earner. Allowing for the latter component, the government imposes a higher tax rate on the second earner with less productivity to deter him or her from entering the labor market. We find this effect contrary to the Ramsey taxation effect, demonstrating that the optimal tax ranking depends on the relative magnitude of these two effects. Our numerical analysis

[2]Feyrer et al. (2008) point out that a longstanding cultural attitude influences the couple's time allocation decisions, including fertility decisions, in developed countries. Fernandez et al. (2004) found, using WWII as a natural experiment, that mothers' female participation changed the marital patterns of sons to exogenously increase the female labor supply in the US. Gay (2018) attempts to identify which channel influences the second generation's marriage patterns—their parents, their parents-in-law, or others.

[3]The collective model basically assumes that households achieve Pareto-efficient allocation. This framework was originally invented by Apps and Rees (1988) and Chiappori (1988, 1992), and has been tested in many empirical studies for its tractability.

demonstrates that the optimal income tax rate for the household member with higher (lower) productivity is lower (higher) if the required tax revenue is relatively small and the income disparity of married couples has a moderate effect on their bargaining power. This result contradicts the Ramsey tax rule, indicating that the optimal tax system is affected by the bargaining power of marital couples. The study does not necessarily propose a higher tax rate for female labor as the only new result of our model. In the presence of endogenous bargaining power, men with higher bargaining power enjoy more private consumption than their partners, contrary to the government objective of equalizing private consumption between spouses. Thus, we also find another reasonable property of the optimal tax rule that favors a lower tax rate for labor supply of the second earner (typically female) so as to correct this intra-couple inequality created through the endogenously determined bargaining power. The optimal tax ranking, therefore, depends on the relative magnitudes of these opposing effects.

The contribution of this paper is twofold: First, and most importantly, we consider endogenous bargaining power as the result of decisions made within marriage. Using a graphical analysis, Chiappori (1992) found that the Ramsey rule does not necessarily guarantee Pareto optimality because taxation can influence each family member's welfare by determining their bargaining position through changes in distribution factors; i.e., the bargaining power is simply determined according to after-tax wage rates. Alesina et al. (2011) formally analyze the optimal income taxation in a two-earner household Nash bargaining setting, showing that the distribution issue should also be resolved with the policies. Their model has demonstrated that the Ramsey rule holds if the government can resolve the distribution issue with a gender-specific lump-sum tax/subsidy and that women are inclined to engage in domestic production because of comparative advantages or social norms. The study endogenizes the bargaining positions of spouses, allowing them to invest in their human capital before the marriage negotiations. While they suppose that the bargaining power is endogenously determined with premarital investment by each family member, we focus alternatively on the decisions made during the marital period, which in turn affect the bargaining positions of other couples. Although each couple regards the bargaining power as exogenous, it is actually endogenous at the social level, and is influenced by the interactions with other households. The social norm of Alesina et al (2011), on the other hand, was considered exogenous and as given in the model. Moreover, although it is always ideal for the government to have multiple policy instruments to correct distortions caused by different factors, a gender-specific lump-sum tax/subsidy in reality is not always feasible. Thus, the second feature of the study is that we attempt to identify the optimal taxation for a two-earner household when income tax is the only available policy tool. This work tries to complement the insights from existing studies, using the standard and familiar optimal tax approach.

The rest of paper is organized as follows. In section 2, we present a model of an income taxation system for a household with two family members. Section 3 investigates the optimality of the taxation system in more specific functions. In section 4, we present our argument with numerical examples. Section 5 concludes our paper.

2 Model

Suppose an economy consists of a government and a household with two individuals. The government imposes income tax on each household member's labor income to finance government spending. Following the collective model framework (Apps and Rees 1988; Chiappori 1988, 1992), the couple bargains over household resource allocation. The model follows Basu (2006), wherein the couple's power is affected by the household decisions. In the model, each couple takes their power as given, according to the social norm or through peer pressure, but their own and the other couple's decisions interdependently change the spouses' power at the social level.

2.1 Household

Consider identical households with two members, the first earner and the second earner. Each spouse is endowed with one unit of time. They allocate it between market work, l_i, and leisure, $1-l_i$, at the wage rate w_i. The spouses have their own utility function, wherein they perceive utility from the consumption of a common good x, their own consumption x_i, and their own leisure $1 - l_i$, as

$$u_i(x, l_i) = x + h_i(x_i) + v_i(1 - l_i), \quad i = 1, 2, \tag{1}$$

where $h_i' > 0$, $h_i'' < 0$, $v_i' > 0$, and $v_i'' < 0$. The prime denotes a derivative. Both husband and wife enjoy the common good (household public good), such as housing, a car, child-rearing commodities, and reserve for the future. This partially captures the scale economy, which is factor in two individuals becoming a family.

The objective function of a household is the weighted average of these two functions, with the weights capturing the balance of power in the household as in the collective model approach originally invented by Apps and Rees (1988) and Chiappori (1988, 1992).[4] The household's maximand is then given by

$$\begin{aligned} \Omega(x, x_1, x_2, l_1, l_2) &\equiv (1-\theta)\, u_1 + \theta u_2 \\ &= x + (1-\theta)\left[h_1(x_1) + v_1(1-l_1)\right] + \theta\left[h_2(x_2) + v_2(1-l_2)\right], \end{aligned} \tag{2}$$

where $\theta \in (0, 1)$ captures the second earner's power that the social norm or peer pressure imposes on the couples. The budget constraint of the household is

$$x + x_1 + x_2 = \rho_1 + \rho_2, \text{ where } \rho_i \equiv (1 - t_i) w_i l_i. \tag{3}$$

In (3), ρ_i is the after-tax income and t_i is the income tax rate for the labor income of spouse i.

Following Basu (2006, p.570), we assume that each couple takes its bargaining power as given when it solves a problem. The interpretation of this assumption is that the couple decides to maximize its own household welfare by simply accepting the tax rates set by the government and facing the peer pressure on gender inequality imposed by society. Therefore, they do not care nor do they realize that their decisions affect other

[4] The bargaining framework was originally introduced to the family literature by Manser and Brown (1980) and McElroy and Horney (1981). They alternatively consider the intra-household distribution of a couple in a Nash bargaining setting.

households through these social norms or peer pressure. The explanation of the formal definition of the power of spouses at the social level is left until the next subsection.

Thus, given θ, each couple solves the following problem:

$$\underset{x,x_1,x_2,l_1,l_2}{\text{Max}} \ \Omega(x, x_1, x_2, l_1, l_2), \quad \text{s.t.} \quad x + x_1 + x_2 = \rho_1 + \rho_2.$$

Solving the problem, we find that the first-order condition with respect to x sets the Lagrange multiplier on the budget constraint of the household to one. Allowing this, we obtain the first-order conditions with respect to x_1, x_2, l_1, and l_2 as

$$x_1 \ : \ (1-\theta)\, h_1' = 1, \tag{4}$$

$$x_2 \ : \ \theta h_2' = 1, \tag{5}$$

$$l_1 \ : \ (1-\theta)\, v_1' = (1-t_1)w_1, \tag{6}$$

$$l_2 \ : \ \theta v_2' = (1-t_2)w_2. \tag{7}$$

(4) and (5) show that the weighted marginal utility from private good consumption is equal to the marginal cost. (6) and (7) show that the weighted marginal utility of each individual's leisure is equalized to its marginal cost measured by the opportunity cost. When $\theta \in (0,1)$, these conditions yield

$$\frac{dx_1}{d\theta} = \frac{h_1'}{(1-\theta)\, h_1''} < 0 \quad \text{and} \quad \frac{dx_2}{d\theta} = -\frac{h_2'}{\theta h_2''} > 0. \tag{8}$$

$$\frac{dl_1}{d\theta} = -\frac{v_1'}{(1-\theta)\, v_1''} > 0 \quad \text{and} \quad \frac{dl_2}{d\theta} = \frac{v_2'}{\theta v_2''} < 0. \tag{9}$$

(8) and (9) characterize the model. From (8), it is straightforward that spouse i's private consumption increases with his or her bargaining power. In addition, from (9), an increase in the power of spouse i induces her/his partner to work more to purchase household good x. For instance, if the second earner's power increases, the working hours decrease for the second earner but increase for the first earner. This is simply because each person prefers more leisure and at the same time larger good consumption. Under this situation, one of the partners could enjoy more consumption by having the other work more in the market instead of increasing her/his own labor supply. Moreover, each individual tries to increase her/his own private consumption as well as the household consumption of the common good x. This causes conflict, so the couple makes household decisions over resource allocation based on bargaining. The above argument is formally summarized as follows.

Lemma 1. *An increase in the power of a spouse increases her/his private good consumption and induces one partner to work more, reducing her/his own working hours.*

2.2 Tax Effects on Labor Supply

Let us define bargaining power. We follow the assumption of Basu (2006) that the bargaining power of one couple is affected by another couple's household decisions through social norms or peer pressure. Specifically, bargaining power is determined by the gender gap of their average disposable (after-tax) incomes. Each household's time allocation defines this gap, since labor supply is one of the determinants of individuals' incomes. Indeed, many empirical studies have documented that the spouses' economic power affects this power balance (Chiappori 1988, 1992; Schultz 1990; Thomas, 1990; Phipps and Burton 1998; Bourguignon et al. 1993; Lundberg et al. 1997; Browning 2000; Aura 2005). However, a newly formed couple cannot know its relative economic position before negotiation because its disposal income is determined by the bargaining decisions of the spouses, including the time allocation between them. Thus, they anticipate their bargaining positions based on observation from the viewpoint of the relative economic power of a typical couple in the society, influenced by the previous generations or peers in the same generation. These situations, where the new couple refers back to the behaviors of other couples, are indeed confirmed by empirical evidence on the social norm of gender roles (Fernandez et al. 2004; Feyrer et al. 2008; Gay 2018). Considering the social interactions among households in the analysis of optimal taxation, it is noteworthy that not only are the new couple's decisions influenced by other couples' decisions but its own decisions also unconsciously influence other couples' bargaining outcomes. Consequently, the power balance of a couple is exogenous for each couple, but endogenous in the whole economy.

The function of power θ depends on the relative after-tax income of the spouses in the society:

$$\theta = \theta(\rho_2 - \rho_1; \gamma), \text{ where } \theta' \geq 0. \tag{10}$$

At the society level, we assume that the power balance between the two household members is determined by two factors: (i) the average after-tax income gap between the spouses, $\rho_2 - \rho_1$; and (ii) determinants exogenous to the couple, γ. The former is a natural argument—that the power within a couple is strengthened if the typical spouse from the same gender group is economically independent. In addition, it is also well known that the power balance is influenced by custom, women's rights, the inheritance tax system, and family law, which are all exogenous to the economic position of women in the society. However, to focus on the economic factors that affect the endogenous bargaining power, we assume that $\theta = 0.5$ if $\rho_2 = \rho_1$.

Solving equations (4)-(7) with (10) allows us to derive each spouse's demand and labor supply in the equilibrium. Formally, we obtain the demand and labor supply functions in the equilibrium, depending not only on one's own tax rates but also on the partner's, as $x_i(t_1, t_2)$ and $l_i(t_1, t_2)$ for $i = 1, 2$. Let us denote $x_{it_j} \equiv \partial x_i/\partial t_j$ and $l_{it_j} \equiv \partial l_i/\partial t_j$ for $i, j = 1, 2$. The effects of the tax changes on demands for private goods and labor supplies in the equilibrium are obtained as (see Mathematical Appendix A)

$$x_{1t_1} = \frac{h_1'\theta' w_1 v_2'' \left(v_1''l_1 - v_1'\right)}{(1-\theta)h_1'' \left(v_1''v_2'' - \theta' v_1' v_1' v_2'' - \theta' v_2' v_2' v_1''\right)} \leq 0, \tag{11}$$

$$x_{2t_1} = -\frac{h_2'\theta' w_1 v_2'' \left(v_1''l_1 - v_1'\right)}{h_2''\theta \left(v_1''v_2'' - \theta' v_1' v_1' v_2'' - \theta' v_2' v_2' v_1''\right)} \geq 0, \tag{12}$$

$$l_{1t_1} = \frac{w_1 \left[\left(1-\theta' v_1' l_1\right) v_2'' - \left(v_2'\right)^2 \theta'\right]}{(1-\theta)\left(v_1''v_2'' - \theta' v_1' v_1' v_2'' - \theta' v_2' v_2' v_1''\right)}, \tag{13}$$

$$l_{2t_1} = \frac{\theta' v_2' w_1 \left(v_1''l_1 - v_1'\right)}{\theta \left(v_1''v_2'' - \theta' v_1' v_1' v_2'' - \theta' v_2' v_2' v_1''\right)} \leq 0, \tag{14}$$

$$x_{1t_2} = -\frac{h_1' v_1'' \theta' w_2 \left(v_2''l_2 - v_2'\right)}{h_1'' (1-\theta) \left(v_1''v_2'' - \theta' v_1' v_1' v_2'' - \theta' v_2' v_2' v_1''\right)} \geq 0, \tag{15}$$

$$x_{2t_2} = \frac{h_2'\theta' w_2 v_1'' \left(v_2''l_2 - v_2'\right)}{h_2''\theta \left(v_1''v_2'' - \theta' v_1' v_1' v_2'' - \theta' v_2' v_2' v_1''\right)} \leq 0, \tag{16}$$

$$l_{1t_2} = \frac{v_1'\theta' w_2 \left(v_2''l_2 - v_2'\right)}{(1-\theta)\left(v_1''v_2'' - \theta' v_1' v_1' v_2'' - \theta' v_2' v_2' v_1''\right)} \leq 0, \tag{17}$$

$$l_{2t_2} = \frac{w_2 \left[\left(1-\theta' l_2 v_2'\right) v_1'' - \left(v_1'\right)^2 \theta'\right]}{\theta \left(v_1''v_2'' - \theta' v_1' v_1' v_2'' - \theta' v_2' v_2' v_1''\right)}. \tag{18}$$

The inequalities follow from $v_i' > 0$, $v_i'' < 0$, $h_i' > 0$, $h_i'' < 0$, and $\theta' \geq 0$. Note that $v_1''v_2'' - \theta' v_1' v_1' v_2'' - \theta' v_2' v_2' v_1''$ in the denominators of all equations are always positive. The numerators in (11), (12), (15), and (16) are positive when $\theta' > 0$, and are zero when $\theta' = 0$. The numerators in (14) and (17) are negative when $\theta' > 0$, and are zero when $\theta' = 0$. The signs of the numerators in (13) and (18) are negative when $\theta' = 0$, and ambiguous when $\theta' > 0$. Thus, we can specify the sign of the tax effects as follows.

Proposition 1

(i) *If the bargaining power is exogenous ($\theta' = 0$), then $x_{it_i} = x_{it_j} = l_{it_j} = 0$ ($i \neq j$) and $l_{it_i} < 0$.*

(ii) *If the bargaining power is endogenous ($\theta' > 0$), $x_{it_i} < 0$, $x_{it_j} > 0$, and $l_{it_j} < 0$ ($i \neq j$) and the sign of l_{it_i} is ambiguous.*

The effect of t_i on x_i is very intuitive. The household welfare function (2) implies that the tax change affects private consumption x_i only through the power function θ.[5] Thus, if $\theta' = 0$, the private consumption is independent of the tax changes. If $\theta' > 0$, the increase in t_i lowers the bargaining power of spouse i and hence reduces her/his private consumption. On the other hand, the increases in t_i raises the bargaining power of spouse j, while it reduces the household income. These have opposite effects on x_j. However, the result $x_{jt_i} > 0$ ($i \neq j$), where $\theta' > 0$, shows that the former effect overcomes the latter effect.

[5] Note that the utility function of each spouse is quasi-linear and the household welfare function is the weighted average of the individual utility.

In our model, a change in t_i affects labor supply of spouse i through two channels. The first is the standard price effect—raising the income tax rate lowers net return to labor and, thus, reduces labor supply.[6] The second is the bargaining power effect, which shows that the increase in income tax rate on spouse i, for instance, strengthens spouse j's power through the changes in the relative net income, so working hours decrease for spouse j and increase for spouse i.

If the bargaining power is exogenous, $\theta' = 0$, the power effect does not appear, so that $l_{jt_i} = 0$ $(j \neq i)$ and $l_{it_i} < 0$ hold. Raising the income tax rate for spouse i lowers her/his working hours with price effects, represented by the lower opportunity cost of leisure. However, the working hours of spouse j are not affected by the income tax rate for spouse i, because the household welfare function is the weighted average of the individual utility.

If the power is endogenous $(\theta' > 0)$, an increase in t_j reduces spouse i's working hours; that is, $l_{it_j} < 0$. This is because the increase in t_j strengthens spouse i's power, and hence, the working hours of spouse i decrease, while those of spouse j increase. However, the sign of l_{it_j} are ambiguous if the power is endogenous $(\theta' > 0)$. This is because the two effects work in the opposite direction. The increase in t_j lowers the power of spouse j, and therefore increases the working hours of spouse j (bargaining power effect). At the same time, as shown in the case of exogenous power, a rise in t_j lowers her/his working hours (price effect). Therefore, the sign of change in the tax rate of a spouse on her/his own labor supply depends on the relative magnitude of the bargaining power effect and the price effect.

2.3 Government

We next consider the government's problem. The government obtains revenue from income taxes on spouses to finance a certain amount of public spending, so that the target of the tax revenue is given at a fixed level, g. The budget constraint of the government is then

$$t_1 w_1 l_1(t_1, t_2) + t_2 w_2 l_2(t_1, t_2) = g. \tag{19}$$

Since we assume many identical couples, g corresponds to the required tax revenue per couple. We assume that the government has two properties. First, it has sufficient foresight—it knows the feedback effect of the bargaining power in the economy. Second, it is utilitarian, as proposed by Apps and Rees (1988) and Cremer et al. (2016, 2017), who analyze the optimal taxation system based on a collective model.[7] Its objective function in the utilitarian manner is then given by $\Phi \equiv 0.5u_1 + 0.5u_2$. Therefore, the government's

[6] Note that there is no income effect because of the quasi-linear utility function.

[7] Apps and Rees (1988) and Cremer et al. (2016, 2017) take a paternalistic approach, and assume the utilitarian optimum based on equal weights between husband and wife. Apps and Rees (1988) examine the welfare effects of the tax reforms. Cremer et al. (2016) study the optimal income taxation of a couple in the Mirrlees tradition, where information on individual incomes is unobservable. Although they treat information friction in their model, the bargaining power is exogenous. Cremer et al. (2017) examine the optimal commodity taxation in the absence of information friction.

maximization problem is[8]

$$\begin{aligned}
\underset{t_1,\, t_2}{\text{Max}} \quad & \Phi = (1-t_1)w_1 l_1 + (1-t_2)w_2 l_2 - x_1 - x_2 \\
& \qquad + \frac{1}{2}\left[h_1(x_1) + v_1(1-l_1)\right] + \frac{1}{2}\left[h_2(x_2) + v_2(1-l_2)\right], \\
\text{s.t.} \quad & t_1 w_1 l_1 + t_2 w_2 l_2 = g,
\end{aligned}$$

where $x_i = x_i(t_1, t_2)$ and $l_i = l_i(t_1, t_2)$. The Lagrangian is

$$\begin{aligned}
\Im \;\equiv\; & (1-t_1)w_1 l_1 + (1-t_2)w_2 l_2 - x_1 - x_2 + \frac{1}{2}\left[h_1(x_1) + v_1(1-l_1)\right] \\
& + \frac{1}{2}\left[h_2(x_2) + v_2(1-l_2)\right] + \mu(g - t_1 w_1 l_1 - t_2 w_2 l_2),
\end{aligned}$$

where μ is the Lagrange multiplier for the government budget constraint. With (4)–(7), the first order conditions with respect to t_1 and t_2 are

$$\begin{aligned}
\frac{\partial \Im}{\partial t_1} \;=\; & -(1+\mu)w_1 l_1 + \left[\frac{(1-2\theta)(1-t_1)}{2(1-\theta)} - \mu t_1\right] w_1 l_{1t_1} \\
& + \left[\frac{(2\theta-1)(1-t_2)}{2\theta} - \mu t_2\right] w_2 l_{2t_1} + \left[\frac{2\theta-1}{2(1-\theta)}\right] x_{1t_1} + \left(\frac{1-2\theta}{2\theta}\right) x_{2t_1} = 0 \qquad (20)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \Im}{\partial t_2} \;=\; & -(1+\mu)w_2 l_2 + \left[\frac{(1-2\theta)(1-t_1)}{2(1-\theta)} - \mu t_1\right] w_1 l_{1t_2} \\
& + \left[\frac{(2\theta-1)(1-t_2)}{2\theta} - \mu t_2\right] w_2 l_{2t_2} + \left[\frac{2\theta-1}{2(1-\theta)}\right] x_{1t_2} + \left(\frac{1-2\theta}{2\theta}\right) x_{2t_2} = 0 \qquad (21)
\end{aligned}$$

The optimal tax systems in our model are obtained by solving (20) and (21) with (19).

3 Optimality

3.1 Specification

In this section, we obtain the optimal taxation system. To understand more about the properties of optimal taxation systems, we further specify the functional forms for our analysis. We first specify the power function of our main interest as

$$\theta = 0.5 + \alpha \cdot (\rho_2 - \rho_1), \qquad \alpha \geq 0. \tag{22}$$

In (22), $\alpha(=\theta')$ is the marginal power for the second earner's relative income size, $\rho_2 - \rho_1$, and is assumed to be constant. If the after-tax income is the same ($\rho_1 = \rho_2$), we see that

[8] We implicitly assume that the government uses its tax revenue to purchase a public good G and provides it to consumers. Then, $G = ng$, where n is the number of households. In addition, we assume that the public good is additive-separable, that is, $u_i = x + h_i + v_i + G$. Then, the precise expression for the government's objective function is $\Phi + G$. From this functional form and constant G due to a fixed revenue requirement, we find that the optimal conditions (20) and (21) are not disturbed by G. Therefore, the results obtained in this paper are valid even if the public good is explicitly introduced to the utility functions.

$\theta = 0.5$, which implies that the couple's power is balanced. When $\alpha = 0$, the bargaining power remains constant; that is, $\theta' = 0$.

We next specify the utility function of each spouse as follows:

$$h_i(x_i) = \frac{1}{\phi} \ln x_i \text{ and } v_i(1 - l_i) = \frac{1}{\beta} \ln(1 - l_i), \tag{23}$$

which satisfy $h_i' > 0$, $h_i'' < 0$, $v_i' > 0$, and $v_i'' < 0$. Under (22) and (23), the demand and labor supply functions are given by

$$x_1 = \frac{(1-\theta)}{\phi}, \; x_2 = \frac{\theta}{\phi}, \; l_1 = 1 - \frac{(1-\theta)}{\beta(1-t_1)w_1}, \text{ and } l_2 = 1 - \frac{\theta}{\beta(1-t_2)w_2}. \tag{24}$$

Then, the comparative statics yields

$$x_{1t_1} = -\frac{\alpha w_1 x_1}{(1-\theta)\left(1 + \frac{2\alpha}{\beta}\right)} \leq 0, \tag{25}$$

$$x_{2t_1} = \frac{\alpha w_1 x_2}{\theta\left(1 + \frac{2\alpha}{\beta}\right)} \geq 0, \tag{26}$$

$$l_{1t_1} = -\frac{w_1 \left[(1-l_1)\beta + (1-2l_1)\alpha\right](1-l_1)}{(1-\theta)\left(1 + \frac{2\alpha}{\beta}\right)}, \tag{27}$$

$$l_{2t_1} = -\frac{\alpha w_1 (1-l_2)}{\theta\left(1 + \frac{2\alpha}{\beta}\right)} \leq 0, \tag{28}$$

$$x_{1t_2} = \frac{\alpha w_2 x_1}{(1-\theta)\left(1 + \frac{2\alpha}{\beta}\right)} \geq 0, \tag{29}$$

$$x_{2t_2} = -\frac{\alpha w_2 x_2}{\theta\left(1 + \frac{2\alpha}{\beta}\right)} \leq 0, \tag{30}$$

$$l_{1t_2} = -\frac{\alpha w_2 (1-l_1)}{(1-\theta)\left(1 + \frac{2\alpha}{\beta}\right)} \leq 0, \tag{31}$$

$$l_{2t_2} = -\frac{w_2 \left[(1-l_2)\beta + (1-2l_2)\alpha\right](1-l_2)}{\theta\left(1 + \frac{2\alpha}{\beta}\right)}. \tag{32}$$

(25)-(32) show that the argument summarized in Proposition 1 holds in our specifications.

3.2 Optimal Taxes

In this section, we provide the optimal income tax formula for the couple with bargaining. Before doing so, let us define $r_i \equiv t_i/(1-t_i)$, $\omega \equiv -\mu^{-1}$, and $\varepsilon_{ij} \equiv (\partial l_i/\partial W_j)(W_j/l_i)$, where $W_i \equiv (1-t_i)w_i$. r_i is a transform of the tax rate t_i. From the definition of r_i, we find that $r_1 - r_2 = (t_1 - t_2)\,/\,(1-t_2)(1-t_1)$, which shows that $r_1 \lesseqgtr r_2 \Longleftrightarrow t_1 \lesseqgtr t_2$, since $1 > t_i$ for $i = 1, 2$. ε_{ii} (ε_{ij}) is the elasticity of spouse i's labor supply with respect to the after-tax wage rate for spouse i (j). Using the definitions, we obtain the following proposition.

Proposition 2. *The optimal income tax rates satisfy*

$$\begin{aligned} r_1 &= (1-\omega)\frac{(\alpha+\beta)\left[\beta\varepsilon_{22}+\alpha\left(\varepsilon_{22}-\varepsilon_{11}-1\right)\right]}{\beta\varepsilon_{11}\varepsilon_{22}\left(2\alpha+\beta\right)-\alpha^2\left(\varepsilon_{11}+\varepsilon_{22}+1\right)}+\omega\frac{\theta-\frac{1}{2}}{1-\theta} \\ &\quad +\omega\frac{\alpha\beta}{\phi}\frac{\left(\frac{1}{2}-\theta\right)}{\theta\left(1-\theta\right)}\frac{\left(\varepsilon_{11}+1\right)\left(\alpha+2\alpha\varepsilon_{22}+\beta\varepsilon_{22}\right)}{\beta\varepsilon_{11}\varepsilon_{22}\left(2\alpha+\beta\right)-\alpha^2\left(\varepsilon_{11}+\varepsilon_{22}+1\right)}, \end{aligned} \tag{33}$$

$$\begin{aligned} r_2 &= (1-\omega)\frac{(\alpha+\beta)\left[\beta\varepsilon_{11}+\alpha\left(\varepsilon_{11}-\varepsilon_{22}-1\right)\right]}{\beta\varepsilon_{11}\varepsilon_{22}\left(2\alpha+\beta\right)-\alpha^2\left(\varepsilon_{11}+\varepsilon_{22}+1\right)}+\omega\frac{\frac{1}{2}-\theta}{\theta} \\ &\quad +\omega\frac{\alpha\beta}{\phi}\frac{\left(\theta-\frac{1}{2}\right)}{\theta\left(1-\theta\right)}\frac{\left(\varepsilon_{22}+1\right)\left(\alpha+2\alpha\varepsilon_{11}+\beta\varepsilon_{11}\right)}{\beta\varepsilon_{11}\varepsilon_{22}\left(2\alpha+\beta\right)-\alpha^2\left(\varepsilon_{11}+\varepsilon_{22}+1\right)}. \end{aligned} \tag{34}$$

Proof. See Mathematical Appendix B.

The second terms in (33) and (34) correct the price distortion. They correspond to the labor response effects under endogenous bargaining power. Note that the labor supply responses in this model contain the effects of weight change, as shown by (27), (28), (31), and (32). The second terms in RHS of (33) and (34) are the individual weight terms. It describes the discrepancy of government weight (0.5) and the actual household weight (θ). For example, when $\theta < 0.5$, the first term in the optimal tax rate r_1 is negative, and that in r_2 is positive. The third terms in (33) and (34) show the response of private good consumption when the bargaining power is endogenous.

For a useful discussion hereafter, we suppose that $\omega \in (0,1)$ and $\beta\varepsilon_{11}\varepsilon_{22}\left(2\alpha+\beta\right) - \alpha^2\left(\varepsilon_{11}+\varepsilon_{22}+1\right) > 0$.[9] The latter holds when α is not extremely large.[10] To understand our results more clearly, we consider the case of $\alpha = 0$ ($\theta' = 0$), wherein the balance of power is exogenous. Notice that $\theta = 1/2$ when $\alpha = 0$, leading that the second and third terms in (33) and (34) disappear. Using these properties, the optimal tax rates (33) and (34) are reduced to

$$r_1 = \frac{1-\omega}{\varepsilon_{11}} \text{ and } r_2 = \frac{1-\omega}{\varepsilon_{22}} \quad \rightarrow \quad r_1 - r_2 = \frac{(1-\omega)(\varepsilon_{22}-\varepsilon_{11})}{\varepsilon_{11}\varepsilon_{22}}. \tag{35}$$

These expressions are the so-called Ramsey's inverse-elasticity rule itself: The higher tax rate should be imposed on the income of the individual with less elasticity. That is, a simple Ramsey rule holds as in (35) if the bargaining power is exogenous. However, the optimal tax rule is not that simple if the bargaining power is endogenous. From (33) and (34), we have the ranking of the optimal tax rates as follows:

$$\begin{aligned} r_1 - r_2 &= (1-\omega)\frac{\left(\varepsilon_{22}-\varepsilon_{11}\right)\left(2\alpha+\beta\right)\left(\alpha+\beta\right)}{\beta\varepsilon_{11}\varepsilon_{22}\left(2\alpha+\beta\right)-\alpha^2\left(\varepsilon_{11}+\varepsilon_{22}+1\right)}+\omega\frac{\theta-\frac{1}{2}}{\theta\left(1-\theta\right)} \\ &\quad +\omega\frac{\alpha\beta}{\phi}\frac{\left(\frac{1}{2}-\theta\right)}{\theta\left(1-\theta\right)}\frac{2\varepsilon_{11}\varepsilon_{22}\left(2\alpha+\beta\right)+\left(\varepsilon_{11}+\varepsilon_{22}\right)\left(3\alpha+\beta\right)+2\alpha}{\beta\varepsilon_{11}\varepsilon_{22}\left(2\alpha+\beta\right)-\alpha^2\left(\varepsilon_{11}+\varepsilon_{22}+1\right)}. \end{aligned} \tag{36}$$

The first term reflects the government's objective to correct the distortion of labor supply in terms of the Ramsey rule as modified by the bargaining power effect, α. The government

[9] We have confirmed that $\omega \in (0,1)$ in our numerical examples in the next section.

[10] We have numerically confirmed that this term is positive in almost all cases, including the numerical examples in the next section.

imposes the higher tax rate on the individual whose labor supply is less elastic with respect to after-tax wage in order to repress the price distortions: $r_1 \gtreqless r_2 \iff \varepsilon_{22} \gtreqless \varepsilon_{11}$. The second term shows that the government tries to maximize the total income of the household considering its bargaining-based decision making. Suppose the case that the second earner's bargaining power is sufficiently low (i.e., the value of θ is sufficiently small). In this case, the second earner's labor supply is large as a consequence of the couple's time allocation resolved through intra-household bargaining. However, as the second earner's productivity (wage) is low compared with the first earner's, the labor supply of the first earner with higher productivity (i.e., wage), rather than that of the second earner with lower productivity, is increased, in practice, to achieve a larger total income. Therefore, the government sets a lower tax for the first earner and a higher tax for his/her partner to induce the first earner to work more, whereas the second earner works less. The third term shows the distribution of welfare determined by private consumptions. The concavity of the function h_i ($h_i'(x_i) > 0$, $h_i''(x_i) < 0$) and the equal weights in the government's objective function for the first and second earners imply that the government wants to achieve equal private consumption for the spouses as $x_1 = x_2$. The term captures this effect. Suppose now that θ is sufficiently small. In this case, the first earner with higher bargaining power enjoys more private consumption than the second earner, $x_1 > x_2$. The government then seeks to reduce (increase) the first (second) earner's consumption to a level close to his/her partner's by imposing a higher tax rate on the first earner than the second earner. This consideration is strengthened by the smaller value of ϕ, which means that the couple assigns a higher weight to private goods consumption.

Considering that the three terms lead to the optimal tax rate in the opposite direction, it is ambiguous whether the first or second earner faces a higher tax rate. We may expect that if g is relatively small, the endogenous bargaining power consideration is likely to dominate the Ramsey consideration, suggesting that a lower (higher) tax rate will be imposed on the first (second) earner. In the next section, the ranking of spouses' optimal tax rates will be demonstrated with numerical examples.

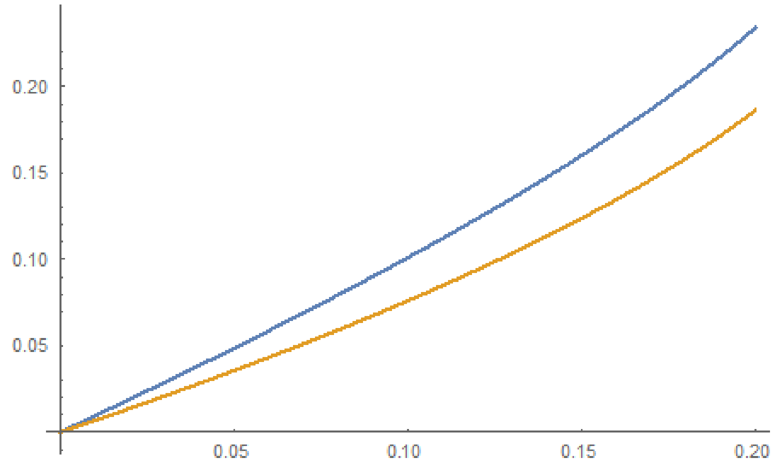
4 Numerical Analyses

This section presents numerical examples of the optimal tax rates for each member in the household. In the previous section, we have seen that the optimal taxation in our setting is composed of two parts: (i) Ramsey taxation consideration, determined by the relative size of the elasticities of labor supply of two household members; and (ii) endogenous bargaining power consideration, arising from the feedback effect of gap in their after-tax disposable incomes. To visually understand the argument, we make use of numerical analyses of the two effects. The relevant parameters are set in the simulation as $\beta = 1.0$, $\phi = 1.0$, $w_1 = 1.2$, and $w_2 = 1.0$.

4.1 Constant Bargaining Power

First, as a benchmark, let us look at Figure 1, in the case of $\alpha = 0$, where bargaining power is exogenous. In this case, the bargaining power coincides with the weight in the government's objective function ($\theta = 1/2$). Figure 1 shows the relationship between the optimal tax rates t_1 and t_2, and the required tax revenue g. Here, the y-axis represents the tax rate, and the x-axis represents the required tax revenue, g. The blue line shows the optimal income tax rate for the first earner, and the red line is the optimal income tax rate for the second earner. The optimal tax rates in this case are given by (35). When the government does not require the revenue (i.e., $g = 0$), the optimal tax rates for the two parties are zero, since taxing policies distort the household decisions.

Figure 1: A numerical example of the optimal tax rates ($\alpha = 0.0$ and $\phi = 1.0$)



Note. X–axis shows required tax revenue g and Y–axis shows tax rate t_i.

On the other hand, when the government does require the tax revenue (i.e., $g > 0$), it further increases the tax rates for the spouses. From Figure 1, we see that the gap between the optimal tax rates widens as g increases. Consequently, when $\alpha = 0$, the optimal tax rate for the labor income of the first earner, whose labor is less elastic, is higher than that of the second earner in all areas.[11] The figure shows that the Ramsey tax rule holds.

4.2 Endogenous Bargaining Power

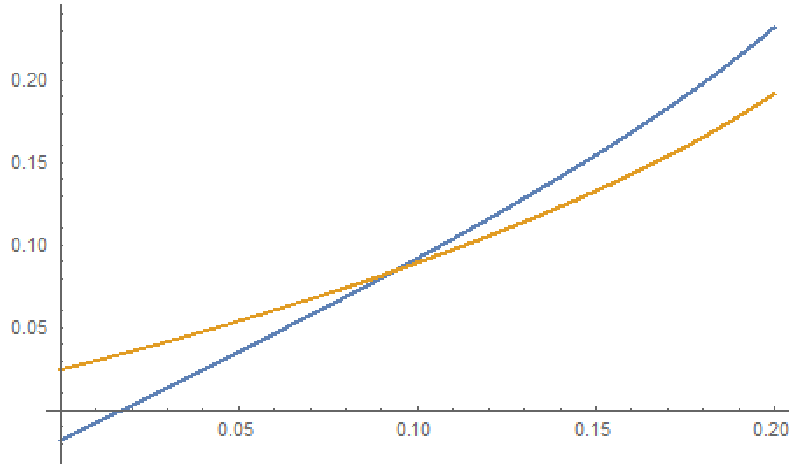
Next, we look at Figure 2, in the case of $\alpha = 0.1$, as an example of the bargaining power being endogenous. Again, Figure 2 indicates the relationship between the optimal tax rates, which are given by (33) and (34), in the y-axis and the required tax revenue, g, in the x-axis. The blue line is the optimal tax rate for the first earner, and the red line is that for the second earner.

As shown in Figure 2, even when $g = 0$, the government formulates tax policies for the couple's labor incomes. This is because the gap between the disposable incomes of the two household members causes the actual bargaining power to differ from the weight assigned in the government's objective function. To correct this, the government implements tax policies. Specifically, when the power is endogenous, it is considered favorable for the individual with higher after-tax income. As a result, the first earner, who has higher

[11]We have confirmed that $\varepsilon_{11} < \varepsilon_{22}$ in the interval of g in Figure 1.

bargaining power, will demand that the second earner work more to increase her/his consumption while working less.[12] To correct this effect, the government imposes a lower (higher) tax rate for the labor income of the first (second) earner. In particular, when $g = 0$, the tax rate for the first earner becomes negative (meaning a subsidy for the first earner's labor supply).

Figure 2: A numerical example of the optimal tax rates ($\alpha = 0.1$ and $\phi = 1.0$)



Note. X–axis shows required tax revenue g and Y–axis shows tax rate t_i.

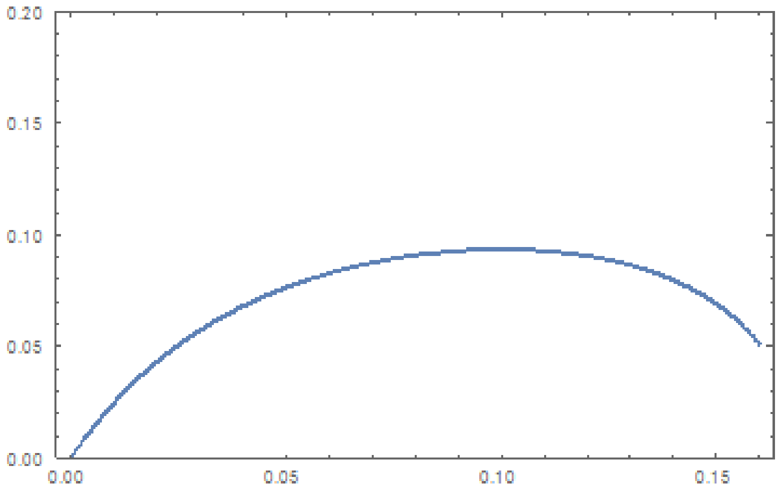
However, when the government requires a large tax revenue, it needs to set higher tax rates. Higher taxation causes large price distortions in household decisions, compromising the welfare of the couple. Thus, the government envisages a tax structure built upon the Ramsey taxation rule. As shown in Figure 2, when the required tax revenue is relatively small, tax policies are used to resolve the couple's distribution problems. However, when the required tax revenue is relatively large, the Ramsey tax consideration dominates the endogenous bargaining power consideration, and the government inclines toward the Ramsey rule. Consequently, the optimal taxation system for spouses' labor incomes involves a relationship reversal at a certain level of tax revenue, which is represented by the intersection of the two lines in Figure 2.

In Figure 3, the curve plots the configuration of g (x-axis) and α (y-axis), wherein the same tax rate for the two household members is optimal ($t_1 = t_2$). In the area above the curve, $t_1 > t_2$ holds, while in the area below the curve, $t_1 < t_2$. In line with the argument illustrated in Figures 1 and 2, when the required tax revenue is sufficiently high (i.e., larger g), the Ramsey tax consideration dominates the endogenous bargaining power consideration in order to minimize the household's distortions. Figure 3, viewed from left to right, shows that the relationship between the two optimal taxes is not monotonous in a certain range. The reason for this can be explained intuitively as follows. When the marginal bargaining power effect is sufficiently small (i.e., smaller α), $t_1 > t_2$ since the Ramsey effect is greater than the bargaining power effect. As α increases, the

[12]We have confirmed that $\theta < 0.5$ in the interval of g in Figure 2.

government's objective inclines toward distribution by mitigating the bargaining power effect due to the income gap between the spouses, i.e., $t_1 < t_2$. When α is sufficiently large, the distribution of welfare, which determined by private consumption represented by the third term in (36), matters for the government. The income gap between the spouses creates a large consumption gap. Specifically, the first earner enjoys higher private consumption than the second earner, and the gap in private consumption widens as α increases. An optimum policy for the government is to lower the bargaining power of the first earner by increasing her/his tax rate in order to reduce the consumption difference between married couples.

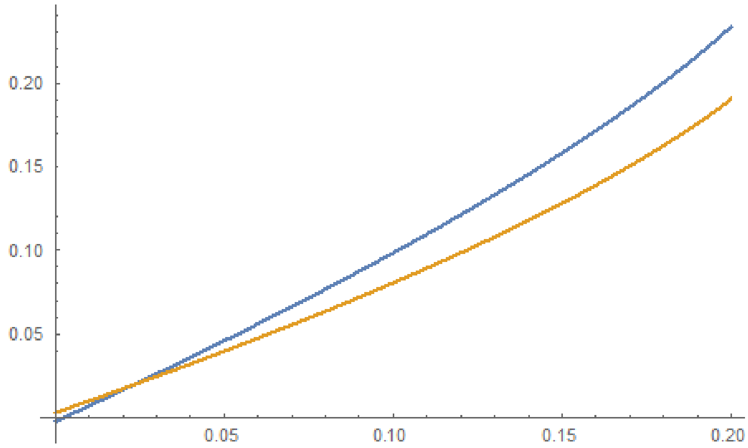
Figure 3: Combinations of α and g when $t_1 = t_2$



Note. X–axis shows the marginal bargaining power effect α and Y–axis shows the.required tax revenue g.

We next numerically demonstrate the effects of ϕ, which can be regarded as the weight assigned to private consumption in the individual's preference ranking, on the optimal tax rates. When the value of ϕ is small, private consumption holds a major significance in determining the optimal taxation system. Figure 4 plots the configuration of g in the x-axis and the optimal tax rates in the y-axis, given $\phi = 5.0$ with the other parameters set the same value as those of Figure 2. Comparing Figures 2 and 4, we see that the smaller ϕ is, the larger the range where $t_1 > t_2$ holds as the optimal tax ranking. This is consistent with the interpretation for the third term in (36).

Figure 4: A numerical example of the optimal tax rates ($\alpha = 0.1$ and $\phi = 5.0$)



Note. X–axis shows required tax revenue g and Y–axis shows tax rate t_i.

5 Conclusion

We developed a theoretical model, wherein a couple bargains over their consumption, its time allocation between leisure and market work, and the bargaining position depends on the gap between the after-tax labor incomes of the typical couple in the society. The model shows that a higher after-tax income results in a higher bargaining position for one spouse and, consequently, higher labor supply by the other. Under this situation, we investigate the optimal taxation system the government sets to maximize the utilitarian social welfare function. The model shows that the conditions of the optimal taxation system are characterized by the Ramsey tax consideration and the endogenous bargaining power consideration. The former is the so-called Ramsey tax rule (inverse elasticity rule), where it is optimal for the government to set a higher tax rate for the labor income of the individual with lower wage elasticity, typically the first earner. However, when the bargaining power is endogenous, as in our model, the government also takes into consideration the divergence from the bargaining decision reflected by the taste of the spouse who is in the favorable position, leading to the opposite relationship of the taxation system: a lower tax rate for the first earner's labor income and a higher rate for the second earner. This increases the household consumption of the good because the productivity of the first earner is higher than that of the second earner. The direction of the optimal tax system depends on the relative magnitudes of these two effects, and the relative magnitude depends on the marginal bargaining power and the required tax revenue.

Before closing our paper, we need to mention the limitations and future directions of our study. First, we assume married individuals, and do not consider people who decide to remain single. Nevertheless, we have treated common consumption as a feature that connects two individuals in a family. Second, we do not consider domestic production, which also relates to the first point. One can consider a domestic model that specializes in market or domestic work, which provides the ground for two individuals to behave as a couple. Apps and Rees (1988) indeed argue that the introduction of domestic production can alter the implication of policy redistribution. Specifically, consider a case where family members allocate their time between these two tasks instead of having leisure, and that each member has a different preference for domestic production goods. Our results can be considered to hold substantially where the first earner's wage is higher, the two individuals' domestic production time is substitutable, and the second earner assigns a higher weight to domestic production than his/her partner does. In the opposite case where the first earner assigns a higher weight to domestic production than his/her partner, one can expect multiple equilibria, as well as different optimal taxation systems depending on which equilibrium the economy belongs to. The latter case may be interesting because social norms accelerate the decline or increase in female labor participation. Third, we defined bargaining power in our analysis as endogenous at the social level, but assumed that the individuals within a household would treat it as exogenous. One can alternatively expect the choices of individuals to affect their bargaining power. In this case, each spouse can improve his/her bargaining position by increasing her/his labor supply. If we incorporate this strategic effect, our result may be mitigated because higher bargaining power allows a spouse to pursue further autonomy by increasing her/his labor supply even as the individual seeks to enjoy larger leisure as predicted in our model. These extensions should be an interesting research direction in the future.

Mathematical Appendices

Appendix A. Totally differentiating (4)-(7) and (10) with respect to x_1, x_2, l_1, l_2, t_1, and t_2 yields

$$\begin{bmatrix} (1-\theta)h_1'' & 0 & \theta' h_1'(1-t_1)w_1 & -\theta' h_1'(1-t_2)w_2 \\ 0 & \theta h_2'' & -\theta' h_2'(1-t_1)w_1 & \theta' h_2'(1-t_2)w_2 \\ 0 & 0 & (1-\theta)\, v_1'' - \theta' v_1'(1-t_1)w_1 & \theta' v_1'(1-t_2)w_2 \\ 0 & 0 & \theta' v_2'(1-t_1)w_1 & \theta v_2'' - \theta'(1-t_2)w_2 v_2' \end{bmatrix} \begin{pmatrix} dx_1 \\ dx_2 \\ dl_1 \\ dl_2 \end{pmatrix}$$

$$= \begin{bmatrix} -\theta' w_1 l_1 h_1' \\ \theta' w_1 l_1 h_2' \\ \theta' w_1 l_1 v_1' - w_1 \\ -\theta' w_1 l_1 v_2' \end{bmatrix} dt_1 + \begin{bmatrix} \theta' w_2 l_2 h_1' \\ -\theta' w_2 l_2 h_2' \\ -\theta' w_2 l_2 v_1' \\ \theta' w_2 l_2 v_2' - w_2 \end{bmatrix} dt_2.$$

Solving this, we get (11)-(18).

Appendix B. Substituting (22) into (24), and solving for l_i and x_i, we get

$$l_1 = \frac{2\beta(\alpha+\beta)W_1 + 2\alpha\beta W_2 - 2\alpha - \beta}{2\beta W_1 (2\alpha+\beta)}, \tag{A1}$$

$$l_2 = \frac{2\beta(\alpha+\beta)W_2 + 2\alpha\beta W_1 - 2\alpha - \beta}{2\beta W_2 (2\alpha+\beta)}, \tag{A2}$$

$$x_1 = \frac{2\alpha+\beta+2\alpha\beta(W_1 - W_2)}{2\phi(2\alpha+\beta)}, \tag{A3}$$

$$x_2 = \frac{2\alpha+\beta+2\alpha\beta(W_2 - W_2)}{2\phi(2\alpha+\beta)}. \tag{A4}$$

Substituting (A1) and (A2) into (22), we get

$$\theta = \frac{1}{2} - \frac{\alpha\beta(W_1 - W_2)}{2\alpha+\beta}. \tag{A5}$$

Using (A1)-(A4), the comparative statics yield

$$\frac{\partial l_1}{\partial W_1} = \frac{2\alpha+\beta-2\alpha\beta W_2}{2\beta W_1^2 (2\alpha+\beta)}, \tag{A6}$$

$$\frac{\partial l_1}{\partial W_2} = \frac{\alpha}{W_1(2\alpha+\beta)}, \tag{A7}$$

$$\frac{\partial l_2}{\partial W_2} = \frac{2\alpha+\beta-2\alpha\beta W_1}{2\beta W_2^2 (2\alpha+\beta)}, \tag{A8}$$

$$\frac{\partial l_2}{\partial W_1} = \frac{\alpha}{W_2(2\alpha+\beta)}, \tag{A9}$$

$$\frac{\partial x_1}{\partial W_1} = \frac{\partial x_2}{\partial W_2} = \frac{\alpha\beta}{\phi(2\alpha+\beta)}, \tag{A10}$$

$$\frac{\partial x_2}{\partial W_1} = \frac{\partial x_1}{\partial W_2} = -\frac{\alpha\beta}{\phi(2\alpha+\beta)}. \tag{A11}$$

(A6) and (A8) can be rewritten as

$$\frac{\partial l_1}{\partial W_1}\frac{W_1}{l_1} \equiv \varepsilon_{11} = \frac{2\alpha+\beta-2\alpha\beta W_2}{2\beta(\alpha+\beta)W_1 + 2\alpha\beta W_2 - 2\alpha - \beta}, \tag{A12}$$

$$\frac{\partial l_2}{\partial W_2}\frac{W_2}{l_2} \equiv \varepsilon_{22} = \frac{2\alpha+\beta-2\alpha\beta W_1}{2\beta(\alpha+\beta)W_2 + 2\alpha\beta W_1 - 2\alpha - \beta}. \tag{A13}$$

Solving (A12) and (A13) for W_1 and W_2, we get

$$W_1 = \frac{(\varepsilon_{11}+1)(2\alpha+\beta)(\alpha-\beta\varepsilon_{22})}{2\beta\left(\alpha^2+\alpha^2\varepsilon_{11}+\alpha^2\varepsilon_{22}-\beta^2\varepsilon_{11}\varepsilon_{22}-2\alpha\beta\varepsilon_{11}\varepsilon_{22}\right)}, \tag{A14}$$

$$W_2 = \frac{(\varepsilon_{22}+1)(2\alpha+\beta)(\alpha-\beta\varepsilon_{11})}{2\beta\left(\alpha^2+\alpha^2\varepsilon_{11}+\alpha^2\varepsilon_{22}-\beta^2\varepsilon_{11}\varepsilon_{22}-2\alpha\beta\varepsilon_{11}\varepsilon_{22}\right)}. \tag{A15}$$

Now, we simplify (20) and (21). Substitution of (A6)-(A11) and using the definition of ε_{11} and ε_{22}, i.e., (A12) and (A13), yield

$$\begin{aligned}\left(1-\frac{1}{\omega}\right) l_1 + l_1 \left[\frac{(1-2\theta)}{2(1-\theta)} + \frac{r_1}{\omega}\right] \varepsilon_{11} + \left[\frac{(2\theta-1)}{2\theta} + \frac{r_2}{\omega}\right] \frac{\alpha}{(2\alpha+\beta)} \\ + \frac{2\theta-1}{2(1-\theta)} \frac{\alpha\beta}{\phi(2\alpha+\beta)} - \frac{1-2\theta}{2\theta} \frac{\alpha\beta}{\phi(2\alpha+\beta)} = 0,\end{aligned} \tag{A16}$$

$$\begin{aligned}\left(1-\frac{1}{\omega}\right) l_2 + l_2 \left[\frac{(2\theta-1)}{2\theta} + \frac{r_2}{\omega}\right] \varepsilon_{22} + \left[\frac{(1-2\theta)}{2(1-\theta)} + \frac{r_1}{\omega}\right] \frac{\alpha}{(2\alpha+\beta)} \\ + \frac{1-2\theta}{2\theta} \frac{\alpha\beta}{\phi(2\alpha+\beta)} - \frac{2\theta-1}{2(1-\theta)} \frac{\alpha\beta}{\phi(2\alpha+\beta)} = 0.\end{aligned} \tag{A17}$$

Substituting (A14) and (A15) into (A1) and (A2), we get

$$l_1 = \frac{1}{2\alpha+\beta} \frac{\alpha+\beta}{\varepsilon_{11}+1} \text{ and } l_2 = \frac{1}{2\alpha+\beta} \frac{\alpha+\beta}{\varepsilon_{22}+1}. \tag{A18}$$

Substituting (A18) into (A16) and (A17), we get

$$\begin{aligned}\left(1-\frac{1}{\omega}\right) \frac{1}{2\alpha+\beta} \frac{\alpha+\beta}{\varepsilon_{11}+1} + \frac{1}{2\alpha+\beta} \frac{\alpha+\beta}{\varepsilon_{11}+1} \left[\frac{(1-2\theta)}{2(1-\theta)} + \frac{r_1}{\omega}\right] \\ + \left[\frac{(2\theta-1)}{2\theta} + \frac{r_2}{\omega}\right] \frac{\alpha}{(2\alpha+\beta)} + \left[\frac{2\theta-1}{2(1-\theta)}\right] \frac{\alpha\beta}{\phi(2\alpha+\beta)} - \frac{1-2\theta}{2\theta} \frac{\alpha\beta}{\phi(2\alpha+\beta)} = 0,\end{aligned} \tag{A19}$$

$$\begin{aligned}\left(1-\frac{1}{\omega}\right) \frac{1}{2\alpha+\beta} \frac{\alpha+\beta}{\varepsilon_{22}+1} + \frac{1}{2\alpha+\beta} \frac{\alpha+\beta}{\varepsilon_{22}+1} \left[\frac{(2\theta-1)}{2\theta} + \frac{r_2}{\omega}\right] \\ + \left[\frac{(1-2\theta)}{2(1-\theta)} + \frac{r_1}{\omega}\right] \frac{\alpha}{(2\alpha+\beta)} + \left(\frac{1-2\theta}{2\theta}\right) \frac{\alpha\beta}{\phi(2\alpha+\beta)} - \frac{2\theta-1}{2(1-\theta)} \frac{\alpha\beta}{\phi(2\alpha+\beta)} = 0.\end{aligned} \tag{A20}$$

The optimal tax rule satisfies these two conditions. Solving these for r_1 and r_2, we obtain (33) and (34).

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