88-F-15

Bertrand Competition with Free Entry and the Theory of Contestable Markets*

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August 1988 (Revised Version)

PREPARED UNDER
THE PROJECT ON ECONOMIC THEORY
RESEARCH INSTITUTE FOR THE JAPANESE ECONOMY

ABSTRACT

The contestable market theory predicts a long-run industry configuration which corresponds with the classical Marshallian competitive equilibrium, but without the assumption of price-taking firms. This paper provides a game-theoretic interpretation of this recent development. In particular, we extend Bertrand's model of price-setting oligopolists to industries with free entry and with U-shaped and more general cost functions. We prove that the unique Nash equilibrium in pure strategies is the competitive outcome. Unlike the conventional treatment of price-setting games, our construction permits the existence of pure strategy equilibrium and yet does not necessitate Grossman [1981]'s supply function equilibria.

1. INTRODUCTION

Given a common U-shaped average cost curve for an industry the classical Marshallian theory of perfect competition and the recent contestable market theory by Baumol, Panzar and Willig [1982] both predict that the industry equilibriates in the long run at a point where price equals the minimum average cost and each incumbent firm operates at the minimum-average-cost level of output. This long-run outcome is efficient from social welfare point of view. The former obtains the result by the assumption of price-taking firms and free entry, and the latter by the assumption of a contestable market. This paper provides a game-theoretic interpretation of the latter theory.

The contestable market theory in a single product context is summarized as follows. Consider an industry for a good. Let $D(\cdot)$ be the industry demand function, and let $C(\cdot)$ be the cost function which applies to every firm. A contestable market is defined to be a market with costless entry and costless exit. Costless exit needs no explanation. Costless entry means that potential entrant faces no disadvantage vis à vis incumbents with respect to either the available production techniques (i.e., the cost function) or consumers' perception of the attractiveness of the product. Assuming that the market is contestable in this sense, a tuple $(p; m; y_1, \cdots, y_m)$, in which p denotes a price, m denotes the number of incumbent firms, and $y_1, \cdots, y_m$ denote positive output quantities of the m firms, is said to be a sustainable equilibrium if

(a) $\sum_{i=1}^{m} y_i = D(p)$;

(b) $p\, y_i - C(y_i) \geq 0$ for $i = 1, \cdots, m$;

(c) $p^e\, y^e - C(y^e) \leq 0$ for all $p^e < p$ and $y^e \leq D(p^e)$.

That is, the contestable market theory depicts a stable industry configuration in which (a)total demand equals total supply (market clearing), (b)each incumbent firm obtains nonnegative profit (costless exit), and (c)no further entry is possible. They then show that these three conditions imply the efficient industry outcome with the socially "right" amount of entry and with each incumbent firm operating at the minimum average cost. The classical assumption of price-taking firms plays no role in this new theory, and thus they claim that the threat of entry can serve to discipline the market without the presence of a sufficient number of firms; hence the name _the new invisible-hand theorem_.

The prediction of the contestable market theory that optimality of industry performance is attained even with a small number of firms is similar to that of Bertrand [1883] who modelled a market with price-setting oligopolists. But Baumol, Panzar and Willig claim that this resemblance is only superficial:

> "However, though the _result_ corresponds precisely with that of our small group case, the underlying forces in the Bertrand model are fundamentally quite distinct from ours. The basic distinction once again resides in the crucial role of potential entrants in our analysis, as contrasted with that of incumbents in Bertrand's.[1/]

They further argue:

> "As is well known, the Bertrand's result is highly dependent upon the assumption that marginal costs are constant _and_ equal to average cost throughout the relevant range. If marginal cost begins to rise, or lies below average cost at (what might otherwise be) an equilibrium output, serious problem arise for the existence and uniqueness of equilibrium price and output. Furthermore, the zero profit property tends

to evaporate, and with it the optimality of the quantity of resource allocated to the industry."[2/]

Thus, the authors of the contestable market theory claim that the theory is quite different from Bertrand's classical work, because (1)the threat of potential entrants is the key to their result while competition among the incumbents is the key to Bertrand's, and (2)their efficiency result holds for any form of the cost function while Bertrand's result holds only in the very restrictive case of universal constant-returns-to-scale.

The key assumption for the contestable market theory is condition (c), the no-profitable-entry condition. This condition reflects the contestability assumption: A new entrant incurs the same cost $C(\cdot)$ as the incumbents (free access to the present technology), and if an entrant offers a price below the price quoted by incumbents, he can sell any quantities he wishes, constrained only by the market demand at his price (free access to the market). If an entrant cannot find an entry plan (p^e, y^e) which yields sufficient revenue to cover the production cost $C(y^e)$, then the industry is thought to be immune against new entry. A no-entry condition of this form is manifested in condition (c).

An important premise of condition (c) is that possibility of a new entry is evaluated relative to the incumbents' pre-entry price p. Taken literally, it implies that the entrant supposes the incumbents would not react to a new entry by undercutting prices.

However, Baumol et al. claim that such an assumption is irrelevant to motivate condition (c). They argue that the condition is a natural consequence of the assumption of costless exit. Namely, a firm will enter the market (without any concern about the incumbents' possible countermoves) if he expects a profit opportunity under the current price, since it takes at least a while until the incumbents recover their market share by cutting prices down. And, when the incumbents react by undercutting prices and preclude all further profit to the entrant, the entrant could readily exit from the market without loss of investment. However, we think this argument does not endorse condition (c). Such hit-and-run incursions of new firms into the market would only cause temporal loss of incumbents' profits. One can imagine a long-run equilibrium of an industry that allows temporal entries and exits of this sort at the margin.

The authors are correct in saying that the implicit competition between incumbents and potential entrants plays a crucial role in their result, but their assertion that the mode of competition between the two parties is irrelevant is not well-founded. In our view, condition (c) must be interpreted as reflecting the assumption that the potential entrants evaluate the profitability of entry with the conjecture that his entry will not affect the price of the incumbent firms.[3/] But this is not a point of concern in this paper.

Regardless of the story that underlies condition (c), its feature that possibility of a new entry is evaluated relative to the incumbents' pre-entry price p is essential to deduce the efficiency result. Mathematically, it means that potential entrants act in accordance with the Bertrand-Nash price-setting game. And this observation suggests that the main body of the contestable market theory may well be given a reformulation as a noncooperative game in the spirit of Bertrand's classical work.

Our paper is not the first to investigate this theoretical connection. Grossman [1981] considered an industry producing a homogeneous product and with free entry. By constructing a game in which each firm's strategy is a supply function, he proved that, unlike the Cournot model of oligopoly, the unique Nash equilibrium of his game is the competitive outcome.

To what extent Grossman thought of his work in relation to the contestable market theory is questionable. The basic motive of our paper is the intuition that his conclusion should follow from a straightforward extension of Bertrand's game of price competition to situations with free entry. In our view, his construction using the supply function equilibrium is an unnecessary complication. In this regard, Mirman, Tauman and Zang [1984] is more in line with our paper, but with little success. They dealt with the rather special case of natural monopoly. Yet, they could show the coincidence of their equilibrium with the prediction of the contestable market theory only by imposing a peculiar set of assumptions that either there is a continuum of potential entrants or the demand curve is tangent to the average cost curve at a point and uniformly below elsewhere.

In the next section we illustrate the basic structure of our game. Essentially, we construct a one-shot game in which all firms announce their prices. Consumers seek to purchase at the lowest price. But, the lowest price firm(s) produces only up to the point of maximum profit. Consumers who could not purchase at the lowest price, then, visit the firm(s) with the second lowest price and purchase the good if the price is acceptable. This process continues until no remaining consumers can find goods sold at prices lower than their reservation prices. Although this story may suggest a formulation as a game with two stages, consumers' moves and firms' productions are treated as part of the outcome function of the game, <u>not</u> strategies.

Section 3 establishes the main result assuming that the average cost curve is U-shaped. Namely, we prove that there is a unique Nash equilibrium of the game in pure strategies, and that it is the same as the Marshallian competitive equilibrium and the prediction of the contestable market theory. In section 4 we show that the analysis is easily extended to the case of natural monopoly and to the case of large "residual demands", whose meaning will be clarified in the main text. We also show that the Marshallian outcome is the Nash equilibrium even when, due to the integer problem, the sustainable equilibrium does not exist. The contestable market theory fails to predict the long-run industry configuration if industry demand at the competitive (= minimum average-cost) price is not an integer multiple of the competitive level of output. To get around this nonexistence problem, they resort on the casual observation that the average cost curve is "flat-bottomed". Our game theoretic formulation accommodates such situations without the assumption of flat-bottomed average cost curve. It predicts the Marshallian equilibrium with unfulfilled demand on the margin.

2. FORMULATION OF THE GAME

Consider an industry producing a single homogeneous good. In the main body of this paper we assume that the average cost curve, which applies to every firm, is U-shaped as depicted in Fig. 1. A typical story would be that a firm must incur some fixed cost to produce a positive quantity of output. If y is the quantity of output, total cost is $C(y)$ with $C(0) = 0$. The unique minimum point of the average cost curve is denoted by (p_c, y_c). The marginal cost curve is assumed to be strictly increasing in the range $y > y_c$. No firm earns positive profit at prices less than p_c, and hence we restrict the admissible price range to $[p_c, \infty)$.

For the price range $p \in [p_c, \infty)$, define $\bar{Y}(p)$ and $\underline{Y}(p)$ by

$$\bar{Y}(p) = \arg\max_{y > 0} \{p\, y - C(y)\}, \tag{1}$$

and

$$\underline{Y}(p) = \inf \{ y > 0 \mid p\, y - C(y) \geq 0\}. \tag{2}$$

$\bar{Y}(p)$ is the profit-maximizing quantity of output for a firm who takes price p as given, so it corresponds to the marginal cost curve in the range $y \geq y_c$. $\underline{Y}(p)$ is the zero-profit level of output for a given price p; it corresponds to the downward-sloping portion of the average cost curve. By construction we have $\bar{Y}(p_c) = \underline{Y}(p_c) = y_c$. Given price $p \geq p_c$ a price-taking firm would choose to produce in the range $[\underline{Y}(p),\bar{Y}(p)]$ if demand to him is no less than $\underline{Y}(p)$. Note that a firm's profit is increasing in y in the range $[\underline{Y}(p),\bar{Y}(p)]$. Hence, he will choose to produce the maximum amount in this output range subject to his demand constraint. We assume that a firm prefers serving the market with zero profit if his only other alternative is to produce nothing.

Fig. 1

There are N potential firms. The market demand is given by a downward-sloping demand function D(p). For the moment we imagine that each consumer wants at most one unit and D(p) is the number of consumers willing to pay p or more for the product (i.e., the number of consumers whose reservation price is at least p).

Although we formulate a one-shot game, the story proceeds in two stages. In the first stage all the N firms announce their prices. In the second stage, consumers' demands are allocated to each firm depending on the preannounced prices and the firms' actual productions.

The demand allocation process proceeds as follows. Assume that consumers have brand image in a limited sense. Namely, firms $i = 1,\cdots,N$ are numbered according to the strength of brand in the sense that if two or more firms announce the same price consumers purchase the good from the firm with smaller i. We will motivate this assumption in the next section. Consumers are assumed not to move strategically and firms must make production decisions taking his price as given. Thus, the second stage contains no strategic element. It can be formulated as an outcome function of our game as follows.

Let $\mathbf{P} = (p_1,p_2,\cdots,p_N)$ be the announced prices. For each i let $I_i(\mathbf{P})$ denote the set of firms whose prices are lower than p_i or whose prices are p_i but whose brand is stronger than i. That is,

$$I_i(\mathbf{P}) = \{ j \mid p_j < p_i, \text{ or } p_j = p_i \text{ and } j < i\}. \qquad (3)$$

Consumers buy first from the cheapest firm, and when several firms offer the same price they buy first from the firm with the strongest brand. Firms produce taking his price as given and to maximize profit. Hence, if $D_i(\mathbf{P})$

denotes the number of consumers who visit firm i and $x_i(\mathbf{P})$ denotes i's actual production, we have

$$D_i(\mathbf{P}) = \max \{ D(p_i) - \sum_{j \in I_i(\mathbf{P})} x_j(\mathbf{P}) , \quad 0 \}. \tag{4}$$

and

$$x_i(\mathbf{P}) = \begin{cases} \bar{Y}(p_i) & \text{if } D_i(\mathbf{P}) > \bar{Y}(p_i), \\ D_i(\mathbf{P}) & \text{if } \underline{Y}(p_i) \leq D_i(\mathbf{P}) \leq \bar{Y}(p_i), \\ 0 & \text{if } D_i(\mathbf{P}) < \underline{Y}(p_i). \end{cases} \tag{5}$$

The set $I_i(\mathbf{P})$ is empty for the firm who is most preferred by consumers, namely the firm who has the strongest brand among those with the cheapest price. Eq.(4) must be read as $D_i(\mathbf{P}) = \max \{D(p_i), 0\} = D(p_i)$ for this firm. Starting with this firm, (4) and (5) recursively determine $D_i(\mathbf{P})$ and $x_i(\mathbf{P})$ following the obvious order of i.

Given prices $\mathbf{P} = (p_1, p_2, \cdots, p_N)$, the profit of firm i, i = 1,2,$\cdots$,N, is

$$\Pi_i = p_i \; x_i(\mathbf{P}) - C(x_i(\mathbf{P})). \tag{6}$$

Firms independently and simultaneouly choose prices to maximize profit. We focus on the Nash equilibrium of this one-shot game. When N is not too small, some firms would produce nothing in equilibrium. These firms will be called the <u>outsiders</u>. It should be clear that the above formulation embodies the Bertrand's notion of price competition in a straightforward fashion to the case with free entry.

We have chosen a specific scenario to interpret the demand function. Kreps and Scheinkman [1983] uses this scenario to show that the perfect equilibrium outcome of a two stage oligopoly game, in which firms, first, make capacity (production) decisions and, second, after capacity levels are made public, choose prices, is the Cournot outcome. Unlike their result, our main theorems do not depend on this interpretation of the demand function. The present choice of the demand rationing story is merely to give the proofs in a simplest formulation.

3. CHARACTERIZATION OF THE UNIQUE NASH EQUILIBRIUM

We start our analysis with the following basic observation. Suppose that a certain number of firms set their prices at p_c. Clearly the profit-maximizing (positive) quantity of output at this price is y_c. The total market demand at this price is $D(p_c)$. Therefore, the maximum number of firms who can operate at this price is an integer m_c which satisfies

$$m_c\ y_c \leq D(p_c) < (m_c+1)\ y_c. \qquad (7)$$

Assume that the residual market demand, $D(p)-m_c y_c$, is not large enough to cover the production cost for any level of output. This is shown in Fig. 2. When firms $i = 1,2,\cdots,m_c$ choose price p_c, other firms are indifferent to their prices, since no matter what their price levels be (in the range $[p_c, \infty)$) they will come out to produce nothing. And, if at least one among those outsiders announce p_c, no one among the first m_c firms can gain by increasing his price. This is because it would only shift his current market share to the outsider and by assumption no profit opportunity would be left in the residual demand. Thus, there exists at least one equilibrium in which m_c + 1 or more firms choose price p_c. In this equilibrium m_c firms produce y_c units of output and other N - m_c firms act as outsiders. This essentially corresponds to the Marshallian long-run equilibrium and also to the sustainable equilibrium outcome predicted by the contestable market theory.

<u>Fig. 2</u>

We now illustrate why we have introduced the <u>brand</u> <u>name</u> story to our formulation. Consider the case where N = 2 and the average cost curve and the industry demand curve are as depicted in Fig. 3. The theory of long-run perfect competition and the contestable market theory both predict that the industry stabilizes at a state where one of the firms set price at p_c and produces y_c units. This is one of the situations which we later call natural monopoly. Now suppose that demand is allocated equally to the two firms when they charge the same price. Then, the competitive outcome is never a Nash equilibrium. This is because at $p_1 = p_2 = p_c$, both firms get half of demand which is too small to cover production costs. If $p_1 > p_2 = p_c$, then firm 2 can raise price a little and increase his profit. In fact, one can easily show that this game has no Nash equilibrium in pure strategies.

Fig. 3

Thus, if we assume that demand is allocated equally in case of price ties, then the natural monopoly outcome is never a game-theoretic equilibrium in a model with prices as strategic variables and with potential entry.

This observation leads to two different directions to pursue further. Grossman maintained the equal-demand-for-equal-price rule and changed the strategy space to the space of supply functions. We instead assume a lexicographic rationing rule when several firms offer the same price. First of all, the latter is exactly what happens in the Marshallian equilibrium. In the case of Fig. 3 it predicts an outcome where one firm operates at (p_c, y_c) and other potential firms are excluded from the market even if they announce price p_c. Secondly, it conforms to reality: a potential entrant is unable to capture a portion of demand if he charges the prevailing market price. We treat the lexicographic ordering as exogenously given, since our analysis do not intend to predict who will become insiders. The brand name has only a limited role in that if two firms charge the same price consumers first visit the one with stronger brand. If they charge differing prices they first visit the one with lower price. Thirdly, with this assumption we can get around constructing the artificial game with supply functions equilibria. We believe that this is the way one should extend Bertrand's classical idea of oligopolistic price competition to technologies without constant returns to scale and industries with free entry.

We now state the assumptions that we have implicitly used in the foregoing discussion.

Assumption 1. $N \geq m_c + 1$, where m_c is a positive integer defined by (7).

Assumption 2. $D(p) - m_c y_c < \underline{Y}(p)$ for all $p \geq p_c$.

Assumption 1 embodies the threat of potential entry by requiring that the number of potential firms is at least $m_c + 1$. Assumption 2 says that when m_c firms each charges price p_c and supplies y_c units the residual demand curve is uniformly to the left of the average cost curve (recall Fig. 2). We will examine the consequence of deleting Assumption 2 in the next section.

We have already shown that under these assumptions any strategy pair $\mathbf{P} = (p_1, p_2, \cdots, p_N)$ in which $m_c + 1$ or more prices are p_c constitutes an equilibrium point of our game. The first m_c firms among those with price p_c produce y_c units of output and other $N - m_c$ firms become outsiders. The rest of this section shows that there is no other equilibria in pure strategies.

We first observe the following. For any $\mathbf{P} = (p_1, p_2, \cdots, p_N)$, $i = 1, 2, \cdots, N$ and $k \notin I_i(\mathbf{P}) \cup \{i\}$, we have

$$D(p_k) - \sum_{j \in I_k(\mathbf{P})} x_j(\mathbf{P}) \leq D(p_i) - [D(p_i) - D(p_k)] - \sum_{j \in I_i(\mathbf{P})} x_j(\mathbf{P}) - x_i(\mathbf{P})$$

$$\leq [D(p_i) - \sum_{j \in I_i(\mathbf{P})} x_j(\mathbf{P})] - x_i(\mathbf{P})$$

$$\leq D_i(\mathbf{P}) - x_i(\mathbf{P}),$$

in which the first inequality uses $I_k(\mathbf{P}) \supset I_i(\mathbf{P}) \cup \{i\}$ and $x_j(\mathbf{P}) \geq 0$, the second uses $D(p_i) \geq D(p_k)$, and the third uses (4). Then, using (4) again, we get

$$D_k(\mathbf{P}) = \max \{D(p_k) - \sum_{j \in I_k(\mathbf{P})} x_j(\mathbf{P}) \ , \ 0\}$$

$$\leq \max \{D_i(\mathbf{P}) - x_i(\mathbf{P}) \ , \ 0 \}.$$

The last inequality simply says that the number of consumers who visit firm k is at most the number of consumers who visit firm i less the number of consumers who are served by firm i. Therefore, if $\underline{Y}(p_i) \leq D_i(\mathbf{P}) \leq \overline{Y}(p_i)$, then, from (5), $x_i(\mathbf{P}) = D_i(\mathbf{P})$ and hence $D_k(\mathbf{P}) = 0$. Taking the contraposition, $D_k(\mathbf{P}) > 0$ implies that either $D_i(\mathbf{P}) < \underline{Y}(p_i)$ or $D_i(\mathbf{P}) > \overline{Y}(p_i)$ holds. In the former case i's demand is not sufficient to cover production cost. In the latter case, using (5) we obtain $x_i(\mathbf{P}) = \overline{Y}(p_i)$. Verbally, it states that (1)if firm i's profit is nonnegative ($D_i(\mathbf{P}) \geq \underline{Y}(p_i)$) and (2)if at least one firm either announces a higher price than i or announces the same price but has weaker brand, and yet obtains positive demand, then firm i must produce on the marginal cost curve. To summarize:

LEMMA 1. <u>For any</u> $\mathbf{P} = (p_1, p_2, \cdots, p_N)$, <u>if</u> $D_i(\mathbf{P}) \geq \underline{Y}(p_i)$ <u>and there is at least one</u> $k \notin I_i(\mathbf{P}) \cup \{i\}$ <u>for which</u> $D_k(\mathbf{P}) > 0$, <u>then</u> $x_i(\mathbf{P}) = \overline{Y}(p_i)$.

PROPOSITION 1. <u>No firm earns positive profit in any pure-strategy equilibrium</u>.

PROOF. Let $(p_1, \cdots, p_N)$ be a Nash equilibrium. We first observe that if at least one firm earns positive profit then every other firm must also earn positive profit at this equilibrium. To see this, let i be the firm who earns positive equilibrium profit, i.e., $p_i x_i(\mathbf{P}) - C(x_i(\mathbf{P})) > 0$. Let j be currently earning no profit, i.e., $p_j x_j(\mathbf{P}) - C(x_j(\mathbf{P})) = 0$. Then, by changing price from p_j to $p_j' = p_i - \epsilon$ with $\epsilon > 0$, he can capture at least $x_i(\mathbf{P})$ units of demand. If he chooses ϵ sufficiently small, he is guaranteed of a positive profit by producing $x_i(\mathbf{P})$. Hence, denoting $\mathbf{P'} = (p_1, \cdots, p_{j-1}, p_j', p_{j+1}, \cdots, p_N)$ we have $p_j' x_j(\mathbf{P'}) - C(x_j(\mathbf{P'})) > 0$. This implies that $(p_1, \cdots, p_N)$ cannot be an equilibrium.

Suppose all N firms earn positive profits at an equilibrium $(p_1, \cdots, p_N)$. Then $D_i(\mathbf{P}) > \underline{Y}(p_i)$ for all $i = 1, \cdots, N$. Let $\hat{i}$ be the firm who announces the highest price, or if there are several such firms, has the weakest brand. From Lemma 1, $x_i(\mathbf{P}) = \overline{Y}(p_i)$ for all i except $\hat{i}$. We now focus on the demand to $\hat{i}$. Noting that $I_{\hat{i}}(\mathbf{P}) = N - \{\hat{i}\}$, (4) yields

$$D_{\hat{i}}(\mathbf{P}) = \max \{ D(p_{\hat{i}}) - \sum_{j \neq \hat{i}} x_j(\mathbf{P}) \ , \ 0 \}$$

$$= \max \{ D(p_{\hat{i}}) - \sum_{j \neq \hat{i}} \overline{Y}(p_j) \ , \ 0 \}. \qquad (8)$$

Since $\overline{Y}(p_j) \geq y_c$, it follows that

$$\sum_{j \neq \hat{i}} \overline{Y}(p_j) \geq (N - 1)y_c \geq m_c y_c,$$

where the second inequality uses Assumption 1. On the other hand, using Assumption 2 we have

$$D(p_{\hat{i}}) - m_c y_c < \underline{Y}(p_{\hat{i}}).$$

Combining these two inequalities we get

$$D(p_{\hat{i}}) - \sum_{j \neq \hat{i}} \bar{Y}(p_j) < \underline{Y}(p_{\hat{i}}),$$

and, using (8),

$$D_{\hat{i}}(P) < \underline{Y}(p_{\hat{i}}).$$

From (5) this implies $x_{\hat{i}}(P) = 0$; namely, firm $\hat{i}$ earns zero profit. Thus we obtain a contradiction. ■

PROPOSITION 2. Any firm who sets his price above p_c has no market share in any pure-strategy equilibrium.

We use the following lemma to prove this proposition.

LEMMA 2. At most one firm can charge price above p_c and produce a positive quantity of output in any pure-strategy equilibrium.

PROOF. Suppose that a firm i charges a price $p_i > p_c$ and produce a positive quantity $x_i(P) > 0$ in equilibrium. From Proposition 1 his profit must be zero, so his price-output pair $(p_i, x_i(P))$ must lie on $\underline{Y}(\cdot)$, the downward-sloping portion of the average cost curve. Let A denote the point at which i operates (see Fig. 4). Then he is operating below $\bar{Y}(p_i)$, and from Lemma 1 it implies that his price must be highest among all active firms and that all other active firms, including the ones who also charge the highest price, must be producing on $\bar{Y}(\cdot)$. But, no firm except i can charge the highest price, since if there were a firm who operates at $(p_i, \bar{Y}(p_i))$ then it would contradict Proposition 1. Further, no firm can set his price in the open interval (p_c, p_i), since, from Proposition 1, he must produce on $\underline{Y}(\cdot)$ and to produce below $\bar{Y}(\cdot)$ would contradict Lemma 1. Thus all other active firms must operate at the bottom of the average cost curve, B. ■

Fig. 4

PROOF OF PROPOSITION 2. Suppose a firm sets his price above p_c and produce a positive quantity in equilibrium. From Lemma 2, all other active firms must set their prices at p_c. From (7), there are m_c firms in the second group of firms. Then, from Assumption 2, the residual demand to the first firm must be insufficient to cover his production cost. This means that the first firm's profit is negative, a contradiction. Therefore, all active firms must charge price p_c in any equilibrium. ■

Proposition 2 immediately implies that there is no pure-strategy equilibrium to our game except the one in which at least $m_c + 1$ firms announce p_c and others announce higher prices. The demand rationing scheme and the firms' production decisions, Eqs.(4) and (5), imply that m_c firms who has stronger brand each produce y_c units and others stay out of the industry in this equilibrium. This obviously corresponds to the Marshallian equilibrium outcome under long-run perfect competition.

If one carefully reads the above proofs, it should be clear that our result does not depend on the specific rationing rule that we defined in Section 2. The essential step in the proofs was Lemma 1. It would result from other rationing schemes, too, as long as one maintains the brand name story in the case of price ties. Naturally, Assumption 2, on the magnitude of the redisual demands, must be stated with a proper modification. We chose not to generalize the formulation, merely because we didn't want to confuse the readers with nonessential notational complexities.

4. EXTENSIONS OF THE RESULT

The result of the previous section can be extended to other situations. Here, we examine three of them. The first and economically most important is the case in which the industry turns out to be monopolized by a single firm in equilibrium . This is usually called the case of natural monopoly. The second is the case in which the average cost curve is flat-bottomed. The third is the case in which residual demands are large enough to intersect the average cost curve.

(1) Natural monopoly

The exposition in the last section already included a case of natural monopoly: namely, if m_c defined by (7) is 1 then a single firm operates in the industry with the price-output pair (p_c, y_c). In fact we discussed this case in Fig. 3.

Here we assume that either the average cost curve is U-shaped and m_c is zero or the average cost curve is monotone decreasing (i.e., y_c is $+\infty$). In the latter case we assume that the average cost curve lies above the demand curve for sufficiently large levels of output. We consider a situation in which these two curves intersect as exhibited in Fig.5. We replace Assumption 1 by $N \geq 2$ and delete Assumption 2.

Fig. 5

Let p_ℓ denote the minimum price at which the average cost curve and the industry demand curve intersect. And let $y_\ell = D(p_\ell)$. An obvious equilibrium for this case is the one in which at least two firms announce p_ℓ and others announce higher prices. In this equilibrium, only one firm, the monopolist, operates at (p_ℓ, y_ℓ) and other N - 1 firms stay out of the market. This outcome is usually called the Ramsey second-best solution, and it again coincides with the prediction of the contestable market theory. Note that the presence of a potential entrant quoting the incumbent's price is critical to sustain this outcome as an equilibrium of our game. It is straightforward to prove that no other (pure-strategy) equilibrium exists in this situation.

PROPOSITION 3. The unique pure-strategy equilibrium of the game in Fig. 5 is the one in which at least two firms announce p_ℓ and others announce higher prices. Only one firm produce y_c units in this equilibrium and other N-1 firms stay out of the market.

PROOF. The industry demand is such that the average cost of production exceeds the price for any level of output if price is set below p_ℓ. Thus the relevant price range is $[p_\ell, \infty)$. The average cost is declining at least in the output range $y \leq y_\ell$. Hence, the unit margin (price minus average cost) strictly increases in y for $y \leq y_\ell$ if price is fixed. So, profit-maximizing level of output, $\bar{Y}(p)$, exceeds y_ℓ , and hence it exceeds the total industry demand D(p) for any $p \geq p_\ell$. This implies, from Eqs. (4) and (5), that for any price tuple $\mathbf{P} = (p_1,\cdots,p_N)$ the industry demand will concentrate on one firm; namely, on the one who has the strongest brand among those with the lowest price. For other firms, $x_i(\mathbf{P}) = 0$.

Let i = 1 be the monopolist, without loss of generality. All we have to show is that his price, p_1, must be p_ℓ if $\mathbf{P}$ is an equilibrium. Suppose the contrary that $p_1 > p_\ell$. Then, any of the remaining N-1 firms can change his price below p_1 and capture the whole industry demand with positive profit. This contradicts to equilibrium. If p_1 equals p_ℓ and all other p_i's are higher than p_ℓ, firm 1 has incentive to increase his price a bit and earn positive profit. Hence, at least two firms must charge p_ℓ to sustain the equilibrium. ■

(2) Flat-bottomed Average Cost Curve

The contestable market theory, summarized in Introduction, uses an equilibrium notion which requires market-clearing. (Note that it is not a game-theoretic equilibrium). Thus, if the total industry demand at the equilibrium price $D(p_c)$ is not an integer multiple of y_c (the minimum-average-cost level of output), then there is no equilibrium. Baumol et. al. resolved this nonexistence problem, alternatively called the integer problem, by resorting to the casual observation that the average cost curve is nearly flat around the bottom. Fig. 6 exhibits a typical flat-bottomed average cost curve.

<u>Fig. 6</u>

Our construction accommodates the case in which $D(p_c)$ is not an integer multiple of y_c. As seen in the previous section, our equilibrium may permit the presence of unfulfilled demands. Therefore, we need not resort on a flat-bottomed average cost curve in such a case.

It should also be clear that the discussion in the previous section is fully compatible with a flat-bottomed average cost curve.[4/] That is, all the statements remain valid if we change the definition of y_c to

$$y_c = \max \{\text{argmin } C(y)/y\}.$$

(3) Large Residual Demand

The final situation we consider is the one in which, given that m_c firms each supply y_c units of the good at price p_c, the residual market demand is large enough to cover the production cost for some output level. This is a case where Assumption 2 is violated. Fig. 7 exhibits such a case. We must replace Assumption 1 to $N \geq m_c + 3$ to deal with this situation..

Fig. 7

It is easy to offer one equilibrium for this game. Let p_ℓ denote the price level at which the residual demand curve intersects the average cost curve. If there are multiple intersections, let p_ℓ be the minimum price. Let $y_\ell = D(p_\ell)$. Then, an equilibrium is a price tuple $\mathbf{P} = (p_1, \cdots, p_N)$ in which $m_c + 1$ firms' prices are p_c, two or more firms prices are p_ℓ, and other prices are higher than p_ℓ.

To see if this is an equilibrium, consider any firm who charges p_c and produces y_c. He has no chance of earning positive profit, since if he raises price the firm who charges p_c and remain inactive will come in and take away his current customers. Note that the latter firm currently produces nothing, since his current demand is insufficient to cover production cost at price p_c. The same reasoning applies to the firm who operates at (p_ℓ, y_ℓ), since he is faced with the potential challenge from the one who charges p_ℓ and currently remain inactive. No chance is left to the currently inactive $N - (m_c+1)$ firms for earning positive profits, either. Here, a sort of natural monopoly occurs for the residual demand among the firms who charges more than p_c.

It is again straightforward to show that this is the only pure-strategy equilibrium of the game. The zero-equilibrium-profit property is proved by repeating the argument made in Proposition 1 and using the assumption that N $\geq m_c + 3$. Lemma 2 of the previous section establishes that at most one firm can get a positive market share with price higher than p_c. Clearly, the minimum of the zero-profit prices, p_ℓ, is the only price that he can charge. Thus, m_c firms must charge p_c (producing y_c units) and one firm must charge p_ℓ (producing y_ℓ units) for any equilibrium. And, at least two additional firms, one with price p_c and one with price p_ℓ, are required to sustain this equilibrium.

FOOTNOTES

1/ Baumol, Panzar and Willig, op. cit., p.44.

2/ Ibid.

3/ Theoretically there could be an infinite sequence of strategic actions and counteractions between incumbents and entrants. So the validity of this conjecture should be tested in repeated game formulations. Maskin and Tirole [1988] is such an example. They show an interesting result that the (Markov) perfect equilibrium of their alternating-move dynamic game coincides with the prediction of the contestable market theory if the time discount factor is near one; that is, if firms are sufficiently far-sighted or one period (commitment time) is very short.

4/ This case is not covered in Grossman's formulation [1981]. We can extend our results to cases with more oddly-shaped average cost curves. The critical assumption in our proof is that the average cost is non-increasing in the range where output is smaller than y_c.

REFERENCES

[1] Baumol, W.J., J.C. Panzar and R.D. Willig: Contestable Markets and the Theory of Industry Structure, Harcourt Brace Jovanovich Inc., 1982.

[2] Bertrand, J.: "Review of Thèorie Mathematique de la Richesse Sociale and Recherches sur les Principles Mathematique de la Thèorie des Richesses," Journal des Savants, (1883), 499-508.

[3] Grossman, S.J.: "Nash Equilibrium and the Industrial Organization of Markets with Large Fixed Costs," Econometrica, 49 (1981), 1149-1172.

[4] Kreps, D.M. and J.A. Scheinkman: "Quantity precommitment and Bertrand competition yield Cournot outcomes," The Bell Journal of Economics, 14 (1983), 326-37.

[5] Maskin, E. and J. Tirole: "A Theory of Dynamic Oligopoly, I: Overview and Quantity Competition with Large Fixed Costs," Econometrica, 56 (1988), 549-569.

[6] Mirman, L.J., Y. Tauman and I. Zang: "Monopoly and Sustainable Prices as a Nash Equilibrium in Contestable Markets," Discussion Paper No.562, The Center for Mathematical Studies in Economics and Management Science, 1984.

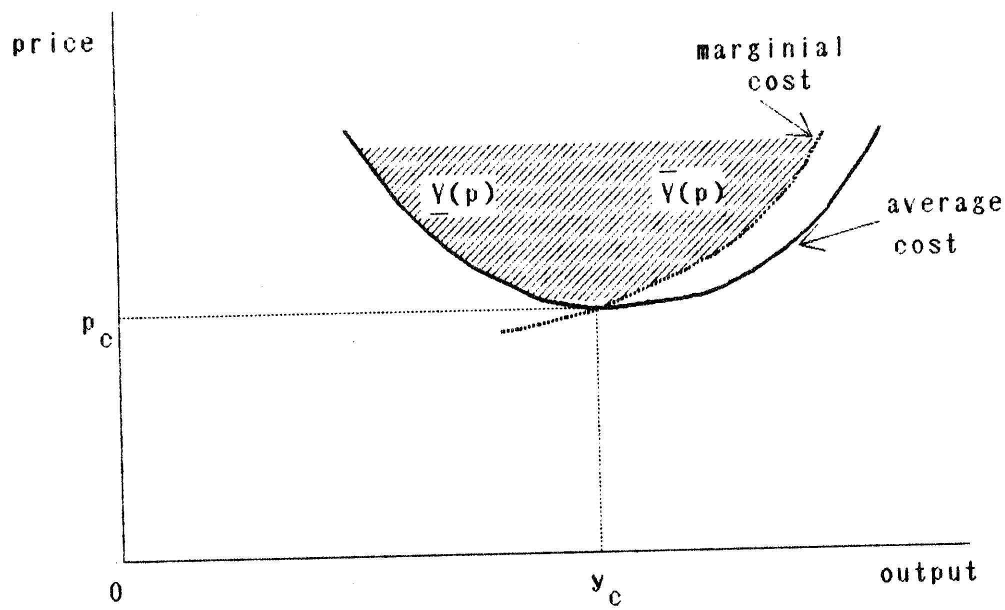
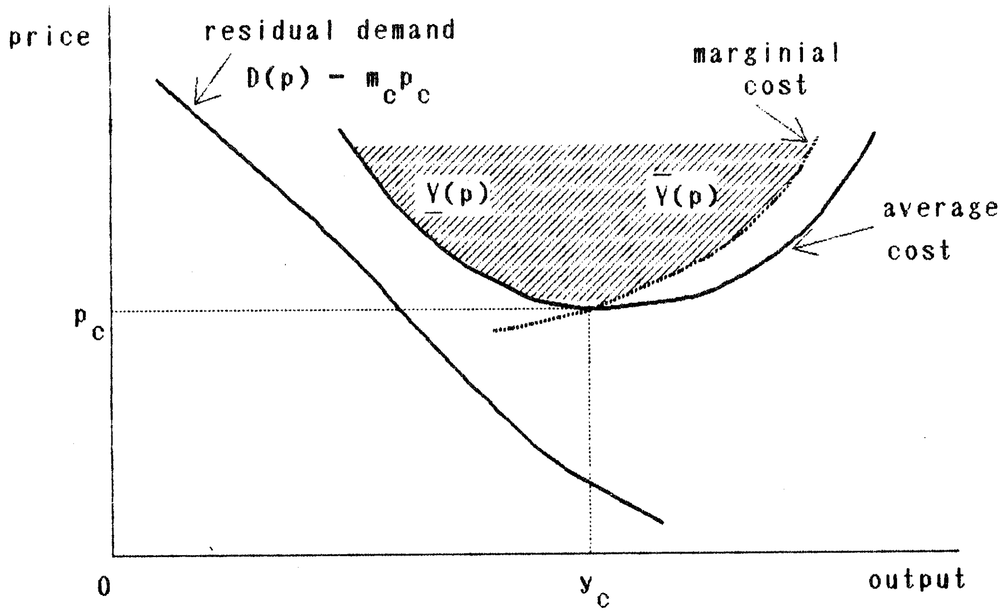


Fig. 1



price
residual demand
D(p) - m c p c
marginial
cost
Y(p)
Y(p)
average
cost
p c
0
y c
output

Fig. 2

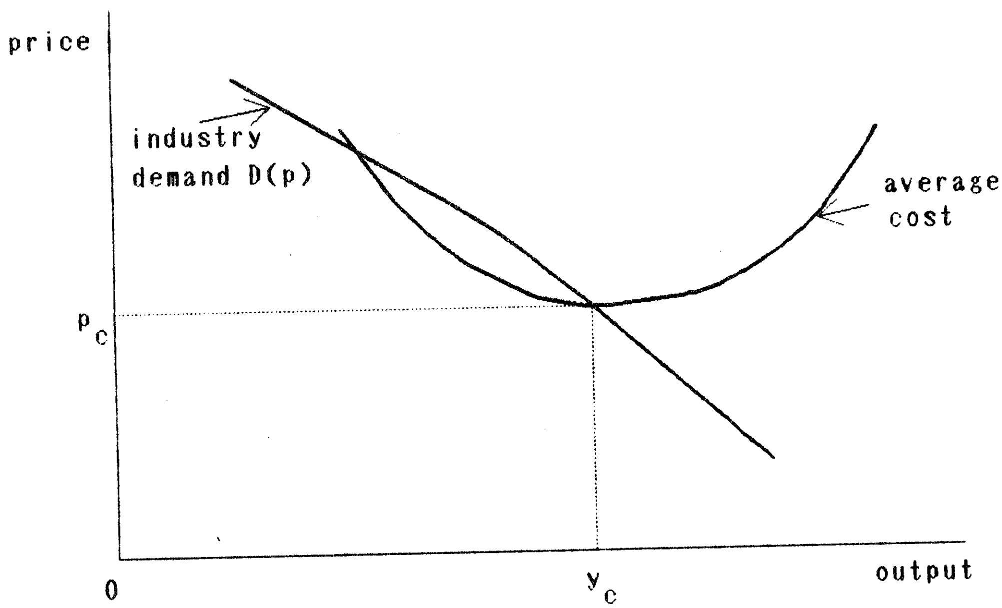
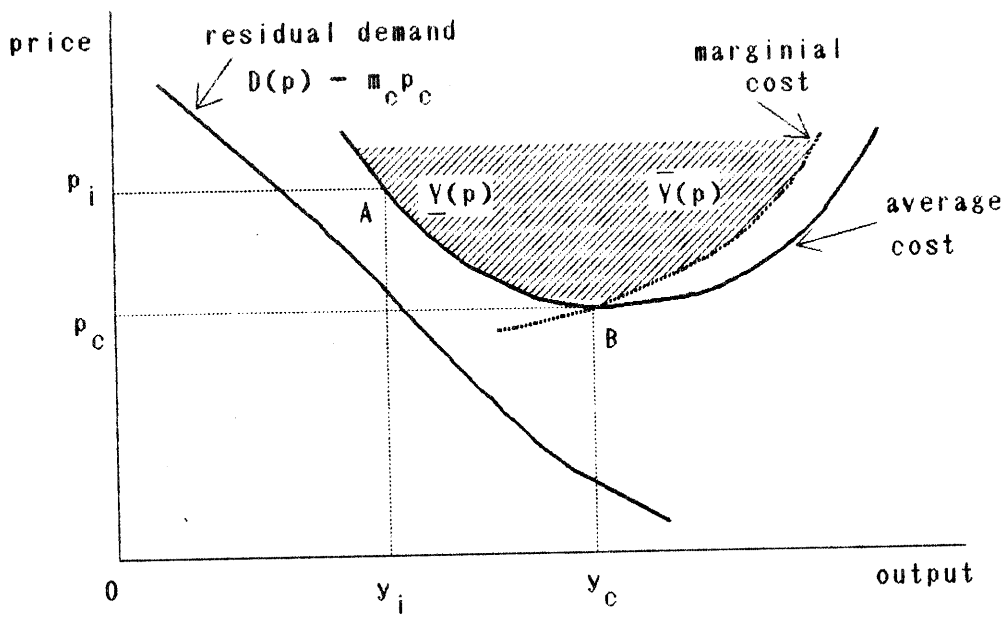


Fig. 3



price
residual demand
D(p) − m_c p_c
marginial
cost
p_i
Y(p)
Y(p)
average
cost
A
p_c
B
0
y_i
y_c
output

Fig. 4

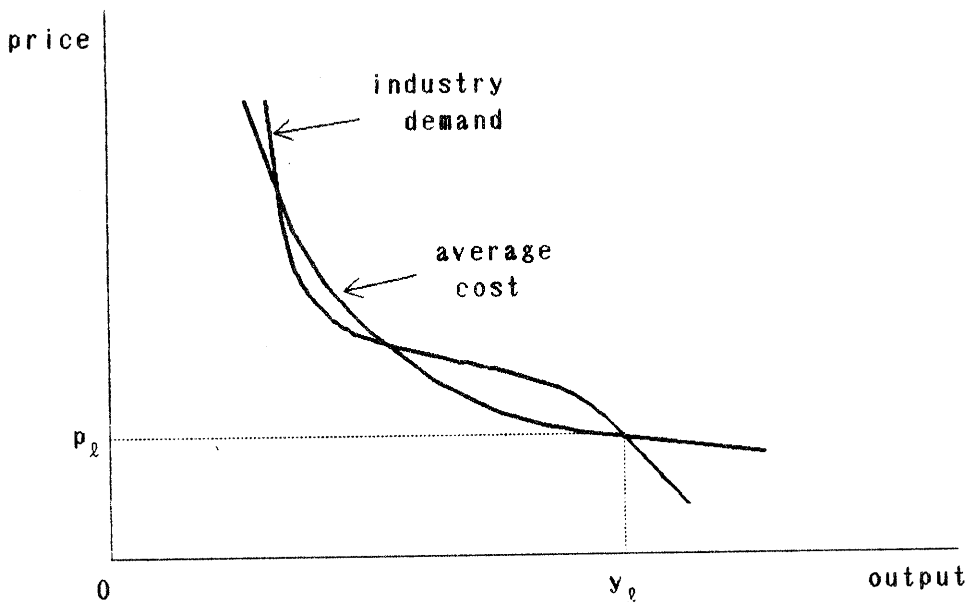


Fig. 5

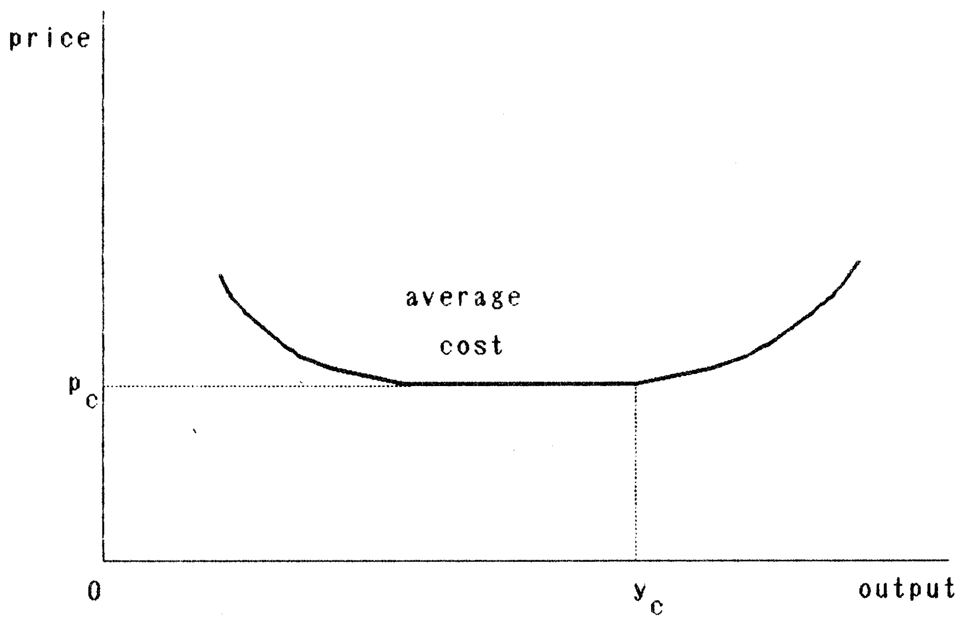


Fig. 6

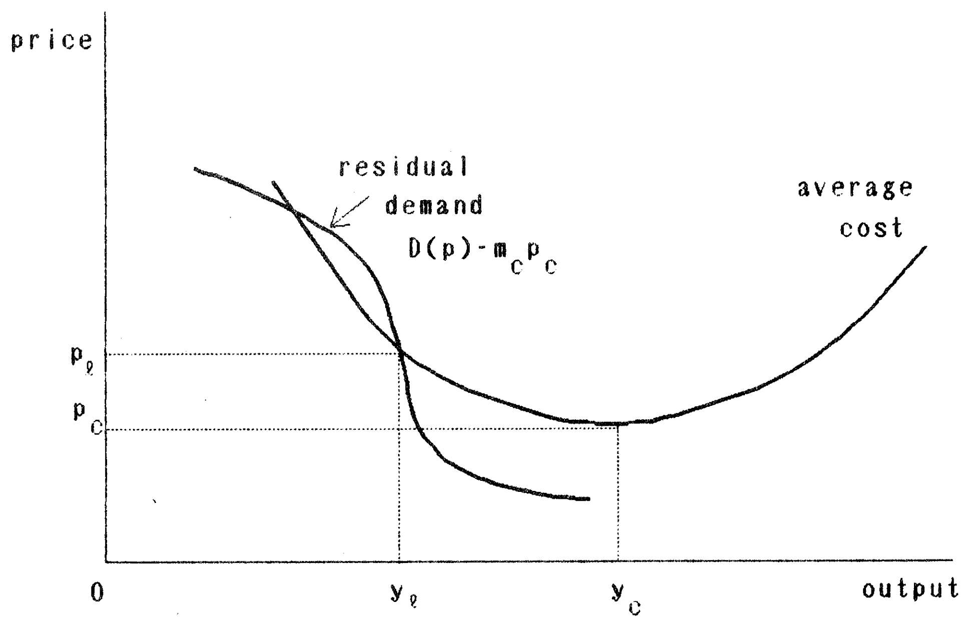

price
residual
demand
D(p)-$m_c p_c$
average
cost
p_ℓ
p_c
0
y_ℓ
y_c
output

Fig. 7