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Competition and Macroeconomic Adjustment

by

Kiyohiko G. Nishimura
University of Tokyo

and

Hiroyuki Toda
Yale University

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Kiyohiko G. Nishimura

Faculty of Economics

University of Tokyo

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Hiroyuki Toda

Department of Economics

Yale University

ABSTRACT

This paper investigates the adjustment of a monopolistically competitive economy to an infrequent, unexpected, and permanent change in money supply. Information is imperfect because the magnitude of the change is not fully revealed to the public, and it is also incomplete because one firm does not know other firms' prices. The impact effect of the change is asymmetric: prices are downward rigid but upward flexible. Competition makes prices rigid initially, but accelerates the adjustment later. Thus increased competition implies the short-run inflexibility of prices, but at the same time brings about their long-run flexibility.

1. INTRODUCTION

This paper investigates the adjustment process of a monopolistically competitive economy to an infrequent, unexpected, and permanent change in aggregate demand. Specifically we concentrate the change in money supply, because it has been a focal point of the macroeconomic discussion in the last several decades. However, the analysis can easily be extended to other unexpected permanent changes in, for example, preferences and technology.

The model is a macroeconomic version of Nishimura (1986), whose choice-theoretic microfoundation is presented in Nishimura (1987a, 1987c). In this economy, products are differentiated and firms have a partial monopoly power over their customers. Firms are assumed to be price-makers and quantity-takers (no rationing). The basic difference of our model from Nishimura's (1986) is that our model is concerned with the infrequent unexpected permanent change in money supply, while Nishimura analyzes the frequent, partially anticipated, temporary changes. We analyze the adjustment of the economy from one steady state to another steady state. Specifically (1) the economy is initially in a steady state, (2) then an unexpected, and permanent change in money supply occurs, and (3) after that the economy gradually moves toward a new steady state as firms learn about the change. We analyze both the impact effects (such as the magnitude and the direction of induced price changes in initial several periods) and the persistent effects (such as the speed of adjustment of firms' expectations about money supply). In order to make analysis tractable, we assume the structure of the model is log-linear, and that disturbances are log-normally distributed.

In this economy, information is imperfect and incomplete for firms. Information is imperfect because the magnitude of the change is not revealed fully to the public. Information is incomplete because one firm does not know other firms' conditions and actions including their prices. Firms have information only about quantities they sold in the past. They determine their prices based on this imperfect and incomplete information. On the contrary, consumers have all relevant information they need, that is, they know all prices firms offer, when they buy from them.

Firms are assumed to know the structure of the economy. Their knowledge includes the distribution of the past unexpected permanent changes as well as the parameters of preferences and technology. Firms are assumed to be Bayesian, use this distribution as the prior distribution, and update their prior using newly available information (their sales) each period.

The Bayesian learning of these firms is different from the one of one independent decision maker, because it necessarily involves learning of endogenous variables. Because the model is monopolistically competitive, the demand at one firm is dependent on other firms' prices or the average price among them in the particular model of this paper. The average price is determined by the interplay of these ignorant firms' price decisions. This leads in general to a well-known infinite regress problem even if all firms know all the structural parameters of preferences and technology.

We avoid this problem by assuming that all firms knows the distribution of the past changes and use this distribution as their prior. Thus in effect we make the common prior assumption. This assumption is justified if no other information about the change is available to firms. Under this common-prior assumption, it is possible to form rational (or equilibrium or consistent) expectations about unknown variables in the economy using the Bayesian method. Thus the model is one of rational learning (see Blume, Bray, and Easley (1983) and Townsend (1978, 1983)). One important consequence of rational learning is that we can analyze uncertainty associated with firms' expectations, which is endogenously determined in the economy.

We obtain two basic results. First, the impact effect of the monetary change is asymmetric: prices are downward rigid but upward flexible. This is due to the asymmetric effect of uncertainty on prices.

Note that in this economy, the firm's monopoly power is limited by competition among other firms. This checking mechanism works through the negative effect of the average price (the other firms' prices) on the demand at the firm. However, uncertainty due to imperfect and incomplete information introduces the possibility of local-global confusion similar to the one in the island models of macro rational expectations such as Lucas's (1973). This implies firms do not respond to the change on the average as much as in the perfect and complete information case. The effect is essentially the same as that of the increase in the degree of monopoly power, because firms become less worried about the effect of the average price on the demand they face. Thus firms raise prices, so that the average price goes up when uncertainty due to imperfect and incomplete information is increased. This effect of uncertainty does not depend on whether the unexpected change is positive or negative. Thus the impact effect becomes asymmetric, making prices rigid downward and flexible upward, although the direct effect of the unexpected change through firms' expectations about the change is symmetric as in other macroeconomic models. In the case of a positive change, the effect of uncertainty raises prices further, while the same effect checks the decline in prices if the change is negative.

Second, competition makes prices rigid initially, but accelerates the adjustment later. Thus increased competition implies the <u>short-run</u>

inflexibility of prices, but at the same time brings about its long-run flexibility.

The reason competition makes prices rigid in the short run is the same as in Nishimura (1986, 1987a), which is also based on the possibility of local-global confusion due to imperfect and incomplete information. An increased competition implies firms have to put more weight on their expected average price than on their own conditions, in determining their prices. Note that the expected average price is computed based on their own conditions, which consist of the economy-wide conditions and the firm-specific ones. This introduces the possibility of local-global confusion. Because of this possibility, their expected average price is less sensitive to the monetary change on the average than their own conditions. Consequently the shift in weights from their expected average price from their own conditions implies that their optimal prices, and ultimately the actual average price, become less sensitive to the monetary change.

The long-run flexibility of prices is in fact the consequence of the short-run inflexibility in our learning model. Firms learn the monetary change through their actual sales. Thus the more sensitive their sales are to the change, the more rapid their learning process is. The sales become responsive to the monetary change when prices are rigid, because in this case quantities bear the burden of adjustment. This means that if prices are more rigid in initial several periods, then the process of learning during these periods speeds up, making rapid overall adjustment possible. Thus contrary to the traditional view that price inflexibility is bad for economic welfare, the result of this paper suggests the possibility that price rigidity is beneficial to the long-run level of welfare, though our analysis is not intended to do welfare analysis.

Although the model is constructed for purely theoretical purposes, the results obtained have some relevance to recent empirical studies in macroeconomics. First, consider the asymmetry of price responses to permanent demand shocks. The asymmetric effects of aggregate demand changes have been emphasized, though not well documented, in debates over stabilization policies, since Keynes's idea of the downward rigidity and upward flexibility of wages. This paper shows that such asymmetry may be present when unexpected, permanent changes occur to the economy. The source is not wages, but prices, because in our model labor inputs are determined efficiently in the bilateral monopoly relations. However, the model can be extended to the economy with monopolistically competitive unions, without changing the results qualitatively (see Nishimura (1987c)).

Recent developments in non-linear dynamics has renewed the interest about the possibility of asymmetric response. The asymmetry or non-linearity in time series data has been investigated in recent years by many authors (see, for example, Brock and Sayers (1987)). They find the strong evidence for the asymmetry in many economic time series though the exact nature of the asymmetry is still elusive.

Second, recent inter-country studies about the macroeconomic adjustment to demand as well as cost shocks reveal a large difference even among industrial countries (see, for example, Sachs (1979), Gordon (1983), and Schultze (1986)). A consensus seems to be emerged from the studies, which states that the United States has more rigid (nominal) prices than other countries (European countries and Japan). However, the United States economy is considered by laymen (and many economists, I suspect) as more competitive than other industrialized economies. This is not fitted very well to the traditional view that competition leads the economy to an idealized Arrow-Debreu economy in which prices are completely flexible. This evidence is consistent with the models of imperfect and incomplete information in this paper and Nishimura (1986, 1987a).

Third, as for the speed of adjustment, Schultze (1986) finds that, though the United States economy suffers from short-run price rigidity, it adjusts to the changes in demand and cost much faster than European counterparts. This long-run flexibility is often stressed in popular discussion about the strength of the United States economy. This is also consistent with the imperfect/incomplete information view.

The plan of this study is as follows. In Section 2, the basic model is presented and the scenario of the adjustment process is given informally. Section 3 contains the main results. It consists of five parts. In the first subsection, we explain the formation of rational expectations in a general way. In the second, the initial steady state is depicted. In the third the unexpected change is introduced and the Bayesian learning process is explicitly presented. The fourth subsection deals with the impact effects. There the asymmetric effect of the change on prices and the effect of increased competition are analyzed and evaluated. The last subsection explains the persistent effects of the change. There we investigate the speed of the expectational adjustment (the adjustment of the firms' expectations about the unknown change in money supply to the true value). The pattern of the output adjustment is also analyzed. The output adjustment is more complex than the expectational adjustment, so that we utilize simulation to explain the major characteristics. Section 4 contains concluding remarks.

2. THE MODEL

In this paper we analyze a simple monopolistically competitive economy with real balance effects. The model is a macroeconomic version of Nishimura (1986). The choice-theoretic foundation of the model is presented in Nishimura (1987a, 1987c). The model can be derived from either (a log-linear approximation of) the economy inhabited by homogeneous consumer-workers having CES utility functions (as in Dixit and Stiglitz (1979), see Nishimura (1987a)), or the economy with heterogeneous consumers with Leontief utility functions whose parameters are distributed in a specific way (as in Houthakker (1974) and Sattinger (1986), see Nishimura (1987c)). We do not go into details of the derivation of the model in this paper, but instead we start with the reduced-form equations.

Consider an economy with homogeneous firms except for individual disturbances specified later. Consumers as workers supply labor to firms, and consumers as capitalists own firms' stocks. Goods are not storable in this economy. The product market is monopolistically competitive, while workers in this economy are unionized firm by firm, and the firm and its union are in a bilateral monopoly in the labor market. The firm is assumed to maximize joint benefit of its stockholders and its union.

We assume there is no investment nor government expenditure. The sole source of demand is consumption. We assume that each consumer's consumption is solely dependent on his real balances. Thus in this model disturbances in nominal money supply are the only source of macroeconomic demand disturbances. In this paper, all equations are linear in logarithm, and disturbances are normally distributed in logarithm. The average of a variable is the log of the geometric average or the arithmetic average in logarithm.

Let $\bar{y}$ be the average of consumers' demand for goods, m money supply, and $\bar{p}$ the price level (the average price). The average demand of consumers for goods is assumed to be

$$(1) \qquad \bar{y} = b(m - \bar{p}),$$

where b is the real-balance elasticity of the average demand, which satisfies $b > 0$.

We assume product differentiation, so that firms has a partial monopoly with their customers. Specifically we assume the following form of demand for a particular firm's products,

$$(2) \qquad q = -k(p - \bar{p}) + \bar{y} + u,$$

where q is the demand for the firm's products, p its price, and u the demand disturbance specific to this firm. The parameter k is the firm's own-price elasticity of the demand, which is assumed to be sufficiently large to satisfy $k > b + 1$.[1] The disturbance u is assumed to be normally distributed, satisfying $Eu = 0$ and $Eu^2 = \sigma_u^2$. Except for informational difference described below, firms are different only with respect to u. Thus the above distributional assumption implies that the number of firms is normalized to unity.

The firm has to announce its price at the beginning of each period in order to inform its customers and potential buyers of its being in business in the current period. The firm has to satisfy all demand his announcement creates (price-making, and quantity-taking). The implicit assumption here is that information diffusion is sluggish in an economy in which continuing search and monitoring are costly (see, for example, Nishimura (1987)). We assume the length of the period is long, and cannot be ignored. Thus our model is one variant of non-market clearing models in which current prices do not in general reflect fully the market conditions of the current period.[2]

Let us now characterize information available to the firm. The demand for a particular firm's products is, combining (1) and (2),

$$q = -k(p - \bar{p}) - b\bar{p} + \alpha, \tag{3}$$

where

$$\alpha = bm + u, \tag{4}$$

which is the individual nominal demand condition. We assume that <u>the firm knows only the past history of their own q in the periods relevant to our analysis</u>. Thus the firm does not know not only current $\bar{p}$, m, u, and α, but also their past values in our model. The firm forms its subjective distribution of $\bar{p}$ and α based on available information about q. How such expectations are formed will be discussed later.

Two remarks on this assumption about information availability may be due. First, this assumption may at the first glance seem to be restrictive in an economy where the government as well as private agencies collects economic data and release them. However, such data are often available with substantial delays and, more importantly, they usually contain substantial errors. (Even the government statistics undergo substantial revision processes for a long period of time.) Thus it is rather unrealistic to assume that economic agents can obtain accurate economic data in a short period. We assume the other extreme of complete unavailableness, in order to highlight the effect of learning and adjustment in macroeconomics. Although it is possible to introduce in this model outside noisy information about α, m, u, and $\bar{p}$, which may be updated as time passes, such complication makes analysis quite cumbersome with little difference. Thus we stick to the complete unavailableness assumption throughout this paper.

Second, we assume that the firm does not know α. This assumption is different from that of Nishimura (1986, 1987a, 1987b) in which current α is observable. The difference is due to the nature of disturbances to be analyzed. Nishimura (1986, 1987a, 1987b) is concerned mostly with transitory, at least partly predictable changes in demand and cost, while we analyze permanent, unexpected changes in money supply. Consequently for the purpose of this study, it seems appropriate to assume away the possibility of observing α before price determination.

In this economy, the firm maximizes the real joint benefit of its stockholders and its union. The disutility of the union (in terms of goods) when its members have to produce q, is assumed to be

$$(5) \qquad z = -\log(c_1 + 1) + (c_1 + 1)q.$$

The coefficient c_1 satisfies $c_1 > 0$. c_1 depends on the degree of returns to scale in production and the degree of increasing marginal disutility of labor. Note that positive c_1 still allows to some extent increasing returns to scale, though it is dominated by increasing marginal disutility of labor.

Under the above assumptions, the firm's problem amounts to maximize $E_{\bar{p},\alpha}\Pi$, where $E_{\bar{p},\alpha}$ is the expectation operator with respect to the firm's subjective distribution of $\bar{p}$ and α (which will be specified later), and Π is the real joint benefit such that

$$(6) \qquad \Pi = \exp(-\bar{p})\{\exp(p)\exp(q)\} - \exp(z),$$

subject to (3) and (5).

In the following, we consider the adjustment of prices and quantities to a permanent, unexpected change Δm in money supply. Specifically we analyze the adjustment from the steady state with m^* to that with $m^* + \Delta m$. We assume that the permanent, unexpected change Δm occurs in an economy at period 0 which has been in a steady state with m^* before period 0. Firms are informed of the occurrence (but not the magnitude) of the change at the end of period 0. Firms are assumed to know the objective distribution of Δm in the past. This becomes their prior distribution for Δm. Firms update their expectations about m, u, $\bar{p}$, and α for period 1, using their available information, that is, their q, at the end of period 0. Using these expectations they determine their optimal prices at the beginning of period 1. At the end of period 1, they again update their expectations using newly available information q, and prepare for the next period. The whole process repeats itself until new steady state is reached. The next section describes the details of the adjustment process.

3. THE MACROECONOMIC ADJUSTMENT PROCESS

3.0. Formation of Rational Expectations and the Optimal Pricing

Let us first consider the formation of rational expectations. We describe the process in a general way which is applicable not only in the initial steady state, but also in the adjustment process. Let Ω be information available to the firm at the beginning of the period, which includes the knowledge of the structure of the economy and the past history of its sales. The firm assumes that $\bar{p}$ and α are jointly normally distributed with the mean $(e[\bar{p}|\Omega], e[\alpha|\Omega])$ and the variance-covariance matrix $\hat{\Sigma}(\Omega)$. Here $e[x|\Omega]$ is the linear least squares regression of x on Ω, which is the same as the conditional expectation of x based on Ω under our linear Gaussian assumption. $\hat{\Sigma}(\Omega)$ is the error variance-covariance matrix of $(e[\bar{p}|\Omega], e[\alpha|\Omega])$ conditional on information Ω such that

$$\hat{\Sigma}(\Omega) = \begin{bmatrix} \hat{V}_{\bar{p}}(\Omega) & \hat{V}_{\bar{p}\alpha}(\Omega) \\ \hat{V}_{\bar{p}\alpha}(\Omega) & \hat{V}_{\alpha}(\Omega) \end{bmatrix}$$

where $\hat{V}_{\bar{p}}(\Omega) = E[\{\bar{p} - e[\bar{p}|\Omega]\}^2|\Omega]$, $\hat{V}_{\bar{p}\alpha}(\Omega) = E[\{\bar{p} - e[\bar{p}|\Omega]\}\{\alpha - e[\alpha|\Omega]\}|\Omega]$, and $\hat{V}_{\alpha}(\Omega) = E[\{\alpha - e[\alpha|\Omega]\}^2|\Omega]$.

Under the above expectational assumption, the optimal pricing formula is, from the first order condition of optimality,

$$(7) \qquad p = (1 + c_1k)^{-1}[a + c_1b\cdot e[m|\Omega] + \{1 + c_1(k - b)\}e[\bar{p}|\Omega]]$$

in which

$$(8) \qquad a = \log\{k/(k-1)\} + (1/2)[w^t\cdot\hat{\Sigma}(\Omega)\cdot w - z^t\cdot\hat{\Sigma}(\Omega)\cdot z]$$

where $w^t = [(c_1 + 1)(k - b), (c_1 + 1)]$, $z^t = [(k - b - 1), 1]$, and t denotes the transpose. Here we use the well-known property of log-normal distributions. Under our assumptions, the second-order condition is also satisfied.

Note that, since (i) α, m, and u are all unobservable and (ii) u is not serially correlated, the past history of the sales is irrelevant in estimating current u. This implies the estimate of u is the same and equal to zero for all firms. Thus $e[\alpha|\Omega] = b \cdot e[m|\Omega]$ for all Ω. This characteristic is utilized in the above optimal price formula. Another implication is that the firm at the beginning of the period is characterized by $e[m|\Omega]$ and $e[\bar{p}|\Omega]$ only.

In order to make analysis simple, we focus our attention to the case in which $e[\bar{p}|\Omega] = e[\bar{p}|e[m|\Omega], \hat{\Sigma}(\Omega)]$. This implies that firms having the same subjective distribution of m get the same expectations about the average price. This is a natural assumption because $e[u|\Omega] = 0$. Under the above assumption the firm is identified with a pair $(e[m|\Omega], \hat{\Sigma}(\Omega))$. Then the actual average price is

$$(9) \qquad \bar{p} = E_{e[m|\Omega],\ \hat{\Sigma}(\Omega)}\ p.$$

Here $E_{e[m|\Omega],\ \hat{\Sigma}(\Omega)}$ is the expectation operator with respect to the actual distribution of $e[m|\Omega]$ and $\hat{\Sigma}(\Omega)$ among firms. This distribution is generated by the past history of firms' sales, which in turn generated by the past history of the firm-specific disturbances and the economy-wide changes.

The firm forms rational (or equilibrium or consistent) expectations about $\bar{p}$ and α, using the complete knowledge about the economy just described and taking it into account that other firms also have this knowledge.

The major problem of rational expectations of this kind is about the formation of expectations about other firms' expectations. One firm's expectations depend on its expectations about other firms' expectations. This leads to a so-called infinite regress problem in which firms forecast the forecasts of forecasts of others, and so on. This study, like other rational expectations studies, circumvents the problem in two ways. First, the firm is assumed to know the structure of the economy including prior distribution about Δm other firms have. In fact, the prior distribution is homogeneous, which is the past distribution of Δm. Because all firms come to know permanent changes in money supply eventually through learning, the past history of Δm is common knowledge, so is the prior distribution. Second, we are concerned only with expectational (Bayesian-Nash) equilibrium. Under the first assumption, the second assumption is justified rather strongly (see, for example, Harsanyi (1967-1968) and Binmore and Dasgupta (1986)). These two assumptions are sufficient to get rational expectations, the derivation of which is discussed later in this section.

3.1. The Initial Steady State

Let us first consider as a frame of reference the steady state in which money supply is equal to m^* for a long time. Hereafter superscript * represents the steady state value. In the steady state the firm has sufficient information to estimate m^* correctly. Thus their Ω can be treated as the same, having the form $\Omega = \{m^*\}$ for all firms. Then we have $e[\alpha^*|\Omega] = e[\alpha^*|m^*] = bm^*$ for all firms. Because of the homogeneity of expectations, firms post the same price, as shown below.

<u>The optimal pricing formula</u>

Under the above expectational assumption, the optimal pricing formula in the initial steady state is from (7),

$$p^* = (1 + c_1k)^{-1}[a^* + c_1bm^* + \{1 + c_1(k - b)\}e[\bar{p}^*|m^*]]. \tag{10}$$

where

$$a^* = \log\{k/(k-1)\} + (1/2)[w^t\cdot\hat{\Sigma}(m^*)\cdot w - z^t\cdot\hat{\Sigma}(m^*)\cdot z]. \tag{11}$$

Note that in determining its price, each firm has to know, in addition to $e[\bar{p}^*|m^*]$, the value of a^*, which depends on the error variance-covariance matrix $\hat{\Sigma}(m^*)$. However, because $\hat{\Sigma}(m^*)$ can be calculated with certainty and is common to all firms, the firm can compute a^* correctly. We will compute a^* explicitly later in this subsection. For time being, let us treat a^* as the parameter which each firm knows.

<u>Formation of rational expectations</u>

We (and the firm) use the undetermined coefficient method to obtain $e[\bar{p}^*|m^*]$. Let $e(\bar{p}^*|m^*) = H^* + J^*m^*$. Substituting this into the optimal pricing formula, and averaging over all firms, we obtain

$$\bar{p}^* = (1 + c_1k)^{-1}[\ a^* + c_1bm^* + \{1 + c_1(k - b)\}(H^* + J^*m^*)]. \tag{12}$$

Apply $e[\cdot|m^*]$ on both sides, and equate the coefficients to obtain the rational expectations values for H^* and J^*. This procedure yields $H^* = (c_1b)^{-1}a^*$ and $J^* = 1$, so that $e[\bar{p}^*|m^*] = (c_1b)^{-1}a^* + m^*$. Substituting these into the optimal pricing formula, and averaging over all firms, we obtain

$$\bar{p}^* = (c_1b)^{-1}a^* + m^*. \tag{13}$$

Thus the firm predicts correctly the average price and all firms post the same price.

Calculation of the error variance-covariance matrix and a^*

Using the foregoing results, we can calculate the error variance-covariance matrix $\Sigma(m^*)$. Because the firm predict $\bar{p}^*$ correctly, we have $\hat{V}_{\bar{p}}(m^*) = 0$ and $\hat{V}_{\bar{p}\alpha}(m^*) = 0$. Moreover, because m^* is known with certainty, we get $\hat{V}_{\alpha}(m^*) = \sigma_u^2$. Thus we obtain

$$(14) \qquad a^* = \log\{k/(k-1)\} + \psi\cdot\sigma_u^2,$$

where

$$(15) \qquad \psi = \frac{1}{2}\{(1 + c_1)^2 - 1\}.$$

It is clear that a^* is common for all firms and can be calculated without knowing α.

At the end of each period, the firm observes its demand:

$$(16) \qquad q^* = - k\{p^* - \bar{p}^*\} - b\bar{p}^* + \alpha^*.$$

Consequently, the average output is

$$(17) \qquad y^* = b(m^* - p^*).$$

The characteristics of the steady state

Let us briefly summarize the characteristics of the steady state. First, although $\bar{p}^*$ is not observable, it can be computed by the firms with certainty. This is due to the fact that information the firms possess can be treated as homogeneous in the steady state. Second, the degree of uncertainty (the variance-covariance matrix $\hat{\Sigma}$) in the economy is also completely known by the firms and becomes common knowledge. Third, the degree of competition, k, influences the average price through a^* only.

3.2. An Unexpected Permanent Change in Money Supply in Period 0

Let us assume that money supply changes permanently from m^* to $m = m^* + \Delta m$ in period 0. However, firms do not know the change when they decide their period 0 prices. They continue to assume that money supply is m^* as in the previous periods. Thus their prices do not change, so that $p(0) = p^*$ and $\bar{p}(0) = \bar{p}^*$ (they are in fact the same). At the end of period 0, the firm observes the demand for its products. For later reference, let us define

$$(18) \qquad \beta(0) \equiv q(0) - [-k\{p(0) - \bar{p}(0)\} - b\bar{p}(0)].$$

By definition, $\beta(0) = \alpha(0) = bm + u(0)$. Because $\bar{p}(0)$ is estimated correctly, $\beta(0)$ can be calculated from the available data.

Firms are informed, by an outside informational agency such as newspapers, of the occurrence of the change (not the magnitude of the change) only <u>after</u> the observation of the demand for their products, q(0).[3] We assume that firms know the way such permanent changes in money supply occur from the past experience. Specifically firms are assumed to have common prior distribution of the change, Δm. For notational convenience, we use the prior distribution of $m = m^* + \Delta m$, which is derived from the prior distribution of Δm. Let the mean and variance of the prior distribution of m be $\bar{m}$ and $\bar{V}_m$. In the following we describe how firms form optimal expectations about m after observing q(0), or equivalently, $\beta(0)$.

Expectation Updating

At the end of Period 0, the firm observes $\beta(0) = bm + u(0)$. It is well-known that the appropriate procedure of updating its expectations about m is the following two-step method based on the least squares estimation or the Kalman filtering (see, for example, Bertsekas (1976) and Athans (1974)). Let $\hat{V}_m(1)$ and $\hat{m}(1)$ be the variance and the mean of the posterior distribution of m for the firm observing $\beta(0)$. First, the variance is updated in the following variance-update equation.

$$(19) \qquad \hat{V}_m(1) = \bar{V}_m - \frac{b^2\bar{V}_m^{\,2}}{b^2\bar{V}_m + \sigma_u^{\,2}} .$$

Second, the mean-update equation is

$$(20) \qquad \hat{m}(1) = \bar{m} + \theta_1\{\beta(0) - b\bar{m}\}, \quad \text{where } \theta_1 = b\hat{V}_m(1)/\sigma_u^{\,2}.$$

A remarkable aspect of the above update equations is that the posterior variance is homogeneous among firms and can be calculated with certainty, so long as the prior distribution is the same. (In fact, the common prior is not necessary. What we need is the knowledge of the distribution of other firms' priors.) This implies that <u>firms agree on the degree of uncertainty about m, though they do not on the estimate of m</u>. The immediate consequence of this characteristic is that θ_1, which represents the speed of expectational adjustment, is common to all firms and computed with certainty. These characteristics of the linear Gaussian system is exploited in the following discussion. Thus the second characteristic of the steady state described earlier (known uncertainty) is carried over to the adjustment process, while the first characteristic (common expectations) is not.

<u>The average expectations about m</u>

Since the firm knows that other firms are also updating their expectations about m according to the above updating equations, the firm is certain that the average expectations of the economy about m in period 1, $E_{u(0)}\hat{m}(1)$, is the function of unknown true money supply m:

$$(21) \qquad E_{u(0)}\hat{m}(1) = (1 - b\theta_1)\bar{m} + b\theta_1 m = (1 - \lambda_1)\bar{m} + \lambda_1 m, \quad \text{where } \lambda_1 = b\theta_1.$$

Note that λ_1 represents the extent to which the average expectations at the beginning of period 1 are close to the true value of money supply. By construction, λ_1 depends on b and σ_u^2, and is independent of k, the degree of competition.

3.3. The First Period of Adjustment: The Impact Effects

As in period 0, each firm forms the expectations about the average price $\bar{p}(1)$ and individual demand condition $\alpha(1)$ to determine its optimal price. Note that unlike in period 0 firms are different from one another in their subjective estimate of m, $\hat{m}(1)$, reflecting different individual demand conditions in period 0. However, except for this difference, firms are informationally identical. (In particular, $\hat{V}_m(1)$ is the same for all firms.) Thus the firm is identified by its subjective estimate of m, $\hat{m}(1)$. This implies that firm-specific information is summarized in $\hat{m}(1)$. Thus Ω can be expressed as $\Omega = \{\hat{m}(1)\}$, because other information can be treated as identical for all firms. Because $\hat{m}(1)$ is determined by $u(0)$ through $\beta(0)$ (see (20)), the average price $\bar{p}(1)$ is defined as $\bar{p}(1) = E_{\hat{m}(1)}\, p(1) = E_{u(0)}p(1)$.

<u>The optimal pricing formula</u>

The optimal pricing formula for the firm having $\hat{m}(1)$ is, taking $e[\alpha(1)|\hat{m}(1)] = b\hat{m}(1)$ into account,

$$(22) \qquad p(1) = (1 + c_1k)^{-1}[a(1) + c_1b\hat{m}(1) + \{1 + c_1(k - b)\}e[\bar{p}(1)|\hat{m}(1)]].$$

where

$$(23) \qquad a(1) = \log\{k/(k-1)\} + (1/2)[w^t\cdot\hat{\Sigma}(\hat{m}(1))\cdot w - z^t\cdot\hat{\Sigma}(\hat{m}(1))\cdot z].$$

In the following we can treat $a(1)$ as a known parameter common to all firms, as in the initial steady state. Its computation will be given later.

<u>Formation of rational expectations</u>

Consider now the formation of $e[\bar{p}(1)|\hat{m}(1)]$. Using the same undetermined coefficient method,[4] we obtain

$$(24) \qquad e[\bar{p}(1)|\hat{m}(1)] = \frac{a(1)}{c_1b} + \bar{m} + \xi(\lambda_1)[\hat{m}(1) - \bar{m}],$$

where $\xi(\lambda)$ is such that

$$(25) \qquad \xi(\lambda) = \xi(\lambda;\, k,\, b,\, c_1) = \frac{c_1b\lambda}{(1 + c_1k)(1 - \lambda) + c_1b\lambda}\,.$$

By definition, ξ satisfies $\partial\xi/\partial\lambda > 0$, $\lim_{\lambda\to 0} \xi = 0$, $\lim_{\lambda\to 1} \xi = 1$, $\partial\xi/\partial k < 0$, $\partial\xi/\partial b > 0$, and $\partial\xi/\partial c_1 > 0$. FIGURE 1 shows this function with $k = 5$, $b = 0.5$, and $c_1 = 0.8$. (These values are arbitrary and only for the illustrative purpose. They will be used in illustrative examples later in this section.) The figure reveals that ξ is highly non-linear, which is insensitive when λ is small, but becomes elastic as λ increases.

The average price

Each firm obtains its optimal price by substituting the above expectations to the optimal pricing formula (22). This implies

$$(26) \qquad p(1) = \frac{a(1)}{c_1 b} + \bar{m} + \frac{c_1 b}{(1 + c_1 k)(1 - \lambda_1) + c_1 b\lambda_1} [\hat{m}(1) - \bar{m}].$$

Then averaging over all firms, we obtain because of (21),

$$(27) \qquad \bar{p}(1) = E_{u(0)}p(1) = \frac{a(1)}{c_1 b} + \{1 - \xi(\lambda_1)\}\cdot\bar{m} + \xi(\lambda_1)\cdot m.$$

The average price equation implies that the average price is the weighted sum of the prior mean $\bar{m}$ and the true value of money supply, m. The weight $\xi(\lambda_1)$ also represents the sensitivity of the first-period average price with respect to the unknown true money supply m.

Recall that λ_1 is the degree of completion of expectational adjustment. Thus the characteristics of ξ depicted in Figure 1 shows that the sensitivity of the average price to m is very small when the expectational adjustment is far from completion, but becomes large very rapidly as the expectational adjustment approaches to completion.

Calculation of the error variance-covariance matrix

FIGURE 1
The Shape of ξ

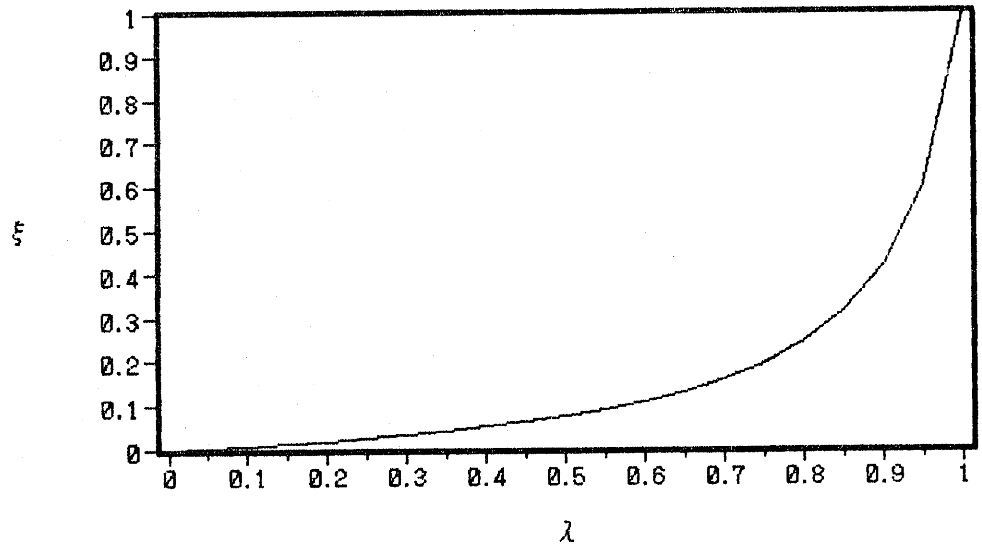
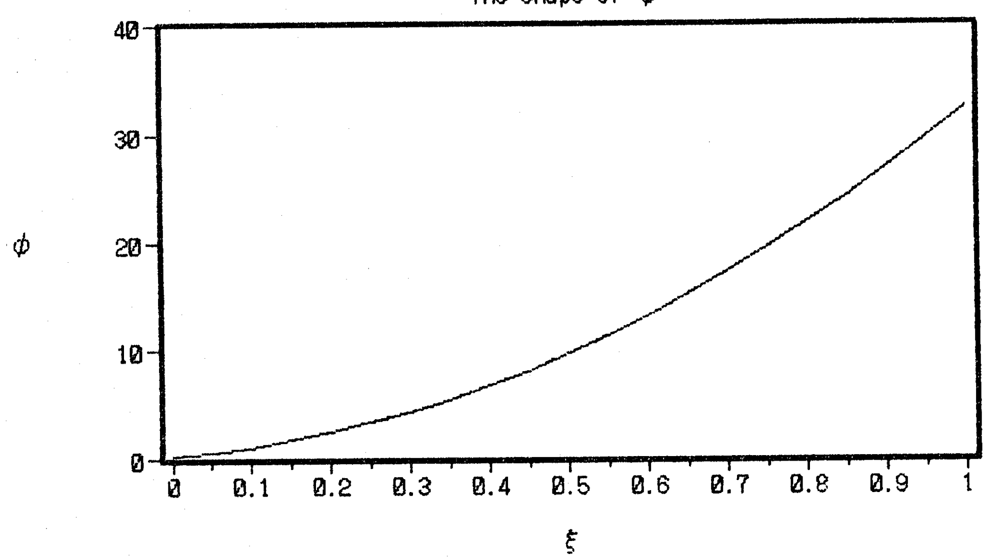


FIGURE 2
The Shape of ϕ



The firm calculates $\hat{\Sigma}(\hat{m}(1))$ and a(1), using the foregoing results. We obtain $\hat{V}_{\bar{p}}(\hat{m}(1)) = \{\xi(\lambda_1)\}^2\hat{V}_m(1)$, $\hat{V}_{\bar{p}\alpha}(\hat{m}(1)) = b\cdot\xi(\lambda_1)\cdot\hat{V}_m(1)$, $\hat{V}_\alpha(\hat{m}(1)) = b^2\hat{V}_m(1) + \sigma_u^2$. Recall that $\hat{V}_m(1)$ is the same for all firms. Consequently we have

$$(28) \qquad a(1) = a^* + \phi(\xi(\lambda_1))\cdot\hat{V}_m(1),$$

where $\phi(\xi)$ is such that

$$(29) \quad \phi(\xi) = \phi(\xi;\ k,\ b,\ c_1) = \frac{1}{2}[\{(1 + c_1)(k - b)\}^2 - (k - b - 1)^2]\xi^2$$

$$+ \{(1 + c_1)^2(k - b) - (k - b - 1)\}b\xi + \frac{1}{2}\{(1 + c_1)^2 - 1\}b^2.$$

Note that $\phi(\xi;\ k,\ b,\ c_1)$ represents the effect of uncertainty about the average price on the prices. $\phi(\xi;\ k,\ b,\ c_1)$ satisfies $\partial\phi/\partial\xi > 0$, $\partial\phi/\partial k > 0$, and $\partial\phi/\partial c_1 > 0$, for all $\xi > 0$. (Note that $\xi(\lambda_1) > 0$.) The sign of $\partial\phi/\partial b$ is ambiguous. ϕ is depicted in FIGURE 2, with $k = 5$, $b = 0.5$, and $c_1 = 0.8$.

The impact effects of the unexpected permanent change in money supply

Using the results obtained so far, we obtain two major propositions about the impact effect of the unexpected change in money supply.

First, the impact effect of the unexpected permanent change on prices is asymmetric. The positive change in money supply ($\Delta m > 0$) increases the average price much more than the negative change ($\Delta m < 0$) decreases the average price. In other words, prices are less flexible in the downward direction than in the upward direction (upward flexibility and downward rigidity). The following PROPOSITION 1 is easily proved from (13) and (27).

PROPOSITION 1: (Upward Flexibility and Downward Rigidity of Prices)

Consider the economy in a symmetric condition such that $m^* = \bar{m}$. Consider following two cases of unexpected permanent changes in money supply: $\Delta m = m - \bar{m} = \mu$ in one case, and $\Delta m = m - \bar{m} = -\mu$ in the other, where $\mu > 0$. Let $\bar{p}_{\mu}(1)$ be the average price in the former case, and $\bar{p}_{-\mu}(1)$ be that in the latter. Then

$$(30) \qquad [\bar{p}_{\mu}(1) - p^*] - [p^* - \bar{p}_{-\mu}(1)] = (c_1 b)^{-1} 2\phi(\xi(\lambda_1))\hat{V}_m(1) > 0.$$

Proof. Note that λ_1 is the same in both cases. Consequently, $\xi(\lambda_1)$ and $\phi(\xi(\lambda_1))$ are also the same in the two cases. Because we have $\bar{p}_{\mu}(1) - p^* = (c_1 b)^{-1}\phi(\xi(\lambda_1))\hat{V}_m(1) + \xi(\lambda_1)\mu$ and $p^* - \bar{p}_{-\mu}(1) = -(c_1 b)^{-1}\phi(\xi(\lambda_1))\hat{V}_m(1) + \xi(\lambda_1)\mu$ under the assumption of this proposition, (30) holds.

This asymmetric response is due to the effect of uncertainty on prices. The optimal pricing formula (22) reveals the unexpected change in money supply influences the price in two ways: (i) through $\hat{m}(1)$ and $e[\bar{p}(1)|\hat{m}(1)]$ and (ii) through $\hat{\Sigma}$. Thus the unexpected change alters the expected mean of the average price and at the same time the accompanying error variance. Although the effect through the former route is symmetric due to the linear structure of our model, that of the latter is not. Uncertainty about $\bar{p}(1)$ increases the optimal price through an increase in $a(1)$, regardless of whether the change is positive or negative. This causes the asymmetry.

Note that in a monopolistically competitive market the firm has a partial monopoly power, though such a monopoly power is limited by competition from other firms through the negative effect of the average price in the profit function. However, in the case of incomplete information, the average price is unobservable, which should be estimated. The average price depends on the economy-wide change which is unknown to the firm. Thus the firm estimates the economy-wide change through their own conditions. This introduces the possibility of local-global confusion, as in the island models of macro rational expectations models such as Lucas's (1973). Each firm corresponds to one island in such models. Because of the possibility of local-global confusion, firms become in general less responsive to economy-wide changes in the incomplete information case than in the complete information case. This implies the increased possibility that the firm's action is not matched by other firms' actions. Thus uncertainty due to incomplete information is in essence the same as an increase in the partial monopoly power, which raises the prices.[5]

Second, an increase in competition makes prices in the first adjustment period more rigid with respect to the unexpected permanent change in money supply. We have

PROPOSITION 2: (Competition Makes Prices Rigid)

An increase in the degree of competitiveness, k, always reduces the elasticity of the average price to money supply changes. That is, we have, taking $\Delta m = m - \bar{m}$,

$$(31) \qquad \frac{d}{dk} \frac{\partial \bar{p}(1)}{\partial(\Delta m)} < 0.$$

Proof. Note that we know from (27) that $\partial\bar{p}(1)/\partial(\Delta m) = \varepsilon(\lambda_1; k, b, c_1)$. Because λ_1 is independent of k, we obtain $d[\partial\bar{p}(1)/\partial(\Delta m)]/dk = \partial\varepsilon/\partial k$. Because (25) reveals $\partial\varepsilon/\partial k < 0$, we have (31).

The reason of this price rigidity with respect to m is similar to that in Nishimura (1986, 1987a), which is based on the local-global confusion due to incomplete information described above. The optimal pricing formula (22) can be transformed into the weighted average (plus a constant) of the expected money supply and the expected average price such that

$$(32) \qquad p(1) = \frac{a(1)}{1 + c_1 k} + \frac{c_1 b}{1 + c_1 k} \hat{m}(1) + \left(1 - \frac{c_1 b}{1 + c_1 k}\right) e[\bar{p}(1) | \hat{m}(1)].$$

Because $\bar{p}(1)$ is the average of $p(1)$ it is clear that $\bar{p}(1)$ depends on the average of $\hat{m}(1)$ times $\{c_1 b/(1 + c_1 k)\}$. Thus $e[\bar{p}(1)|\hat{m}(1)]$ depends on the expectations about the average of $\hat{m}(1)$, $e[E_{u(0)}\hat{m}(1)|\hat{m}(1)]$, times $\{c_1 b/(1 + c_1 k)\}$. However, (21) reveals that $e[E_{u(0)}\hat{m}(1)|\hat{m}(1)] = (1 - \lambda_1)\bar{m} + \lambda_1 \hat{m}(1)$. Because $\lambda_1 < 1$ (due to the possibility of local-global confusion), $e[E_{u(0)}\hat{m}(1)|\hat{m}(1)]$ is less sensitive to the change in m than $\hat{m}(1)$. Consequently $e[\bar{p}(1)|\hat{m}(1)]$ is less sensitive with respect to m than $\hat{m}(1)$. However, (32) shows that increased competition (an increase in k) shifts the weight from $\hat{m}(1)$ to $e[\bar{p}(1)|\hat{m}(1)]$, which is less sensitive to m than $\hat{m}(1)$. Thus the shift in weights toward $e[\bar{p}(1)|\hat{m}(1)]$ induces the stickiness of prices with respect to m.

<u>Observation of $\beta(1)$</u>

At the end of period 1 the firm observes its demand:

$$(33) \qquad q(1) = -k\{p(1) - \bar{p}(1)\} - bp(1) + \alpha(1).$$

If the firm could observe the average price, $\bar{p}(1)$, it could deduce the level of money supply m from (33), because all parameter values, including $a(1)$ and λ_1, are known to the firm. However, the firm is assumed to be unable to observe $\bar{p}(1)$ in our model. Hence it must infer m from its own demand q(1). Substituting (27) into (33) we obtain

$$(34)\quad q(1) = -kp(1) + (k-b)\left\{\frac{a(1)}{c_1 b} + \frac{(1+c_1k)(1-\lambda_1)}{(1+c_1k)(1-\lambda_1)+c_1 b\lambda_1}\bar{m} + \xi(\lambda_1)m\right\} + \alpha(1).$$

Define $\beta(1)$ as

$$(35)\quad \beta(1) \equiv q(1) - \left[-kp(1) + (k-b)\left\{\frac{a(1)}{c_1 b} + \frac{(1+c_1k)(1-\lambda_1)}{(1+c_1k)(1-\lambda_1)+c_1 b\lambda_1}\bar{m}\right\}\right].$$

By definition, we have

$$(36)\qquad \beta(1) = A(\lambda_1)m + u(1),$$

where $A(\lambda)$ is such that

$$(37)\qquad A(\lambda) = A(\lambda;\, k,\, b,\, c_1) = \frac{b(1 + c_1k - \lambda)}{(1 + c_1k)(1 - \lambda) + c_1 b\lambda}.$$

Note that $\beta(1)$ is computable with certainty because q(1), $\bar{m}$, and λ_1 as well as other parameters are known. $\beta(1)$ is the only available information to the firm. Using this, the firm updates its expectations about m.

3.4. The s-th period of adjustment: The Persistent Effects

Because the same process will be repeated for period s such that $s \geq 2$, we discuss the s-th period as the representative period in the adjustment process.

Expectation Updating

At the end of period s-1, each firm observes

(38) $$\beta(s-1) = A(\lambda_{s-1})m + u(s-1).$$

Each firm then updates its expectations on m through the Kalman filtering algorithm. The variance update equation is

(39) $$\hat{V}_m(s) = \hat{V}_m(s-1) - \frac{\{A(\lambda_{s-1})\}^2 \hat{V}_m(s-1)^2}{\{A(\lambda_{s-1})\}^2 \hat{V}_m(s-1) + \sigma_u^2},$$

and the mean update equation is

(40) $$\hat{m}(s) = \hat{m}(s-1) + \theta_s\{\beta(s-1) - A(\lambda_{s-1})\hat{m}(s-1)\} \text{ where } \theta_s = A(\lambda_{s-1})\hat{V}_m(s)/\sigma_u^2.$$

<u>The average expectations</u>

Note that the average expectations in the previous period was

(41) $$E_{u(0),\cdots,u(s-2)}\, \hat{m}(s-1) = (1 - \lambda_{s-1})\bar{m} + \lambda_{s-1}\, m.$$

Using this and (40), we obtain the average expectations about m in the s-th period such that

(42) $$E_{u(0),\cdots,u(s-1)}\hat{m}(s) = (1 - \lambda_s)\bar{m} + \lambda_s m,$$

where

(43) $$\lambda_s = \{1 - \theta_s A(\lambda_{s-1})\}\lambda_{s-1} + \theta_s A(\lambda_{s-1}).$$

Expectation formation about $\bar{p}(s)$ and the actual average price $\bar{p}(s)$

The formation of rational expectations about $\bar{p}(s)$ and α is the same as in the first period of adjustment. Then using the same undetermined coefficient method, we obtain the rational expectations of $\bar{p}(s)$ for a firm having $\hat{m}(s)$ such that

$$(44) \qquad e[\bar{p}(s)|\hat{m}(s)] = (c_1 b)^{-1}a(s) + \{1 - \xi(\lambda_s)\}\bar{m} + \xi(\lambda_s)\hat{m}(s).$$

where the following a(s) is obtained in the same argument as in the first period.

$$(45) \qquad a(s) = a^* + \phi(\xi(\lambda_s))\hat{V}_m(s).$$

Consequently, we obtain the average price such that

$$(46) \qquad \bar{p}(s) = \frac{a^* + \phi(\xi(\lambda_s))\hat{V}_m(s)}{c_1 b} + \{1 - \xi(\lambda_s)\}\bar{m} + \xi(\lambda_s)m.$$

At the end of the s-th period, each firm observes its own demand q(s). Then it calculates

$$(47) \quad \beta(s) \equiv q(s)-[-kp(s)+(k-b)\{\frac{a(s)}{c_1 b}+\frac{(1+c_1k)(1-\lambda_s)}{(1+c_1k)(1-\lambda_s)+c_1b\lambda_s}\bar{m}\}] = A(\lambda_s)m + u(s).$$

Using this the firm updates its expectations about m for the next period.

The Persistent Effects

Let us now consider the persistent effects of the unexpected monetary change beyond the first period. As for the speed of expectational adjustment we have the following proposition (the proof will be given in APPENDIX). Note that λ_s represents the degree of closeness of the average expectations to the true value of money supply, m. Thus the proposition implies that the more competitive the economy is, the faster the expectational adjustment is.

PROPOSITION 3: (Competition Makes Expectational Adjustment Fast)

λ_s is increasing in k, i.e., $d\lambda_s/dk > 0$, for all $s \geq 2$.

Although the formal proof is rather cumbersome, the intuition behind this proposition is simple. When the economy becomes more competitive, the prices in the first period are more rigid than before (PROPOSITION 2). Consequently the quantities bear the burden of adjustment. This implies that the demand at the firm changes with m more than before and reveals more information about m. In fact, (39) is rewritten as

$$(48) \qquad A(\lambda;\, k,\, b,\, c_1) = b + (k - b)\xi(\lambda;\, k,\, b,\, c_1).$$

Consequently $\partial A/\partial k > 0$. Because $\beta(2) = A(\lambda_1)m + u(1)$, an increase in k makes information $\beta(2)$ more informative about m. Thus λ_2 in the next period is larger than before, since the firms form expectations about m based on superior information. From (43) we have

$$(49) \qquad \lambda_2 = \lambda_1 + (1 - \lambda_1)\theta_s A(\lambda_1) = \lambda_1 + (1 - \lambda_1)\frac{\hat{V}_m(1)}{\hat{V}_m(1) + [\sigma_u^2/\{A(\lambda_1)\}^2]},$$

implying $\partial\lambda_2/\partial k > 0$ because λ_1 is independent of k and $\partial A/\partial k > 0$. A similar argument holds for λ_s; $s \geq 3$. Note that in this proposition the factor enabling the economy to adjust itself rapidly is the initially sluggish price adjustment. Thus the <u>short-run price inflexibility</u> is the necessary condition of the rapid overall adjustment, which is often considered as a benchmark of <u>the long-run flexibility of prices.</u>

Because of the gradual nature of the Kalman filter algorithm, the time path of λ_s is monotonically converging to unity, though it is highly non-linear. However, the time path of the average price, and consequently, that of the average output is not in general monotone. This is because the average price depends on not only the average expectations about m (whose time path is monotone) but also the error variance-covariance matrix in a very non-linear way (see (46)). In the following we examine the time path of the average output in several typical cases.

<u>The Adjustment of the Average Output</u>

Several examples are shown in FIGURE 3 through 7 in which the time paths of the average expectations and the average output are drawn for particular cases. In all cases, we are concerned with symmetric cases in which $m^* = \bar{m} = 0$. The values of the other parameters are, if not otherwise stated, $k = 5$, $b = 0.5$, $c_1 = 0.8$, $\bar{V}_m = 5$, and $\sigma_u^2 = 5$. These values are arbitrary ones for the illustrative purpose. However, the economy with $k = 5$ and $b = 0.5$ seems to be pretty competitive.

Among these examples, FIGURE 3 is particularly interesting. This example shows the case of pure uncertainty, in which $m = \bar{m} = 0$ so that $\Delta m = m - m^* = m - \bar{m} = 0$. This is the case in which the firms still think no change is likely (and their expectations are turned out to be correct), but they are not sure about their expectations.

In this example, the average expectations about m is always equal to zero (the true value). However, because of the imaginary uncertainty about m, the average price increases and the average output decreases after the introduction of uncertainty (period 0). The impact effect appears in period 1. (There is no effect in period 0 because firms do not expect the change at all and the change does not occur.) Moreover, the effect of uncertainty feeds itself and grows for some time after period 1, so that the average price continues to rise and the average output keeps declining. This self-feeding effect of uncertainty stems from the first term in (46), $\phi(\xi(\lambda_s))\cdot\hat{V}_m(s)$, because the second term and the third one in (46) are zero since we have $\bar{m} = m = 0$ in FIGURE 3.

The degree of expectational adjustment, λ_s, increases as time passes. This implies that the average price is increasingly sensitive to unknown m, and thus $\xi(\lambda_s)$ increases. Note that m is unknown, so that an increase in the sensitivity of the average price to m means an increase in uncertainty about the average price. This raises the firm's optimal price as explained earlier. Thus the increase is translated into an increase in $\phi(\xi(\lambda_s))$, which is the sensitivity of the average price with respect to the source of uncertainty, $\hat{V}_m(s)$. Although $\hat{V}_m(s)$, the uncertainty about unknown m, decreases as time passes, this reduction may be offset by an increase in $\phi(\xi(\lambda_s))$. This happens in FIGURE 3 initially. However, the effect of the reduction in $\hat{V}_m(s)$ dominates eventually, so that the average price begins to decline and return to the steady state.

FIGURES 4 and 5 are normal examples with a large change in m. In these cases, we have $\Delta m = 30$ so that $m = 30$ under our assumption. FIGURE 4 depicts the adjustment in a competitive economy with $k = 5$, while FIGURE 5 shows the non-competitive economy with $k = 1.51$ (recall that $k > b + 1$). In

FIGURE 3
Pure Uncertainty

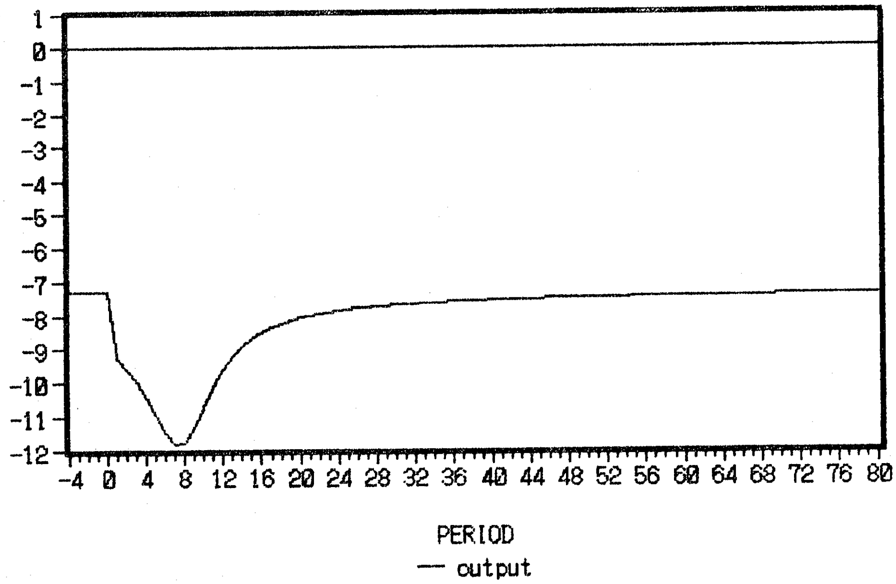


FIGURE 4
Competitive Adjustment

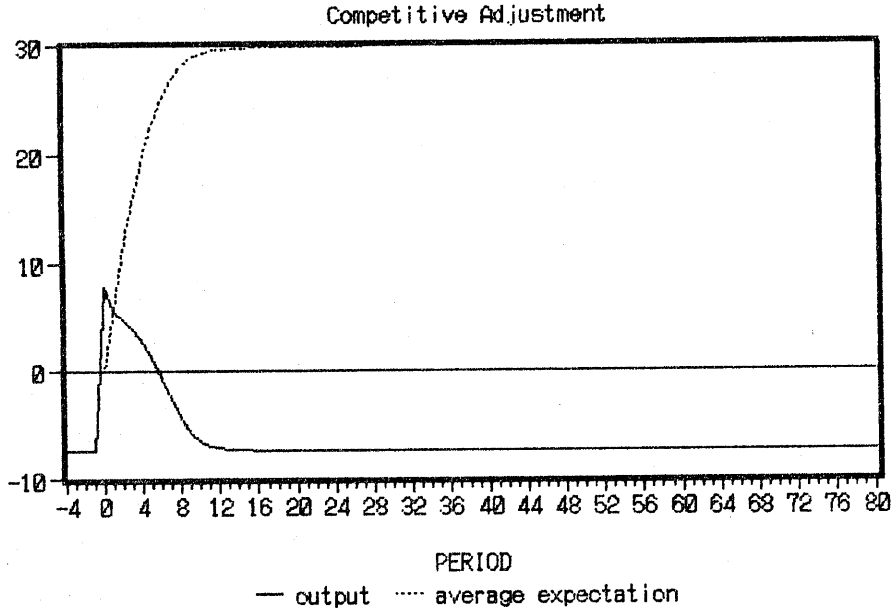


FIGURE 5
Non-Competitive Adjustment

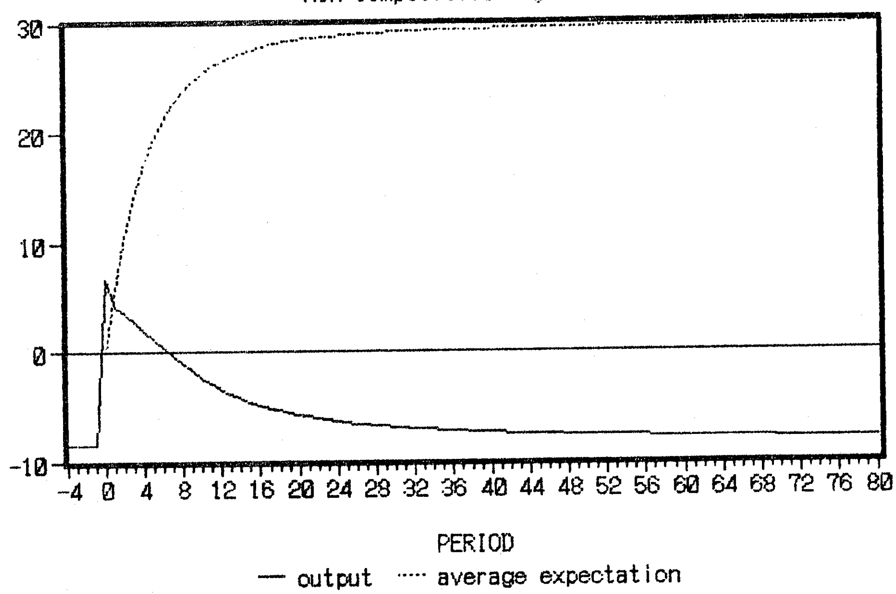


FIGURE 6
Small Changes: Positive vs. Negative

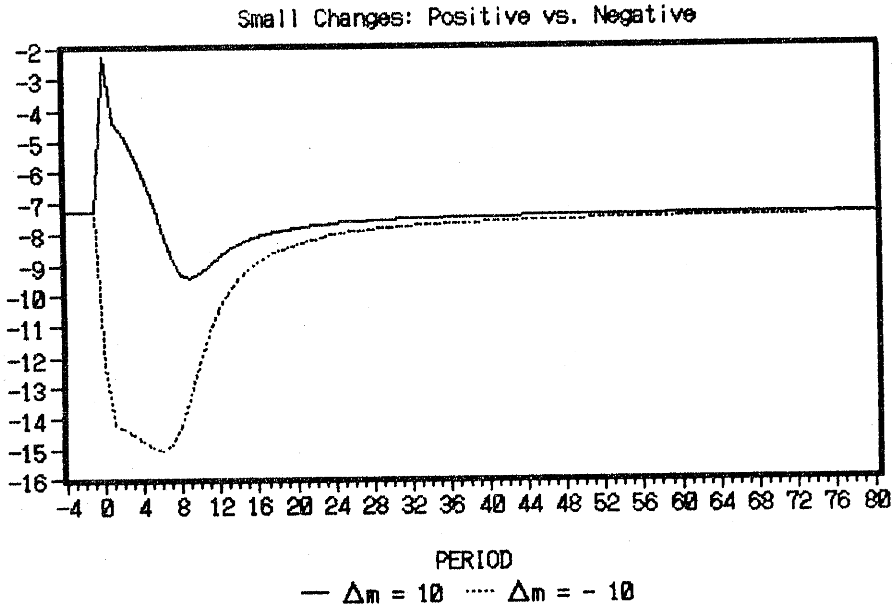
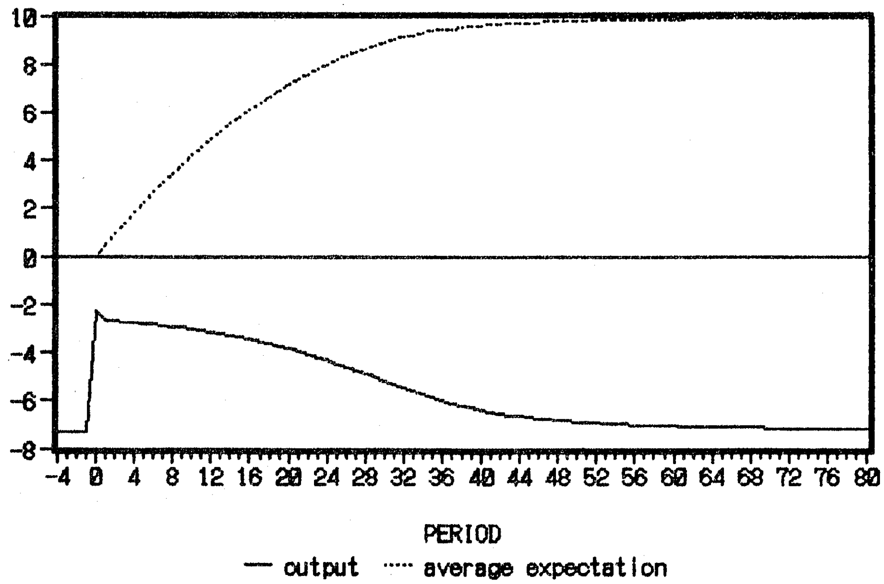


FIGURE 7

The Unprecedented Change



these cases, because Δm is large, the second term and the third one in (46) dominate the first term. This implies the adjustment of the average output is monotone like the average expectations.

FIGURES 4 and 5 show that the average output path as well as the average expectation path converges to a new steady state much more rapidly in the competitive case than in the non-competitive case. Thus PROPOSITION 3, which states the rapid convergence of the average expectations in a competitive economy, holds for the average-output adjustment. The speed of convergence for the output is relatively slow at the beginning in the competitive case, and then accelerated afterwards. Thus the competitive economy has short-run price inflexibility but long-run price flexibility compared with the non-competitive economy.

FIGURE 6 compares the effects of the positive monetary change and the negative one in the case of small changes. Specifically, the case that $\Delta m = 10$ is compared with the case that $\Delta m = -10$. This figure illustrates dramatically the asymmetric response of the economy to monetary changes. Because of the effect of uncertainty on prices, the economy is more vulnerable to deflationary shocks than to inflationary ones.

This figure also shows complicated response of the average output to the small change in money supply. The case of the positive change ($\Delta m = 10$) reveals the possibility that unexpected injection of money designed to pump up economic activities may backfire. The positive effect on output is relatively small and dissipates quickly. The average output declines below the steady state value for some periods.

The last FIGURE 7 depicts the effect of an unexpected, unprecedented change. In this example, $\bar{V}_m$ is set to unity, not five as in the previous examples. This means that past changes in money supply are, if happened, very small. However, we have $\Delta m = 10$ in FIGURE 7, so that this change is very large compared with the expectations held by firms. The result is a very slow expectational adjustment, which also makes the output adjustment very sluggish. In initial several periods, the adjustment is so slow so that outside observers may erroneously conclude the economy is almost in hysteresis.

4. CONCLUDING REMARKS

In this paper we have shown that in a monopolistically competitive economy under imperfect and incomplete information, (1) prices are rigid downward but flexible upward, and (2) competition makes prices rigid in the short run but flexible in the long run. These results are obtained under the assumption of rational learning. Although the model presented in this paper is log-linear, the intuitive explanation given in this paper suggests that these characteristics are likely to hold in more general cases of preference and technology, so long as the framework of rational learning is justified.

The robustness of the results thus rests on (1) the applicability of of the imperfect and incomplete information assumption and (2) that of the rational learning assumption. The justification of the first assumption is presented in Nishimura (1987b), in which public information provision and private information sharing are not likely to be materialized. When information the government has contain substantial errors, The direct policy that keeps the information secret and controls the change directly by utilizing the information, is generally better than the indirect policy that makes it public and does nothing other than that. Private information sharing is not likely because after knowing their own conditions, some firms may find it profitable to prevent such information sharing from being agreed, because information sharing reduces the degree of imperfect and incomplete information about the average price. Such reduction implies the more competitive economy, and less profits for them.

The second assumption of rational learning is often criticized because it requires that firms have an enormous amount of structural information about the economy, and that they can utilize this information very effectively. In a real economic situation, such kind of information is not accessible to economic agents, and they simply do not have ability to do complex Bayesian learning described in this paper.

The real issue, however, is not whether or not economic agents literally know the structure, but how the working of the economy is different from the model described in the paper if they have only limited information about the structure and try to use such information in the learning process. In this perspective, the rational learning analysis is the frame of reference. Moreover, the other learning assumptions (see, for example, Bray and Savin (1986)) are more or less arbitrary at the present state of knowledge. Thus if (1) one is not so certain about the learning process economic agents employ but (2) relatively confident that economic agents are not so ignorant about the structure, the rational learning assumption is justified at least as a first approximation.

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APPENDIX

In this appendix the proof of PROPOSITION 3 is given. First, note that the dynamics of the economy is governed by the following system of two difference equations (see (39) and (43)):

$$\text{(a1)} \quad \lambda_t = \lambda_{t-1} + \frac{A_{t-1}^2 \hat{V}_m(t-1)}{A_{t-1}^2 \hat{V}_m(t-1) + \sigma_u^2}(1 - \lambda_{t-1}) \text{ for } t \geq 1, \text{ where } \lambda_0 = 0.$$

$$\text{(a2)} \quad \hat{V}_m(t) = \frac{\hat{V}_m(t-1)\,\sigma_u^2}{A_{t-1}^2 \hat{V}_m(t-1) + \sigma_u^2} \text{ for } t \geq 1, \text{ where } \hat{V}_m(0) = \bar{V}_m.$$

Here A_t is such that

$$\text{(a3)} \quad A_t = b\frac{1 + c_1 k - \lambda_t}{(1 + c_1 k)(1 - \lambda_t) + c_1 b \lambda_t} \text{ for } t \geq 1, \text{ and } A_0 = b.$$

The following claim excludes the possibility of overshooting expectations. The proof is trivial, thus omitted. This characteristic of the expectation formation is due to the gradual nature of the Kalman filter learning.

<u>CLAIM</u> (No Overshooting Expectations)

Let λ_t be the solution to the system of difference equations (a1) and (a2). If $0 < \sigma_u^2 < \infty$ and $0 < \bar{V}_m < \infty$, then we have (i) $0 < \lambda_t < 1$, and (ii) $\lambda_t < \lambda_{t+1}$, for all $t \geq 1$.

Next we prove the main proposition that λ_t is increasing in k for all $t \geq 2$, that is, that the more competitive the economy is, the faster the average expectations converge to the true value of money supply. The following two lemmas are utilized in the proof of the proposition.

Lemma 1

Let λ_t, $\hat{V}_m(t)$ and A_t be the solutions of (a1) through (a3). Then

(a4) $\lambda_t = 1 - \Phi_t(1 - \lambda_1)$ for $t \geq 2$, where $\Phi_t = \dfrac{\sigma_u^{2(t-1)}}{\Pi_{h=1}^{t-1}(A_h^2 \hat{V}_m(h) + \sigma_u^2)}$.

Proof

In the case of t = 2, from (a1) we have

$$\lambda_2 = \lambda_1 + \left(1 - \frac{\sigma_u^2}{A_1^2\hat{V}_m(1) + \sigma_u^2}\right)(1 - \lambda_1) = 1 - \frac{\sigma_u^2}{A_1^2\hat{V}_m(1) + \sigma_u^2}(1 - \lambda_1).$$

Therefore (a4) holds at t = 2.

Next suppose that (a4) holds at t = s - 1. Then in the case of t = s, we have from (a1)

$$\lambda_s = \lambda_{s-1} + \frac{A_{s-1}^2\hat{V}_m(s-1)}{A_{s-1}^2\hat{V}_m(s-1) + \sigma_u^2}(1 - \lambda_{s-1})$$

$$= \{1 - \Phi_{s-1}(1 - \lambda_1)\} + \frac{A_{s-1}^2\hat{V}_m(s-1)}{A_{s-1}^2\hat{V}_m(s-1) + \sigma_u^2}[1 - \{1 - \Phi_{s-1}(1 - \lambda_1)\}]$$

$$= 1 - \left(1 - \frac{A_{s-1}^2\hat{V}_m(s-1)}{A_{s-1}^2\hat{V}_m(s-1) + \sigma_u^2}\right)\Phi_{s-1}(1 - \lambda_1)$$

$$= 1 - \frac{\sigma_u^2}{A_{s-1}^2\hat{V}_m(s-1) + \sigma_u^2} \frac{\sigma_u^{2(s-2)}}{\Pi_{h=1}^{s-2}(A_h^2 \hat{V}_m(h) + \sigma_u^2)} (1 - \lambda_1)$$

$$= 1 - \frac{\sigma_u^{2(s-1)}}{\Pi_{h=1}^{s-1}(A_h^2 \hat{V}_m(h) + \sigma_u^2)} (1 - \lambda_1) = 1 - \Phi_s(1 - \lambda_1)$$

Therefore (a4) also holds in the case of $t = s$. This completes the proof of Lemma 1.

Lemma 2

Let λ_t, $\hat{V}_m(t)$ and A_t be the solutions of (a1) through (a3). Then we have

(a5) $\Pi_{h=1}^{t}(A_h^2\hat{V}_m(h) + \sigma_u^2) = \sigma_u^{2(t-1)}\{\hat{V}_m(1)\Sigma_{h=1}^{t}A_h^2 + \sigma_u^2\}$ for $t \geq 1$.

Proof

In the case of $t = 1$, (a5) clearly holds. In the case of $t = 2$, using the equation (a2) we have

$$\Pi_{h=1}^{2}(A_h^2\hat{V}_m(h) + \sigma_u^2) = (A_1^2\hat{V}_m(1) + \sigma_u^2)(A_2^2 \frac{\hat{V}_m(1)\sigma_u^2}{A_1^2\hat{V}_m(1) + \sigma_u^2} + \sigma_u^2)$$

$$= A_2^2\hat{V}_m(1)\sigma_u^2 + (A_1^2\hat{V}_m(1) + \sigma_u^2)\sigma_u^2 = \sigma_u^2\{\hat{V}_m(1)\Sigma_{h=1}^{2}A_h^2 + \sigma_u^2\}.$$

Therefore (a5) holds at $t = 2$.

Next suppose that (a5) holds true at $t = s - 2$ and $t = s - 1$. In the case of $t = s$, using (a2), we obtain

$$\Pi_{h=1}^{s}(A_h^{\,2}\hat{V}_m(h) + \sigma_u^{\,2})$$

$$= A_s^{\,2}\cdot\frac{\hat{V}_m(s-1)\cdot\sigma_u^{\,2}}{A_{s-1}^{\,2}\hat{V}_m(s-1) + \sigma_u^{\,2}}\cdot(A_{s-1}^{\,2}\hat{V}_m(s-1) + \sigma_u^{\,2})\cdot\Pi_{h=1}^{s-2}(A_h^{\,2}\hat{V}_m(h) + \sigma_u^{\,2})$$

$$+ \sigma_u^{\,2}\cdot\Pi_{h=1}^{s-1}(A_h^{\,2}\hat{V}_m(h) + \sigma_u^{\,2})$$

$$= [\frac{A_s^{\,2}\sigma_u^{\,2}}{A_{s-1}^{\,2}}(A_{s-1}^{\,2}\hat{V}_m(s-1) + \sigma_u^{\,2}) - \frac{A_s^{\,2}\sigma_u^{\,4}}{A_{s-1}^{\,2}}]\cdot\Pi_{h=1}^{s-2}(A_h^{\,2}\hat{V}_m(h) + \sigma_u^{\,2})$$

$$+ \sigma_u^{\,2}\cdot\Pi_{h=1}^{s-1}(A_h^{\,2}\hat{V}_m(h) + \sigma_u^{\,2})$$

$$= \frac{A_s^{\,2}\sigma_u^{\,2}}{A_{s-1}^{\,2}}\cdot\Pi_{h=1}^{s-1}(A_h^{\,2}\hat{V}_m(h) + \sigma_u^{\,2}) - \frac{A_s^{\,2}\sigma_u^{\,4}}{A_{s-1}^{\,2}}\cdot\Pi_{h=1}^{s-2}(A_h^{\,2}\hat{V}_m(h) + \sigma_u^{\,2})$$

$$+ \sigma_u^{\,2}\cdot\Pi_{h=1}^{s-1}(A_h^{\,2}\hat{V}_m(h) + \sigma_u^{\,2})$$

$$= \frac{A_s^{\,2}\sigma_u^{\,2}}{A_{s-1}^{\,2}}\cdot\sigma^{2(s-2)}\cdot\{\hat{V}_m(1)\Sigma_{h=1}^{s-1}A_h^{\,2} + \sigma_u^{\,2}\} - \frac{A_s^{\,2}\sigma_u^{\,2}}{A_{s-1}^{\,2}}\cdot\sigma^{2(s-2)}\cdot\{\hat{V}_m(1)\Sigma_{h=1}^{s-2}A_h^{\,2} + \sigma_u^{\,2}\}$$

$$+ \sigma_u^{\,2(s-1)}\{\hat{V}_m(1)\ \Sigma_{h=1}^{s-1}A_h^{\,2} + \sigma_u^{\,2}\}$$

$$= A_s^{\,2}\sigma_u^{\,2(s-1)}\hat{V}_m(1) + \sigma_u^{\,2(s-1)}\{\hat{V}_m(1)\ \Sigma_{h=1}^{s-1}A_h^{\,2} + \sigma_u^{\,2}\}$$

$$= \sigma_u^{\,2(s-1)}\{\hat{V}_m(1)\ \Sigma_{h=1}^{s}A_h^{\,2} + \sigma_u^{\,2}\}.$$

Therefore (a5) holds in the case of $t = s$. This ends the proof.

The Proof of PROPOSITION 3

First, note that λ_1 and $\hat{V}_m(1)$ do not depend on k, because from (a1) and (a2), we have

$$\lambda_1 = \frac{b^2 \bar{V}_m}{b^2 \bar{V}_m + \sigma_u^2} \quad \text{and } \hat{V}_m(1) = \frac{\bar{V}_m \sigma_u^2}{b^2 \bar{V}_m + \sigma_u^2}.$$

Since Lemma 1 and Lemma 2 hold true, λ_t can be expressed for $t \geq 2$ as

$$\lambda_t = 1 - \frac{\sigma_u^2}{\hat{V}_m(1) \quad \Sigma_{h=1}^{t-1} A_h^2 + \sigma_u^2} (1 - \lambda_1).$$

Thus in the case of t = 2 we have

$$\lambda_2 = 1 - \frac{\sigma_u^2}{\hat{V}_m(1) A_1^2 + \sigma_u^2} (1 - \lambda_1) \text{ where } A_1 = b\frac{1 + c_1 k \ - \lambda_1}{(1 + c_1 k)(1 - \lambda_1) + c_1 b \lambda_1}.$$

Because we have $dA_1/dk > 0$, we obtain $d\lambda_2/dk > 0$.

Now suppose that $d\lambda_t/dk > 0$ for $t = 2, 3, \cdots, s-1$. Consider the case of $t = s$. Because we have

$$\lambda_s = 1 - \frac{\sigma_u^2}{\hat{V}_m(1) \Sigma_{h=1}^{s-1} A_h^2 + \sigma_u^2} (1 - \lambda_1),$$

it is sufficient to show $dA_t/dk > 0$ for $t = 1, 2, \cdots, s-1$, in order to prove $d\lambda_s/dk > 0$, As for A_1, we have already shown $dA_1/dk > 0$. Note that for $t = 2, 3, \cdots, s-1$, we have from (a3).

$$A_t = b\frac{1 + c_1k - \lambda_t}{(1 + c_1k)(1 - \lambda_t) + c_1b\lambda_t}$$

Taking derivative with respect to k we obtain

$$\frac{dA_t}{dk} = \frac{c_1b}{\Delta_t^2}\left[(1 - \lambda_t)\lambda_t + c_1b\lambda_t + (k - b)(1 + c_1k)\frac{d\lambda_t}{dk}\right]$$

where $\Delta_t = (1 + c_1k)(1 - \lambda_t) + c_1b\lambda_t$. Under our assumptions, we have $k > b$ and $\frac{d\lambda_t}{dk} > 0$ for $t = 2, \cdots, s-1$, and from CLAIM 1, $0 < \lambda_t < 1$ for all t. Therefore $dA_t/dk > 0$ for $t = 2, \cdots, s-1$. Thus we obtain $d\lambda_s/dk > 0$. This completes the proof of Proposition 2.

NOTES

1 This assumption corresponds to the standard one in the industry monopolistic competition models that the individual demand curve is flatter than the market demand curve. See Nishimura (1987a). Note that the condition is not $k > b$ but $k > b + 1$, because firms are concerned with <u>real</u> profits (nominal profits divided by the average price) in our macroeconmic framework, while in the traditional industry framework firms' ojective is to maximize <u>nominal</u> profits.

2 Thus in our model firms cannot use money supply figures announced within the period in determining their prices of this period. Thus the criticism raised by Boschen and Grossman (1982) against market-clearing incomplete-information rational expectations models does not apply to our model.

3 In practice, the firm is likely to discern the occurrence of the change without any outside information from the past series of β only. If β is large for a long time, the firm suspects that the change in money supply has occurred. Because to incorporate expectations about the timing of the change is very cumbersome, we are concerned only with expectations about its magnitude in the text.

4 Let $e[\bar{p}(1)|\hat{m}(1)] = H_1 + J_1\hat{m}(1)$. Inserting this into the optimal pricing formula, and averaging over all firms, we get

$$\bar{p}(1) = \frac{a(1) + c_1 b\{(1-\lambda_1)\bar{m} + \lambda_1 m\} + \{1+c_1(k-b)\}[H_1+J_1\{(1-\lambda_1)\bar{m} + \lambda_1 m\}].}{1 + c_1 k}$$

where we use (21) and the definition of $\bar{p}(1)$. Applying $e[\cdot|\hat{m}(1)]$ on both sides of the above expression and rearranging the terms, we obtain

$$e[\bar{p}(1)|\hat{m}(1)] = \frac{a(1) + [c_1 b + \{1+c_1(k-b)\}J_1](1-\lambda_1)\bar{m} + \{1+c_1(k-b)\}H_1}{1 + c_1 k} + \frac{[c_1 b + \{1+c_1(k-b)\}J_1]\lambda_1}{1 + c_1 k}\hat{m}(1).$$

Equate the coefficient of this with the original expression to find the rational expectation values for H_1 and J_1. This procedure yields (24).

5 The same effect of incomplete information makes the long-run output level depend on the variance of temporary monetary changes (long-run non-neutrality of money) in the model of Nishimura (1987a).