

86-F-6

Wages, Work Intensity and Labor  
Hoarding under Uncertain Demand:  
A State-Contingent Recontracting Model

by

Herbert Gintis  
University of Massachusetts at Amherst

and

Tsuneo Ishikawa  
University of Tokyo

October 1986

PREPARED UNDER  
THE PROJECT ON LABOR AND INCOME DISTRIBUTION  
RESEARCH INSTITUTE FOR THE JAPANESE ECONOMY

\*Discussion Papers are a series of manuscripts in their draft form. They are not intended for circulation or distribution except as indicated by the author. For that reason Discussion Papers may not be reproduced or distributed without the written consent of the author.

October, 1986

Wages, Work Intensity and Labor  
Hoarding under Uncertain Demand:  
A State-Contingent Recontracting Model

by

Herbert Gintis and Tsuneo Ishikawa\*

Introduction

In this paper we shall extend the analysis of Shapiro-Stiglitz [1984] and Gintis and Ishikawa [forthcoming] of the relationship between wages, productivity and unemployment to a situation in which firms face stochastic fluctuations in aggregate demand. Firms offer a state-contingent contract consisting of wages, work discipline parameter(s) and layoff probabilities for different states of nature to workers and are supposed to maximize profits in the long-run. On the other hand, workers who accept the offer and are actually hired by the firm are supposed to decide ex post (namely, after the state of nature is revealed) how hard they work for that period. In this setting we will show that firms exhibit labor hoarding behavior in the sense that they will choose not to layoff workers (who are retained after possible disciplining) but rather let them work with low work intensity when demand for products is low. Thus labor productivity moves pro-cyclically, and we can also show that real wages move pro-cyclically as well.

While some of the important implications are similar with those of the so-called implicit contract models, this model clearly differs from them in that workers are ex post utility maximizers rather than expected utility maximizers. To be sure, the usual risk sharing arrangement could be

considered in the present model and thus alleviating the risk of layoff among workers. This is, however, not the source of labor hoarding feature in the present model. The point is that there is another element of risk for workers which employers deliberately choose not to be insurable. Indeed, it is precisely by imposing the risk of work discipline (i.e., being dismissed) for not working with enough intensity that firms are able to extract work effort from workers. The risk of being dismissed thus remaining, workers cannot afford simply to become expected utility maximizers.

Actual execution of worker dismissal (and thus preserving the risk to be real) is not uncostly to the firm. Workers may protest collectively and demand for a fair treatment (to be more specific, for workers who worked no less hard than those retained but nevertheless were fired). In fact, this has been the focal point of our previous analysis. The intuitive reason for our principal conclusion then appears naturally: By laying off workers, firms dilute the effectiveness of the costly work discipline.

The present model can then be looked at as an alternative formulation of Arthur Okun's notion of the customer's market for labor that does not rely on risk sharing incentives. Our version of the "customer's market", while it avoids layoffs, is not tranquil, but is rather characterized by much tension between employers and workers.

The following part of the paper is devoted to a mathematical demonstration of the property described above. Although the assumption chosen to specify the nature of stochastic fluctuations in the present model is the simplest that is imaginable we believe the main conclusion should carry over to a more general specification.

1. Wages, Discipline and Work Intensity: The Basic Recontracting Model

Suppose that the labor market is organized by a sequential option market, where each firm offers a contract consisting of wage,  $w$  (in output units) and a conditional option for renewal in the next period. In line with our previous paper the criterion for renewal is such that the proportion  $f$  ( $0 < f < 1$ ) of workers who are regarded by the employer as having supplied the least effort are not renewed (i.e., dismissed). Evaluation of individual work effort is governed by imperfect monitoring, with inexactness of observation expressed by parameter,  $m$  (which can be decreased at a cost). This parameter is identified as the width of each tail of a triangular distribution according to which evaluation errors are supposed to occur. Both  $f$  and  $m$  are specified by the contract. Such a worker control environment naturally induces workers to compete (or emulate) with each other, but at the same time, it may generate the feeling of "extended sympathy" among workers, thus inviting collective protest over dismissed workers. In fact, the model depicts individual workers as multifaceted existence, being subject to complex moral forces of defending and pursuing individual interests even vis-a-vis other workers, yet at the same time, willing to act collectively with other workers when their notion of fairness is encroached. Explicit incorporation of such multifacetedness into theory is a necessary procedure whenever we view workers' collective forces, such as trade unions or spontaneous "wild-cat" activities, as something endogenously created (therefore subject to expansion or erosion) rather than exogenously given. Traditional collective bargaining theories (Nash [1950], Hicks [1963]) seem to lack this essential quality, for they take collective behavior as something exogenously given and maintained.<sup>1</sup>

We shall assume a homogeneous work force and a uniform wage, and adopt

a discrete time framework. Each worker, once hired for a period, receives the wage at the end of the period, and at that moment is determined whether or not his/her contract is renewed in the next period. If we denote by  $d$  the probability of dismissal (i.e., non-renewal) for an individual worker and denote by  $V_u$  the present utility value of not having a contract in this firm, then the present utility value  $V_e$  of having a contract in this firm, both  $V_u$  and  $V_e$  evaluated in the beginning of the period under a stationary circumstance, is expressed by:

$$V_e = \frac{u(w, i) + (1 - d) \cdot V_e + d \cdot V_u}{1 + r} \quad (1)$$

where  $r$  is the constant rate of discount and  $u(w, i)$  is the current period utility flow of the wage and the work intensity  $i$ , with  $u_w > 0$ , and  $u_{ii} < 0$ . In order for workers to accept the contract offer  $V_e$  must be no less than  $V_u$ , and this condition is easily verified from (1) to be equivalent to  $u(w, i) > rV_u$ . We therefore call  $rV_u$  the reservation utility. It is an exogenous variable at the level of individual firms. The individual dismissal probability  $d$  naturally becomes a function of  $f$  and  $m$ , which are given by the employer in the contract, and of how far the work intensity of an individual,  $i$ , deviates from that of the worker group average (denoted by  $i_{gr}$ ). Each worker tries to maximize  $V_e$  given the terms of the contract ( $w, f, m$ ), the reservation utility  $rV_u$  and the group average work intensity  $i_{gr}$  (assumed to be uncontrollable directly by an individual). In the previous paper (Gintis-Ishikawa [forthcoming]) we have proved under the assumed monitoring technology the existence of a unique symmetric Nash equilibrium where each worker's (true) work intensity,  $i$ , equaling the group average ( $i_{gr}$ ), is determined by the condition:

$$\frac{(u(w, i) - rV_u)}{-u_i(w, i)} = \frac{m}{\phi} (r+f) \equiv m \cdot a(f) \quad (2)$$

where  $\phi = \sqrt{2f}$  and  $f$  is assumed to be positive.<sup>2</sup> The term  $(m/\phi)$  expresses the reciprocal of the marginal decrease in the individual dismissal probability ( $d$ ) as a result of increasing the work intensity a little, evaluated at the symmetric group effort equilibrium. By substituting (2) into (1), noting the fact that  $d = f$  ex post, we obtain an alternative expression for (2) which admits more direct interpretation:

$$V_e - V_u = \frac{m}{\phi} (-u_i(w, i)) \quad (2')$$

Namely, workers determine their work intensity such that the marginal utility gain of working a little harder defined by the marginal increase in the renewal rate (= marginal decrease in the dismissal rate) times the net benefit of having a job in this firm ( $V_e - V_u$ ) equals the marginal current utility loss due to the same increased work intensity. Thus the positive wedge between  $V_e$  and  $V_u$  lies at the heart of inducing workers to work more than what they would voluntarily do, namely either a positive  $i$  such that  $u_i(w, i) = 0$  or  $i = 0$  in case  $u_i(w, i) < 0$  for all  $i > 0$ . Although this essential property is common with the models of Calvo [1979], Shapiro-Stiglitz [1974] and Bowles [1975] our present formulation goes beyond their models in specifying the manner by which the level of work intensity is endogenously determined.

The newly introduced function  $a(f)$  in (2) has the property  $a'(f) = -(r-f)/\phi^3 < 0$  and  $a''(f) = (3r-f)/\phi^5 > 0$  under an assumption  $f < r$ , which is in itself justified later by the firm behavior. From this condition it is easy to observe that the work intensity is increasing with respect to the dismissal rate ( $f$ ) and the monitoring intensity ( $1/m$ ), and decreasing with respect to the reservation utility,  $rV_u$ . (The direction of the effect of an increase in the wage level,  $w$ , is, in general, not determinate, but the

first order effect is certainly positive, as expected. See expression (6) below.)

The profit maximization for the firm involves balancing of the gains from a higher work intensity and its cost. More specifically, it involves minimization of the unit cost per work intensity

$$e = \{w + c(i)f + s(m)\}/i$$

subject to the "production function" of the work intensity implicitly defined by (2). This is a common property of all the so-called efficiency wage models. In this expression,  $c(i)$  represents the frictional cost of dismissals (per number of dismissed workers) incurred by the employer. As described in the introduction, when the workers realize ex post that everyone worked with similar intensity (which is known among fellow workers but is not known to the employer, due to monitoring errors) and yet some workers got dismissed they may protest collectively leading to overt or covert labor disputes. It is supposed that  $c(i)$  is a strictly convex function of  $i$ , embodying the idea that the workers' sense of fairness is more strongly invoked the harder they work. Similarly  $s(m)$  expresses the monitoring cost per worker with  $s'(m) < 0$  and  $s''(m) > 0$ . Under a linearly approximated utility function  $u(w, i) = w - bi$  ( $b > 0$ ), the necessary first-order conditions for profit maximization (corner solutions avoided) are given by:

$$w - bi = (c'(i)i - c(i))f - s(m) \tag{3}$$

$$c(i) + bma'(f) = 0 \tag{4}$$

$$s'(m) + ba(f) = 0 \tag{5}$$

$$w - bi - rV_u = bma(f) \tag{6},$$

where (6) is a restatement of (2) under the linearly approximated utility function. The four equations (3)-(6), when the proper second order condi-

tions are satisfied, determine the terms of the contract  $(w, f, m)$  and the work intensity,  $i$ , for a given level of the reservation utility,  $rV_u$ . From (4) the contract must satisfy  $a'(f) < 0$ , implying  $f < r$  and justifying the assumption made above. Depending on how we specify the generation of the reservation utility  $rV_u$  in the macro economy we obtain as one of possible states an involuntary unemployment equilibrium or a dualistic labor market with a Walrasian secondary market characterized by distinct wage differentials (See Yellen [1984] for a survey of macroeconomic implications). The case of involuntary unemployment, together with other possible macroeconomic states, is discussed in full in Gintis-Ishikawa [forthcoming].

## 2. A State-Contingent Recontracting Model under Uncertain Demand

We now modify the basic model to take account of the feature of uncertain demand for the firm's product. First, we shall suppose that the technology of production is such that output is proportional to work intensity-adjusted labor input

$$Q = \gamma \cdot iL \tag{7}$$

and that the output level of the firm is determined by demand, which, in turn, is given as a fixed share of the aggregate demand. Second, the aggregate demand is subject to the simplest possible recurrent stochastic shocks; namely, there are two states of nature,  $s = g$  (good) and  $s = b$  (bad), that occur serially independently with probabilities  $p$  and  $1-p$  (i.e., the Bernoulli process). The aggregate demand for a bad state is reduced by  $100q$  per cent from that for a good state. The nature of this stochastic shock is assumed to be understood by everyone in the economy.

The immediate consequence of this modification is that we must now allow for possible differences in values of each variable for each state.



In particular, we are interested in the possible fluctuations in the volume of employment between different states. Let us denote the value of each variable introduced in the previous section for each state by adding a superscript  $g$  or  $b$ , e.g.,  $w^g, w^b$ . Also denote each firm's volume of employment by  $L^g$  and  $L^b$ . Then it follows from the above that

$$\gamma \cdot i^b L^b = (1-q) \gamma \cdot i^g L^g \quad (8).$$

We will normalize the level of output for  $s = g$ ,  $\gamma \cdot i^g L^g$ , as unity.

A more substantive change introduced by the above modification concerns the behavior of the employer as well as that of the worker. As to the employer, we assume that each employer offers a package of a state-contingent contract  $(w^g, w^b, m^g, m^b, f^g, f^b, \theta^{gb})$  to the worker and maximizes the long-run expected profit, whose details are to be specified in the next section. Here we have introduced an important new variable  $\theta^{gb}$  ( $0 < \theta^{gb} < 1$ ) which expresses the rate of non-disciplinary layoff for currently employed workers in state  $g$  when the state turns from good to bad.<sup>3</sup>

For the worker, he/she is in one of two by two states in the beginning of each period; either being employed in  $s=g$  or  $s=b$ , or being unemployed in  $s=g$  or  $s=b$ . The present value of being employed in  $s=g$  and that of being in  $s=b$  are respectively expressed (under a stochastic steady state) as:

$$\begin{aligned} v_e^g = \frac{1}{1+r} & \left( u(w^g, i^g) + p \{ (1 - d^g) v_e^g + d^g v_u^g \} \right. \\ & \left. + (1-p) \{ (1 - d^g - \theta^{gb}) v_e^b + (d^g + \theta^{gb}) v_u^b \} \right) \quad (9) \end{aligned}$$

$$\begin{aligned} v_e^b = \frac{1}{1+r} & \left( u(w^b, i^b) + p \{ (1 - d^b) v_e^g + d^b v_u^g \} \right. \\ & \left. + (1-p) \{ (1 - d^b) v_e^b + d^b v_u^b \} \right) \quad (10). \end{aligned}$$

The meaning of these expressions must be evident. An employed worker

receives the current flow of utility (at the end of the period) plus the present value of being left at either of the four possible states in the beginning of the next period multiplied by respective probabilities.

Clearly these expressions form a simultaneous equations system in  $v_e^g$  and  $v_e^b$ . In a matrix notation they become:

$$\begin{vmatrix} (1+r)-pd^{g*} & -p*(d^{g*}-\theta^{gb}) \\ -pd^{b*} & (1+r)-p*d^{b*} \end{vmatrix} \begin{vmatrix} v_e^g \\ v_e^b \end{vmatrix} = \begin{vmatrix} u(w^g, i^g) + d^g Ev_u + p*\theta^{gb} v_u^b \\ u(w^b, i^b) + d^b Ev_u \end{vmatrix} \quad (11),$$

where  $Ev_u \equiv pv_u^g + (1-p)v_u^b$  and a short-hand convention of  $x^* \equiv 1 - x$  for each variable (therefore  $p^* = 1-p$ ,  $d^{g*} = 1 - d^g$  and so forth) is used.<sup>4</sup> The values of  $v_u^g$  and  $v_u^b$  (and hence,  $Ev_u$ ) are assumed to be given exogeneously within the confines of individual experiments about which this paper is mainly concerned. They are, however, assumed to satisfy the inequality  $v_u^g > v_u^b$ . (This reflects a macroscopically higher current expected utility flow for the unemployed for  $s=g$  than for  $s=b$ . See the discussion at the end of Section 4.)

As we already explained in the introduction, the employed workers, having observed the state of nature, act as ex post maximizers. Hence in state  $g$ , workers determine their work intensity  $i^g$  to maximize  $v_e^g$  under a certain expected level of  $i^b$  which is historically formed, and mutatis mutandis for state  $b$ . By solving (11) for  $v_e^g$  and  $v_e^b$  and by partial differentiation, we obtain the following first-order conditions for each state.

$$\frac{p \cdot u(w^g, i^g) + p \cdot k^{gb} \cdot u(w^b, i^b) - (r - p \cdot (k^{gb} - 1)) Ev_u + pp \cdot \theta^{gb} v_u^b}{-u_i(w^g, i^g)} = \frac{m^g}{\phi^g} A(f^g, f^b, k^{gb}) \quad \text{for given } i^b \quad (\text{at } s = g) \quad (12)$$

$$\frac{p \cdot u(w^g, i^g) + p^* k^{gb} \cdot u(w^b, i^b) - (r - p^*(k^{gb} - 1)) E v_u + p p^* \theta^{gb} v_u^b}{-u_i(w^b, i^b)}$$

$$= \frac{m^b}{\phi^b} A(f^g, f^b, k^{gb}) \quad \text{for given } i^g \quad (\text{at } s = b) \quad (13)$$

where

$$A(f^g, f^b, k^{gb}) \equiv (1+r) - p f^{g*} - p^* k^{gb} f^{b*}$$

$$k^{gb} \equiv 1 - \frac{p \theta^{gb}}{1+r}$$

It should be clear that (12) and (13) are immediate generalization of the formula (2) (or [2']) in the basic model. Thus the marginal expected gains from working a little harder for each state (given the work intensity of the other state), which is now defined by the expected marginal increase in the renewal rate (taking account of the possibility of layoff when the state turns from good to bad) times the expected net wealth advantage of having a job in the next period, just equals the marginal current utility loss due to an increased work intensity.

By combining (12) and (13), and noting that the numerators of the LHS of these equations as well as the term  $A(f^g, f^b, k^{gb})$  are identical, we obtain the relationship:

$$\frac{m^g}{\phi^g} u_i(w^g, i^g) = \frac{m^b}{\phi^b} u_i(w^b, i^b) \quad (14)$$

Recalling the interpretation of the expression  $(m/\phi)$  in the basic model, (14) means that the marginal current utility loss due to an increased effort per marginal increase in the renewal probability for both states must equal. The interpretation becomes much clearer if we divide both sides by  $E v_e$ , say, defined by  $p \cdot v_e^g + p^* \cdot v_e^b$ . Because of serial independence, the unconditional expected present value of having a job in the next period as of today

does not depend on the current state of the economy. Then (14) states that the marginal rate of substitution between the current utility flow and the expected future present utility value as governed by the choice of work intensity must be the same between the two states. This condition must necessarily be satisfied in the workers' expectational equilibrium, that is, an equilibrium such that each worker's decisions of  $i^g$  and  $i^b$  are mutually consistent and does not lead to any change in the behavior.

Since the existence of a stochastic stationary firm equilibrium obviously requires workers' expectational equilibrium, the conditions (12) and (13), or equivalently, (13) and (14), provide two independent constraints to the firm's profit maximization behavior, to which we now turn. In what follows we shall approximate the current utility function by a linear function  $u(w,i) = w - b \cdot i$ , just as in the case of the basic model. In this case the condition (14) reduces to:

$$(m^g/\phi^g) = (m^b/\phi^b) \quad (14')$$

while the condition (13) becomes

$$\begin{aligned} p(w^g - bi^g) + p^*k^{gb}(w^b - bi^b) - (r - p^*(k^{gb}-1))Ev_u + pp^*\theta^{gb}v_u^b \\ = \frac{m^b}{\phi^b} b \cdot A(f^g, f^b, k^{gb}) \end{aligned} \quad (13')$$

We shall consider (13') and (14') as the constraints imposed on the firm.<sup>5</sup>

### 3. Labor Hoarding in Firm Equilibrium

The employer maximizes the expected profit between the two states subject to the constraints (13') and (14'). The expected profit has an expression (utilizing the normalization on output introduced above):

$$E\pi = p\left(1 - \frac{1}{\gamma}e^g\right) + p^*(1-q)\left(1 - \frac{1}{\gamma}e^b\right) \quad (15)$$

where

$$e^j = \frac{w^j + c(i^j)f^j + s(m^j)}{i^j} \quad (j = g, b) \quad (16).$$

By forming a Lagrangian

$$\begin{aligned} L(w^g, w^b, i^g, i^b, m^g, m^b, f^g, f^b, \theta^{gb}) \\ \equiv E\pi - \lambda \{ p(w^g - bi^g) + p^*k^{gb}(w^b - bi^b) - (m^b/\phi^b)bA(f^g, f^b, k^{gb}) \\ - (r - p^*(k^{gb} - 1))Ev_u + pp^*\theta^{gb}v_u^b \} \\ - \mu \{ (m^g/\phi^g) - (m^b/\phi^b) \} \end{aligned} \quad (17)$$

we set  $L_w$  through  $L_f$  equal to zero and set  $L_{\theta^{gb}} < 0$ ,  $\theta^{gb} > 0$ . After eliminating the Lagrangian parameters  $\lambda$  and  $\mu$  and after rearrangement we obtain:

$$\frac{i^b}{i^g} = \frac{1 - q}{k^{gb}} \quad (18)$$

$$w^b - bi^b = \{c'(i^b)i^b - c(i^b)\}f^b - s(m^b) \quad (19)$$

$$w^g - bi^g = \{c'(i^g)i^g - c(i^g)\}f^g - s(m^g) \quad (20)$$

$$-\{ps'(m^g)\phi^g + p^*k^{gb}s'(m^b)\phi^b\} = bA(f^g, f^b, k^{gb}) \quad (21)$$

$$c(i^g) + \frac{bm^g}{\phi^g} = - \frac{s'(m^g)m^g}{2f^g} \quad (22)$$

$$pf^g c(i^g) + p^*k^{gb}f^b c(i^b) = \frac{bm^g}{\phi^g} \left( \frac{(1+r) - p(1+f^g) - p^*k^{gb}(1+f^b)}{2} \right) \quad (23)$$

$$\begin{aligned} (w^b - bi^b) + b(m^b/\phi^b)f^{b*} + Ev_u - (1+r)v_u^b > 0, \quad \theta^{gb} > 0 \text{ and} \\ \{(w^b - bi^b) + b(m^g/\phi^g)f^{b*} + Ev_u - (1+r)v_u^b\} \theta^{gb} = 0 \end{aligned} \quad (24).$$

The conditions (18) through (24) and the constraints (13') and (14') determine nine variables  $(w^g, w^b, i^g, i^b, m^g, m^b, f^g, f^b, \theta^{gb})$  for given  $v_u^g$  and  $v_u^b$ . It should be evident that (19) and (20) correspond to (3), that (21) corresponds to (5) and that (23) corresponds to (4), respectively, in the basic model. (22) regulates the margins by which the firm balances the monitoring and dismissal policies. We should also note that  $\theta^{gb}$  does not appear at all in the first inequality in (24) (as derived from  $L_{\theta^{gb}} < 0$ ).

In fact, the Lagrangian is linear with respect to  $\theta^{gb}$ , and hence, there is a possibility that the optimum lies at the corner of  $\theta^{gb} = 0$ .

This possibility of a corner solution, we call the case of complete labor hoarding. Indeed, if  $\theta^{gb} = 0$ ,  $k^{gb}$  becomes unity, and, from (18),  $i^b = (1-q)i^g$ . By recalling (8), this implies that  $L^g = L^b$ . In other words, downward aggregate demand shock is totally absorbed by the decline in work intensity (or labor productivity) and there is no change in the volume of employment.<sup>6</sup>

The question then becomes if complete labor hoarding ever occurs. We can then state our main proposition:

Proposition 1.

Complete labor hoarding arises if and only if the workers' reaction to layoffs, other terms of the contract remaining intact, is such that

$$\partial i^g / \partial \theta^{gb} \text{ and } \partial i^b / \partial \theta^{gb} < 0 \quad (25).$$

A sufficient (but by no means necessary) condition for this is:

$$\frac{v_u^g - v_u^b}{v_u^b} > \frac{r}{p} \quad (26).$$

The proof of this proposition is immediate by differentiating (13') with respect to  $\theta^{gb}$ , and confirming that both  $\partial i^g / \partial \theta^{gb}$  and  $\partial i^b / \partial \theta^{gb}$  always have the same sign as  $L_{\theta^{gb}}$ . (See the first inequality relationship in (24) for the expression of  $L_{\theta^{gb}}$ ). The expression (26) immediately follows from the relation  $E v_u - (1+r)v_u^b > 0$ . Since the first two terms of  $L_{\theta^{gb}}$  is positive, we understand this to be a very weak sufficient condition.<sup>7</sup>

The economic meaning of this proposition must be clear. Complete labor hoarding occurs if and only if the act of layoff dilutes the effectiveness of the costly monitoring and disciplining, and hence, discourages work intensity. And this confirms the intuitive discussion given in the introduction.

4. Complete and Partial Hoarding with Procyclical Real Wages under Invariant Monitoring Intensities Across States

Aside from the proportional difference in  $i^g$  and  $i^b$  under complete hoarding, the relative magnitudes of wages and the worker control variables ( $m$  and  $f$ ) across states are not immediately clear. Some clearer ideas can be obtained if we additionally suppose that the monitoring intensity ( $1/m$ ) is constrained not to vary across states. Such a case occurs if the monitoring technology is deeply rooted in the organizational structure of the firm such as management hierarchy that is not easily varied in the short-run.

When  $m^g = m^b$ , the conditions for a firm equilibrium are considerably simplified. An immediate consequence from (14') is  $f^g = f^b$ , meaning that the dismissal rate is also the same across states. We shall denote these common values as  $m$  and  $f$ , respectively, just as in the basic model. The simplified conditions are then given by:

$$p(w^g - bi^g) + p^*k^{gb}(w^b - bi^b) - (r - p^*(k^{gb}-1))Ev_u + pp^*\theta^{gb}v_u^b$$

$$= \frac{m}{\phi} b \cdot A(f, k^{gb}) \quad (\text{where } A(f, k^{gb}) \equiv (1+r) - (p+p^*k^{gb})f^*) \quad (13'')$$

$$\frac{i^b}{i^g} = \frac{1 - q}{k^{gb}} \quad (18'')$$

$$w^b - bi^b = \{c'(i^b)i^b - c(i^b)\}f - s(m) \quad (19'')$$

$$w^g - bi^g = \{c'(i^g)i^g - c(i^g)\}f - s(m) \quad (20'')$$

$$-(p + p^*k^{gb})s'(m) = \frac{1}{\phi} bA(f, k^{gb}) \quad (21'')$$

$$pc(i^g) + p^*k^{gb}c(i^b) = \frac{mb}{\phi} \left( \frac{(1+r) - (p+p^*k^{gb})(1+f)}{2f} \right) \quad (23'')$$

$$(w^b - bi^b) + b(m/\phi)f^* + Ev_u - (1+r)v_u^b > 0, \quad \theta^{gb} > 0 \text{ and}$$

$$\{(w^b - bi^b) + b(m/\phi)f^* + Ev_u - (1+r)v_u^b\}\theta^{gb} = 0 \quad (24'').$$

These conditions are far easier to interpret than the general case.

We can then derive the following property.

Proposition 2. Under the assumption of  $v_u^g > v_u^b$ , both the wage and the work intensity are higher in the good state than in the bad state, i.e.,

$$w^g > w^b \quad (27)$$

$$i^g > i^b \quad (28)$$

It is important to note that this property is valid independent of whether or not the case of complete hoarding, as discussed in the previous proposition, holds. Since (28) implies that the decline in work intensity absorbs (at least) a part of the demand shock, thus alleviating the impact on the size of employment, we shall hereafter refer to the circumstance where complete hoarding does not occur as the case of partial labor hoarding. (27) then says that the economy exhibits pro-cyclical real wages no matter whether labor hoarding is complete or not.<sup>8</sup>

The proof of this proposition is immediate for the case of complete hoarding, namely for the case when the first inequality in (24'') holds with strict inequality, implying  $\theta^{gb} = 0$ . We have already seen that  $i^b/i^g = 1-q$ . (27) results by comparing the expressions in (19'') and (20''), thereby recalling our assumption that  $c''(1) > 0$ .<sup>9</sup>

Now for the proof of the case where the first inequality in (24'') holds with equality and thus  $\theta^{gb} > 0$  (i.e., some layoffs occur). By multiplying both sides of this inequality (now an equation) by  $p^*k^{gb}$  and subtracting the resulting expression from (13''), and after rearrangement, we obtain:

$$w^g - bi^g = \frac{m}{\phi} b \frac{(1+r-pf^*)}{p} + rv_u^g + p^* \frac{(Ev_u - v_u^b)}{p} \quad (29)$$

Again by subtracting the LHS of the first inequality of (24'') (which is set



to zero) again from (29), we observe:

$$\begin{aligned} & (w^g - bi^g) - (w^b - bi^b) \\ &= \frac{m}{\phi} b \frac{(1+r)}{p} + r(v_u^g - v_u^b) + \frac{1}{p} (Ev_u - v_u^b) > 0 \quad (30) \end{aligned}$$

under the assumed condition that  $v_u^g > v_u^b$ . We then see from (19'') and (20''), again using the fact that  $c''(i) > 0$ , that  $i^g > i^b$  and, moreover,  $w^g > w^b$ . This proves Proposition 2. It is easy to see from (30) that our assumption  $v_u^g > v_u^b$  is not a necessary condition for our proposition to hold.

The remaining question is whether a macroeconomic configuration of  $(v_u^g, v_u^b)$  that satisfies the condition  $v_u^g > v_u^b$  or a little stronger condition (26) is actually conceivable. Various alternative specifications are possible in defining  $v_u^g$  and  $v_u^b$ ; however, we shall take as an example the case where dismissed workers flow into the reserve pool of the unemployed. Then an unemployed worker in the beginning of a period has a chance to become employed in the current period (i.e., turn into the status of the employed); yet if he/she did not actually get the job he/she will collect an (external or social) unemployment compensation, say  $\underline{w}$ , at the end of the period, and will be left in the status of the unemployed in either of the two possible states for the next period.

The case of complete hoarding is easy to illustrate. Since in this case there is no layoff and also the dismissal rate is common ( $f$ ) between the two states the probability for being rehired for the unemployed, say  $\alpha$ , is the same between the two states. Hence the occurrence of the complete hoarding case implies that

$$v_u^g = \alpha v_e^g + \frac{(1 - \alpha)}{1 + r} (\underline{w} + p v_u^g + p^* v_u^b) \quad (31)$$

$$v_u^b = \alpha v_e^b + \frac{(1 - \alpha)}{1 + r} (\underline{w} + p v_u^g + p^* v_u^b) \quad (32).$$

Thus the difference between  $v_u^g$  and  $v_u^b$  is a simple multiple of the difference between  $v_e^g$  and  $v_e^b$ .<sup>10</sup> On the other hand, equations (9) and (10), together with the condition of no layoffs, under our linear utility function imply that

$$v_e^g - v_e^b = \frac{1}{1 + r} \{(w^g - b_i^g) - (w^b - b_i^b)\} \quad (33).$$

This expression states that the difference in the present utility value of being employed across states is only a multiple of the difference in current utility flows across states. By Proposition 2 we observe that this is strictly positive. Hence it is indeed possible to have an economy in which (26) is actually satisfied. In the case of partial hoarding, we have an additional effect that the probability of being rehired is also higher on average in the good state than in the bad state.<sup>11</sup> Such an effect makes the assumption  $v_u^g > v_u^b$  all the more likely to be actually satisfied.

## 5. Summary and Discussion

The major implications of our model can be summarized as follows:

(i) When the work intensity is conditioned by worker monitoring and disciplining at a cost to employer, layoffs due to recurrent downward demand shocks will be deterred to the extent that layoffs dilute the effectiveness of the costly monitoring and disciplining. A complete labor hoarding policy with a reduced work intensity (labor productivity) for economic downturn may arise under a certain relatively weak condition. (Proposition 1.) We have

also shown that in a somewhat limited framework where the monitoring intensity is constrained to be invariant across the states of nature (due to some technological or organizational reasons) either complete or partial labor hoarding must occur. Adjustment of work intensities across states is accomplished through a pro-cyclical movement in the real wage. (Proposition 2.) To an outside (and uninformed) observer it appears as if workers are buying job security in return for wage flexibility.

(ii) The real wage flexibility is compatible with the existence of rationing in the labor market. More concretely, it can coexist with natural involuntary unemployment (of the kind discussed by Calvo [1979], Solow [1979], Shapiro-Stiglitz [1984], Bowles [1985] and Gintis-Ishikawa [forthcoming]) or a dualistic labor market structure (of the kind discussed by Yellen [1984] and Summers-Bulow [1985]), which have previously been associated with real wage rigidities endogenously generated. This implies that the rigidity per se does not lie at the heart of involuntary unemployment nor of the dualistic labor market structure. At the same time, it gives us a warning against interpreting real wage flexibility as a mere reflection of the competitive labor market in the usual Walrasian sense.

These results and implications derive from our assumption that firms face recurrent stochastic shocks in demand, and moreover, that the nature of the shocks is understood and agreed by all parties involved so that a state-contingent contract may develop. The rather striking contrast in implications that exists between our work and the work of Akerlof-Yellen [1985], which shows that employers have little incentive to change wages in the face of small demand shocks and thus tends to resort to pure "quantity adjustment," originates from the difference in supposition as to whether or not there exists objective and "social" bases for the development of a

state-contingent contract.<sup>12</sup> Note that a state-contingent contract can be regarded as a minimal form of a long-term contract, and as have been already discussed in the implicit contract literature (see Holmstrom [1983] and Bull [1983] for reviews on "enforceability"; see also the discussion of asymmetric information originating from Hall-Lillien [1979]), it requires for its sustenance some form of basic trust on the part of workers as to the employer commitment to the contract. Also required is a room for agreement between the employer and workers in their perception ex post as to what state of nature has actually occurred in each period.

On the other hand, recontracting models have been criticized for its neglect of the possibility that "posting of bonds" by workers or a "seniority wage" contract (as argued by Lazear [1979]) can easily eliminate the rationing feature and thus avoid inefficiencies. (On this point see Yellen [1984], and an exchange between Carmichael [1985] and Shapiro-Stiglitz [1985].) While logically valid, this argument in reality faces various impediments as aptly observed already in an early paper by Becker-Stigler [1974]. One of the impediments (besides the well-noted imperfection in the capital market) is the lack of basic trust towards employers on the part of workers. Therefore there is a certain degree of commonality in the preconditions for development of a state-contingent contract and that of a pure seniority wage contract, and to that extent our state-contingent contract may partly be accompanied by a seniority wage contract. (This provides one of possible empirically testable hypotheses of the model.) The conditions surrounding the development of these alternative forms of contracts clearly relate to the differences in the nature of the organizations for collective bargaining or wage determination, and more fundamentally to the underlying differences in social, cultural and historical realities. This invites an important and intriguing cross-country study.<sup>13</sup>

Footnotes

\* University of Massachusetts at Amherst and University of Tokyo, respectively. This paper is a revised version of a paper "Wages, Work Discipline and Labor Hoarding under Stochastic Demand," November, 1984. The authors are indebted to Masahiro Okuno and participants of seminars at the Tokyo Center for Economic Research and Nagoya University for stimulating comments and discussions. The authors also gratefully acknowledge the financial support provided by the Grant from the International Research Exchange Program of the University of Tokyo.

1. Of course, we could imagine a strong workers' collective, which totally encapsules workers' individualistic motives and regulates the work intensity of each worker, ousting the "rate busters." But such a workers' collective, if it existed, must be rare. A somewhat weaker yet possibly effective form of collective behavior arises when adherence to a certain social norm is morally encouraged, and individual adherence is mutually reinforcing (namely the strength of adherence depends on how much of the members actually adhere to the norm). Such a case is analyzed well by Akerlof [1980]. In fact, Akerlof's model may be adapted to explain, though yet in a somewhat mechanistic way, the rise and fall of trade unions. Our model below is a crude first attempt to formalize the interaction of individual and collective aspects of worker behavior. More refined game-theoretic formalization is certainly to be developed.

2. When the employer's choice of  $f$  is zero, then maximization of  $V_e$  reduces to maximization of the current utility flow  $u(w, i)$  given  $w$ . Thus the first order condition becomes:  $u_i(w, i) \cdot i = 0$ . It implies either a positive voluntarily supplied work intensity ( $u_i(w, i) = 0$ ) or a zero work intensity depending on the nature of the utility function. The original efficiency wage hypothesis that relates the level of wage to loyalty feel-

ings of workers to the employers or that invokes a sense of fair exchange (as argued by Adam Smith and, again more recently by Solow [1979]) can be interpreted as an argument that the utility function of workers actually has a segment such that  $u_i(w, i) > 0$ . The case of  $f=0$  may, in fact, become the optimal choice for employers when the cost of dismissal ( $c(i)$  function below) is sufficiently high. For details, see our previous paper.

3. In principle, we can similarly introduce  $\theta^{bg}$ , namely, the rate of layoff when the state turns from bad to good. However, it is clearly superfluous, and we set  $\theta^{bg} \equiv 0$ .

4. The determinant of the matrix on the left hand side of (11) is shown to equal  $(1+r) \cdot A(f^g, f^b, k^{gb})$ , where  $A(f^g, f^b, k^{gb})$  is defined in (12) and (13) below. As can be seen by (12) and (13), in order for a meaningful firm equilibrium to exist, the determinant (as well as  $A(f^g, f^b, k^{gb})$ ) must be positive.

5. An insightful reader may note that (13') and (14') in themselves are not sufficient to determine the work intensity for both states,  $i^g$  and  $i^b$ , under given terms of the contract. This is because (14') is a degenerate relationship that involves neither  $i^g$  nor  $i^b$ , and hence, workers' expectational equilibrium as conditioned by (13') and (14') is compatible with a continuum of different  $(i^g, i^b)$ 's. This fact, however, need not worry us, since, after all, workers' expectational equilibrium regarding work intensity reactions is only a partial ingredient of the entire set of conditions for a stochastic stationary firm equilibrium which must be satisfied simultaneously. Determination of the specific levels of work

intensity ( $i^g, i^b$ ) just has to wait until other conditions pertaining to the firm are specified, which is the question that we deal with in the next section.

6. The circumstance that this economy is still capable of producing involuntary unemployment (or alternatively a dualistic labor market) is unchanged from the basic model. Complete labor hoarding just means that each firm adjusts the level of output solely in terms of work intensities.

We can also extend our present analysis to the case where laid-off workers are not simply put into the pool of the unemployed but are subject to recall in good times. Let  $v_o^g$  and  $v_o^b$  denote the present utility value of a laid-off worker in the good state and the bad state, respectively. Recall means  $v_o^g = v_e^g$ .  $v_o^b$  lies in between  $v_e^b$  and  $v_u^b$ . This change leads to a modification of the first element of the RHS vector in (11) to  $u(w^g, i^g) + d^g E v_u + p^* \theta^g v_o^b$ , but other parts of our system are not affected. Accordingly the fourth term in the LHS of the first inequality in (24) is changed to  $-(1+r)v_o^b$  in stead of  $-(1+r)v_u^b$ . While the condition for the occurrence of complete hoarding becomes a little stringent, there is no substantive change in our analysis.

7. One may at a first blush be tempted to argue that the condition  $u(w, i) > r v_u$  in the basic model may carry over to our extended model, thereby obtaining  $u(w^b, i^b) > r v_u^b$ . If that were the case the LHS of the first inequality of (24) would always become positive, implying that the case of complete hoarding must occur. However, this argument is not valid, for (as is easily observed from (11)) the condition  $v_e^j > v_u^j$  ( $j=g, b$ ) do not simply translate into the state by state relationship between the current

utility flow  $u(w^j, i^j)$  and the "rental income" of being unemployed  $rv_u^j$ . The difference  $(v_e^j - v_u^j)$  depends also on the utility flow and rental income of the other state. Thus the case where layoffs occur do remain as a genuine possibility.

8. Whether the relative change in the real wage rate is greater or less than the relative change in work intensity (labor productivity) depends upon the nature of the  $c(i)$  and  $s(m)$  functions, and is not known a priori.

9. Alternatively, if the cost of dismissal function had been linear ( $c''(i) \equiv 0$ ), then we would have invariant real wages across the states. However, this statement depends critically on the assumed linearity of the utility function. In the case of a general utility function procyclicality may be compatible with a linear cost of dismissal function.

10. For determination of the value of  $\alpha$  we must specify further the structure of the economy; however, this does not raise any new issue than as already discussed in Shapiro-Stiglitz [1984] and our previous paper, and hence, will be omitted.

11. We have a little complication here, because the probability in question differs in magnitude according as whether the previous period was a good state or a bad state. Hence we use the qualifier on average.

12. We can also present a different version to the argument that employers resort to pure quantity adjustment. Namely, in the context of our basic model, we can easily verify that so long as the demand shock is of an



unanticipated and temporary nature it is optimal (not just near-rational as in Akerlof-Yellen) for the individual employers to maintain the same (w, m, f) contract and resort to pure employment-size adjustment. (Of course, when the level of the reservation utility ( $rV_u$ ) is altered macroscopically, employers must adjust the contract accordingly.) See Gintis-Ishikawa [1986] for more discussion.

13. In fact, one of the present authors has compared the organization of the Japanese and the U. S. labor markets from this perspective and argued that the Japanese market has many features that are expected to be held by the state-contingent recontracting model, namely, the dualistic labor market structure, real wage flexibility, labor hoarding in the primary sector, and moreover, relatively strong seniority wage elements (all of which are by now well-established facts about the Japanese labor market). The U. S. labor market seems much closer to the original recontracting model with a rigid wage feature. For a detailed discussion, see Ishikawa [1986].

#### References

- Akerlof, G., "A Theory of Social Custom, of Which Unemployment May Be One Consequence," Quarterly Journal of Economics, 94, June, 1980.
- Akerlof, G. and Yellen, J., "A Near-Rational Model of the Business Cycle, with Wage and Price Inertia," Quarterly Journal of Economics, 99, 1985.
- Becker, G. and Stigler, G., "Law Enforcement, Malfeasance, and Compensation of Enforcers," Journal of Legal Studies, 3, January, 1974.
- Bull, C., "Implicit Contracts in the Absence of Enforcement and Risk Aversion," American Economic Review, 73, September, 1983.
- Calvo, G., "Quasi-Walrasian Theories of Unemployment," American Economic Review, 69, May, 1979.

- Carmichael, L., "Can Unemployment Be Involuntary?: Comment," American Economic Review, 75, December, 1985.
- Gintis, H. and Ishikawa, T., "Wages, Work Intensity, and Unemployment in Competitive Labor Markets: A Comparative Analysis," Journal of Japanese and International Economies, forthcoming.
- Gintis, H. and Ishikawa, T., "The Theory of Production and Price in Contingent Recontracting Markets," mimeographed, University of Massachusetts at Amherst, August, 1986.
- Hall, R. and Lillien, D., "Efficient Wage Bargains Under Uncertain Supply and Demand," American Economic Review, 69, 1979.
- Holmstrom, B., "Equilibrium Long-Term Contracts," Quarterly Journal of Economics, 97, 1983.
- Ishikawa, T., "On the Sources of Wage Flexibility in the Japanese Labor Market," Discussion Paper, #86-F-5, University of Tokyo, September, 1986.
- Hicks, J., The Theory of Wages, Second edition, London: Macmillan, 1963.
- Lazear, E., "Why Is There Mandatory Retirement?" Journal of Political Economy, 87, 1979.
- Nash, J., "The Bargaining Problem," Econometrica, 18, April, 1950.
- Okun, A. Prices and Quantities: A Macroeconomic Analysis, Washington, D. C.: Brookings Institution, 1971.
- Shapiro, C. and Stiglitz, J., "Involuntary Unemployment as a Worker Discipline Device," American Economic Review, 74, June, 1984.
- Shapiro, C. and Stiglitz, J., "Can Unemployment Be Involuntary?: Reply," American Economic Review, 75, December, 1985.
- Solow, R., "Alternative Approaches to Macroeconomic Theory: A Partial View," Canadian Journal of Economics, 12, August, 1979.
- Summers, L. and Bulow, J., "A Theory of Dual Labor Markets with Application to Industrial Policy, Discrimination and Keynesian Unemployment," mimeographed, May, 1985.
- Yellen, J., "Efficiency Wage Models of Unemployment," American Economic Review, 74, May, 1984.