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Chapter 9

Jevons and Edgeworth

1. Hermann Heinrich Gossen

As we started Chapter 7, i.e., the chapter on Walras, with the contributions made by Cournot, we may perhaps begin this chapter with the discussion of the significance of Gossen's work. This is because Jevons clearly admitted that Gossen had completely anticipated him as regards the general principles and method of the theory of economics, though at the same time he insisted that he never saw nor so much as heard any hint of the existence of Gossen's book, published in German in 1854, before 1878, i.e., before the publication of his own book in 1871 (Jevons[ ], pp. xxxviii - xli). Particularly we wish to emphasize the fact that Gossen anticipated not only the theory of marginal utility of which Jevons shared the fame with Walras and Menger, but also Jevons's theory of exchange in the market, which was based on a view of market quite different from that of Cournot and Walras and was succeeded and developed by Edgeworth. After Gossen's work is reviewed in this section, sections 2 and 3 are devoted to discuss, respectively, economic theories of Jevons and Edgeworth. We shall consider, finally in section 4, the significance which these theories have to the modern theories of mathematical economics and industrial organization.

Heinrich Wilhelm Joseph Hermann Gossen was born in 1810 at Duren, a small town near Cologne, which was then part of the French Napoleonic Empire. His father was a government official, tax collecter in the French administration and also in the Prussian government reinstated after the collapse of the French Empire. Gossen studies in Universities of Bonn and Berlin law and public administration, since his father insisted that he should also be a government official. Gossen's career of public service was, however, not successful, with misplaced effort and disappointments. This may be attributed to his predilection to abstract studies. He retired into private life after 1947. He was first occupied with projects of insurance, but afterwards with his book entitled Entwickelung der Gesetze des menschlichen Verkehrs, und der darous fliessenden Regeln für menschliches Handeln, which was published in 1854. Although Gossen claimed honors in economic science equal to those of Copernicus in astronomy ([9], p. cxlvii), this book was ignored even in Germany before the death of the author in 1858, let alone in England. It was in 1879 that Jevons recognized Gossen as one of the predecessors of the marginal revolution in general and of his own theory in particular.[1)]

Gossen's Entwicklung is originally not divided into parts or chapters. As Walras[21] saw it, however, in a natural manner it falls into two parts of approximately equal length. The first part is devoted to pure theory and comprises the laws of enjoyment and of work, the laws of exchange and the theory of rent (Gossen[9], Chapters 1 - 13, pp. 1 - 139). The second part is devoted to applied theory and comprises rules of conduct pertaining to desires and pleasures, and the refutation of certain social errors concerning education, money, credit and property ([9], Chapters 14 - 25, pp. 140 - 299). In the first part, Gossen started with the so-called Gossen's first law of enjoyment that "The magnitude [intensity] of pleasure decreases continuously if we continue to satisfy one and the same enjoyment without interruption until satiety is ultimately reached" and derived from it the so-called Gossen's second law of enjoyment that "Man obtains the maximum of life pleasure if he allocates all his earned money E between the various pleasures and determines the e in such a manner that the last atom of money spent for each pleasure offers the same amount [intensity] of pleasure."[2)]

Walras[21] agreed with Jevons to recognize Gossen's laws of enjoyment and his theory of exchange which follows in the first part of Entwicklung as one of the predecessors of the marginal revolution. Walras insisted, however, that he can, unlike Jevons, protect the priority of a good part of his discoveries, since neither Gossen nor Jevons has as much as touched the problem of the determination of the equilibrium price under the mechanism of free competition with the unity of the market price. According to Walras, the competitive equilibrium price is established as a result of the equalization of demand and supply, both of which are functions of the market price, common and identical to all exchangers in the market. The second part of Entwicklung is also evaluated high by Walras[21], since it contains discussions on the nationalization of land.

From the point of view of this chapter, however, what is most interesting is Gossen's theory of exchange, which can be considered as the predecessor of the theory of Jevons and also of Edgeworth.[3)] Gossen considered the case of two individual two good exchange, with highly special assumptions on the shape of the marginal utility curves and on the initially available quantities of goods to each individuals. It clearly shows, however, a view of markets, which is different from that of Cournot and Walras discussed in Chapter 7, and is similar to those of Jevons and Edgeworth to be discussed in the following sections.

Suppose an individual A has initially the quantity ad of good I and another individual B owns the quantity a'd' of good II. In the lower part of Figure 1, the line c'b' signifies the marginal utility for A of good II which is obtained through the exchange with B, while the kinked line a'bc shows the marginal utility for A of good I which he gives up to obtain good II. It is assumed that the utility of good I is satiated for the individual A at the quantity of ab which is less than the quantity ad. Similarly, in the upper part, the line cb is the marginal utility curve for B of good I which is obtained through the exchange with A, while the kinked line ab'c' signifies the marginal utility lost for b of good II which he gives to A in exchange for good I. It is assumed that the utility of good II is satiated for the individual B at the quantity of a'b' which is less than the quantity a'd'.[4)]

Gossen first assumes that one unit of good I is exchanged against one unit of good II and that each individual trades off exactly what is superfluous to him. In other words, the individual A offers the quantity bd of good I which is equal to the difference between his initial holding ad and the quantity ab at which his utility is satiated, while the individual B offers the quantity b'd' of good II which is equal to the difference between his initial holding a'd' and the quantity a'b' at which his utility is satiated. A's utility gains from exchange can be seen in the lower part of Figure 1 as the area of the trapezoid a'c's'b, since he has now the quantity bd of good II, which is equal to the quantity b'd'. Similarly, in the upper part of Figure 1, the utility gains from exchange of B is the area of the trapezoid acsb', since he has now the quantity b'd' of good I, which is equal to the quantity bd.

Gossen recognizes "that the gain from barter is by no means exhausted by each individual trading off what is superfluous to him" ([9], p. 97), since in Figure 1 it is clear that the marginal utility of the good to be received is higher, for both of A and B, than that of the good to be surrendered if one more unit of good I is exchanged against one more unit of good II. From the point of view of the individual A, the most advantageous trade is at the point h' where two marginal utility lines cross, since "up to this point, the value of the commodity given up remains smaller than what is obtained in exchange. Were the exchange to go beyond this, to the point l', the relation would reversed." "When equal quantities are exchanged, the barter remains advantageous for A until the value of the last atoms of both objects now owned by A have become equal" (Gossen[9], p. 99).

From the point of view of the individual B, however, the point f which corresponds to the point h' is by no means the most advantageous, since the marginal utility fr of good I is still higher than the marginal utility fg of good II. B will be able to continue the equal quantity barter of two goods advantageously beyond the point at which the barter is most advantageous for A. "Such a continuation, however, would run contrary to the interest of A : yet the barter need not come to an end at h'. The extraordinary advantage that its continuation would bring to B may induce B to compensate A for the disadvantage suffered by the latter by giving up a greater quantity to A than B receives" ([9], p. 99). Suppose A gives up to B, in addition to the quantity a'h' of the good I, also the quantity h'l'. The utility lost by this additional offer is the area of the trapezoid h'l'm'k'. If A is to receive for this quantity of good I only an equal quantity of good II, its value for A would be represented by the area of the trapezoid h'l'q'k'. In this case, A would clearly suffer a loss of value equal to the area of the triangle k'q'm'. A will be compensated for this loss, however, if B, instead of giving A just the quantity h'l', actually gives him a quantity h'n' such that the area of trapezoid l'n'p'q' $\geq$ the area of the triangle k'q'm'. "In the present case, B can proceed in this way with great advantage" ([9], pp. 99), since the quantity h'n' = fn of good II represents for B the value of the trapezoid fnpg, while the quantity fl = h'l' of good I, which B receives for its return, has for B the value represented by the area of the trapezoid flmr, and clearly the area of the trapezoid gqmr is larger than the area of the trapezoid lnpq, in the upper part of Figure 1.

After this interesting argument that A and B can continue the process of exchange by changing the relative price of two goods, Gossen's argument becomes somewhat confused, as is pointed out by Jaffé [12] and Georgescu-Roegen ([9], p. cxi). Then he jumped to a conclusion that "after the exchange, each of the two commodities must be distributed between A and B in such a manner that the last atom one receives from the other will provide the same amount of value for both" ([9], p. 100). Unfortunately, this conclusion is neither interesting nor correct, since it does not make sense unless one admits utilitarian international comparison of utility, and it is not necessarily possible to reach such a distribution of commodities through the process of exchange described by Gossen.

What is important is, however, that Gossen clearly offered a view of market which is different from that of Cournot and Walras, in the sence that the existence of the unique market prices is not presupposed and the relative price is changing in the process of exchange even between the same pair of individuals. Of course, Gossen did not state correctly the final outcome of such a process of exchange. Nor he made clear the relation between different views of market. These are left to be considered to Jevons, who admitted that Gossen is the predecessor to him, and Edgeworth, who succeeded Gossen-Jevons view of market, as will be seen in the following sections of this chapter.

2. Jevons's Theory of Exchange.

William Stanley Jevons was born at Liverpool in 1835 as the ninth child of Thomas, who was in the iron trade and interested in the engineering innovations, and Mary Ann, a poetess, who was the daughter of a learned historian, William Roscoe. Stanley was raised in Unitarian environment in which social and economic problems were often discussed. In 1850, he entered University College in London and studied mathematics, biology, chemistry and metallurgy. Because of the bankruptcy of the parental business, however, he accepted a position as an assayer to the mint in Sydney, where he remained five years. In Australia, the study of meteorology attracted him strongly. As was pointed out by Keynes[14], furthermore, he was struck with the original ideas on economics that he later developed into an important part of the marginal revolution.

On his return to London in 1859, he resumed his studies at University College, but changed his specialization from mathematics and chemistry into logics and economics. Jevons sent a short paper entitled <u>Notice of a General Mathematical Theory of Political Economy</u> to Section F of the British Association for the Advancement of Science, which was read in his absence before the 1862 meeting at Cambridge, but attracted no attention and was not printed. In 1866, he was appointed professor of logic and mental and moral philosophy and Cobden lecturer on political economy in the Owens College, Manchester. In 1876, after nervous and physical exhaustion, Jevons left Manchester, and was elected professor of political economy in University College, London, a post he retained till 1880. Two years later, unfortunately, he met his death in a drawning accident.

Jevons's first book on economics is <u>The Coal Question ; an Inquiry</u>

concerning the Progress of the Nation and the Probable Exhaustion of our Coal Mines (1865). His natural law of social growth is based on an analogy to Malthus's law of population and in a geometrical progression of industries coal plays the role played in Malthus's theory by corn. The appearance of articles by Fleeming Jenkin prompted the publication of Jevons's Theory of Political Economy (1871), which is developed from his paper read before the 1862 meeting of the British Association. Jevons is also well known by his life long inductive studies of commercial fluctuations and prices, which are collected in Investigations in Currency and Finance (1884). His sunspot theory of commercial crises integrated earlier works on economic fluctuations and prices with his life long interest in astronomy and meteorology. Finally, Jevons wrote many books on logic, the largest of which is The Principles of Science, A Treatise on Logic and Scientific Method (1874).

Theory of Political Economy ([13]) consists of Preface of to the First Edition, Preface to the Second Edition, eight Chapters and two appendices. In Preface to the first edition (1871), Jevons declared "to treat Economy as a Calculus of Pleasure and Pain" ([ ], p. vii), while in Preface to the Second Edition (1879), he stated that "the question is not so much whether the theory given in this volume is true, but whether there is really any novelty in it" ([ ], p. xviii) and frankly admitted the existence of many predecessors including Gossen. Chapter I is Introduction, which starts with the declaration that "value depends entirely upon utility" ([13], pp. 1 - 2) against the then prevailing labor theory of value. Jevons argues "that Economics, if it is to be a science at all, must be a mathematical science, --- simply because it deals with quantities" ([13], 3 - 4). "The theory consists in applying the differential calculus to the familiar notions of wealth, utility, value, demand, supply, capital, interest, labour, and all the other quantitative notions belonging to the daily operations of industry" ([13], p. 3).

Following two Chapters are respectively entitled Theory of Pleasure and Pain and Theory of Utility. "Whatever can produce pleasure or prevent pain _may_ possess utility" ([13], p. 41). Jevons made the distinction between total and marginal utilities. "We must now carefully discriminate between the _total utility_ arising from any commodity and the utility attaching to any particular portion of it" ([13], p. 49). The marginal utility is called _final degree of utility_ and is defined "as meaning the degree of utility of the last addition, or the next possible addition of a very small, or infinitely small, quantity to the existing stock" ([13], p. 55). Then, the so-called second law of Gossen is derived in the discussion on the distribution of commodity in different uses. "We must --- have the _final degrees of utility_ in the two uses equal" ([13], p. 65).

Chapter IV of Jevons's _Theory_ is Theory of Exchange. After the definition of trading body and the argument on the law of indifference, Jevons demonstrates the proposition which is called "the keystone of the whole Theory of Exchange, and of the principal problems of Economics" ([13], p. 103), by using the so-called equations of exchange which we shall discuss detailedly in the below. The supply of labor is considered in Chapter V in terms of the painfulness of labor and the utility of the commodity produced by the application of labor. This is, therefore, not so much the theory of labor supply as the theory of production.[5)] After Theory of Rent is discussed in Chapter VI, Chapter VII is devoted to consider Theory of Capital. The rate of interest of capital is derived as "the rate of increase [with respect to "the interval of abstinence"] of the produce divided by the whole produce" ([13], p. 267). Chapter VIII is Concluding Remarks.

Although Jevons first believed that he was the original discoverer of the concept of the marginal utility, later he recognized the existence of the predecessors like Dupuit and Gossen. Therefore, the original contribution, if any, of Jevons should be sought, not in the theory of utility (Chapter III of his Theory), but in the theory of exchange (Chapter IV of Theory). According to Fisher ([8], pp. 68, 98, 155), however, Jevons's model in his discussion on distribution of commodity in different uses (in Chapter III of Theory) is the initial model with the hard core of his research programme, from which a series of other models are derived.

"Let's be the whole stock of some commodity, and let it be capable of two distinct uses. Then we may represent the two quantities appropriated to these uses by x_1 and y_1, it being a condition that $x_1 + y_1 = s$. ---- Let Δu_1, Δu_2, be the increments of utility, which might arise respectively from consuming an increment of commodity in the two different ways. When the distribution is completed, we ought to have $\Delta u_1 = \Delta u_2$; or at the limit we have the equation

$$[\ (1)\ ] \qquad du_1/dx = du_2/dy$$

which is true when x, y are respectively equal to x_1, y_1. We must, in other words, have the final degree of utility in the two uses equal" (Jevons[13], pp. 64 - 65).

Soon we shall see that Jevons's model for the equations of exchange is based on this initial model for Gossen's second law, in such a way that Jevons's view of the equilibrium of exchange in the market is similar to that of Gossen but different from those of Cournot and Walras. Before considering Jevons's equations of exchange, however, we have to discuss two concepts, i.e., the trading body and the law of indifference, both of which are very important if we are to understand the true implications of Jevons's theory of exchange.

"By a trading body I mean, in the most general manner, any body either of buyers or sellers. The trading body may be a single individual in one case; it may be the whole inhabitants of a continent in another; it may be the individuals of a trade diffused through a country in a third. England and North America will be trading bodies if we are considering the corn we receive from America in exchange for iron and other goods. The continent of Europe is a trading body as purchasing coal from England. The farmers of England are a trading body when they sell corn to the millers, and the millers both when they buy corn from the farmers and sell flour to the bakers" (Jevons[13], pp. 95 - 96).

Though some historians of economic thought are critical of Jevons's use of trading body (Blaug[2], p. 312, and Howey[11], p. 52), similar concepts have been used very often in various fields of economic theory. For example, countries are regarded, as Jevons himself suggested, as trading bodies and the utility function, or indifference map, of a country is considered in the theory of international trade.[6)] Another example concerns the models used in theories of the microeconomic foundations of macroeconomics, in which the representative or aggregate household and representative or aggregate firm exchange labor services and consumers' goods (Benassy[1], and Malinvaud[16]).

The reason why Jevons uses such an artificial invention to explain exchange is that the behavior of the aggregate or average person is much more stable than that of an individual person. In other words, differential calculus can be used only for the case of the aggregate or average person.

"The use of an average, or, what is the same, an aggregate result, depends upon the high probability that accidental and disturbing cases will operate, in the long run, as often in one direction as the other so as to neutralize each other. Provided that we have a sufficient number of independent cases, we may then detect the effect of any tendency, however slight. Accordingly, questions which appear, and perhaps are quite indeterminate as regards individuals, may be capable of exact investigation and solution in regard to great masses and wide averages" ([13], p. 17).

"A single individual does not vary his consumption of sugar, butter, or eggs from week to week by infinitesimal amounts, according to each small change in the price. He probably continues his ordinary consumption until accident directs his attention to a rise in price, and he then, perhaps, discontinues the use of the articles altogether for a time. But the aggregate, or what is the same, the average consumption, of a large community will be found to vary continuously or nearly so" ([13], pp. 96 - 97).

The law of indifference, which insists that there is only one price for each commodity in equilibrium, is necessary, as we shall see, to derive Jevons's equations of exchange from his initial model of distribution of commodity in different uses. Jevons explains that the law is established only at the equilibrium through the arbitrage behavior of sellers and buyers. In other words, it is not to be presupposed, in the case of markets he has in mind, unlike in the case of the well organized markets considered by Cournot and Walras.

"If, in selling a quantity of perfectly equal and uniform barrels of flour, a merchant arbitrarily fixed different prices on them, a purchaser would of course select the cheaper obes ---- Hence follows what is undoubtedly true, with proper explanations that in the same open market, at any one moment, there cannot be two prices for the same kind of article" ([13], 98 - 99).

"It follows that the last increments in an act of exchange must be exchanged in the same ratio as the whole quantities exchanged. Suppose that two commodities are bartered in the ratio of x for y; then every m-th part of x is given for the m-th part of y, and it does not matter for which of the m-th parts ---- even an infinitely small part of x must be exchanged for an infinitely small part of y, in the same ratio as the whole quantities. This result we may express by stating that the increments concerned in the process of exchange must obey the equation $dy/dx = y/x$" ([13], pp. 102 - 103).

Behind this "statical view of the equation" there must be a dynamic process of trading. What Jevons had in mind is a piecemeal exchange process, since "dynamically we could not treat the ratio of exchange otherwise than as the ratio of dy and dx, infinitesimal quantities of commodity" ([13], p. 102). In other words, the equation of the law of indifference

(2) $$dy/dx = y/x$$

is an equilibrium condition which is not established "when equilibrium is not attained."

Now we are ready to consider the proposition which contains "the keystone of the whole theory of exchange, and of the principal problems of economics."

"The ratio of exchange of any two commodities will be the reciprocal of the ratio of the final degrees of utility of the quantities of commodity available for consumption after the exchange is completed" ([13], p. 103).

"Let us now suppose that the first [trading] body, A, originally possessed the quantity a of corn, and that the second [trading] body, B, possessed the quantity b of beef. As the exchange consists in giving x of corn for y of beef, ---- the quantities exchanged satisfy two equations, ----

[(3)] $$F_1(a - x)/G_1(y) = y/x = F_2(x)/G_2(b - y)$$

----- The two equations are sufficient to determine the results of exchange; for there are only two unknown quantities concerned, namely, x and y, the quantities given and received" ([13], pp. 107 – 108).

The equations (3) are the so called Jevons's equations of exchange, where the cumbersome notations originally used by Jevons are replaced by more ordinary ones and F_1 and G_1, and F_2 and G_2, respectively, denote A's final degrees of utility of corn and beef, and B's final degrees of utility of corn and beef. These equations are derived from the equations of the second law of Gossen, (1), which can be rewritten in this case as

(4) $$F_1(a - x)/G_1(y) = dy/dx$$

and

(5) $$F_2(x)/G_2(b - y) = dy/dx,$$

since the term "different uses can be expanded to include exchange as a possibility (Jevons[13], pp. 107 - 108 Fisher[8], p. 159). In terms of Edgeworth box diagram, which we shall explain in the next section, equations (4) and (5) are conditions for x and y to be located on the so-called contract curve The equations of exchange (3) is, then, derived from these conditions and the law of indifference (2) and determines the equilibrium point (x, y) on the contract curve.

Walras's regard for Jevons's equations of exchange (3) was not high, since, in his letter to Jevons, Walras pointed out that Jevons had failed to derive "the equation of effective demand as a function of price, which ---- is so indispensable for the solution of the problem of the determination of equilibrium price" (Walras[23], p. 397 [22] pp. 205 - 206). This is no wonder, since the view of market of Walras who followed Cournot is different from that of Jevons who followed Gossen. Walras presupposed the existence of market prices which competitive traders always accept as data, while Jevons tried to justify this supposition by explaining market prices as the equilibrium ratio of exchange resulting from a process of freely competitive exchange.

Walras defines the equilibrium as the equality of demand and supply, both of which are functions of the given market price. Since the law of indifference is simply presupposed, all the individual traders in the market take the identical price even in disequilibrium situations. One might suppose a well organized, highly institutionalized market in which the specialized auctioneer determines the market price and changes it according to the excess demand or supply generated by price taking traders, as the incarnation of the law of supply and demand.

For Jevons, on the other hand, demand and supply are trivially equal even in disequilibrium situations. "Mill's equation --- states that the quantity of a commodity given by A is equal to the quantity received by B. This seems at first sight to be a mere truism, for this equality must necessarily exist if any exchange take place at all" ([13], p. 109). "We may regard x as the quantity demanded on one side and supplied on the other; similarly, y is the quantity supplied on the one side and demanded on the other" ([13], p. 110). Equilibrium is defined by Jevons by the law of indifference, which is established by arbitrage of different exchange ratios, which exist in a single market at disequilibrium.

One may argue that Cournot and Walras also emphasized the importance of arbitrage (Morishima[17], pp. 18 - 19). In the case of three or more commodities, Walras certainly insists that "we do not have perfect or general market equilibrium unless the price of one of any two commodities in terms of the other is equal to the ratio of the prices of these two commodities in terms of any third commodity" (Walras[22], p. 157). As Morishima admitted, however, in the two commodity case the arbitrage theory is trivial, because there is, of course, no arbitrage via a third commodity" ([17], p. 20). In other words, Walras considered only arbitrage among different markets and did not consider arbitrage in a single market where two commodities are exchanged.

Although Walras and Jevons viewed the process of market differently, the resulted equilibrium is identical, since the identical set of x and y is determined both by Walras's condition of demand and supply equality and by Jevons's condition of equations of exchange (3). In other words, in spite of Walras, Jevons arrived the identical equilibrium which Walras considered, even though Jevons failed to derive the equation of effective demand as a function of price, which is, therefore, not necessarily indispensable for the solution of the equilibrium price or equilibrium exchange ratio y/x. While Walras discussed detailedly the process of tâtonnement by which the equilibrium is established (Chapter 7, section 4 of this book), however, Jevons unfortunately did not make clear how the equilibrium (3) is established by the process of arbitrage among traders. The problem is left to Edgeworth whose contributions we shall consider in the next section.

3. Edgeworth's Mathematical Psychics

While Walras and Menger founded, respectively, Lausanne and Austrian schools in the marginal revolution, it has been said that Jevons, who revolted against the classical school in its home country, was isolated and established no school of his own. The problems raised by Jevons were, however, succeeded and solved by Edgeworth who is now regarded, along with Walras, as one of the two founding fathers of the equilibrium theory of the modern mathematical economics.[7)]

Francis Ysidro Edgeworth was born in 1845 in Edgeworthstown in County Longford, Ireland, where the famous family, whose name was taken from Edgeware, formerly Edgeworth, in Middlesex, had settled in the reign of Elizabeth I. His father died when he was two years old. After he studied at Trinity College, Dublin, Edgeworth entered Exeter College, Oxford in 1867, and then transferred to Magdalen Hall and to Balliol, where he obtained a First Class in Literae Humaniares. In 1877, he was called to the bar by the Inner Temple, and his first book, New and Old Methods of Ethics: or 'Physical Ethics' and 'Methods of Ethics' was published. While applying unsuccessfully for a professorship of Greek and that of philosophy, he gave lectures on English language, literature, logic, mental and moral sciences, and metaphysics. In 1881 he published Mathematical Psychics: An Essay on the Application of Mathematics to the Moral Sciences. Marshall reviewed this book -- "one of the two only reviews which Marshall ever wrote, the other being of Jevons's Theory of Political Economy" (Keynes[15], p. 255).

During 1880s, Edgeworth worked at a considerable rate on mathematical statistics so that he received more recognition for his statistical works than for his writings on economics. His third and final book

Metreike: or the Method of Measuring Probability and Utility was published in 1887. In 1890, he succeeded Thorold Rogers in the Tooke Chair of Economics and Statistics at King's College, London. In the next year, 1891, he again succeeded Rogers, this time to become Drummond Professor of Political Economy at Oxford, and was elected a Fellow of All Souls, where he remained for the rest of his life. He retired from the Oxford professorship in 1922. The British Economic Association, which in 1902 became the Royal Economic Society, was founded in 1890 and the publication of Economic Journal was started under the first editorship of Edgeworth. Edgeworth had been continuously responsible for it as editor or co-editor from the first issue in 1891 to his death in 1826. For the last 15 years, Keynes was a co-editor to Edgeworth.[8)]

Although he publisheds many articles and some monographs like Mathematical Psychics, Edgeworth never attempted to write and publish a treatise, like Marshall's Principles of Economics. When Keynes "asked him why he had never ventured on a treatise he answered, with his characteristic smile and chuckle, that large scale enterprise, such as treatise and marriage, had never appealed to him" (Keynes[15], p. 262). In 1925 the Royal Economic Society published under Edgeworth's own editorship his Papers Relating to Political Economy in three substantial volumes, which preserve the whole of Edgeworth's papers -articles and reviews- on economics, which he himself wished to see preserved. Articles are grouped into six sections, Value and Distribution, Monopoly, Money, International Trade, Taxation and Mathematical Economics. Reviews included are those on books written by, among others, N.Keynes, Marshall, Sidgwich, Böhm-Bawerk, I.Fisher, Bortkiewicz, Pigou and Cassel.

In his New and Old Methods of Ethics (1877), Edgeworth attempted first to synthesize the various approaches and then to apply mathematical techniques to the problem of determining the optimal utilitarian distribution. Creedy ([4], p. 34) pointed out an important fact that Edgeworth already used Lagrangean multipler methods and the calculus of variations extensively in this book, a fact which has been overlooked in the literature on the early history of mathematical economics. Keynes commented on this book that "quotations from the Greek tread on the heels of the differential calculus, and the philistine reader can scarcely tell whether it is a line of Homer or a mathematical abstraction which is in course of integration" ([15], p. 257).

The same comment may also be applied to Edgeworth's Mathematical Psychics (1881). Unlike the contemporary readers of Edgeworth, modern readers may find it difficult to read, not mathematically, but literarily, since Edgeworth often used Greek, and occasionally Latin, in the middle of arguments and also often quoted short passages from English literature, without giving the sources. Fortunately, however, Creedy ([4], pp. 135 - 160) provides excellent notes on Mathematical Psychics, in which Greek or Latin phrases are translated into English and the sources of most of literary quotations are traced. After a brief introductory description of contents, Part 1 of Mathematical Psychics was devoted to justify the use of mathematics in economics where numerical data are not available. Part 2 is concerned with the Calculus of Pleasure and "may be subdivided, namely Economics and Utilitarian Ethics. The economical Calculus investigates the equilibrium of a system of hedonic forces each tending to maximum individual utility; the Utilitarian Calculus the equilibrium of a system in which each and all tend to maximum universal utility" (Edgeworth[6], pp. 15 - 16).

In Economical Calculus, Edgeworth started with the definition that the first principle of Economics is that every agent is actuated only by self interest" ([6], p. 16). Contract is defined as the species of action that the agent acts with "the consent of others affected by his actions" ([6], pp. 16 - 17). "The problem to which attention is specially directed ---- is: <u>How far contract is indeterminate</u> -- an inquiry of more than theoretical importance, if it show not only that indeterminateness tends to prevent[9)] widely, but also in what direction an escape from its evils is to be sought" ([6], p. 20).

"Demonstrations. -- The general answer is -- (α) Contract without competition is indeterminate, (β) Contract with <u>perfect</u> competition is perfectly determinate, (γ) Contract with more or less perfect competition is less or more indeterminate. (α) Let us commence with almost the simplest case of contract, -- two individuals, X and Y, whose interest depends on two variable quantities, which they are agreed not to vary without mutual consent. Exchange of two commodities is a particular case of this kind of contract. Let x and y be the portions interchanged, as in Professor Jevons's example" ([6], p. 20).

To consider "Professor Jevons's example," Edgeworth first introduced the new famous concepts of the contract curve and indifference curves. Since "his presentation can hardly be described as trnsparent" and "the notation used by Edgeworth is not helpful" (Creedy[4], p. 55), however, let us rather consider the problem by using the so-called Edgeworth box diagram, which was actually adumbrated by Pareto "twelve years after the publication of Edgeworth's <u>Mathematical Psychics</u>" (Jaffé[12]).

Jevons's equations of exchange to determine the quantities exchanged, which are discussed in the previous section, are satisfied at point E in Figure 2, the so-called Edgeworth box diagram, where the quantity of corn is measured horizontally, that of beef vertically, the quantities of commodity available to the trading body A are measured with the origin at A, those available to the trading body B, with the origin at B, and point C denotes the initial allocation of commodities. Curves I, II, etc. are indifference curves of trading body A and curves 1, 2, etc. are those of trading body B. The common tangent to two indiffernce curves at E passes through point C, so that Jevons's equations of exchange is satisfied at E. The curve DEF is the contract curve which is a locus of points where indifference curves of the two bodies are tangent each other. "At what point then will they refuse to move at all? When their lines of indifference are coincident (and lines of preference not only coincident, but in opposite directions" (Edgeworth[6], p. 22).[10)]

If each trading body consists of only a single individual, i.e., in the case of isolated exchange or bilateral monopoly, the equilibrium point is indeterminate in the sense that any point on the contract curve between D and F is a stable outcome of exchange. "This simple case brings clearly into view the characteristic evil of indeterminate contract, deadlock, undecidable opposition of interest, ---- It is interest of both parties that there should be some settlement, one of the contracts represented by contract-curve between limits. But which of these contracts is arbitrary in the absence of arbitration, the interests of the two adversa pygnantia fronte all along the contract curve" (Edgeworth[6], p. 29).[11)] There is no reason why only E can be an equilibrium and Jevons's equations of exchange cannot be always satisfied in this case. Since Jevons said that "the trading body may be a single individual in one case" and considered the indeterminateness of equilibrium only in the case where a commodity is indivisible,[12] we have to admit that Jevons did not recognize the indeterminateness of bilateral monopoly. In view of Jevons's reason for using the trading body, i.e., the fact that the behavior of a group of persons is more stable than that of a single individual, however, Jevons should not have admitted the case of a single-individual trading body. Edgeworth was quite generous in this respect.

" ---- the Jevonian 'law of indifference' has place only where there is competition, and, indeed, perfect competition. Why, indeed, should an isolated couple exchange every portion of their respective commodities at the same rate of exchange? Or what meaning can be attached to such a law in their case? ---- This consideration has not been brought so prominently forward in Professor Jevons's theory of exchange, but it does not seem to be lost sight of. His couple of dealers are, I take it, a sort of typical couple, clothed with the property of 'indifference,' whose origin in an 'open market' is so lucidly described; not naked abstractions like the isolated couples imagined by a De Quincey or Courcelle-Seneuil in some solitary region. Each is in Berkleian phrase a 'representative particular;' an individual dealer only is presented, but there is presupposed a class of competitors in the background" (Edgeworth[6], p. 109, see also p. 31)

Having demonstrated that (α) Contract without competition is indeterminate, Edgeworth proceeded to show that (γ) Contract with more or less perfect competition is less or more indeterminate. Let us follow his discussions, again using Figure 2 and translating them into the language of the modern theory of games.[13] Suppose first that there are two identical (in taste and initial holding) individual traders in each trading body, i.e., trader A_1 and trader A_2 of type A and trader B_1 and trader B_2 of type B. It is evident that equal quantities of goods should be allocated to identical traders of the same type, after, for example, A_1 traded with B_1 and A_2 traded with B_2. "It is evident that there cannot be equilibrium unless (1) all the field is collected at one point; (2) that point is on the contract-curve. For (1) if possible let one couple be at one point, and another couple at another point. It will generally be the interest of the X of the one couple and the Y of the other to rush together, leaving their partner in the lurch. And (2) if the common point is not on the contract-curve, it will be the interest of all parties to descend to the contract-curve" (Edgeworth[ ], p. 35)

We can, therefore, still use Figure 2, by interpreting that it describes the trade between the representative traders of each trading body. Allocation D on the contract curve, which can be a stable outcome of exchange in the case of single individual trading bodies, can now be blocked by a coalition of A_1, A_2 and either of B_1 and B_2. Suppose A_1 and, say, B_1 who joins the coalition keep the contract D between them while A_2 cancels the contract D with B_2 who does not join the coalition and returns to C. Thus, A_1, A_2 and B_1 can arrange by themselves in such a way that, whilst B_1 remains at point D, A_1 and A_2 reach the mid-point M between D and C, thereby achieving a higher level of utility than at D. With some side-payments to B_1, all the traders joining coalition (A_1, A_2, B_1) can be better off by themselves than they are at D, and the allocation D is blocked. Any allocation on the contract curve and close to D, say, G, can be blocked similarly by the same coalition. Similarly, allocation F or those allocations on the contract curve and close to F can be blocked by a coalition of both B_1 and B_2 and any one of A_1 and A_2.

It is clear that the case of two-individual trading bodies is more competitive than the case of the bilateral monopoly of a single individual trading bodies. The range of possible contracts in the former is less indeterminate than in the latter, since some of the allocations possible in the latter can be blocked in the former. Allocations like H in Figure 3, however, cannot be blocked by the coalitions of three traders suggested by Edgeworth and belong, therefore, to the core, i.e., the set of stable allocations which cannot be blocked by coalitions of traders. The reason why H cannot be blocked is that at the mid-point between H and C traders of type A cannot achieve a higher level of utility than at H. "In general for any number short of the practically infinite (if such a term be allowed) there is a finite length of contract curve ---- at any point of which if the system is placed, it cannot by contract or recontract be displaced; that there are an indefinite number of final settlements, a quantity continually diminishing as we approach a perfect market. We are brought back again to case (β)" (Edgeworth[6], p. 39).

Let us now consider the case where there are infinitely many identical traders of type A and infinitely many identical traders of type B, to show that (β) Contract with perfect competition is perfectly determinate. In this case, any allocation on the contract curve, except allocation E, can be blocked by a coalition of traders considered by Edgeworth. For example, consider the allocation H in Figure 3. This allocation can be blocked by a coalition formed by all the A traders and more than half but less than all the B traders. In the coalition some A traders still continue trade with B traders in the coalition and are located at H, while the rest of A traders having no trade partners in the coalition are located at C. By increasing the numbers of B traders joining the coalition sufficiently and therefore increasing the number of A traders located at H, we can make the average allocation of A traders (some at H, some at C) sufficiently close to H, on the line CH, that it is located like J above the indifference curve passing through H. By reallocating among themselves, therefore, all the traders joining the coalition can be better off than they are at the allocation H, so that H is blocked by such a coalition. Similarly, any point between D and F where the common tangent to two indifference curves does not pass through the point C can be blocked by a coalition of traders, if necessary, by changing the role of A traders and B traders from those in the case of the allocation H. Obviously only the point E belongs to the core, i.e., the set of allocations which are not blocked by such a coalition, and therefore the equilibrium is determinate in this case.

"Thus, proceeding by degrees from the case of two isolated bargainers to the limiting case of a perfect market, we see how contract is more or less indeterminate according as the field is less or more affected with the first imperfection, limitation of numbers" (Edgeworth[6], p. 42).[14)]

Since equilibrium is thus indeterminate in Edgeworth's theory of exchange except for the limiting case of infinitely large number of traders, "competition requires to be supplemented by arbitration, and the basis of arbitration between self interested contractors is the greatest possible sum-total utility. Thus the economical leads up to the utilitarian calculus" (Edgeworth[6], p. 56).[15)] Unlike the case of Pigou who also considered utilitarian social welfare function,[16)] in Utilitarian Calculus, Edgeworth did not insist the equal distribution as the optimal income distribution. "Thus the distribution of means as between the equally capable of pleasure is equality; and generally is such that the more capable of pleasure shall have more means and more pleasure" ([6], p 64). " ---- in the minds of many good men among the moderns and the wisest of the ancients, there appears a deeper sentiment in favour of aristocratical privilege -- the privilege of man above brute, of civilized above savage, of birth, of talent, and of the male sex. This sentiment of right has a ground of utilitarianism in supposed difference of capacity. Capacity for pleasure is a property of evolution, an essential attribute of civilization" ([6], p. 77).

4. Reflections on Edgeworth's Theorem

As we saw in the previous section, Edgeworth proved a theorem that the equilibrium of exchange is determinate only in the case of the perfect competition. This important theorem is called Edgeworth's equivalence theorem, since it shows the equivalence of two different approaches to the problem of exchange in the market, i.e., Cournot-Walras approach and Jevons-Edgeworth approach. In Cournot-Walras approach, the law of indifference, i.e., the existence of the uniform market price is presupposed and competitive traders are assumed to be price takers. In Jevons-Edgeworth approach, however, the law of indifference is established only at the equilibrium, through arbitrage activities of individual traders to take advantage of the existence of different prices in the same market. Individual traders are, therefore, not price takers, and free to make contract at whatever price they like, to cancel it to make recontract at more favourable terms, and to organize coalition to block the existing contracts.

The assumptions made in Cournot-Walras approach may not be realistic, unless there is an auctioneer as in the case of well organized markets. Edgeworth's equivalence theorem justifies, however, Walrasian assumption, since it is not the assumption but the outcome that matters for a theory and we can assume that traders behave as if they are price takers even though traders are actually not price takers, provided that we have the same outcome as assured by Edgeworth. Walrasian theory based on the demand and supply functions can be safely applied to situations where traders are not price takers and free to form and break coalitions in the process of bargaining as in Jevons-Edgeworth approach, so that demand and supply functions do not, strictly speaking, make sense.

Walras, who criticized Jevons for the lack of clear concept of demand functions in the latter's equations of exchange, is thus helped by Edgeworth, who followed Jevons, to increase the relevancy of his theory.

Edgeworth's equivalence theorem is also called a limit theorem, since it is proved only for the limiting case where the number of traders is infinitely large on both sides of the market. In other words, the infinitely large number of traders is shown to be a sufficient condition for the outcome of the Jevons-Edgeworth bargaining process to be equivalent with that of perfect competition. A natural question is, then, whether it is also a necessary condition or not.

Following Farrell[7], consider the case of duopoly where there are only two traders of one type and infinitely many traders of another type, though the total quantities of two goods are finite in the economy. Since equal quantities of goods should be allocated to identical traders of the same type, we can still use Edgeworth box diagrams, i.e., Figures 2 and 3, which now describe the half of the economy, one of the duopolists and its infinitely many customers.

Suppose first that there are two B traders, B_1 and B_2 and infinitely many traders of type A. In Figures 2 and 3, BC is the quantity of the second good initially held by a B trader and AC is the sum of quantities of the first good initially held by the half of A traders. Curves I, II, etc. in Figure 2 are aggregate indifference curves of A traders, as well as individual ones, which can be constructed if the identical individual indifference curves are homothetic so that the marginal rate of substitution between two goods depends only on the ratio of the quantities of goods and Engel curve is a line through the origin (Chipman[3]).

An allocation H in Figure 3 can now be blocked by a coalition of one B trader and more than half but less than all of infinitely many A traders. All the A traders currently trading with B_1 trader which joins the coalition also join the coalition and keep the contract H with B_1. Some A traders currently trading with B_2 which does not join the coalition join the coalition and cancels the contract with B_2 to return to the initial point C. By decreasing the number of the latter type A traders joining the coalitions sufficiently and therefore increasing the number of A traders located at H, relative to those A traders located at C, we can make the average allocation of individual A traders in the coalition sufficietntly close to that at the point H, on the line CH, that it is like an allocation at J located above the indifference curve passing through H. By reallocating among themselves, therefore, all the A traders in the coalitions are better off than they are at the allocation H. With some side payments to B_1 which is located at H, all the traders joining the coalition can be better off than they are at H, so that H is blocked.

Suppose next that there are two A traders, A_1 and A_2 and infinitely many traders of type B. In Figures 2 and 3, then, AC is the quantity of the first good initially held by an A trader and BC is the sum of quantities of the second good initially held by the half of B traders. Curves 1 and 2 in Figure 2 are aggregate as well as individual indifference curves of B traders. In this case an allocation H in Figure 3 can be blocked by a coalition of one A trader and the less than half of infinitely many B traders. Suppose A_1 joins the coalition. Those B traders also joining the coalition can keep trade with A_1 unchanged so that they can keep the same level of utility as enjoyed at the allocation H. A_1

cancels trade with those B traders who are not permitted to join the coalition, so that A_1 moves on HC from H toward C. Unless it cancels too many contracts with B traders, A_1 can be located like at J above the indifference curve passing through H. By the reallocation among themselves then, all the traders joining the coalition can be made better off than they are at the allocation H, so that H is blocked.

Similarly, any point between D and F on the contract curve where the common tangent to two indifference curves does not pass through the point C can be blocked by a coalition of traders, if necessary, by changing the role of A traders and B traders from those in the case of the allocation H. Again, it is the point E only which belongs to the core allocation. In other words, even a duopoly market ends up with an equilibrium identical to that of the perfect competition, if customers are infinitely many and free to organize coalitions.

If infinitely many customers behave as price takers so that their demand can be expressed as a function of price, duopolists will take advantage of it so that the outcome is different from that of the perfect competition, as was discussed by Cournot (See Chapter 7, section 1). In our case, however, even at point E where not only customers but also duopolists seem to behave as if they are price takers, customers are actually not price takers, so that there is no possibility of price-making behavior of some traders to take advantage of price-taking behavior of other traders, a problem that is considered by Roberts and Postlewait [20].

Edgeworth's equivalence theorem is thus extended to the case of duopoly in which the number of traders at one side of the market is only two, though the number of customers at the other side of the market is infinitely large. In Jevons-Edgeworth process of bargaining, of course, it is assumed that there are no costs for traders to know about the rate of exchange in contracts made among other traders, to make contracts and to cancel them, and to organize coalitions to block contracts already established. If these costs of information, communication, transaction and organization are taken into consideration, the outcome of Jevons-Edgeworth process may not be equivalent to that of Walrasian perfect competition. What is essential for Walrasian equilibrium with all its optimal properties is, therefore, not so much the infinity of the number of small traders on both sides of the market as the assumption of no cost for information, communication, transaction and organization.

In Cournot's model of oligopoly (Chapter 7, section 1), an increase in the number of firms in the industry results in lower equilibrium price and larger consumers' surplus.[17)] Allocative efficiency of the industry is therefore, inversely related to the degree of concentration. The discussion in the above suggests, however, that the efficiency may be independent of the degree of concentration, if firms can make (and cancel) freely any type of contract directly with customers, so that customers need not behave as price-takers. In other words, the above conclusion obtained from Cournot's model is a result of its too specific assumption that firms can offer only the contract of the uniform price to their customers, can have only indirect contacts with their customers through the price of their products. This is, of course, a result of Cournot-Walras view of the market.

To separate further Edgeworth's equivalence theorem from his limit theorem, let us return to the case where there are two traders of type A, i.e., A_1 and A_2 and two traders of type B, i.e., B_1 and B_2. Consider the allocation H on the contract curve DEF in Figure 4, which is an essentially identical reproduction of Figure 3. As we saw in the previous section, the point H cannot be blocked by the coalition of A_1, A_2 and B_1, considered by Edgeworth. We can show, however, that H is not a stable outcome of exchange of two two-individual trading bodies, if Jevons's law of indifference and his view of trading process are taken into consideration.

Jevons had in mind a piecemeal exchange process, since "dynamically we could not treat the ratio of exchange otherwise than as the ratio of dy and dx, infinitesimal quantities of commodity" (Jevons[13], p. 102). Let us rule out, therefore, the possibility of an indivisible lump-sum transaction and assume that each single transaction is divisible, so that every portion of a homogeneous commodity is treated indifferently, i.e., exchanged against the other commodity at the same rate of exchange. In other words, the dynamic exchange process is a piecemeal one, as considered by Jevons, and finite, i.e., not infinitesimal transaction is permissible only when it is divisible.

There must be, then, at least two successive transactions and the ratio of exchange should vary in the course of exchange between C and H, in Figure 4, if allocation H can ever be reached by exchange starting from C. Otherwise, if there is only a single transaction and the exchange ratio remains unchanged throughout the exchange process, it must be equal to the slope of line CH and exchange must proceed on line CH and, starting from C and moving toward H. Such an exchange process has to be terminated at J, however, since the transaction is divisible and it is unfavourable for A to go beyond J. Further, we can see that the variable ratio of exchange between C and H must be identical to the slope of the tangent to the indifference curve passing through H, for the last infinitesimal or finite transaction arriving at H.

Suppose, therefore, that exchange proceeds like CKH, i.e., the first transaction to be done with exchange ratio favourable to B, and unfavourable to A, like CK, and the second transaction, with exchange ratio favourable to A, and unfavourable to B, like KH, though the average ratio is CH. If we suppose that this is the case, not only with exchange between A_1 and B_1 but also with exchange between A_2 and B_2, allocation H can be blocked by the following arbitrage of different exchange ratios.[18] B_1 cancels a part of the transaction with A_1, with the ratio of exchange KH, and proposed some new transaction to A_2. Suppose, for the moment, that B_1 offers A_2 the exchange ratio CK. A_2 may accept this proposal of B_1 by cancelling a part of his transaction with B_2 at the exchange ratio CK, since "provided that A gets the right commodity in the proper quantity, he does not care whence it comes, so that we need not —— distinguish the source or destination" (Jevons[13], p. 127). In any event, if B_1 offers A_2 an exchange ratio strictly between CK and KH, both parties can be strictly better off as a result of cancelling parts of their earlier provisional contracts. Not only B_1 but also B_2, A_1 and A_2 may take the initiative to exploit different exchange ratios.

Such a coalition of, say, B_1 and A_2 to block a contract H is certainly different from the coalition of, say, A_1, A_2 and B_1 considered by Edgeworth. The latter coalition is a closed coalition that its member can be better off by themselves than at H, irrespective of the reaction of trader B_2. The former coalition is, on the other hand, an open coalition that its member can be better off than at H, only if non-member traders B_2 and A_1 keep respectively the transactions, which traders B_1 and A_2 do not cancel, unchanged. In view of the fact that B_2 and A_1 are, respectively, in competition with B_1 and A_2, and the assumption of divisible trade, it may be natural for the member of a coalition to expect unchanged behavior of non-member traders, when only a small part of transactions with them is cancelled. We have, of course, admit that our discussion is based on a more stringent assumption than that of Edgeworth.[19)]

Be that as it may, this kind of arbitrage activity is the basis of Jevons's law of indifference and an allocation E, i.e., the solution of Jevons's equations of exchange, is the only allocation which is not blocked by arbitrage activities leading to the law of indifference. Thus, Edgeworth's equivalence theorem holds not as a limit theorem, if transactions are assumed to be divisible and the competition between identical traders is assured, so as to satisfy Jevons's law of indifference. Even Edgeworth, who did Jevons more than justice, insofar as the latter's concept of a trading body was generously interpreted so as to exclude the case of indeterminate equilibrium of bilateral monopoly (see section 3), did Jevons less than justice here, since the implication of Jevons's law of indifference was not fully exploited in Edgeworth's equivalence theorem as a limit theorem.

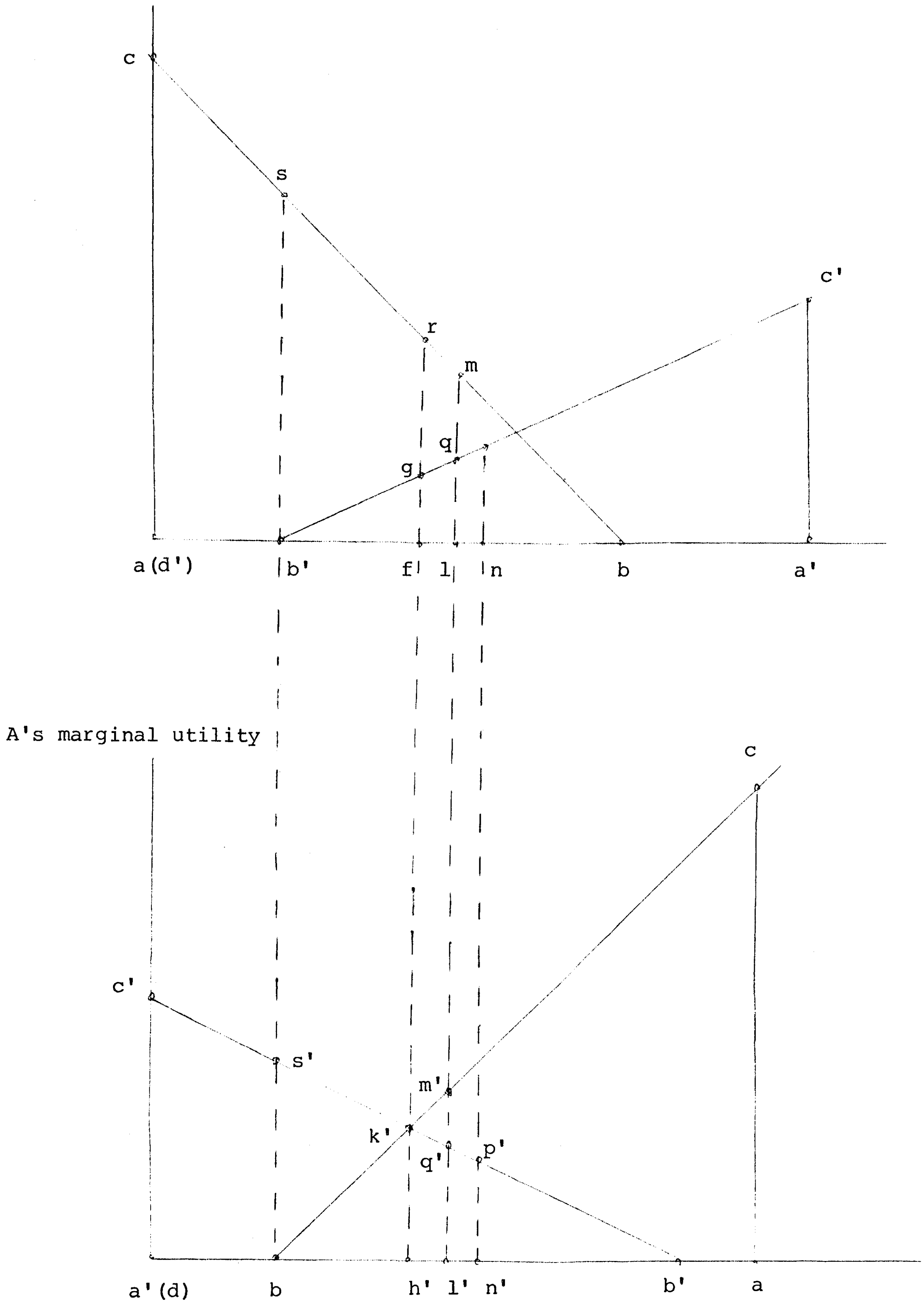

Figure 1

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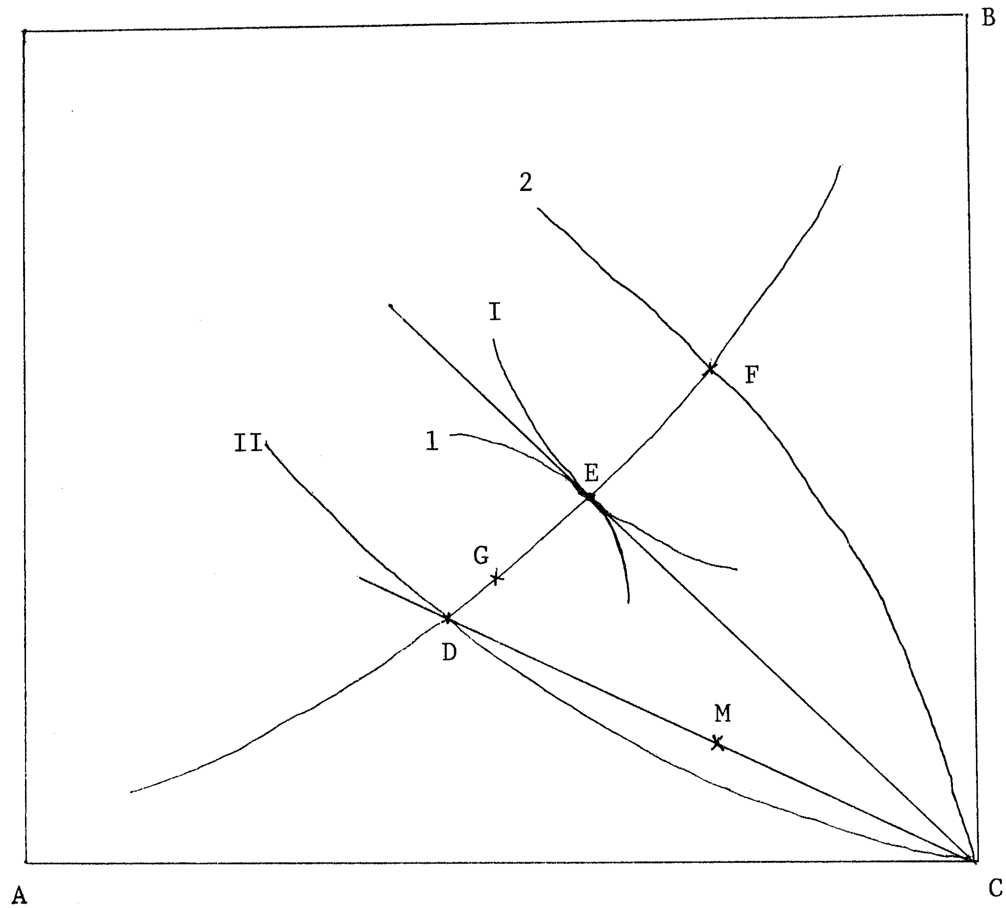

Figure 2

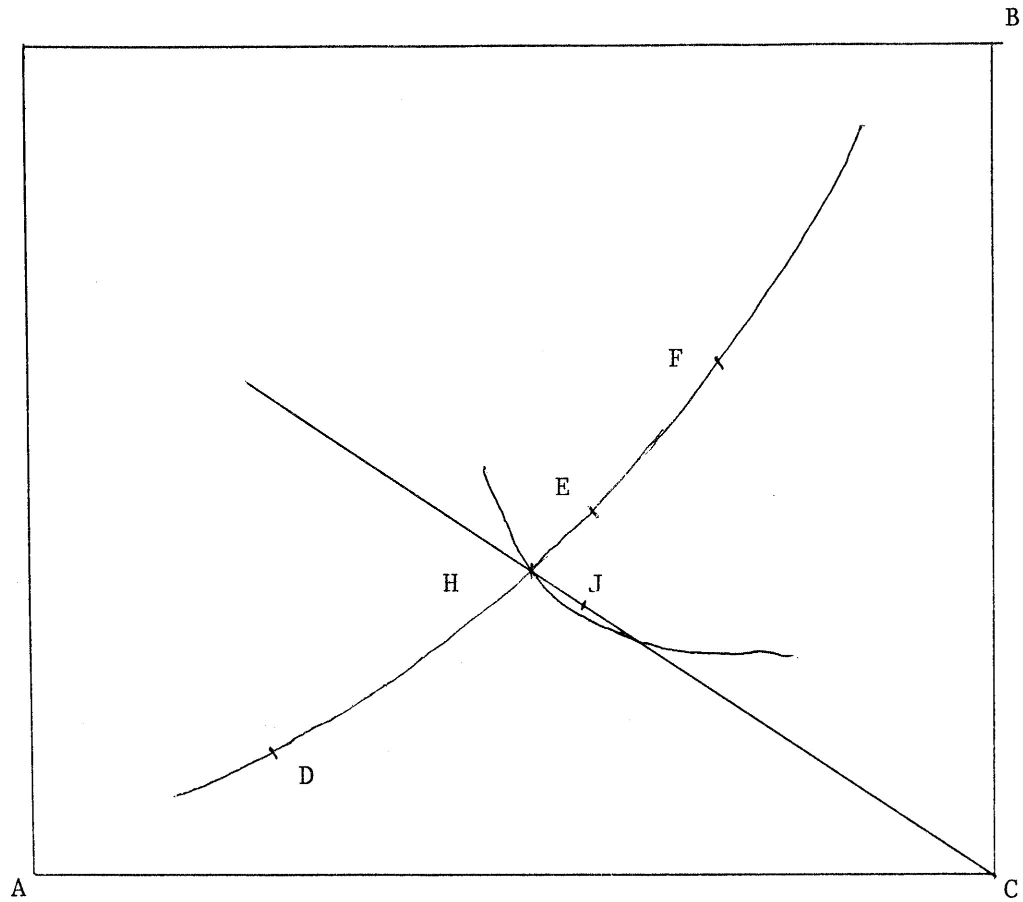

Figure 3

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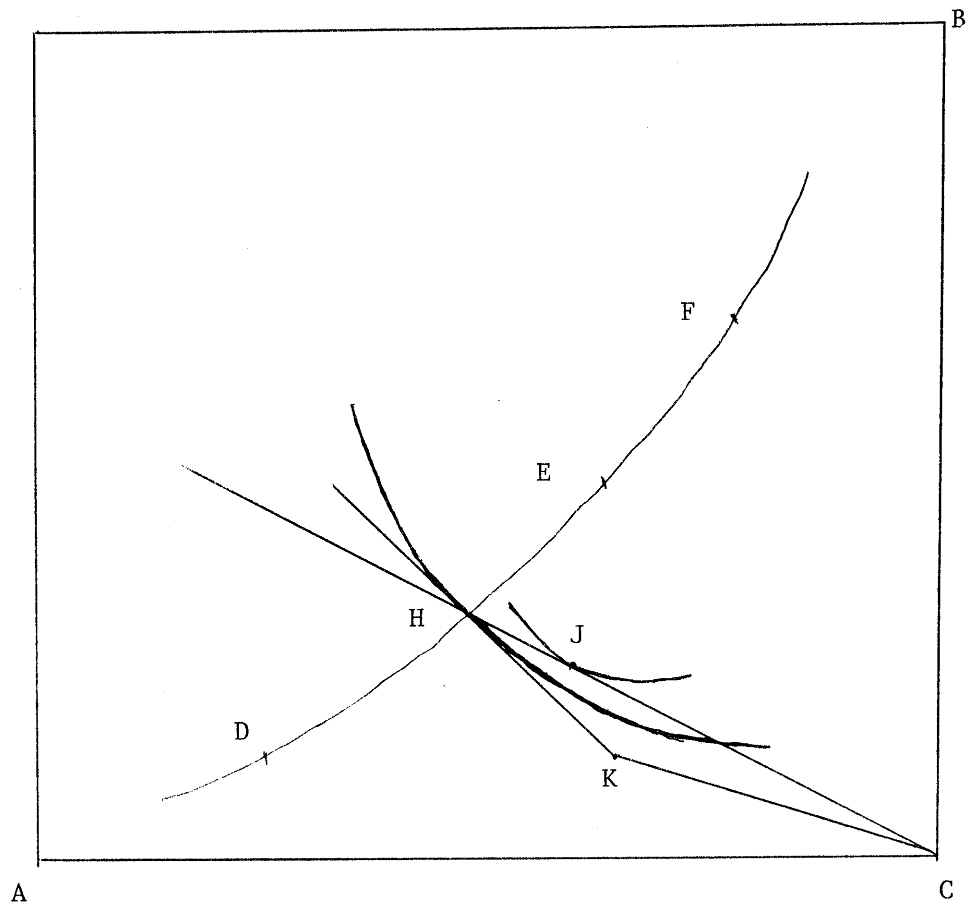

Figure 4

Footnotes

1) See Jevons[13], pp. xxxv - xlii, particularly, p. xxxviii. See also the introduction written by Georgescu-Roegen to Gossen[9], pp. xi - cxlv, and Walras[21].

2) See Gossen[9], pp. 6, 108 - 109, and xciv. The e in the statement of the second law signifies the proportion of income for a specific pleasure. See [9], p. 308.

3) See Gossen[9], pp. 95 - 100. See also Jaffé[12] and Georgescu-Roegen's exposition ([9], pp. cv - cxiii).

4) Figure 1 is based on Figure 7.2 in page 98 of Gossen[9]. Notations are slightly changed so as to make the exposition simplified. Corresponding changes are also made in the quotations from Gossen[9] in the below.

5) For Jevons's theory of production, see Pagano[18], pp. 77 - 81, and Walras[21].

6) See Chipman[3] for conditions necessary for the existence of social indifference curves.

7) "We show how two different approaches to the solution of the exchange problem lead, in large economies, to essentially the same result. The two approaches derive from ideas developed by Edgeworth and Walras" (Hildenbrand and Kirman[10], p. v).

8) For the life and work of Edgeworth, see Creedy[4], pp. 7 - 22, and Keynes[15], pp. 251 - 266.

9) 'to prevent' should read 'to be present'. See Creedy[4], p. 138.

10) The line of indifference is the slope of the indifference curve and defined in page 21 of Edgeworth[6]. "The ---- <u>line of preference</u> ---- is perpendicular to the line of indifference" (Edgeworth[6], p. 22).

11) 'adversâ pugnantia fronte' means 'fighting face to face'. Creedy[4], p 139.

12) See Jevons[13], pp. 96 and 130 - 134.

13) For modern expositions of Edgeworth's theorem, see Debreu and Scarf[5] and Hildenbrand and Kirman[10], pp. 18 - 23.

14) Besides the limitation of numbers, Edgeworth also discussed other imperfections like the existence of combinations. Incidentally, 'contract is more or less' should read 'contract is less or more' in this quotation.

15) For Edgeworth's justification of utilitarianism and its relation to contractarian neo-utilitarianism, see Creedy[4], pp. 81 - 85.

16) "The economic welfare of a community is likely to be greater ---- the larger is the average share of the national dividend that accurues to the poor ----" (Pigou[19], p. v).

17) See equation (11) in Chapter 7, section 1, where p is given as a decreasing function of n.

18) Generally, of course, the path of exchange with a varying exchange ratio may be different for different traders of the same type. Still, arbitrage is possible, since for each trader some exchange transactions are with an exchange ratio more favourable than CH and some other transactions are with an exchange ratio less favourable.

19) In the actual world, however, closed coalition is rather rare. For example, the Second World War was fought between Axis Powers (Germany, Italy, Japan, etc.) and Allied Power (France, U.K., U.S.A., U.S.S.R., etc.), but these two coalitions were not closed, since until the last moment diplomatic relations were kept between Japan and the U.S.S.R.

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