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Adam Smith gave several interesting numerical examples in his Early Draft of the Wealth of Nations (1763) to show how the price of the labor product would be diminished and the wages of labor increased, as a result of the division of labor. In view of pre-capitalistic nature of the economy considered there, however, it is difficult to affirm that such examples can really be realized, as was admitted by Smith himself. In his Wealth of Nations (1776), on the other hand, Smith considered fully capitalistic growing economies, where the natural rate of wage is higher than the subsistence level. Our aim in this note is, therefore, to show firstly that in such economies prices are reduced and wages are increased as a result of a further division of labor, and secondly that Smith's numerical example can really be realized if it is considered as an example in the case of a capitalistic growing economy.

A numerical example given in the Early Draft is introduced in section (2) while section (3) is devoted to show that in the Wealth of Nations Adam Smith emphasized effects of the division of labor, which he tried to demonstrate by his numerical example in the Early Draft. We construct a Smithian growth model in section (4), in the balanced growth path of which Smith's natural prices and natural wages are realized. By the use of this model,

section (5) is devoted to show that a further division of labor in the production of a commodity reduces its natural price, and increases the natural rates of wages and profit. Finally in section (6) we demonstrate that Smith's numerical example in the Early Draft can be generated from this growth model of the Wealth of Nations.

In the so-called Early Draft of the Wealth of Nations, Adam Smith gave several numerical examples of effects of the division of labor, among which the most interesting runs as follows.[1)]

Let us suppose, for example, to return to the frivolous instance, which I formerly gave, that pins may be valued at a penny the hundred, which is nearly the price of some particular sorts of them. The pinmaker who, according to the foregoing supposition, could be considered as making two thousand pins a day, produces work to the values of twenty pence. Let five pence be allowed for the price of the wire, the wear of the tools and the profits of the master of the work, there remain fifteen pence for the wages of the artizan, with which he can purchase all the necessaries and conveniences of life. The case here, is the same, as if he gave five hundred pins to his master for affording him the wire the tools and the employment, and kept fifteen hundred to himself, in order to be exchanged for the productions of the other arts which he had occasion for. For it is the same thing, with regard to opulence, whether we consider a person as possessed of a particular merchandize, or of the value of a particular merchandize. Let us suppose, that by still further divisions of labour and improvements of art, a pin-maker could be made to produce four thousand pins a day. In this case, tho' pins were to be valued one forth less, and to be sold for three farthings the hundred, the artizan would produce work to the value of thirty pence a day. His master might have ten pence or the value of thirteen hundred and thirty three pins for his

profits and expenses, and the artizan retain twenty pence or the value of two thousand six hundred and sixty seven pins for his wages. The price of the work would be diminished, and the wages of the labourer increased; the public would be better supplied and the workmen more amply rewarded (Smith, 1763, p.331).

Let us note that a further division of labor causes not only an increase in the labor productivity (from 2000 pins to 4000 pins) but also an increase in wages (from 15 to 20 pence), an increase in gross profits (from 5 to 10 pence) and a reduction of the price of the product (from 1/100 penny to 3/400 pence). Although Smith wished to emphasize that " labour becomes dear and work cheap " (1763, p.332) as a result of a further division of labor, however, he admitted that " I do not mean that the profits are divided in fact precisely in the above manner, but that they may be divided in such manner " (Smith, 1973, p. 331). In other words, Smith had to be satisfied merely to point out a possibility of high wages and low prices, not the necessity.

Perhaps we may interpret this as follows. While the effect on the productivity of labor can be determined technologically within a single production unit, the distribution of the surplus created by the division of labor (which Smith called profits in the above) into the reduction of price of the product and the increase in wages cannot be considered without having recourse to the analysis of changes in equilibrium prices and wages in a whole economy.

For an economy considered in the Early Draft, where the capitalist production does not dominate yet,[2] however, it is

very difficult to construct an equilibrium model of a whole economy by which we can see the effect of the division of labor on equilibrium level of prices and wages, which Smith wished to demonstrate. Although Smith had the system of natural (competitive equilibrium) prices of such an economy (1963, pp. 345-6), for example, his concept of natural price of labor is still limited to the subsistence level of wages, i.e., the cost of living and education, including some insurance.[3] If so, how can such a wage be increased as the result of a further division of labor? One might, furthermore, argue that it is meaningless to consider natural prices and wages in such an economy, since numbers of independent producers are not sufficiently large and their mobility among sectors is rather limited (Morishima and Catephores, 1978, p. 185).

These considerations suggests to leave the world of the Early Draft and proceed to the capitalist economy of the Wealth of Nations to show not only the technological possibility but also the economic necessity of low prices and high wages caused by the division of labor.

It is well known that Adam Smith started his Wealth of Nations also with numerical examples of the divisions of labor. But they are limited to effects of the divisions of labor on the labor productivity. Unlike in the case of the Early Draft, no numerical examples are given to show that " labour becomes dear and work cheap " as a result of the division of labor. Though not quantitatively, however, Smith still emphasizes that " labour becomes dear and work cheap " in his qualitative arguments on the effects of the division of labor.

The same cause, however, which raises the wages of labour, the increase of stock, tends to increase its productive powers, and to make a smaller quantity of labour produce a greater quantity of work. The owner of the stock which employs a great number of labourers, necessarily endeavours, for his own advantage, to make such a proper division and distribution of employment, that they may be enabled to produce the greatest quantity of work possible. - - - - What takes place among the labourers in a particular workhouse, takes place, for the same reason, among those of a great society. The greater their number, the more they naturally divide themselves into different classes and subdivisions of employment. - - - - There are many commodities, therefore, which, in consequence of these improvements, come to be produced by so much less labour than before, that the increase of its price is more than compensated by the diminution of its quantity (Smith, 1776, p.104).

It is the natural effect of improvement, however, to diminish gradually the real price of almost all manufactures. That of the manufacturing workmanship diminishes, perhaps, in all of them without exception. In consequence of better machinery, of greater dexterity, and of a more proper division and distribution of work, all of which are the natural effects of improvement, a much smaller quantity of labour becomes requisite for executing any particular piece of work; and though, in consequence of the flourishing circumstances of the society, the real price of labour should rise very considerably, yet compensate the greatest rise which can happen in the price (Smith, 1776, p.260).

The increase of demand, though in the beginning it may sometimes raise the price of goods, never fail to lower it in the long run. It encourages production, and thereby the competition of the producers, who in order to undersell one another, have recourse to new divisions of labour and new improvements of art which never otherwise have been thought of (Smith, 1776, p.748).

Unlike in the Early Draft, the natural rate of wage (the rate of wage in a long run equilibrium) is not limited to the level of subsistence in the Wealth of Nations.[4] In a growing economy, it is higher than in the stationary economy where it is at subsistence level. It is the rate of wage, which equate the supply of labor to the demand for labor. The latter in turn depends on the wages fund, the stock of products produced by labor and supplied by capitalists to employ labor. " The liberal reward of labour, therefore, as it is the necessary effect, so

it is the natural symptom of increasing national wealth. The scanty maintenance of labouring poor, on the other hand, is the natural symptom that things are at stand " (Smith, 1776, p.91).

Our next task is, therefore, to construct a balanced growth model of national wealth and labor in which we can show that low natural prices and high natural wages are resulted from a further division of labor.[5]

Let us construct our three sector Smithian growth model, in which there are two sectors to produce products and one sector to (re)produce labor.

$$(1) \qquad L(t) = a_{xL}X(t+1) + a_{yL}Y(t+1)$$

$$(2) \qquad X(t) = a_{Lx}L(t+2) + a_{yx}Y(t+1)$$

$$(3) \qquad Y(t) = a_{Ly}L(t+2) + a_{xy}X(t+1)$$

L(t), X(t) and Y(t) denote, respectively, the labor population at period t, output of product x at t and that of product y at t, while a_{Lx}, a_{xL} and a_{xy} signify, respectively, the amount of x necessary to produce one unit of labor, that of labor necessary to produce one unit of x and that of y necessary to produce one unit of x, and the like. The production period of labor is assumed twice as long as those of products,[6] and the consumption of capitalists is assumed away.[7]

If we rewrite (1), (2), (3) into

$$(4) \qquad L(t) = a_{xL}X(t+1) + a_{yL}Y(t+1)$$

$$(5) \qquad X(t) = a_{yx}Y(t+1) + a_{Lx}K(t+1)$$

$$(6) \qquad Y(t) = a_{xy}X(t+1) + a_{Ly}K(t+1)$$

$$(7) \qquad K(t) = L(t+1)$$

and consider a such balanced growth that $L(t) = L(1+g)^t$, $X(t) = (1+g)^t$, $Y(t) = Y(1+g)^t$, and $K(t) = K(1+g)^t$ for

some positive g, L, X, Y and K, Frobenius theorem assures us the existence of such a growth path in which 1+g is the inverse of a positive eigen-value of the matrix A of coefficients in the right hand side of (4), (5), (6), i.e.,

(8)
$$\begin{bmatrix} 0 & AxL & AyL & 0 \\ 0 & 0 & Ayx & ALx \\ 0 & Axy & 0 & ALy \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

and (L, X, Y, K) is a positive eigen-vector of matrix A (Debreu and Herstein, 1953).

On such balanced growth path, relative prices of labor and products have to be unchanged so that we have equations of natural prices

(9) $w = (1 + s)(a_{Lx}p + a_{Ly})$

(10) $p = (1 + r)(a_{xL}w + a_{xy})$

(11) $1 = (1 + r)(a_{yL}w + a_{yx}p)$

in which product y is the numeraire and w, p and r denote, respectively, the natural rate of wage, the natural price of product x and the natural rate of profit. As is seen in (9), the natural rate of wage is different from the subsistent wage ($a_{Lx}p + a_{Ly}$).

From the budget constraint of labor on the balanced growth

path,

(12) $wL(t) = (1+g)^2 L(t)a_{Lx}p + (1+g)^2 L(t)a_{Ly}$

and that of the capitalists of x sector,

(13) $pX(t) = (1+g)X(t)\, a_{xL}\, w + (1+g)X(t)a_{xy}$

we have $(1+g)^2 = 1+s$ and $(1+r) = (1+g)$ in view of (9) and (10). Then, (9), (10) and (11) can be rewritten as

(14) $w = (1+r)q$

(15) $p = (1+r)(a_{xL}w + a_{xy})$

(16) $1 = (1+r)(a_{yL}w + a_{yx}p)$

(17) $q = (1+r)(a_{Lx}p + a_{Ly})$

so that the existence of positive price-vector is assured as an eigen-vector of the transposed matrix A in (8).

On the Smithian balanced growth path, the rate of profit r is equal to the rate of growth g, while the natural rate of wage w is higher than the subsistence level if g is positive, which we so assume.

By using the model constructed in the previous section, we can study effects of a further division of labor, which Smith tried to show by his numerical examples which we introduced in section (2). Suppose x sector is an industry where a further division of labor is introduced, i.e., it is the industry which was represented by pin-makers in Smith's examples. Although the price of pin and wages of labor are given in terms of money in Smith's examples to show "labour becomes dear and work cheap," we consider them in terms of the price of the product y,[8] which represents other goods in general in the production of which the division of labor remains unchanged. This is because, for example, " the labourer is rich or poor, is well or ill rewarded, in proportion to the real, not to the nominal price of his labour " (Smith 1776, p.51).

To consider the effects of a further division of labor on the natural rate of profit r, the natural rate of wage w and the natural price p of the relevant product x, let us differentiate (9), (10) and (11) with respect to A_{xL}.

$$(18) \qquad dw = 2(1+r)(a_{Lx}p + a_{Ly})dr + (1+r)^2 a_{Lx}dp$$

$$(19) \qquad dp = (a_{xL}W + a_{xy})dr + (1+r)a_{xL}dw + (1+r)w\, da_{xL}$$

$$(20) \qquad 0 = (a_{yL}w + a_{yx}p)dr + (1+r)a_{yx}dp + (1+r)a_{yL}dw$$

Rewriting (18), (19) and (20) into

(21) $B\ (\ dp,\ dr,\ dw\ ) = (\ -(1+r)w\ da_{xL},\ 0,\ 0\ )$

where B is a 3 by 3 matrix composed of coefficients of the right hand side coefficients of (19), (18) and (20), we can solve for

(22) $dr/da_{xL} = (1+r)^2 w\ [(1+r)^2 a_{Lx}a_{yL} + a_{yx}\ ]/\ |B|$

(23) $dp/da_{xL} = -w(\ 2wa_{yL} + 1\ )\ /\ |B|$

(24) $dw/da_{xL} = -(1+r)\ w\ ((1+r)A_{Lx} - 2w\ a_{yx}\ )\ /\ |B|.$

In view of (14), (15), (16), (17) and $r > 0$, we can easily confirm that $|B| < 0$. Then, $dr/da_{xL} < 0$, [9] $dp/da_{xL} > 0$, and $dw/da_{xL} < 0$ if

(25) $(1+r)A_{Lx} - 2wa_{yx} < 0.$

In words, the natural rate of profit is increased and the natural price of its product is reduced, if the labor productivity in a sector is increased (a_{xL} is reduced) by a further division of labor. Furthermore, the natural rate of wage is increased if (25) is satisfied.

Similar results are obtained if waste of materials is reduced (a_{xy} is reduced) as a result of a further division of labor.

The final task left to be done is to check whether Smith's numerical examples cited in Section (2) above can be consistent with the natural equilibria in our Smithian growth model developed in section (4) above.

There are two situations. In the first situation, the natural rate of wage w is 15, the natural price p of product x is 1/100, the labor input coefficient a_{xL} of product x is 1/2000, and the gross profit per laborer in the production of x is 5. In the second situation, w is 20, p is 3/400, a_{xL} is 1/4000, and the per capita gross profit in x is 10.

We have to supply numerical values which are not given in Smith's example, i.e., the natural rate of profit r and input coefficient a_{xy} in the two situations, and other a_{ij}'s (i = L, y, j = L, x, y) which are unchanged between two situations. To find numerical values consistent with equations (9), (10) and (11), note that the possible range of the ratio of (1 + r)'s in two situations can be obtained from (9) and (11) so that a_{ij}'s in them can be positive, since (1 + s) in (9) is $(1 + r)^2$. The values, then, of r and a_{xy} in two situations can be determined from (10) so that the gross profit is equalized to the values given by Smith.

An example of such numerical values are, for (9), (10) and (11)

$$(9)' \qquad \frac{15}{w} = [\, 1 + (1/19)]^2 \, [\, 1113 \, (\frac{1/100}{p}) + 2.417 \,]$$

$$(10)' \qquad \underset{P}{\underline{1/100}} = [\,1 + (\,1/19\,)][(\,\underset{r}{\underline{1/2000}}\,)(\underset{a_{xL}}{\underline{15}}) + (\,4/2000\,)]$$

$$(11)' \qquad 1 = [1 + (1/19)][(1/420)(\underset{a_{yL}}{\underline{15}}) + (3200/35)(\underset{a_{yx}}{\underline{1/100}})]$$

in the first situation, and

$$(9)'' \qquad \underset{w}{\underline{20}} = [\,1 + (\,8/22)\,]^2\,[\,1113(\underset{a_{Lx}}{\underline{3/400}}) + 2.417\,]$$

$$(10)'' \qquad \underset{P}{\underline{3/400}} = [\,1 + (\,8/22\,)]\,[(\,\underset{r}{\underline{1/4000}}\,)(\underset{a_{xL}}{\underline{20}}) + (\,2/4000\,)]$$

$$(11)'' \qquad 1 = [\,1 + (8/22)][(1/420)(\underset{a_{yL}}{\underline{20}}) + (3200/35)(\underset{a_{yx}}{\underline{3/400}})]$$

in the second situation, where values given by Smith are underlined,

Since the gross profit per capita in x sector is

$$(26) \qquad r(\,a_{xL}w + a_{xy}\,)(\,1/a_{xL}\,) + a_{xy}(\,1/a_{xL}\,),$$

it can easily be confirmed that it is 5 in the first situation and 10 in the second situation, as was given by smith himself. It can also be easily seen that condition (25) in section (5) is satisfied in two situations. Note that in this example the division of labor works not only through the increase in labor productivity (i.e., the reduction of a_{xL}) but also through the

saving in the use of materials, etc. (i.e., the reduction of a_{xy}).

Thus, Smith's numerical example of the effects of the divisions of labor, which was given in the Early Draft, i.e., the discussion of an economy where the capitalistic production does not dominates yet, can be used as an example of the natural equilibria of capitalistic production in our Smithian growth model of the Wealth of Nations.

Notes

1) Similar numerical examples can also be seen in Smith (1762-3), p.343.

2) While Meek (1956, pp.46-7) considered the economy of the Early Draft as an economy of small independent producers (craftsmen and labourers who still owned their own means of production), Niimura (1986, pp.134-35) regarded the economy as a mixture of such independent producers and capitalistic producers.

3) Smith(1763), p.345. See also Smith(1962-3), pp.495-6

4) This was well recognized in Japanese literature on Smith. See, for example, Tomizuka(1957), pp.251-2. See also, however, Blaug(1985), p.44, and Ekelund and Héebert(1990), p.115.

5) For the details of natural prices and wages in the Wealth of Nations, see Negishi(1989), pp.83-89.

6) This assumption is necessary only in the final section (6).

7) See Negishi(1988) for a model with capitalist consumption and the relation between propensity to consume and rates of growth and profit.

8) The results in the below remain unchanged, if they are considered in terms of some index of all prices and wages.

9) This can also be shown directly by Frobenius theorem. See Debreu and Herstein (1953).

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