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Economic Geography and a Theory of International Currency: Implications of a Random Matching Model

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Abstract

This paper presents a new theory that may explain why the US dollar is the dominant medium of exchange in international transactions. Unlike previous studies, we investigate a model in which economic geography affects the international currency choice. The model is based on a random matching model in which agents trade with foreign agents using a specific currency. We consider a world that consists of two regions, the EU and the USA, each of which has different active time zones. In local transactions, matched agents use their local currency. However, in international transactions, sellers choose either the Euro or the US dollar as the invoice currency to maximize their expected discounted utility. We show that under some reasonable conditions, the US dollar becomes the unique international currency, even if each region is symmetric in all ways except for their locations. Further, when the US dollar is used for international transactions, the expected discounted utility becomes higher in the US than in the EU in the steady-state equilibrium.

JEL code: E41, E42, F33, F41

Key words: international currency, medium of exchange, random matching model

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1. Introduction

The choice of international currency is a topic that has been widely discussed in the field of international finance (McKinnon, 1979; Magee and Rao, 1980; and Broz, 1997). Although the relative importance of the US has recently declined in the world economy, the US dollar still dominates other currencies in terms of its transaction volume in international markets. In the globalized world economy, the US dollar is the dominant vehicle currency on which various international transactions rely.

Table 1 summarizes foreign exchange turnover by currency from 1992 to 2019. The data are based on the BIS's Triennial Central Bank Survey of Foreign Exchange and Derivatives Market Activity in April 2019. Throughout this period, the share of the US dollar was close to 90%, which implies that most currencies in global foreign exchange markets were exchanged into US dollars. The US dollar is the dominant vehicle currency in foreign exchange turnover, even outside the US. Based on the same BIS dataset, Table 2 reports the currency shares of foreign exchange turnover from various countries in 2019. Although these economies are highly globalized, this indicates that the share of turnover against the US dollar was more than 90% in nearly half of them, and exceeded 70% in all cases.

The US dollar is the legal tender, which means that it is the dominant medium of exchange that is recognized in meeting a financial obligation within the US. In local transactions, the US dollar is the currency that extinguishes debt when offered in payment. However, in international transactions, the US dollar is no longer a legal tender, since there is no institutional reason that makes the US dollar a dominant vehicle of cross-border transactions. Therefore, it is important to explore why private agents still choose the US dollar as the dominant vehicle currency in international transactions.

The purpose of this paper is to provide a new theory that may explain why the US dollar is the only medium of exchange used in international transactions. Unlike previous studies, we investigate a model in which economic geography affects the choice of international currency. The model is based on a random matching model in which agents use a specific currency to trade with foreign agents. We consider a world economy that consists of two regions, the EU and the US, each of which has different active time zones. In each region, “sellers” specialize in production, while “buyers” specialize in consumption. In local transactions, matched agents always use their local currency because they are legal tenders in the region. However, in international transactions, sellers choose either the Euro or the US dollar as the invoice currency to maximize their expected discounted utility, taking as given the strategies of other active agents and the distribution of currencies in each region.

A key feature of the model is that the US is located west of the EU, meaning that agents in each region are active at different times. We show that under reasonable conditions, the US dollar becomes the unique international currency, even if the two regions are symmetric in every way except for their locations. We also show that when the US dollar is used for international transactions, the expected discounted utility becomes higher in the US than in the EU in the steady-state equilibrium. A crucial element is that the active time zone moves from the EU to the US every day. Since dynamic programming is solved backward, this derives a sequence of causality that what the US agents determine affect the decision of EU agents. Since US agents use the US dollar for their local transactions, US buyers have more US dollars than Euros. EU sellers thus choose the US dollar for their international transactions because it results in relatively large gains from trade. US sellers also choose the US dollar when trading with EU agents because the Euro becomes useless for local transactions in the following period. The results imply that geographical features of the regions can explain why the US dollar remains the dominant medium of exchange in international transactions.

Except for overlapping daytime transactions in the two regions, our model is based on Lagos and Wright's (2005) random matching model. It also has several common features with previous open economy versions of random matching models, such as Matsuyama, Kiyotaki, and Matsui (1993). Thus, without different active time zones, no specific currency can be the unique equilibrium currency; if there is an equilibrium in which currency j is the international currency, there is also an equilibrium in which currency k ($\neq j$) can be the international currency. However, because two homogeneous regions are asymmetrically linked in this research, our model has the feature that the US dollar becomes the unique international currency.

Without using the random matching models mentioned above, several previous studies have explored the determinants of currency in international trade. For example, Krugman (1980) and Rey (2001) showed that transaction costs could make the US dollar a dominant medium of exchange in international trade. In their model, both "history" and "expectation" are key determinants when choosing an international currency. Once an international currency is established, a large structural change would be necessary for it to be replaced in the economic environment. Thus, if there is inertia of the previous economic superpower of the US economy, their models explain why the US dollar is chosen as the vehicle currency. Friberg (1998) and Goldberg and Tille (2005) investigated why many international trades are invoiced in a third currency; that is, vehicle currency. Assuming that an exporter commits to selling the demanded quantity at the ex-post realized price, they demonstrated that monopolistically competitive exporters set their prices in terms of the third currency when the markets are highly competitive. These previous studies are important in terms of explaining why the US dollar is the dominant vehicle currency in international transactions. However, they suggest that if there is a dramatic structural or expectational change, the other currency may overtake the US dollar as the dominant vehicle of international transactions. In contrast, our model suggests that even if there is a dramatic structural or expectational change, the US dollar could remain dominant in international transactions because the geographical features in the regions will never change.

2. Literature review

In the context of an open economy, Matsuyama et al. (1993) wrote a seminal paper that constructed a two-country, two-currency monetary search model to study the emergence of international currency. However, this work was based on Kiyotaki and Wright's (1997) research, which assumed the indivisibility of goods and money. In the literature, the open economy model was extended in several directions (e.g., Trejos and Wright, 1996; Zhou, 1997; Matsui, 1998; Li and Matsui, 2009).

Kannan (2009) investigated an open economy version of Lagos and Wright's (2005) model to develop a three-country, three-currency model with divisible goods and money, and calibrated the welfare of circulating an international currency. Zhang (2014) also explored an open economy version of the Lagos-Wright model by introducing imperfect recognizability of foreign currency to study its effect on the existence conditions of international currency and welfare implications of inflation. In relation to this model, Lester, Postlewaite, and Wright (2012) analyzed currency competition between two currencies by developing a model consisting of multiple assets with different recognizabilities that determine the liquidity of each asset. Liu, Lu, and Zhang (2017) investigated the role of trade finance and financial intermediation in the choice of invoice currency by incorporating time-to-ship friction and financial market development. Chapter 12 of Rocheteau and Nosal (2017) tackled the problem of indeterminacy in the nominal exchange rate and proposed a mechanism to uniquely determine the nominal exchange rate by introducing a pricing mechanism that reflects the notion that each agent prefers the domestic currency to the foreign currency.

Lagos and Wright (2005) assumed that one period consists of daytime, during which the decentralized market is open, and nighttime, during which the centralized market is open. Except for Matsuyama et al. (1993), the above studies applied a monetary search model in an open economy. Our study similarly analyzes an open economy version of Lagos-Wright model. However, our model is in marked contrast with the aforementioned studies because we only introduce overlapping daytime structures in which international trade takes place in one of the two consecutive daytimes.

Some existing studies have developed a model consisting of two consecutive daytimes and one nighttime. For example, Berensten, Camera, and Waller (2005) assumed that both buyers and sellers can access the centralized market after two rounds of decentralized markets. In the context of asset pricing, Geromichalos et al. (2016) developed a model in which sellers and buyers access the decentralized market during the first daytime, while sellers who obtained assets during the first daytime and investors access the OTC market, that is, the decentralized secondary asset market during the second daytime. In the context of an open economy, Geromichalos and Jung (2018) incorporated the inter-dealer FOREX market as a bilateral OTC market during the first daytime before trading in the decentralized market in the second day. These studies developed a model with two consecutive decentralized markets and one centralized market. However, unlike our study, they did not consider overlapping decentralized markets across two regions. By utilizing the OLG framework, our model incorporates the time difference of the two regions.

Some studies have incorporated search friction into the OLG model. For example, Zhu (2008) constructed a two-period OLG model in which all agents access both the decentralized market with the search friction and the centralized market when young, and then access only the centralized market in the next period when old. Jacquet and Tan (2011) adopted the same two-period OLG settings in which the young access both centralized and decentralized markets, and the old access the centralized market in the next period. Chapter 6 of Rocheteau and Nosal (2017) also adopted the same OLG model using Lagos and Wright's (2005) framework. However, these models are closed-economy models that do not consider second trading in the decentralized market.

To the best of our knowledge, there is no open-economy monetary search model with two consecutive decentralized markets and OLG settings. More importantly, in previous open economy models, multiple equilibria emerge, as is often the case in monetary search models. In contrast, we show that the equilibrium is uniquely determined by incorporating the time difference between two regions. We also show that when the US dollar is used for international transactions in the steady-state equilibrium, the expected discounted utility becomes higher in the US than in the EU.

3. Basic structure of the model

In the following analysis, we consider the world economy, which consists of two regions: the EU (region 1) and the USA (region 2). The USA is located west of the EU, such that EU agents start their transactions earlier than US agents every business day. Apart from their locations, we assume that the two regions are symmetric. Although the symmetric assumption excludes many realistic features in the world, it allows us not only to simplify our model, but also to clarify how the geographical features matter to determine the international currency.

In each region, there are "buyers" that specialize in consumption, and "sellers" that specialize in production. Buyers purchase q units of perishable goods, paying $\tilde{m}$ units of money to obtain utility $u(q)$. On the other hand, sellers produce q units of perishable goods, incurring the disutility of $c(q)$ to obtain $\tilde{m}$ units of money. Denoting the price of money in terms of the goods by ϕ, the gain from the trade is $u(q)$ - $\phi\tilde{m}$ for the buyer, and $\phi\tilde{m}$ - $c(q)$ for the seller.

In accordance with Lagos and Wright (2005), we consider daytime decentralized transactions and nighttime centralized transactions. In a nighttime, centralized transactions take place in a Walrasian market where prices are adjusted to equalize demand and supply. In contrast, during the daytime, each active buyer who has m units of money is randomly matched with an active seller that produces q units of goods. When matched, they solve Kalai's bargaining problem to determine the allocation of their total benefits.[1] If a seller's bargaining power is denoted by θ, the matched seller's gain is $\theta(u(q) - c(q))$ and the matched buyer's gain is $(1-\theta)(u(q) - c(q))$, where $0 < \theta < 1$. When $\tilde{m}$ units of money are paid for the transaction, the bargaining solution leads to $\phi\tilde{m} = \theta\, u(q) + (1-\theta)\, c(q)$.[2]

When the cash-in-advance constraint (hereafter, CIA) is not binding, the equilibrium units of production are q^*, whereby $u'(q^*) = c'(q^*)$. In this case, units of money paid for q^* units of goods are m^*, which satisfy the following equation: $\phi m^* = \theta\, u(q^*) + (1-\theta)\, c(q^*)$. By contrast, when the CIA is binding, the equilibrium units of production q are constrained by the number of units of money the buyer has. When the buyer has m units of money, it holds that $\phi m = \theta\, u(q) + (1-\theta)\, c(q)$ under the bargaining solution. This implies that when the CIA is binding, the equilibrium production is expressed as $q(\phi m)$, where $q'(\phi m) > 0$. In the following analysis, we define the total benefits from the trade when the CIA is binding, as well as those when the CIA is not binding, as follows:

$$v(\phi m) \equiv u(q(\phi m)) - c(q(\phi m)), \tag{1a}$$

$$v^* \equiv u(q^*) - c(q^*). \tag{1b}$$

[1] Studies such as Lester, Postlewaite and Wright (2012), Zhang (2014) and Liu, Lu and Zhang (2017) adopted Kalai's (1977) proportional bargaining to determine the bargaining solutions in the decentralized market.

[2] When $\phi = \theta\, u(q) + (1-\theta)\, c(q)$, the gain from trade is $\phi - c(q) = \theta(u(q) - c(q))$ for matched sellers and $-\phi + u(q) = (1-\theta)(u(q) - c(q))$ for matched buyers.

We assume that the function $v(\phi m)$ satisfies the Inada conditions such that $\lim_{\phi m \to 0} v' = +\infty$ and $\lim_{\phi m \to +\infty} v' = 0$.

Many structures in our model follow those of Lagos and Wright (2005). However, the model is unique in that agents from the two regions are heterogeneous in terms of when they are active. Because of the different locations, the agents in each region are active in different time zones. Figure 1 depicts how transactions take place in each time zone. One day consists of four time zones: three daytimes and one nighttime. Daytime I is "morning of the EU" when only EU agents make decentralized local transactions; Daytime II is "afternoon of the EU" and "morning of the USA" when agents from both the EU and US make decentralized international transactions[3]; Daytime III is the "afternoon of the USA" when only US agents make decentralized local transactions; and Nighttime is "night of the EU and the USA" when both EU and US agents make centralized local and international transactions.

In each decentralized local market, transactions take place if and only if an active seller is matched with an active buyer from the same region. In contrast, in the decentralized international market, transactions take place if and only if an active seller is matched with an active buyer from another region. Sellers and buyers equally populate each market. We assume that the matching probability is p for decentralized local transactions in each region, and p^* for decentralized international transactions. We also assume that the matching probability is higher in each decentralized local market than in the decentralized international market (that is, $p > p^*$).

At the beginning of the day, all agents in the same region are homogeneous. However, each morning, they randomly become sellers and buyers. The randomization patterns are summarized

[3] For analytical simplicity, we assumed that there are no decentralized local transactions during Daytime II. The following results are essentially the same even if EU and US agents make not only decentralized international transactions, but also decentralized local transactions during Daytime II.

in Figure 2. In the morning, each agent becomes a buyer with a probability of 0.5 and a seller with a probability of 0.5).[4] Regardless of transactions in the morning, the buyers and the sellers in the morning become sellers and buyers in the afternoon, respectively. The agents carry over any unexchanged money for the next trading opportunity.

Each region supplies a fixed amount of the local currency (Euros or US dollars). We assume that each local currency is the legal tender in the region, so that all agents need to use it when trading in their local market.[5] However, when trading in a decentralized international market, each seller chooses a currency to maximize his or her expected discounted utility. In the following analysis, we denote the price of the Euro in terms of the goods by ϕ^{E}, and the price of the US dollar in terms of the goods by ϕ^{D}.

4. The choice of the currency by sellers

The purpose of this section is to consider the choice of currency in a decentralized international market. In the market, a transaction takes place if and only if a seller is matched with a buyer from another region during Daytime II. We assume that the matched seller first selects the currency, and then the matched agents solve Kalai's bargaining problem in the selected currency.[6] We then determine which currency is the equilibrium currency within this environment.

In equilibrium, the matched seller chooses either the Euro or the US dollar to maximize his or her expected discounted utility, taking as given the strategies of the other agents and the

[4] Studies such as Lester, Postlewaite and Wright (2012) and Zhang (2014) assumed that each agent becomes a buyer or a seller with a probability of 0.5 in each period.

[5] Many papers including Geromichalos and Simonovska (2014), Liu, Lu and Zhang (2017), and Geromichalos and Jung (2018) adopted the assumption that only the domestic currency is accepted in domestic trading.

[6] The matched buyers have no incentive to reject the selected currency because it leads to no trade gains. Previous studies such as Geromichalos and Simonovska (2014), Liu, Lu and Zhang (2017), Geromichalos and Jung (2018), Lester, Postlewaite and Wright (2012), Zhang (2014) and Jung and Pyun (2016) assumed that bargaining solutions are determined after the currency is chosen.

distribution of currencies. When deriving the expected discounted utility, the Lagos-Wright model has a useful feature in that the value function at the beginning of the nighttime is linear. We define the value function of an agent in region i at the beginning of Nighttime by $W_i(m^E, m^D)$, where the agent holds m^E of the Euro and m^D of the US dollar. In the Lagos-Wright model, it holds that $W_i(m_1^E + m_2^E, m_1^D + m_2^D) = \phi^E m_2^E + \phi^D m_2^D + W_i(m_1^E, m_1^D)$. We use this feature to explore how the expected discounted utility is different when using the US dollar versus the Euro in decentralized international transactions.

We first consider the choice of currency by a matched EU seller during Daytime III. Suppose that the EU seller holds $m_{1,S}^E$ of the Euro and $m_{1,S}^D$ of the US dollar, and that the matched US buyer holds $\overline{m}_{2,B}^E$ of the Euro and $\overline{m}_{2,B}^D$ of the US dollar at the beginning of Daytime II. Note that the CIA is always binding for the matched US buyer during Daytime II.[7] Thus, when the US dollar is used during Daytime II, the value function of the matched EU seller is written as follows:

$$
\begin{aligned}
V_{1,S,D}(m_{1,S}^E, m_{1,S}^D) &\equiv - c(q(\phi^D \overline{m}_{2,B}^D)) + W_1(m_{1,S}^E, m_{1,S}^D + \overline{m}_{2,B}^D), \\
&= \phi^D \overline{m}_{2,B}^D - c(\phi^D q(\overline{m}_{2,B}^D)) + W_1(m_{1,S}^E, m_{1,S}^D), \\
&= \theta\, v(\phi^D \overline{m}_{2,B}^D) + W_1(m_{1,S}^E, m_{1,S}^D). \qquad (2a)
\end{aligned}
$$

In contrast, when the Euro is used during Daytime II, the EU seller's value function is written as:

$$
\begin{aligned}
V_{1,S,E}(m_{1,S}^E, m_{1,S}^D) &\equiv - c(q(\phi^E \overline{m}_{2,B}^E)) + W_1(m_{1,S}^E + \overline{m}_{2,B}^E, m_{1,S}^D), \\
&= \phi^E \overline{m}_{2,B}^E - c(\phi^E q(\overline{m}_{2,B}^E)) + W_1(m_{1,S}^E, m_{1,S}^D), \\
&= \theta\, v(\phi^E \overline{m}_{2,B}^E) + W_1(m_{1,S}^E, m_{1,S}^D). \qquad (2b)
\end{aligned}
$$

[7] All buyers face the CIA each morning, because they have no incentive to hold redundant money in the nighttime transactions.

We therefore obtain the following proposition.

Proposition 1: When $\phi^D \bar{m}_{2,B}{}^D > \phi^E \bar{m}_{2,B}{}^E$, the matched EU sellers choose the US dollar in decentralized international transactions.

Proof: Equations (2a) and (2b) imply that $V_{1,S,D}(m_{1,S}{}^E, m_{1,S}{}^D) - V_{1,S,E}(m_{1,S}{}^E, m_{1,S}{}^D) = \theta\{v(\phi^D \bar{m}_{2,B}{}^D) - v(\phi^E \bar{m}_{2,B}{}^E)\}$. Because $v' > 0$, this indicates that $V_{1,S,D}(m_{1,S}{}^E, m_{1,S}{}^D) > V_{1,S.E}(m_{1,S}{}^E, m_{1,S}{}^D)$ when $\phi^D \bar{m}_{2,B}{}^D > \phi^E \bar{m}_{2,B}{}^E$. This derives the following proposition. [Q.E.D.]

When the matched buyers have more US dollars than Euros, the matched sellers can gain more from trading in US dollars than Euros. The EU sellers, who will make no transactions during Daytime III, choose to trade using US dollars during Daytime II when $\phi^D \bar{m}_{2,B}{}^D > \phi^E \bar{m}_{2,B}{}^E$. This intuitive mechanism underpins Proposition 1.

It is natural to suppose that US buyers have more US dollars than Euros at the beginning of Daytime II, since US agents need to use their US dollars during Daytime III. In Appendix 1, we show that when the two currencies have the same inflation rate, US agents always hold more US dollars than Euros at the beginning of Daytime II. Thus, Proposition 1 suggests that choosing the US dollar in reasonable environments is the dominant strategy for matched EU sellers.

Next, we consider the choice of currency of a matched US seller. Suppose that the matched US seller holds $m_{2,S}{}^E$ of the Euro and $m_{2,S}{}^D$ of the US dollar, and that the matched EU buyer holds $\bar{m}_{1,B}{}^E$ of the Euro and $\bar{m}_{1,B}{}^D$ of the US dollar at the beginning of Daytime II. When the US dollar is used during Daytime II, the CIA is always binding for the matched EU buyer because they did not receive the extra US dollar during Daytime I. However, it is not clear whether the CIA is binding for the matched US buyer during Daytime III when receiving the extra US dollar during Daytime II. When it is binding, the value function of the US seller is written as:

$$\mathrm{V}_{2,\mathrm{S,D}}\,(m_{2,\mathrm{S}}{}^{\mathrm{E}},\, m_{2,\mathrm{S}}{}^{\mathrm{D}})$$

$$= -\,\mathrm{c}(q(\phi^{\mathrm{D}}\overline{m}_{1,\mathrm{B}}{}^{\mathrm{D}})) + p\{u(q(\phi^{\mathrm{D}}(m_{2,\mathrm{S}}{}^{\mathrm{D}} + \overline{m}_{1,\mathrm{B}}{}^{\mathrm{D}}))) + \mathrm{W}_2(m_{2,\mathrm{S}}{}^{\mathrm{E}}, 0)]\} + (1\text{-}p)\mathrm{W}_2(m_{2,\mathrm{S}}{}^{\mathrm{E}}, m_{2,\mathrm{S}}{}^{\mathrm{D}} + \overline{m}_{1,\mathrm{B}}{}^{\mathrm{D}}),$$

$$= \phi^{\mathrm{D}}\overline{m}_{1,\mathrm{B}}{}^{\mathrm{D}} - \mathrm{c}(q(\phi^{\mathrm{D}}\overline{m}_{1,\mathrm{B}}{}^{\mathrm{D}})) + p\{u(q(\phi^{\mathrm{D}}(m_{2,\mathrm{S}}{}^{\mathrm{D}} + \overline{m}_{1,\mathrm{B}}{}^{\mathrm{D}}))) - \phi^{\mathrm{D}}(m_{2,\mathrm{S}}{}^{\mathrm{D}} + \overline{m}_{1,\mathrm{B}}{}^{\mathrm{D}})\} + \mathrm{W}_2(m_{2,\mathrm{S}}{}^{\mathrm{E}}, m_{2,\mathrm{S}}{}^{\mathrm{D}}),$$

$$= \theta\, v(\phi^{\mathrm{D}}\overline{m}_{1,\mathrm{B}}{}^{\mathrm{D}}) + p(1\text{-}\theta)\, v(\phi^{\mathrm{D}}\,(m_{2,\mathrm{S}}{}^{\mathrm{D}} + \overline{m}_{1,\mathrm{B}}{}^{\mathrm{D}})) + \mathrm{W}_2(m_{2,\mathrm{S}}{}^{\mathrm{E}}, m_{2,\mathrm{S}}{}^{\mathrm{D}}). \qquad (3a)$$

When it is not binding, the value function of the US seller is written as:

$$\mathrm{V}_{2,\mathrm{S,D}}\,(m_{2,\mathrm{S}}{}^{\mathrm{E}},\, m_{2,\mathrm{S}}{}^{\mathrm{D}})$$

$$\equiv -\,\mathrm{c}(q(\phi^{\mathrm{D}}\overline{m}_{1,\mathrm{B}}{}^{\mathrm{D}})) + p\{u(q^*) + \mathrm{W}_2(m_{2,\mathrm{S}}{}^{\mathrm{E}}, m_{2,\mathrm{S}}{}^{\mathrm{D}} + \overline{m}_{1,\mathrm{B}}{}^{\mathrm{D}} - m^{*\mathrm{D}})\} + (1\text{-}p)\mathrm{W}_2(m_{2,\mathrm{S}}{}^{\mathrm{E}}, m_{2,\mathrm{S}}{}^{\mathrm{D}} + \overline{m}_{1,\mathrm{B}}{}^{\mathrm{D}}),$$

$$= \phi^{\mathrm{D}}\overline{m}_{1,\mathrm{B}}{}^{\mathrm{D}} - \mathrm{c}(q(\phi^{\mathrm{D}}\overline{m}_{1,\mathrm{B}}{}^{\mathrm{D}})) + p\{u(q^*) - \phi^{\mathrm{D}} m^{*\mathrm{D}}\} + \mathrm{W}_2(m_{2,\mathrm{S}}{}^{\mathrm{E}}, m_{2,\mathrm{S}}{}^{\mathrm{D}}),$$

$$= \theta\, v(\phi^{\mathrm{D}}\overline{m}_{1,\mathrm{B}}{}^{\mathrm{D}}) + p(1\text{-}\theta)\, v^* + \mathrm{W}_2(m_{2,\mathrm{S}}{}^{\mathrm{E}}, m_{2,\mathrm{S}}{}^{\mathrm{D}}). \qquad (3b)$$

where $m^{*\mathrm{D}} \equiv \{\theta\, u(q^*) + (1\text{-}\theta)\, \mathrm{c}(q^*)\}/\phi^{\mathrm{D}}$.

In contrast, when the Euro is used during Daytime II, the CIA is always binding for the matched US buyer during Daytime III. However, it is not clear whether it is binding for the matched EU buyer during Daytime II when receiving the extra Euros during Daytime I. When it is binding, the value function of the US seller is written as:

$$\mathrm{V}_{2,\mathrm{S,E}}\,(m_{2,\mathrm{S}}{}^{\mathrm{E}},\, m_{2,\mathrm{S}}{}^{\mathrm{D}})$$

$$= -\,\mathrm{c}(q(\phi^{\mathrm{E}}\overline{m}_{1,\mathrm{B}}{}^{\mathrm{E}})) + p\{u(q(\phi^{\mathrm{D}} m_{2,\mathrm{S}}{}^{\mathrm{D}})) + \mathrm{W}_2(m_{2,\mathrm{S}}{}^{\mathrm{E}} + \overline{m}_{1,\mathrm{B}}{}^{\mathrm{E}}, 0)\} + (1\text{-}p)\mathrm{W}_2(m_{2,\mathrm{S}}{}^{\mathrm{E}} + \overline{m}_{1,\mathrm{B}}{}^{\mathrm{E}}, m_{2,\mathrm{S}}{}^{\mathrm{D}}),$$

$$= \phi^{\mathrm{E}}\overline{m}_{1,\mathrm{B}}{}^{\mathrm{E}} - \mathrm{c}(q(\phi^{\mathrm{E}}\overline{m}_{1,\mathrm{B}}{}^{\mathrm{E}})) + p\{u(q(\phi^{\mathrm{D}} m_{2,\mathrm{S}}{}^{\mathrm{D}})) - \phi^{\mathrm{D}} m_{2,\mathrm{S}}{}^{\mathrm{D}}\} + \mathrm{W}_2(m_{2,\mathrm{S}}{}^{\mathrm{E}}, m_{2,\mathrm{S}}{}^{\mathrm{D}}),$$

$= \theta\, v(\phi^E \bar{m}_{1,B}{}^E) + p(1\text{-}\theta)\, v(\phi^D m_{2,S}{}^D) + W_2(m_{2,S}{}^E, m_{2,S}{}^D).$ (4a)

When it is not binding, the value function of the US seller is written as:

$$V_{2,S,E}\,(m_{2,S}{}^E, m_{2,S}{}^D)$$

$$= -\,c(q^*) + p\{u(q(\phi^D m_{2,S}{}^D)) + W_2(m_{2,S}{}^E + m^{*E}, 0)\} + (1\text{-}p)W_2(m_{2,S}{}^E + m^{*E}, m_{2,S}{}^D),$$

$$= \phi^E m^{*E} - c(q^*) + p\{u(q(\phi^D m_{2,S}{}^D)) - \phi^D m_{2,S}{}^D\} + W_2(m_{2,S}{}^E, m_{2,S}{}^D),$$

$$= \theta\, v^* + p(1\text{-}\theta)\, v(\phi^D m_{2,S}{}^D) + W_2(m_{2,S}{}^E, m_{2,S}{}^D). \quad (4b)$$

where $m^{*E} \equiv \{\theta\, u(q^*) + (1\text{-}\theta)\, c(q^*)\}/\phi^E$.

We therefore obtain the following proposition.

Proposition 2: When $\bar{m}_{1,B}{}^D > 0$, matched US sellers choose the US dollar in decentralized international transactions if their bargaining power is small enough.

Proof: Since $v^* > v(\phi^D\,(m_{2,S}{}^D + \bar{m}_{1,B}{}^D))$ and $v^* > v(\phi^E \bar{m}_{1,B}{}^E)$, both $V_{2,S,D}\,(m_{1,S}{}^E, m_{1,S}{}^D)$ and $V_{2,S,E}$ $(m_{2,S}{}^E, m_{2,S}{}^D)$ are relatively large when the CIA is not binding. Equations (3a) and (4b) imply that $V_{2,S,D}\,(m_{1,S}{}^E, m_{1,S}{}^D) - V_{2,S,E}\,(m_{1,S}{}^E, m_{1,S}{}^D) > \theta\,\{v(\phi^D \bar{m}_{1,B}{}^D) - v^*\} + p(1\text{-}\theta)\{v(\phi^D\,(m_{2,S}{}^D + \bar{m}_{1,B}{}^D)) - v(\phi^D m_{2,S}{}^D)\}$. When $\bar{m}_{1,B}{}^D > 0$, $v(\phi^D\,(m_{2,S}{}^D + \bar{m}_{1,B}{}^D)) > v(\phi^D m_{2,S}{}^D)$. Therefore, if θ is sufficiently small, then $V_{2,S,D}\,(m_{1,S}{}^E, m_{1,S}{}^D) > V_{2,S,\,E}\,(m_{1,S}{}^E, m_{1,S}{}^D)$. This derives the following proposition. [Q.E.D.]

Unlike EU sellers, US sellers tend to choose the US dollar for international transactions during Daytime II because the Euro is useless for local transactions during Daytime III. This derives

Proposition 2, which holds that US sellers tend to trade with EU agents using the US dollar during Daytime II in reasonable environments.

To the extent that there is a positive probability that US sellers choose the US dollar, any EU buyers hold some amount of US dollars at the beginning of Daytime II, even if the probability is negligible. This indicates that $\bar{m}_{1,B}{}^{D}$ is always positive in a trembling-hand perfect Nash equilibrium (see Appendix 2 for the formal proof). Proposition 2 therefore suggests that US sellers choose the US dollar when their bargaining power is sufficiently small. The assumption that the seller's bargaining power is small enough includes the ultimatum game as a special case, which has been widely used in the literature. This is particularly reasonable in an environment where sellers choose their currency in advance. This line of thinking was followed in the analysis.

Propositions 1 and 2 indicate that both EU and US sellers choose the US dollar during Daytime II in reasonable environments. In other words, the US dollar is likely to be a unique equilibrium currency in the decentralized international market. This result is in marked contrast with those of existing studies, in which multiple equilibria emerged when choosing the international currency. In the following analysis, we assume that the US dollar is the unique equilibrium currency in international transactions during Daytime II.

5. The solution of dynamic programming

In the previous section, we showed that choosing the US dollar is a unique equilibrium currency in the decentralized international market under reasonable environments. Given that all agents use the US dollar in the decentralized international market, this section derives equilibrium money holdings. We solve the dynamic programming for EU and US agents who maximize their expected discounted utility. We denote the daily discount factor by β, where $0 < \beta < 1$. As in the literature, each agent discounts utility in the following day. However, we assume that there is no intra-daily discount for utility, so that the afternoon utility holds equal weight as the morning utility.

Suppose that each EU agent holds $m_{1,t}^{\mathrm{E}}$ of the Euro and $m_{1,t}^{\mathrm{D}}$ of the US dollar and that each US agent holds $m_{2,t}^{\mathrm{E}}$ of the Euro and $m_{2,t}^{\mathrm{D}}$ of the US dollar at the beginning of day t (where the money holdings of the match agent are denoted by those with an upper bar). When the US dollar is used in the decentralized international market, the CIA of US dollars always binding for EU agents. The expected utility of EU agents at the beginning of the day is written as:

$$
\begin{aligned}
&\mathrm{V}_1(m_{1,t}^{\mathrm{E}}, m_{1,t}^{\mathrm{D}}) \\
&= 0.5[p\theta\, v(\phi_t^{\mathrm{E}}\overline{m}_{1,t}^{\mathrm{E}}) + p^*(1-\theta)v(\phi_t^{\mathrm{D}}m_{1,t}^{\mathrm{D}})] \\
&\quad + 0.5[p(1-\theta)v(\phi_t^{\mathrm{E}}m_{1,t}^{\mathrm{E}}) + p^*\theta\, v(\phi_t^{\mathrm{D}}\overline{m}_{2,t}^{\mathrm{D}})] + \mathrm{W}_1(m_{1,t}^{\mathrm{E}}, m_{1,t}^{\mathrm{D}}) \\
&= 0.5[(1-\theta)\{pv(\phi_t^{\mathrm{E}}m_{1,t}^{\mathrm{E}})+p^*v(\phi_t^{\mathrm{D}}m_{1,t}^{\mathrm{D}})\} + \theta\{pv(\phi_t^{\mathrm{E}}\overline{m}_{1,t}^{\mathrm{E}}) +p^*v(\phi_t^{\mathrm{D}}\overline{m}_{2,t}^{\mathrm{D}})\}] \\
&\quad + \mathrm{W}_1(m_{1,t}^{\mathrm{E}}, m_{1,t}^{\mathrm{D}})\ . \qquad (5)
\end{aligned}
$$

In contrast, when the US dollar is used in the decentralized international market, the CIA of US dollars is not necessarily binding for US agents during Daytime III. When it is binding during Daytime III, the expected utility of US agents at the beginning of the day is written as:

$$
\begin{aligned}
&\mathrm{V}_2(m_{2,t}^{\mathrm{E}}, m_{2,t}^{\mathrm{D}}) \\
&= 0.5[p^*\theta\, v(\phi_t^{\mathrm{D}}\overline{m}_{1,t}^{\mathrm{D}}) + p^*p(1-\theta)v(\phi_t^{\mathrm{D}}(m_{2,t}^{\mathrm{D}}+\overline{m}_{1,t}^{\mathrm{D}})) + (1-p^*)p(1-\theta)v(\phi_t^{\mathrm{D}}m_{2,t}^{\mathrm{D}})] \\
&+ 0.5[p^*(1-\theta)v(\phi_t^{\mathrm{D}}m_{2,t}^{\mathrm{D}}) + p^*p\theta\, v(\phi_t^{\mathrm{D}}(\overline{m}_{2,t}^{\mathrm{D}}+\overline{m}_{1,t}^{\mathrm{D}})) + (1-p^*)\, p\theta\, v(\phi_t^{\mathrm{D}}\overline{m}_{2,t}^{\mathrm{D}})] \\
&+ \mathrm{W}_2(m_{2,t}^{\mathrm{E}}, m_{2,t}^{\mathrm{D}}) \\
&= 0.5[(1-\theta)\{(p-pp^*+p^*)v(\phi_t^{\mathrm{D}}m_{2,t}^{\mathrm{D}}) + pp^*v(\phi_t^{\mathrm{D}}(m_{2,t}^{\mathrm{D}}+\overline{m}_{1,t}^{\mathrm{D}}))\} \\
&+ \theta\{p^*v(\phi_t^{\mathrm{D}}\overline{m}_{1,t}^{\mathrm{D}}) + (1-p^*)p\, v(\phi_t^{\mathrm{D}}\overline{m}_{2,t}^{\mathrm{D}}) + pp^*\, v(\phi_t^{\mathrm{D}}(\overline{m}_{2,t}^{\mathrm{D}}+\overline{m}_{1,t}^{\mathrm{D}}))\}]
\end{aligned}
$$

$+ \mathrm{W}_2(m_{2,t}{}^{\mathrm{E}}, m_{2,t}{}^{\mathrm{D}})$. (6a)

When it is not binding, the expected utility of US agents at the beginning of the day is as follows:

$$\mathrm{V}_2(m_{2,t}{}^{\mathrm{E}}, m_{2,t}{}^{\mathrm{D}})$$
$$= 0.5[p^*\theta\, v(\phi_t{}^{\mathrm{D}}\overline{m}_{1,t}{}^{\mathrm{D}}) + p^*p(1\text{-}\theta)v^* + (1\text{-}p^*)p(1\text{-}\theta)v(\phi_t{}^{\mathrm{D}}m_{2,t}{}^{\mathrm{D}})]$$
$$+ 0.5[p^*(1\text{-}\theta)v(\phi_t{}^{\mathrm{D}}m_{2,t}{}^{\mathrm{D}}) + p^*p\theta\, v^* + (1\text{-}\,p^*)\, p\theta\, v(\phi_t{}^{\mathrm{D}}\overline{m}_{2,t}{}^{\mathrm{D}})] + \mathrm{W}_2(m_{2,t}{}^{\mathrm{E}}, m_{2,t}{}^{\mathrm{D}})$$
$$= 0.5[(1\text{-}\theta)\{(p\text{-}pp^*\text{+}p^*)v(\phi_t{}^{\mathrm{D}}m_{2,t}{}^{\mathrm{D}}) + pp^*v^*\}$$
$$+ \theta\{p^*v(\phi_t{}^{\mathrm{D}}\overline{m}_{1,t}{}^{\mathrm{D}}) + (1\text{-}\,p^*)p\, v(\phi_t{}^{\mathrm{D}}\overline{m}_{2,t}{}^{\mathrm{D}}) + pp^*\, v^*\}] + \mathrm{W}_2(m_{2,t}{}^{\mathrm{E}}, m_{2,t}{}^{\mathrm{D}})\ . \tag{6b}$$

In the Lagos-Wright model, the value function of the agent in region i at the beginning of Nighttime is:

$$\mathrm{W}_{\mathrm{i}}(m_{\mathrm{i},t}{}^{\mathrm{E}}, m_{\mathrm{i},t}{}^{\mathrm{D}}) = U(X^*) - X^* + \phi_t{}^{\mathrm{E}}\, m_{\mathrm{i},t}{}^{\mathrm{E}} + \phi_t{}^{\mathrm{D}}\, m_{\mathrm{i},t}{}^{\mathrm{D}}$$
$$+ \max\ \{-\phi_t{}^{\mathrm{E}}m_{\mathrm{i},t+1}{}^{\mathrm{E}} - \phi_t{}^{\mathrm{D}}\, m_{\mathrm{i},t+1}{}^{\mathrm{D}} + \beta\ V_{\mathrm{i}}(m_{\mathrm{i},t+1}{}^{\mathrm{E}}, m_{\mathrm{i},t+1}{}^{\mathrm{D}})\}. \tag{7}$$

Therefore, if we define

$$v_1(m_{1,t}{}^{\mathrm{E}}, m_{1,t}{}^{\mathrm{D}}) \equiv 0.5[(1\text{-}\theta)\{pv(\phi_t{}^{\mathrm{E}}m_{1,t}{}^{\mathrm{E}})\text{+}p^*v(\phi_t{}^{\mathrm{D}}m_{1,t}{}^{\mathrm{D}})\}$$
$$+ \theta\{pv(\phi_t{}^{\mathrm{E}}\overline{m}_{1,t}{}^{\mathrm{E}}) + p^*v(\phi_t{}^{\mathrm{D}}\overline{m}_{2,t}{}^{\mathrm{D}})\}], \tag{8}$$
$$v^a{}_2(m_{2,t}{}^{\mathrm{E}}, m_{2,t}{}^{\mathrm{D}}) \equiv 0.5[(1\text{-}\theta)\{(p\text{-}pp^*\text{+}p^*)v(\phi_t{}^{\mathrm{D}}m_{2,t}{}^{\mathrm{D}}) + pp^*v(\phi_t{}^{\mathrm{D}}(m_{2,t}{}^{\mathrm{D}}\text{+}\overline{m}_{1,t}{}^{\mathrm{D}}))$$
$$+ \theta\{p^*v(\phi_t{}^{\mathrm{D}}\overline{m}_{1,t}{}^{\mathrm{D}}) + (1\text{-}\,p^*)p\, v(\phi_t{}^{\mathrm{D}}\overline{m}_{2,t}{}^{\mathrm{D}}) + pp^*v(\phi_t{}^{\mathrm{D}}(\overline{m}_{2,t}{}^{\mathrm{D}}\text{+}\overline{m}_{1,t}{}^{\mathrm{D}}))\}], \tag{9a}$$
$$v^b{}_2(m_{2,t}{}^{\mathrm{E}}, m_{2,t}{}^{\mathrm{D}}) \equiv 0.5[(1\text{-}\theta)\{(p\text{-}pp^*\text{+}p^*)v(\phi_t{}^{\mathrm{D}}m_{2,t}{}^{\mathrm{D}}) + pp^*v^*)$$

$$+\ \theta\{p^{*}v(\phi_t^{D}\overline{m}_{1,t}^{D}) + (1-p^{*})p\ v(\phi_t^{D}\overline{m}_{2,t}^{D}) + pp^{*}v^{*}\}], \qquad (9b)$$

the value function for the agent in region i is written as:

$$V_i(m_{i,t}^{E}, m_{i,t}^{D}) = \text{constant} + v_i(m_{i,t}^{E}, m_{i,t}^{D}) + \phi_t^{E}\ m_{i,t}^{E} + \phi_t^{D}\ m_{i,t}^{D},$$
$$+ \max\ \{-\phi_t^{E}\ m_{i,t+1}^{E} - \phi_t^{D}\ m_{i,t+1}^{D} + \beta\ V_i(m_{i,t+1}^{E}, m_{i,t+1}^{D})\}. \qquad (10)$$

Here, $v_2(m_{2,t}^{E}, m_{2,t}^{D}) = v^{a}{}_2(m_{2,t}^{E}, m_{2,t}^{D})$ when the CIA is binding and $= v^{b}{}_2(m_{2,t}^{E}, m_{2,t}^{D})$ when it is not binding. It then holds that:

$$V_i(m_{i,0}^{E}, m_{i,0}^{D})$$
$$= max_{m_{t+1}} \sum_{t=0}^{\infty} \beta^{t}\{\text{constant} + v_i(m_{i,t}^{E}, m_{i,t}^{D}) + \phi_t^{E}(m_{i,t}^{E} - m_{i,t+1}^{E}) + \phi_t^{D}(m_{i,t}^{D} - m_{i,t+1}^{D})\}. \qquad (11)$$

Because function v_i satisfies the Inada conditions, the first-order conditions of dynamic programming are as follows:

$$-\phi_t^{E} + \beta\{\partial v_i(m_{i,t+1}^{E}, m_{i,t+1}^{D})/\partial m_{i,t+1}^{E}\} + \beta\varphi_{t+1}^{E} \leq 0, \qquad (12)$$
$$-\phi_t^{D} + \beta\{\partial v_i(m_{i,t+1}^{E}, m_{i,t+1}^{D})/\partial m_{i,t+1}^{D}\} + \beta\varphi_{t+1}^{D} \leq 0, \qquad (13)$$

Here, each inequality holds strictly if and only if it is a corner solution. Assuming that $\phi_t^{D}/\phi_{t+1}^{D} > \beta$, we obtain the following:

$$\beta\phi_{t+1}^{E}[1 + 0.5p(1-\theta)v'(\phi_t^{E}m_{1,t+1}^{E})] = \phi_t^{E}, \qquad (14)$$
$$\beta\phi_{t+1}^{D}[1 + 0.5p^{*}(1-\theta)v'(\phi_t^{D}m_{1,t+1}^{D})] = \phi_t^{D}, \qquad (15)$$

$$m_{2,t+1}{}^{\mathrm{E}} = 0, \tag{16}$$

$$\beta\phi_{t+1}{}^{\mathrm{D}}[1 + 0.5(1-\theta)\{(p-pp^*+p^*)v'(\phi_t{}^{\mathrm{D}}m_{2,t+1}{}^{\mathrm{D}}) + pp^*v'(\phi_t{}^{\mathrm{D}}(m_{2,t}{}^{\mathrm{D}} + \overline{m}_{1,t+1}{}^{\mathrm{D}}))\}] = \phi_t{}^{\mathrm{D}}, \tag{17a}$$

$$\beta\phi_{t+1}{}^{\mathrm{D}}[1 + 0.5(1-\theta)\{(p-pp^*+p^*)v'(\phi_t{}^{\mathrm{D}}m_{2,t+1}{}^{\mathrm{D}})\}] = \phi_t{}^{\mathrm{D}}. \tag{17b}$$

Here, Equation (17a) holds when the CIA is binding and (17b) holds when it is not binding.

The above equations are the money demand functions of the EU and US agents. Because $v'' < 0$, the money demand is decreasing in the nominal interest rate, which is equal to $(1/\beta)(\phi_t{}^{\mathrm{E}}/\phi_{t+1}{}^{\mathrm{E}})$ for the Euro and $(1/\beta)(\phi_t{}^{\mathrm{D}}/\phi_{t+1}{}^{\mathrm{D}})$ for the US dollar in our model. It is also increasing in the matching probability p or p^* and the buyer's bargaining power $1-\theta$.

6. The steady-state equilibrium

In the steady state, where $\phi_t{}^{\mathrm{E}}/\phi_{t+1}{}^{\mathrm{E}} = \phi_t{}^{\mathrm{D}}/\phi_{t+1}{}^{\mathrm{D}} = \pi > \beta$, Equations (14), (15), (16), (17a), and (17b) lead to the steady-state equilibrium money holdings of the EU and US agents as follows:

$$v'(\phi^{\mathrm{E}}m_1{}^{\mathrm{E}}) = 2\{1-(\beta/\pi)\}/\{p(1-\theta)\}, \tag{18}$$

$$v'(\phi^{\mathrm{D}}m_1{}^{\mathrm{D}}) = 2\{1-(\beta/\pi)\}/\{p^*(1-\theta)\}, \tag{19}$$

$$m_2{}^{\mathrm{E}} = 0, \tag{20}$$

$$\{(p^*/p)-p^*+1)\}v'(\phi^{\mathrm{D}}m_2{}^{\mathrm{D}}) + p^*\, v'(\phi^{\mathrm{D}}(m_2{}^{\mathrm{D}} + \overline{m}_1{}^{\mathrm{D}})) = 2\{1-(\beta/\pi)\}/\{p(1-\theta)\}, \tag{21a}$$

$$\{(p^*/p)-p^*+1)\}v'(\phi^{\mathrm{D}}m_2{}^{\mathrm{D}}) = 2\{1-(\beta/\pi)\}/\{p(1-\theta)\}, \tag{21b}$$

where $\phi^{\mathrm{E}}m_1{}^{\mathrm{E}}$, $\phi^{\mathrm{D}}m_1{}^{\mathrm{D}}$, $\phi^{\mathrm{E}}m_2{}^{\mathrm{E}}$, $\phi^{\mathrm{D}}m_2{}^{\mathrm{D}}$, and $\phi^{\mathrm{D}}(m_2{}^{\mathrm{D}} + \overline{m}_1{}^{\mathrm{D}})$ are the steady-state values of $\phi_t{}^{\mathrm{D}}m_{1,t}{}^{\mathrm{D}}$, $\phi_t{}^{\mathrm{E}}m_{1,t}{}^{\mathrm{E}}$, $\phi_t{}^{\mathrm{E}}m_{2,t}{}^{\mathrm{E}}$, $\phi_t{}^{\mathrm{D}}m_{2,t}{}^{\mathrm{D}}$, and $\phi_t{}^{\mathrm{D}}(m_{2,t}{}^{\mathrm{D}} + \overline{m}_{1,t}{}^{\mathrm{D}})$, respectively. Therefore, we obtain the following proposition.

Proposition 3: The steady-state equilibrium money holdings satisfy the following:

$$\phi^D(m_2^D + \overline{m}_1^D) > \phi^D m_2^D > \phi^E m_1^E > \phi^D m_1^D > \phi^E m_2^E = 0. \tag{22}$$

Proof: Because $v'' < 0$ and $p > p^*$, it is easy to see that $v'(\phi^D(m_2^D + \overline{m}_1^D)) < v'(\phi^D m_2^D)$ and $v'(\phi^E m_1^E) < v'(\phi^D m_1^D)$. From Equations (18), (21a), and (21b), it holds that $v'(\phi^E m_1^E) = \{(p^*/p)-p^*+1)\}v'(\phi^D m_2^D) + p^* v'(\phi^D(m_2^D + \overline{m}_1^D))$ when the CIA is binding and $v'(\phi^E m_1^E) = \{(p^*/p)-p^*+1)\}v'(\phi^D m_2^D)$ when it is not binding. Because $1 > p > p^*$, they imply that $v'(\phi^E m_2^E) > v'(\phi^D m_1^D)$. Therefore, it holds that $v'(\phi^D(m_2^D + \overline{m}_1^D)) < v'(\phi^D m_2^D) < v'(\phi^E m_1^E) < v'(\phi^D m_1^D)$. Because $v'' < 0$ and $m_{2,t+1}^E = 0$, we obtain the proposition. [Q.E.D.]

Proposition 3 has an important implication when comparing EU and US agents' expected discounted utilities in the steady-state equilibrium. Note that $\phi^E m_1^E = \phi^E \overline{m}_1^E$, $\phi^D m_1^D = \phi^D \overline{m}_1^D$, $\phi^E m_2^E = \phi^E \overline{m}_2^E$, and $\phi^D m_2^D = \phi^D \overline{m}_2^D$ in the steady-state equilibrium. Then, in the steady-state equilibrium, it holds that

$$V_1(m_{1,t}^E, m_{1,t}^D) \equiv [0.5/\{1-(\beta/\pi)\}][pv(\phi^E m_1^E) + p^*(1-\theta)v(\phi^D m_1^D) + p^*\theta\, v(\phi^D m_2^D)], \tag{23}$$

$$V^a_2(m_{2,t}^E, m_{2,t}^D) \equiv [0.5/\{1-(\beta/\pi)\}]\, [p^*\theta v(\phi^D m_1^D) + \{(p-pp^*+p^*)(1-\theta)+(1-p^*)p\theta\} v(\phi^D m_2^D)$$
$$+ pp^* v(\phi^D(m_1^D+m_2^D))], \tag{24a}$$

$$V^b_2(m_{2,t}^E, m_{2,t}^D) \equiv [0.5/\{1-(\beta/\pi)\}]\, [p^*\theta v(\phi^D m_1^D) + \{(p-pp^*+p^*)(1-\theta)+(1-p^*)p\theta\} v(\phi^D m_2^D)$$
$$+ pp^* v^*], \tag{24b}$$

Here, $V_2(m_{2,t}^E, m_{2,t}^D) = V^a_2(m_{2,t}^E, m_{2,t}^D)$ when the CIA of US dollars is binding and $V_2(m_{2,t}^E, m_{2,t}^D) = V^b_2(m_{2,t}^E, m_{2,t}^D)$ when it is not binding. When the CIA of US dollars is binding, it holds that

$$(V_2(m_{2,t}^E, m_{2,t}^D) - V_1(m_1^E, m_1^D))/[0.5p/\{1-(\beta/\pi)\}]$$

$$= (1-\theta)[pp^*\{v(\phi^D(m_1^D+m_2^D)-v(\phi^E m_1^E)\} + p(1-p^*)\{v(\phi^D m_2^D) - v(\phi^E m_1^E)\}$$

$$+ p^*\{v(\phi^E m_2^D)-v(\phi^D m_1^D)\}]$$

$$+ \theta[pp^*\{v(\phi^D(m_1^D+m_2^D) -v(\phi^E m_1^E)\} + p(1- p^*)\{v(\phi^D m_2^D)-v(\phi^E m_1^E)\}$$

$$- p^*\{v(\phi^E m_2^D)-v(\phi^D m_1^D)\}]$$

$$= pp^*\{v(\phi^D(m_1^D+m_2^D)-v(\phi^E m_1^E)\} + p(1-p^*)\{v(\phi^D m_2^D) - v(\phi^E m_1^E)\}$$

$$+ p^* (1-2\theta)\{v(\phi^D m_2^D)-v(\phi^D m_1^D)\}. \qquad (25a)$$

Similarly, when the CIA is not binding, it holds that:

$$(V_2(m_{2,t}^E, m_{2,t}^D) - V_1(m_{1,t}^E, m_{1,t}^D))/[0.5p/\{1-(\beta/\pi)\}]$$

$$= pp^*\{v^*-v(\phi^E m_1^E)\} + p(1-p^*)\{v(\phi^D m_2^D) - v(\phi^E m_1^E)\}+ p^* (1-2\theta)\{v(\phi^D m_2^D)-v(\phi^D m_1^D)\}. \quad (25b)$$

Equations (24a) and (24b) lead to the following proposition.

<u>Proposition 4</u>: US agents have higher utility than EU agents in the steady state.

Proof: When θ is small, $1-2\theta > 0$. From Proposition 3, it holds that $\phi^D m_2^D > \phi^E m_1^E$ and $\phi^D m_2^D > \phi^D m_1^D$. Because $v'(\phi m) > 0$ when $\phi m < \phi m^*$ and $v^* > v(\phi m)$ for all ϕm, Equations (24a) and (24b) show that $V_2(m_{2,t}^E, m_{2,t}^D) > V_1(m_{1,t}^E, m_{1,t}^D)$. This implies that US agents have higher utility than EU agents in the steady state. [Q.E.D.]

The money market equilibrium is determined by equalizing the supply and demand of the

Euro and the US dollar. In our model, each government supplies a fixed quantity of the Euro $\overline{M_t}^E$ and the US dollar $\overline{M_t}^D$, respectively. If the total population is one in both the EU and the US, we obtain the following money market equilibrium in the steady state:

$$\phi^{E}\overline{M^E} = \phi^{E}m_1^{E} + \phi^{E}m_2^{E}. \tag{26}$$

$$\phi^{D}\overline{M^D} = \phi^{D}m_1^{D} + \phi^{D}m_2^{D}. \tag{27}$$

Here, $\phi^{E}\overline{M^E}$ and $\phi^{D}\overline{M^D}$ are the steady-state equilibria of $\phi_{,t}^{E}\overline{M_t}^E$ and $\phi_{,t}^{D}\overline{M_t}^D$, respectively.

Since Proposition 3 implies that $\phi^{E}m_1^{E} + \phi^{E}m_2^{E} < \phi^{D}m_1^{D} + \phi^{D}m_2^{D}$, it holds that $\phi^{E}\overline{M_t}^E < \phi^{D}\overline{M_t}^D$. Given $\overline{M_t}^E = \overline{M_t}^D$, we can see that $\phi_{,t}^{E} < \phi_{t}^{D}$. This indicates that the US dollar has a larger purchasing power than the Euro in the steady state.

7. Concluding remarks

In this paper, we presented a new theory that may explain why the US dollar is the only medium of exchange in international transactions. Unlike previous studies, we applied a model in which economic geography affects the choice of international currency. The model is based on random matching, in which agents trade with foreign agents using a specific currency. We showed that under reasonable conditions, the US dollar becomes the unique equilibrium international currency, even if each region is symmetric in all ways except their locations. We also showed that when the US dollar is used for international transactions, the expected discounted utility becomes higher in the US than in the EU in the steady-state equilibrium.

What was crucial in the paper is that the active time zone moves from the EU to the USA every day. Since dynamic programming is solved backward, this derived a sequence of causality whereby what US agents determined affects the decision of EU agents. Our results were obtained even though the two regions were symmetric in all ways except for their locations. This implies that the geographical features of the regions can explain why the US dollar remains a dominant medium of exchange in international transactions.

A number of studies have explored how geography affects economic activity (see Krugman, 1991; Fujita, Krugman, and Venables, 2001; and Fujita and Thisse, 2002; among others). However, most of them investigated the locational choices of economic agents, whereas few investigated how geography affects financial markets.[8] In particular, none of them focused on different time zones or explored how they affect the choice of international currency in the world economy. Thus, the contribution of this study to the literature is unique.

Appendix 1. Proof of $\phi_t^{\mathrm{D}}\overline{m}_{2,\mathrm{B}}^{\mathrm{D}} > \phi_t^{\mathrm{E}}\overline{m}_{2,\mathrm{B}}^{\mathrm{E}}$ in section 4

This appendix shows that when the two currencies has the same inflation rate, US agents always hold the US dollar more than the Euro at the beginning of Daytime II (that is, $\phi_t^{\mathrm{D}}\overline{m}_{2,\mathrm{B}}^{\mathrm{D}} > \phi_t^{\mathrm{E}}\overline{m}_{2,\mathrm{B}}^{\mathrm{E}}$). Since it is obvious that US agents hold a relatively large amount of US dollars when they are used for the transactions, we explore whether US agents hold more US dollars than Euros, even when the Euro is used for international transactions during Daytime II. When the Euro is used in a decentralized international market, the CIA is always binding for US agents. The expected utility of US agents at the beginning of the day is written as:

$$
\begin{aligned}
&\mathrm{V}_2(m_{2,t}^{\mathrm{E}}, m_{2,t}^{\mathrm{D}}) \\
&= 0.5[\theta p^* v(\phi_t^{\mathrm{E}}\overline{m}_{1,t}^{\mathrm{E}}) + (1-\theta)p\, v(\phi_t^{\mathrm{D}}m_{2,t}^{\mathrm{D}})] + 0.5[(1-\theta)p^* v(\phi_t^{\mathrm{E}}m_{2,t}^{\mathrm{E}}) + \theta p\, v(\phi_t^{\mathrm{D}}\overline{m}_{2,t}^{\mathrm{D}})] \\
&\quad + \mathrm{W}_2(m_{2,t}^{\mathrm{E}}, m_{2,t}^{\mathrm{D}}) \\
&= 0.5[(1-\theta)\{p\, v(\phi_t^{\mathrm{D}}m_{2,t}^{\mathrm{D}}) + p^* v(\phi_t^{\mathrm{E}}m_{2,t}^{\mathrm{E}})\} + \theta\{p^* v(\phi_t^{\mathrm{E}}\overline{m}_{1,t}^{\mathrm{E}}) + p\, v(\phi_t^{\mathrm{D}}\overline{m}_{2,t}^{\mathrm{D}})\}]
\end{aligned}
$$

[8] A study by Coeurdacier and Martin (2009) is one of the exceptions.

$+ W_2(m_{2,t}{}^{E}, m_{2,t}{}^{D})$. (A1)

When the two currencies have the same inflation rate, $\phi_t^E/\phi_{t+1}^E = \phi_t^D/\phi_{t+1}^D = \pi$. Thus, the first-order conditions of dynamic programming lead to

$$(\beta/\pi)\ [1 + 0.5p^*(1-\theta)\ v'(\phi_{t+1}^E m_{2,t+1}^E)] = 1, \tag{A2}$$

$$(\beta/\pi)\ [1+0.5p(1-\theta)\ v'(\phi_{t+1}^D m_{2,t+1}^D)] = 1. \tag{A3}$$

Equations (A2) and (A3) imply that $v'(\phi_t^E m_{2,t}^E) - v'(\phi_t^D m_{2,t}^D) = \{(p-p^*)/p^*\}\ v'(\phi_t^D m_{2,t}^D)$. This indicates that when $p > p^*$, $v'(\phi_t^E m_{2,t}^E) > v'(\phi_t^D m_{2,t}^D)$, or equivalently $\phi_t^E m_{2,t}^E < \phi_t^D m_{2,t}^D$. Since $\phi_t^D \overline{m}_{2,B}^D = \phi_t^D m_{2,t}^D$ and $\phi_t^E \overline{m}_{2,B}\,E = \phi_t^E m_{2,t}^E$, this implies that $\phi_t^D \overline{m}_{2,B}^D > \phi_t^E \overline{m}_{2,B}{}^E$ in Section 4.

Appendix 2. Proof of $\overline{m}_{1,B}^D > 0$ in section 4

This appendix shows that any EU buyer holds US dollars at the beginning of Daytime II (i.e., $\overline{m}_{1,B}^D > 0$ in the trembling-hand perfect Nash equilibrium).[9] It is obvious that $\overline{m}_{1,B}^D > 0$, when the US dollar is used during Daytime II. Thus, we explore whether $\overline{m}_{1,B}^D > 0$, even when the Euro is used for almost all the transactions that occur during Daytime II.

For simplicity, we consider the case in which EU sellers always use the Euro in the decentralized international market. However, assuming that $\varepsilon \rightarrow 0$, we consider the case where US sellers use the Euro with probability 1-ε, and the US dollar with probability ε in the decentralized international market. When the CIA is binding, the expected utility of EU agents at the beginning of the day is written as:

[9] The trembling-hand perfect Nash equilibrium is a natural refinement of Nash equilibrium due to Reinhard Selten. It is an equilibrium that takes the possibility of off-the-equilibrium play into account by assuming that the players may choose unintended strategies, albeit with negligible probability.

$V_1(m_{1,t}^{E}, m_{1,t}^{D})$

$= 0.5[p\theta\, v(\phi_t^{E}\overline{m}_{1,t}^{E}) + p^*(1-\theta)\{p(1-\varepsilon)v(\phi_t^{E}m_{1,t}^{E}+\overline{m}_{1,t}^{E}) + (1-p)\,(1-\varepsilon)v(\phi_t^{E}m_{1,t}^{E}) + \varepsilon\, v(\phi_t^{D}m_{1,t}^{D})\}]$

$+ 0.5[p(1-\theta)v(\phi_t^{E}m_{1,t}^{E}) + p^*\theta v(\phi_t\overline{m}_{2,t}^{E})] + W_1(m_{1,t}^{E}, m_{1,t}^{D})$

$= 0.5[(1-\theta)(p+ p^*(1-p)\,(1-\varepsilon))v(\phi_t^{E}m_{1,t}^{E}) +(1-\theta)\{pp^*(1-\varepsilon)v(\phi_t^{E}m_{1,t}^{E}+\overline{m}_{1,t}^{E}) + p^*\varepsilon\, v(\phi_t^{D}m_{1,t}^{D})\}$

$+ \theta\,\{pv(\phi_t^{E}\overline{m}_{1,t}^{E}) + p^*v(\phi_t\overline{m}_{2,t}^{E})] + W_1(m_{1,t}^{E}, \phi_t^{D}m_{1,t}^{D}).$ (A4)

Similarly, when the CIA is not binding during Daytime II, the expected utility of EU agents at the beginning of the day is written as:

$V_1(m_{1,t}^{E}, m_{1,t}^{D})$

$= 0.5[p\theta\, v(\phi_t^{E}\overline{m}_{1,t}^{E}) + p^*(1-\theta)\{p(1-\varepsilon)\, v^* + (1-p)\,(1-\varepsilon)\, v(\phi_t^{E}m_{1,t}^{E}) + \varepsilon\, v(\phi_t^{D}m_{1,t}^{D})\}]$

$+ 0.5[p(1-\theta)v(\phi_t^{E}m_{1,t}^{E}) + p^*\theta v(\phi_t\overline{m}_{2,t}^{E})] + W_1(m_{1,t}^{E}, m_{1,t}^{D})$

$= 0.5[(1-\theta)\{p+ p^*(1-p)\,(1-\varepsilon)\}v(\phi_t^{E}m_{1,t}^{E}) + (1-\theta)\,pp^*(1-\varepsilon)\, v^* + (1-\theta)\,p^*\varepsilon\, v(\phi_t^{D}m_{1,t}^{D})$

$+ \theta\,\{pv(\phi_t^{E}\overline{m}_{1,t}^{E}) + p^*v(\phi_t\overline{m}_{2,t}^{E})] + W_1(m_{1,t}^{E}, m_{1,t}^{D}).$ (A5)

Following the same procedure as in Section 5, maximizing the value function with respect to $\phi_t^{D}m_{1,t+1}^{D}$ leads to the following first-order condition:

$$\beta\phi_{t+1}^{D}[1 + 0.5(1-\theta)\,p^*\varepsilon\, v'(\phi_{t+1}^{D}m_{1,t+1}^{D})] = \phi_t^{D}. \quad \text{(A6)}$$

Note that $\phi_t^{D}/\phi_{t+1}^{D} > \beta$. Then, because v satisfies the Inada conditions, this indicates that $\phi_t^{D}m_{1,t}^{D} > 0$ for all t, even if $\varepsilon \to +0$. Since $\phi^{D}\overline{m}_{1,B}^{D} = \phi_t^{D}m_{1,t}^{D}$ in Section 4, this implies that $\overline{m}_{1,B}^{D} > 0$ in the trembling-hand perfect Nash equilibrium. This proves $\overline{m}_{1,B}^{D} > 0$ in Section 4.

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Table 1. Foreign Exchange Turnover by Currency

	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
USD	90	82	83	87	90	88	86	85	87	88	88
EUR	…	…	…	…	38	37	37	39	33	31	32
JPY	28	23	25	22	24	21	17	19	23	22	17
GBP	15	14	9	11	13	16	15	13	12	13	13
AUD	2	2	3	3	4	6	7	8	9	7	7
CAD	1	3	3	4	4	4	4	5	5	5	5
CHF	10	8	7	7	6	6	7	6	5	5	5
CNY	…	…	…	0	0	0	0	1	2	4	4
HKD	1	1	1	1	2	2	3	2	1	2	4
NZD	…	0	0	0	1	1	2	2	2	2	2
SEK	1	1	1	0	2	2	3	2	2	2	2
KRW	…	…	…	0	1	1	1	2	1	2	2
SGD	0	0	0	1	1	1	1	1	1	2	2
NOK	0	0	0	0	1	1	2	1	1	2	2
MXN	…	…	…	0	1	1	1	1	3	2	2
INR	…	…	…	0	0	0	1	1	1	1	2
RUB	…	…	…	0	0	1	1	1	2	1	1
ZAR	…	0	0	0	1	1	1	1	1	1	1
TRY	…	…	…	…	0	0	0	1	1	1	1
BRL	…	…	…	0	0	0	0	1	1	1	1
TWD	…	…	…	0	0	0	0	0	0	1	1
DKK	0	0	1	0	1	1	1	1	1	1	1
PLN	…	…	…	0	0	0	1	1	1	1	1
DEM	26	40	36	30	…	…	…	…	…	…	…
FRF	2	4	8	5	…	…	…	…	…	…	…
XEU	1	3	2	1	…	…	…	…	…	…	…
ITL	1	1	1	1	…	…	…	…	…	…	…
NLG	1	1	1	1	…	…	…	…	…	…	…
ESP	0	1	1	1	…	…	…	…	…	…	…
BEF	0	0	1	1	…	…	…	…	…	…	…
Other	19	13	16	21	7	5	8	5	2	2	2
Total	200	200	200	200	200	200	200	200	200	200	200

Note. Because two currencies are involved in each transaction, the sum of the percentage shares of individual currencies totals 200% instead of 100%.

Table 2. Currency Shares of Foreign Exchange Turnover by Country

	AUD	CHF	EUR	GBP	JPY	USD	OTH
Australia	49.4	0.8	13.3	6.0	11.1	91.7	0.3
Brazil	1.4	0.4	15.7	0.9	2.2	91.7	5.9
Canada	3.9	1.9	16.2	14.2	5.4	94.7	0.9
Chile	0.1	0.3	5.5	0.6	0.6	98.5	93.7
China	0.9	0.5	16.3	0.9	4.1	97.5	0.1
Colombia	0.0	0.0	6.5	1.4	0.5	98.8	91.6
Denmark	4.0	3.3	45.3	9.4	6.5	78.1	0.4
France	3.3	7.2	60.0	14.0	12.9	79.4	1.3
Germany	2.0	9.7	70.9	11.4	10.6	73.1	2.4
Hong Kong SAR	8.3	1.2	14.5	4.6	15.8	96.6	1.5
India	0.8	0.5	7.5	3.4	1.2	97.2	0.6
Indonesia	2.5	0.1	5.4	3.0	2.2	93.4	83.9
Ireland	1.0	3.9	73.0	18.6	1.3	84.1	0.5
Israel	0.0	0.1	7.2	0.6	0.1	91.4	100.0
Italy	1.9	6.3	75.6	11.1	5.3	82.0	1.5
Japan	7.3	1.1	19.9	6.9	77.8	76.1	0.6
Korea	1.2	0.6	5.6	1.4	3.1	92.8	0.2
Malaysia	5.6	0.4	8.5	7.5	3.5	95.8	63.3
Mexico	0.0	0.4	3.7	0.5	0.7	97.6	0.3
Netherlands	1.7	15.1	51.7	15.1	5.2	83.9	1.2
New Zealand	14.5	0.8	6.7	3.0	4.3	89.9	0.1
Norway	0.4	4.9	51.7	6.0	1.2	70.9	0.1
Peru	0.0	0.0	4.8	0.1	0.4	99.2	95.3
Philippines	1.8	0.1	6.1	1.7	6.0	97.6	80.5
Russia	0.3	1.3	35.8	2.4	1.0	86.7	0.2
Singapore	12.0	1.7	16.8	7.4	24.5	93.8	4.2
South Africa	0.9	0.6	15.7	14.9	0.9	93.4	0.1
Spain	2.0	10.2	57.3	15.6	4.8	82.9	3.4
Sweden	0.4	7.9	40.9	11.0	2.2	79.7	0.5
Switzerland	5.1	33.4	36.0	7.6	10.5	80.9	1.3
Thailand	0.6	0.2	8.0	2.3	5.5	93.9	86.4
Turkey	0.1	0.6	32.6	4.8	3.1	89.4	0.1
United Kingdom	6.5	4.9	36.0	16.6	15.4	89.5	2.3
United States	5.9	4.8	35.7	14.5	13.8	88.8	2.3
Total	6.9	5.1	32.4	12.5	17.0	88.7	3.1

Note. Because two currencies are involved in each transaction, the sum of the percentage shares of individual currencies totals 200% instead of 100%.

Figure 1. Four Time Zones and Traders' Activity in a Day

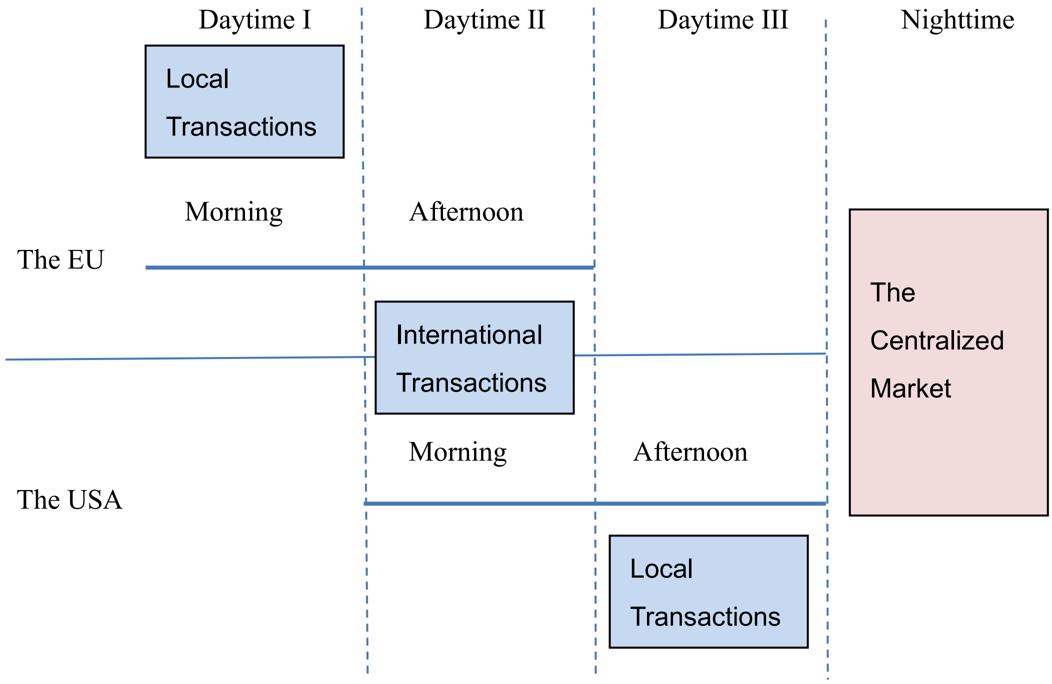

Figure 2. Randomization Patterns of Sellers and Buyers in a Day

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