92-F-2

On the 'Lock-In' Effects of Capital Gains Taxation

by

Yoshitsugu Kanemoto
The University of Tokyo

April 1992

Preliminary, Comments Welcome

On the 'Lock–In' Effects of Capital Gains Taxation

Yoshitsugu Kanemoto

Faculty of Economics, University of Tokyo

7–3–1 Hongo, Bunkyo–ku, Tokyo 113 Japan

Abstract

The most important drawback of taxing capital gains only upon realization rather than on accrual is its "lock–in" effect. This paper uses a simple land development model to examine the distortion that the lock–in effect generates. A surprising result in this model is that the lock–in effect does not arise if the purchase price (i.e, the price at which the current owner bought the land) is sufficiently high. Rather than delaying the sale, the owner sells the land as soon as possible even if development occurs much later. No "real" distortion occurs in this case because the buyer's choice of development timing is not distorted by the capital gains tax. If the purchase price is low, however, the owner sells the land just before development and the development time is in general distorted. A sufficient condition for the *non*–existence of the lock–in effect is that the purchase price was formed with perfect foresight.

1. Introduction

The most important drawback of taxing capital gains only upon realization rather than on accrual is its "lock–in" effect: because taxes are deferred until the asset is sold, investors tend to be locked into previously purchased assets. How big a distortion that the lock–in effect creates cannot be evaluated, however, unless we have a model that links asset holding to real resource allocation. This paper uses a simple land development model to examine the distortion that the lock–in effect generates.

Consider a plot of land which is currently under–utilized, for example, a farm located at a commuting distance from the city center. Converting it into residential use yields much higher rents, but the development costs (i.e., costs of preparing the land for residential use and building a house) are substantial. The lock–in effect will distort the "real" resource allocation if it delays development.

The lock–in effect does not cause any distortion if development does not require the sale of land. If for example there is no efficiency loss from renting the land, the owner can rent the land to a family which builds its house on the rented land. The rental arrangement is often inefficient, however, because the house cannot be separated from the land. As well known in the human capital literature pioneered by Becker (1962), the specificity of capital produces serious market failure when a complete contingent contract is not feasible. It is often the case, therefore, that the owner must sell the land for development to occur. We focus on this situation because the lock–in effect immediately yields a distortion in real resource allocation by causing a delay in land development.

A surprising result in this model is that the lock–in effect does not arise if the purchase price of the land (i.e, the price at which the current owner bought the land) is sufficiently high. In fact, the opposite extreme is obtained. Rather than delaying the sale, the owner sells the land as soon as possible even if development occurs much later. No "real" distortion occurs in this case because the buyer's choice of development timing is not distorted by the capital gains tax. If the purchase price is low, however, the owner sells the land just before development and the development time is in general distorted. A sufficient condition for the non–existence of the lock–in effect is that the purchase price was formed with perfect foresight, i.e., future returns were correctly anticipated at the time of purchase. The lock–in effect arises therefore only when there exist unexpected capital gains.

These results differ markedly from those obtained in models of security trading, e.g., Constantinides (1983).[1] The essence of the argument there can be summarized as follows.

Consider an asset which the current owner purchased at price p_0. Its price is p_1 ($> p_0$) at time t_1, and if the owner sells it then, a capital gains tax of $\tau(p_1-p_0)$ is levied. Alternatively, the owner can postpone the sale until her death at time t_2 when the price is p_2 and a capital gains tax of $\tau(p_2-p_0)$ is levied. In the latter case the after–tax value of the asset at the time of death is

$$W_H = p_2 - \tau(p_2-p_0).$$

In the former case, it is assumed that the owner buys the same asset again at time t_1. The after–tax value of the asset is then $p_2-\tau(p_2-p_1)$ at the time of death. We must subtract from this the value of the tax paid at time 1. If the discount rate is i, the value of the tax at time 2 is $\tau(p_1-p_0)e^{-i(t_2-t_1)}$, and the net worth of the owner is

$$W_S = p_2 - \tau(p_2-p_1) - \tau(p_1-p_0)e^{-i(t_2-t_1)}$$

at that time. Comparing this with W_H above yields

$$W_H - W_S = \tau(p_1-p_0)[e^{i(t_2-t_1)}-1] > 0.$$

[1] Auerbach (1991) which presents a method of capital gains taxation that eliminates the lock–in effect also contains a similar explanation of the lock–in effect.

This inequality implies the lock–in effect: the owner is willing to hold the asset even when there exists another asset that yields a higher rate of return so long as the difference in returns is smaller than $W_H - W_S$ above.

The results in this paper are are based on a different comparison. The point of departure is the observation that the use of discount rate i means that the owner has an access to an asset whose net rate of return is i (for example, a bond with maturity $t_2 - t_1$ with after–tax interest rate i). She can invest in this asset at time t_1 and the net worth at time t_2 is then

$$W_B = [p_1 - \tau(p_1 - p_0)]e^{i(t_2 - t_1)}.$$

Comparing this with W_H yields

$$W_H - W_B = (1-\tau)[p_1 e^{i(t_2-t_1)} - p_2] - \tau p_0[e^{i(t_2-t_1)} - 1].$$

Hence, if the rate of appreciation of the asset price equals the interest rate, then $W_B > W_H$ when the purchase price p_0 is positive. This is the case when the asset yields no income gains. Even in a period when no dividends are paid, the stock price is positive if investors expect future dividends. During such a period the price must rise at the rate of interest in order to attract buyers. The capital gains tax then induces the owner to sell the stock because the after–tax rate of return is lower than the interest rate when p_0 is positive. If the asset yields positive income gains, the rate of price appreciation is lower than the interest rate. Even in such cases however we have $W_B > W_H$ for a sufficiently high p_0.

The organization of this paper is as follows. A model of land development is formulated in Section 2. Section 3 solves for the optimal timing of sale given an arbitrary development time. The optimal solution is either at the initial time or just before development. Section 4 derives conditions for optimal development timing. Combining them with results in Section 3 shows that the owner chooses to sell the land at the initial time when the purchase price is sufficiently high. A corollary of this result is that the lock–in effect arises only when *unexpected* capital gains exist. Section 5 characterizes the optimal solution for the two cases where the tax rates are close to zero and one. Section 6 obtains complete characterizations of the optimal solutions in two special cases: (1) the pre–development rent is constant and the post–development is exponential and (2) both the pre– and post–development rents are exponential and have the same rate of increase. Section 7 contains concluding discussions.

2. The Model

Consider a plot of land which is currently undeveloped (or under–utilized). If it remains undeveloped, the land rent is $v(t)$ at time t. The costs of developing the land and constructing a building are C and do not vary over time. The building is infinitely durable and cannot be reconstructed. After development the property yields (gross) rents $R(t)$ at time t. The net land rent is the gross rents minus the user cost of capital. Under our assumption of perfect durability of capital, the user cost is iC when the (after–tax) interest rate is i. The net rent is then $r(t) = R(t) - iC$.

We assume perfect foresight of rent profiles. Furthermore, rent profiles satisfy the following conditions. First, both pre– and post–development rents are nondecreasing over time: $R'(t) \geq 0$, and $v'(t) \geq 0$ for any t. Second, the rent of developed land rises faster than that of undeveloped land: $R'(t) > v'(t)$ for any t. Third, the net land rent of developed land is lower than that of undeveloped land at the initial time but eventually becomes higher: $r(0) < v(0)$ but $r(\infty) > v(\infty)$.

Capital gains taxation is irrelevant unless selling the land at some point in time dominates owning it forever. We therefore assume that the current owner of the land lacks the ability to manage development of the land and that renting the land is inefficient. The former assumption reflects agency problems. The latter assumption can be justified by specificity of capital. Because a building cannot be removed from the plot of land on which it is built, separating building ownership from land ownership results in inefficient investment similar to that observed in the specific human capital model of Becker (1962).[2] Under these assumptions, the landowner must sell the land to benefit from land development.

The buyer of the land does not have to develop it immediately. We assume that the buyer can also earn the pre–development rent $v(t)$ until development.

In our model, the only choice is when to sell and develop the land. If the landowner sells the land at time S and if the buyer develops it at time T, then land price before tax is

$$(2.1) \qquad q(S,T) = e^{-i(T-S)}\int_T^\infty r(t)e^{-i(t-T)}dt + \int_S^T v(t)e^{-i(t-S)}dt$$

at time S. If the capital gains tax rate is τ and if the landowner acquired the land at price p_0, then land price after tax is $p(S,T;\tau,p_0) = q(S,T) - \tau[q(S,T)-p_0]$. The land value at time 0 is the discounted value of the land price plus the discounted sum of pre–development rents that the owner earns until he sells the land:

$$(2.2) \qquad P(S,T;\tau,p_0) = [(1-\tau)q(S,T)+\tau p_0]\exp(-iS) + \int_0^S v(t)e^{-it}dt.$$

We omit τ and p_0 from the initial land value function $P(S,T:\tau,p_0)$ when this does not cause confusion.

When the landowner sells the land, he will choose the buyer whose offer price is the highest. This means that given the selling time S the development time T is chosen to maximize $q(S,T)$. The landowner then chooses S to maximize the initial land value $P(S,T)$. It is however easier to solve the problem in the opposite order. We first solve for the optimal selling time S given an arbitrary development time T. It will be shown that the optimal selling time is either the initial time 0 or just before the development time T, depending on the initial purchase price p_0. We then solve for the optimal development time for each of these cases, and comparing them

[2] See Kanemoto (1990) for the analysis of rental contracts of land in an imperfect information model.

yields the optimal solution.

3. Timing of Sale

We first solve for the optimal selling time given an arbitrary development time T. The rent profile is determined uniquely by the development time, and a change in selling time changes the tax payment only.

Let us examine the effects of postponing the sale on the effective tax burden. By deferring the tax payment, the owner would receive an implicit interest subsidy if the amount of the tax were constant. Because of land price appreciation however the tax payment increases, and we have to compare the two effects to obtain the net effects. The former interest subsidy is the interest rate times the capital gains tax, $i\tau[q(S,T)-p_0]$. Land price appreciation increases the tax payment by $\tau q_S(S,T)$. In equilibrium the rate of return on land equals the interest rate, $(q_S+v)/q = i$, and the increase in tax payment equals $\tau[iq(S,T)-v(S)]$. Comparing the two effects shows that postponing the sale is profitable if and only if $v(S) > ip_0$, i.e., the rent is higher than the interest rate times the initial purchase price.

If for example the purchase price p_0 is zero, then the owner always chooses to postpone the sale when the rent is positive. In this case the optimal selling time is (just before) the development time. The reason for this result is that with positive rents the rate of land price appreciation is lower than the interest rate. If p_0 is positive and the pre–development rent $v(t)$ is zero, then the owner chooses to sell the land immediately. In this case land price rises at the same rate as the interest rate and the rate of increase of capital gains income is higher than the interest rate.

Because of our assumption that the pre–development rent is non–decreasing over time, it is never optimal to sell at an intermediate time between 0 and T. This can be seen by checking the second order condition for an interior optimum. The first order condition for an interior maximum is

(3.1) $$\frac{\partial P}{\partial S} = \tau[v(S)-ip_0]e^{-iS} = 0,$$

and the selling time that satisfies this condition is unique. At this point however the second order condition is not satisfied:

(3.2) $$\frac{\partial^2 P}{\partial S^2} = \tau v'(S)e^{-S} > 0.$$

Hence, the optimal selling time is one of the corners: either $S = 0$ or T. The optimal selling time follows immediately from comparing these two corner solutions.

Lemma 1. Define

(3.3) $$\phi(T) \equiv [\int_0^T v(t)e^{-it}dt]/(1-e^{-iT}).$$

Then,

(3.4) $$P(0,T) \gtreqless P(T,T) \text{ as } p_0 \gtreqless \phi(T),$$

where $\phi(T)$ satisfies $v(0)/i \leq \phi(T) \leq v(T)/i$ and $\phi'(T) \geq 0$.

Proof:

The two corner solutions satisfy

$$P(0,T) = (1-\tau)\{e^{-iT}\int_T^\infty r(t)e^{-i(t-T)}dt + \int_0^T v(t)e^{-it}dt\} + \tau p_0$$

and

$$P(T,T) = \{(1-\tau)\int_T^\infty r(t)e^{-i(t-T)}dt + \tau p_0\}e^{-iT} + \int_0^T v(t)e^{-it}dt.$$

The difference between them is

$$P(T,T) - P(0,T) = \tau\{\int_0^T v(t)e^{-it}dt - p_0(1-e^{-iT})\}.$$

The lemma then follows immediately.

Q.E.D.

An implication of this lemma is that, if $p_0 < v(0)/i$, then $S = 0$ is optimal. That is, if the initial purchase price p_0 is lower than the capitalized value of the land rent at time 0 (i.e., the value of land that would prevail if the land rent did not rise from the initial rent $v(0)$), then selling at the initial time 0 is optimal. Another implication is that, if $p_0 > v(T)/i$, then $S = T$ is optimal: if the initial purchase price is higher than the capitalized value of rent just before development, then it is optimal to postpone the sale of land until the development time.

4. Timing of Development

We have seen that the owner sells either at the initial time or at the development time. This section examines the optimal development time for each of these two cases. Unfortunately, the latter case is too complicated to yield a complete characterization of the solution. General results are obtained however for a special case where the purchase price p_0 is sufficiently high. An interesting implication of the results is that, when the purchase price was determined under prefect foresight, the lock–in effect never arises. Another implication is that if the lock–in effect does not exist for vacant land where pre–development rent is zero. This section also contains preliminary results that will be used in the following sections.

4.1. Sale at the Initial Time

If the owner sells the land at time 0, the buyer chooses the development time T to maximize q(0,T). This maximization yields the following lemma.

Lemma 2. If the owner sells the land at time 0, then the optimal development time is T^* which satisfies

$$R(T^*) - iC = v(T^*). \tag{4.1}$$

A necessary condition for $(S,T) = (0,T^*)$ to be optimum is

$$p_0 \geq \phi(T^*). \tag{4.2}$$

Development occurs when the post–development rent exceeds the pre–development rent by the interest rate times the development cost. This condition is well known in the literature on durable housing (surveyed in Fujita (1986)).[3] Because the buyer does not pay the capital gains tax, the development time in this case is first best optimum.

4.2. Sale at the Development Time

When the owner sells the land just before development, the value of land at the initial time is

$$(4.3)\quad P(T,T;\tau,p_0) = \{(1-\tau)\int_T^\infty r(t)e^{-i(t-T)}dt + \tau p_0\}e^{-iT} + \int_0^T v(t)e^{-it}dt.$$

In this case the development time T cannot be earlier than the first best development time T^* obtained in Lemma 2. Otherwise $S = T < T^*$, but the buyer then chooses to develop at T^*, which contradicts the claim that development occurs at T ($< T^*$). The optimal development (=sale) time in this case is therefore

$$(4.4)\qquad T^{**}(\tau,p_0) = \operatorname{argmax}_{\{T\}}\{P(T,T;\tau,p_0)\colon T \geq T^*\}.$$

Define $\hat{T}(\tau,p_0)$ as the development time that satisfies the first order condition for an interior maximum:

$$(4.5)\qquad (1-\tau)r(\hat{T}(\tau,p_0)) + \tau i p_0 \equiv v(\hat{T}(\tau,p_0)).$$

The interior solution may not lie in $[T^*,\infty)$ or may not satisfy the second order condition. Furthermore, it may not be unique. It is therefore difficult to obtain a complete characterization of the optimum solution. We at least know however that the optimum development time in this case is either one of the interior solutions, $\hat{T}(\tau,p_0)$, or one of the two corner solutions, T^* and ∞.

Lemma 3. The optimum development time for the case where the owner sells the land just before development is

$$(4.6)\qquad T^{**}(\tau,p_0) = \hat{T}(\tau,p_0) \text{ or } T^* \text{ or } \infty.$$

---

[3] Extensions of this condition appear in the literature on distortionary effects of land value taxation, e.g., Bentick (1979), Skouras (1978), and Wildasin (1982).

4.3. The Optimum Timing for Sale and Development

Although Lemma 3 does not provide a complete characterization of $T^{**}(\tau,p_0)$, combining it with Lemmas 1 and 2 yields the following results on the optimal timing for sale and development.

Proposition 1. If $p_0 \geq \phi(T^{**}(\tau,p_0))$, then the optimum is $(S,T) = (0,T^*)$. If $p_0 \leq \phi(T^*)$, then the optimum is $S = T = T^{**}(\tau,p_0)$.

Proof:

From $T^{**} \geq T^*$, we have $\phi(T^{**}) \geq \phi(T^*)$. From Lemmas 1 and 2, we have

$$P(0,T^*) \geq P(0,T^{**}) \geq P(T^{**},T^{**}) \text{ if } p_0 \geq \phi(T^{**})$$
$$P(0,T^*) \leq P(T^*,T^*) \leq P(T^{**},T^{**}) \text{ if } p_0 \leq \phi(T^*),$$

which yields the proposition.

Q.E.D.

This proposition shows that if the initial purchase price is higher than or equal to $\phi(T^{**})$, then the lock–in effect does not arise. Instead, the owner sells the land at the initial time. The development timing in this case is first best.[4] If the initial purchase price is lower than or equal to $\phi(T^*)$, then the owner delays the sale until the development time. The development time is not in general first best, and the distortion is always in the direction of delaying development.

The following proposition shows that, if the initial purchase price is determined under perfect foresight, then it is always higher than $\phi(T^{**})$. The lock–in effect occurs therefore only when unexpected capital gains exist.

---

[4]This result differs from those in Kanemoto (1985) which showed that capital gains taxation slows down land development. The difference is caused by our implicit assumption on development technology. This paper assumes indivisibility in development so that the only choice variable is the timing of development. In Kanemoto (1985) I assumed that the unit cost of housing capital does not depend on the size of development. Under this assumption, development occurs continuously and the choice variable is how much land to develop at each instant of time.

Corollary 1. If the initial purchase price is determined under perfect foresight, then the optimum is always $(S,T) = (0,T^*)$.

Proof:

If the initial purchase price p_0 is determined under perfect foresight, then

$$\begin{aligned} p_0 &\geq \int_0^\infty v(t)e^{-it}dt \\ &\geq \int_0^T v(t)e^{-it}dt + \frac{v(T)}{i}e^{-iT} \\ &\geq \phi(T)(1-e^{-iT}) + \phi(T)e^{-iT} \\ &= \phi(T) \end{aligned}$$

for any T. In particular, we have $p_0 \geq \phi(T^{**}(\tau,p_0))$.

Q.E.D.

Another implication of Proposition 1 is that if the pre–development rent is zero, then no distortion in development timing arises. If, in addition, the purchase price p_0 is positive, then the owner chooses to sell at time 0. If p_0 is zero, then the owner is indifferent between any points in the interval $[0,T^*]$. The last result corresponds to Proposition 8 in Sinn (1986).

Corollary 2. If $v(t) = 0$ for any t and if $p_0 > 0$, then the optimum is $(S,T) = (0,T^*)$. If $v(t) = 0$ for any t and if $p_0 = 0$, then the optimal timing of sale is any point in $[0,T^*]$ and the optimal development time is T^*.

If $\phi(T^*) < p_0 < \phi(T^{**})$, then comparison between $P(0,T^*)$ and $P(T^{**},T^{**})$ is necessary to obtain the optimum solution. This comparison is difficult to carry out in the general case because we do not have a complete characterization of T^{**}. In the next two sections we examine special cases, and the following three lemmas will be used there.

Lemma 4. Define $\psi(\tau)$ by $P(\hat{T}(\tau,\psi(\tau)),\hat{T}(\tau,\psi(\tau)),\tau,\psi(\tau)) \equiv P(0,T^*)$. Then,

$$P(0,T^*) \gtreqless P(\hat{T},\hat{T}) \quad \text{as} \quad p_0 \gtreqless \psi(\tau),$$

where $\psi(\tau)$ is unique and satisfies

$$\phi(T^*) \leq \psi(\tau) \leq \phi(\hat{T}).$$

Proof:

Let $\Psi(\tau,p_0) \equiv P(\hat{T}(\tau,p_0),\hat{T}(\tau,p_0),\tau,p_0) - P(0,T^*,\tau,p_0)$. Then,

$$\partial\Psi/\partial p_0 = \tau e^{-i\hat{T}} - \tau < 0.$$

Since $\Psi \geq 0$ at $p_0 = \phi(T^*)$, $\psi(\tau)$ which satisfies $\Psi(\tau,\psi(\tau)) = 0$ is unique, and

$$P(0,T^*) \gtreqless P(\hat{T},\hat{T}) \quad \text{as} \quad p_0 \gtreqless \psi(\tau).$$

Since $P(0,T^*) \geq P(0,\hat{T}) \geq P(\hat{T},\hat{T})$ at $p_0 \geq \phi(\hat{T})$ and the opposite inequality holds at $p_0 = \phi(T^*)$, we must have $\phi(T^*) \leq \psi(\tau) \leq \phi(\hat{T})$.

Q.E.D.

Lemma 5. Define $\psi_\infty(\tau)$ by $P(\infty,\infty,\tau,\psi_\infty(\tau)) \equiv P(0,T^*)$. Then,

$$P(0,T^*) \gtreqless P(\infty,\infty) \quad \text{as} \quad p_0 \gtreqless \psi_\infty(\tau),$$

where $\psi_\infty(\tau)$ is unique and satisfies

$$\phi(T^*) \leq \psi_\infty(\tau) \leq \phi(\infty).$$

Proof:

The lemma follows from

$$\begin{aligned}
&P(0,T^*) - P(\infty,\infty) \\
&= (1-\tau)[e^{-iT^*} \textstyle\int_{T^*}^{\infty}(R(s)-iC)e^{-i(s-T^*)}ds + \int_0^{T^*} v(s)e^{-is}ds] \\
&\quad + \tau p_0 - \textstyle\int_0^{\infty} v(s)e^{-is}ds \\
&= \tau[p_0 - \psi_\infty(\tau)].
\end{aligned}$$

Q.E.D.

LEMMA 6. Let $\xi(\tau) = \frac{1}{\tau}\{\int_{T^*}^{\infty} v(t)e^{-i(t-T^*)}dt - (1-\tau)\int_{T^*}^{\infty} r(t)e^{-i(t-T^*)}dt\}$. Then

$$P(T^*,T^*) \gtreqless P(\infty,\infty) \text{ as } p_0 \gtreqless \xi(\tau),$$

where $\xi(\tau)$ is unique and satisfies

$$\xi(\tau) \geq \psi_\infty(\tau).$$

PROOF:

From

$$P(T^*,T^*) = \{(1-\tau)\int_{T^*}^{\infty} r(t)e^{-i(t-T^*)}dt + \tau p_0\}e^{-iT^*} + \int_0^{T^*} v(t)e^{-it}dt$$

and

$$P(\infty,\infty) = \int_0^{\infty} v(t)e^{-it}dt,$$

we obtain

$$P(T^*,T^*) \gtreqless P(\infty,\infty) \text{ as } p_0 \gtreqless \xi(\tau).$$

If $p_0 = \psi_\infty(\tau)$, then $P(0,T^*) = P(\infty,\infty)$ and $p_0 \geq \phi(T^*)$. From Lemma 1, the last inequality implies that $P(0,T^*) \geq P(T^*,T^*)$. Hence $P(T^*,T^*) \leq P(\infty,\infty)$ and $p_0 \leq \xi(\tau)$. This shows that $\psi_\infty(\tau) \leq \xi(\tau)$.

Q.E.D.

5. Limiting Tax Rates

If the tax rate τ is sufficiently close to zero or one, we can obtain the optimal timing for sale and development.

Let us first examine the case where τ is close to 0. The following lemma shows that T^{**} is an interior solution $\hat{T}$ if p_0 is lower than $v(T^*)/i$, and that T^{**} is a corner solution T^* if p_0 is higher than $v(T^*)/i$.

LEMMA 7. If the tax rate τ is sufficiently close to zero, then

$$T^{**}(\tau,p_0) = \hat{T}(\tau,p_0) \text{ if } p_0 < v(T^*)/i\ ,$$

and

$$T^{**}(\tau,p_0) = T^* \text{ if } p_0 > v(T^*)/i\ .$$

Proof:

Define $\Pi(T;\tau,p_0) \equiv P(T,T;\tau,p_0)$. If $\tau = 0$, then

$$\partial\Pi/\partial T = [v(T)-r(T)]e^{-iT} = 0$$

has a unique solution T^*. As shown in Lemma 2, this interior solution maximizes $\Pi(T)$. Now, let us examine what happens when τ is increased slightly.

First, we show that $\Pi(T;\tau,p_0)$ has a unique maximum at $\hat{T}(\tau,p_0)$ defined by (4.5). Let $\pi(T) = v(T)-r(T) + \tau[r(T)-ip_0]$. Then, $\partial\Pi/\partial T = \pi(T)e^{-iT}$ and $\pi(\hat{T}) = 0$. For a sufficiently small τ, $\pi'(T) = v'(T)-(1-\tau)r'(T)$ is negative for any T. Hence, $\pi(T) > 0$ for $T < \hat{T}$, and $\pi(T) < 0$ for $T > \hat{T}$. This implies that $\Pi'(T) > 0$ for $T < \hat{T}$ and $\Pi'(T) < 0$ for $T > \hat{T}$. Hence, $\Pi(T)$ achieves a unique maximum at $\hat{T}$.

Next, from

$$\partial\Pi(\hat{T})/\partial T = \{v(\hat{T})-r(\hat{T}) + \tau[r(\hat{T})-ip_0]\}e^{-i\hat{T}} = 0,$$

We have

$$\partial\hat{T}(\tau,p_0)/\partial\tau = \frac{r(T)-i\,p_0}{(1-\tau)\,r'(T)-v'(T)}.$$

The denominator of the right hand side is positive for small τ. Hence $\hat{T}$ is increasing in τ at T^* if $r(T^*) > ip_0$, and decreasing if $r(T^*) < ip_0$. The lemma follows immediately from $r(T^*) = v(T^*)$.

Q.E.D.

Combining this lemma with Lemmas 1 and 4 yields

Proposition 2. If the tax rate τ is close to zero, then the optimum solution is

$$(S,T) = (0,T^*) \text{ if } p_0 > \psi(\tau)$$

$$S = T = \hat{T}(\tau,p_0) \text{ if } p_0 < \psi(\tau),$$

where $\phi(T^*) < \psi(\tau) < v(T^*)/i$.

Proof:

From $v(T^*)/i > \phi(T^*)$, Lemma 1 implies that, if $p_0 \geq v(T^*)/i$, then $P(0,T^*) \geq P(T^*,T^*)$. Combining this with Lemma 7 shows that the optimum in this case is $(S,T) = (0,T^*)$.

If $p_0 \leq \phi(T^*)$, then Lemmas 1 and 7 yield

$$P(\hat{T},\hat{T}) \geq P(T^*,T^*) \geq P(0,T^*).$$

Hence the optimum in this case is $(S,T) = (\hat{T},\hat{T})$.

If $\phi(T^*) < p_0 < v(T^*)/i$, then Lemmas 1 and 7 yield $P(\hat{T},\hat{T}) \geq P(T^*,T^*) < P(0,T^*)$. Hence, we must compare $P(\hat{T},\hat{T})$ and $P(0,T^*)$ in this case, but this comparison is carried out in Lemma 4.

Q.E.D.

Next, consider the case where τ is close to one. In this case we impose an additional assumption that $v(t)$ is strictly increasing. The following lemma shows that an interior solution $\hat{T}$ is never optimal in this case. If the initial purchase price is low, T^{**} is infinity, and otherwise it coincides with the first best development time T^*.

Lemma 8. If $v(t)$ is strictly increasing and if the tax rate is sufficiently close to one, then

$$T^{**}(\tau,p_0) = T^* \text{ if } p_0 > \xi(\tau),$$
$$T^{**}(\tau,p_0) = \infty \text{ if } p_0 < \xi(\tau).$$

Proof:

If τ=1, then we have

$$\partial\Pi/\partial T = [v(T) - ip_0]e^{-iT}.$$

Since v(T) is strictly increasing in T, $\Pi'(T) < 0$ for $T < \hat{T}$, and $\Pi'(T) > 0$ for $T > \hat{T}$. This implies that $\hat{T}$ yields the minimum land value. Hence the optimum in this case is either $T = T^*$ or $T = \infty$. By continuity, the same is true for τ close enough to 1, and Lemma 6 immediately yields this lemma.

Q.E.D.

Combining this lemma with Lemmas 1 and 5 yields the result that the optimal solution is either $(S,T) = (0,T^*)$ or $S = T = \infty$. Hence, if the tax rate is close to one, then the owner sells the land at the initial time or keeps the land forever. In the former case development occurs at the first best timing and in the latter case development never takes place.

PROPOSITION 3. If v(t) is strictly increasing and if τ is sufficiently close to 1, then

$$(S,T) = (0,T^*) \text{ if } p_0 > \psi_\infty(\tau)$$

$$S = T = \infty \text{ if } p_0 < \psi_\infty(\tau),$$

where $\psi_\infty(\tau)$ satisfies $\phi(T^*) < \psi_\infty(\tau) < \xi(\tau)$, $\psi_\infty'(\tau) > 0$, and

$$\psi_\infty(1) = \int_0^\infty v(s)e^{-is}ds.$$

PROOF:

From

$$\xi(1) = \frac{\int_{T^*}^\infty v(t)e^{-it}dt}{e^{-iT^*}} \geq v(T^*)/i \geq \phi(T^*),$$

we have $\xi(\tau) \geq \phi(T^*)$ for τ close enough to one. Hence, if $p_0 \geq \xi$, then $P(0,T^*) \geq P(T^*,T^*) \geq P(\infty,\infty)$ for any T. In this case $(S,T) = (0,T^*)$ is optimal. If $p_0 \leq \phi(T^*)$, then $P(\infty,\infty) \geq P(T^*,T^*) \geq P(0,T^*)$, which implies that $(S,T) = (\infty,\infty)$ is optimal. In the case where $\phi(T^*) < p_0 < \xi(\tau)$, we must compare $P(\infty,\infty)$ and $P(0,T^*)$. Lemma 5 then yields the proposition.

Q.E.D.

This result shows that even if the tax rate is close to one, the lock–in effect does not arise when the initial purchase price is sufficiently high.

6. Exponential Rents

A complete characterization of the solution is possible in special cases where rents increase at constant rates. This section examines two cases: (1) the post–development rent increases at a constant rate θ but the pre–development rent is constant, and (2) both the post– and pre–development rents rise at the same rate θ. The first case is simple, but the second case is fairly complicated.

6.1. Constant Pre–Development Rent

If $R(t) = Re^{\theta t}$ and $v(t) = v$, then $\hat{T}$ is unique and satisfies

$$\hat{T} = \frac{1}{\theta}\{\ln[v+iC+\frac{\tau}{1-\tau}(v-ip_0)]-\ln(R)\}.$$

At $\hat{T}$ the second order condition for maximum is always satisfied

$$\partial^2\Pi/\partial T^2 = -(1-\tau)r'(\hat{T})e^{-i\hat{T}} < 0.$$

The uniqueness of $\hat{T}$ then implies that $\hat{T}$ maximizes $\Pi(T)$ globally.

Next, it is easy to see that T^* satisfies

$$T^* = \frac{1}{\theta}\{\ln(v+iC)-\ln(R)\}.$$

Hence, $\hat{T} \geq T^*$ only if $p_0 \leq v/i$.

From $\phi(T) = v/i$, Proposition 1 yields the following result.

Proposition 4. Suppose the post–development rent rises at a constant rate θ and the pre–development rent is constant. Then, if $p_0 > v/i$, then the optimum is $(S,T) = (0,T^*)$, if $p_0 < v/i$, then the optimum is $(S,T) = (\hat{T},\hat{T})$, and if $p_0 = v/i$, then both $(S,T) = (0,T^*)$ and $(S,T) = (\hat{T},\hat{T})$ are optimal.

If the pre–development rent is constant, the solution is particularly simple. When the initial purchase price, p_0, is higher than the capitalized value of the pre–development rent, v/i, then the owner sells the land immediately and there is no distortion in development timing. When the opposite inequality holds, the owner delays the sale until the development time. In this case development is later than the first best. As the tax rate rises from zero to one, the development time moves from the first best level T^* to infinity. Unlike in Proposition 3 which assumes that the pre–development rent is strictly increasing over time, the development time is always finite when the tax rate is less than one.

6.2. Equal Rates of Increase

Next, consider the case where the pre–development rent increases at the same rate as the post–development rent: $R(t) = Re^{\theta t}$ and $v(t) = ve^{\theta t}$. In this case T^* and $\hat{T}$ are

$$T^* = \frac{1}{\theta}\ln(X)$$
$$\hat{T} = \frac{1}{\theta}\ln(\hat{X}),$$

where X and $\hat{X}$ are defined as

$$X \equiv \frac{C}{R-v}i$$
$$\hat{X} \equiv \frac{(1-\tau)C-\tau p_0}{(1-\tau)R-v}i.$$

Comparing these two yields

Lemma 9. If $\tau < \frac{R-v}{R}$, then

$$\hat{T} \gtreqless T^* \quad \text{as} \quad p_0 \lesseqgtr C\frac{v}{R-v}.$$

If $\tau > \frac{R-v}{R}$, then

$$\hat{T} \gtreqless T^* \quad \text{as} \quad p_0 \gtreqless C\frac{v}{R-v}.$$

Using this result, we obtain $T^{**}(\tau,p_0)$ as follows.

LEMMA 10. If $\tau < \frac{R-v}{R}$, then

$$T^{**}(\tau,p_0) = \begin{cases} \hat{T}(\tau,p_0) & \text{when } p_0 < C\frac{v}{R-v} \\ T^* & \text{when } p_0 \geq C\frac{v}{R-v}. \end{cases}$$

If $\tau > \frac{R-v}{R}$, then

$$T^{**}(\tau,p_0) = \begin{cases} \infty & \text{when } p_0 < \xi(\tau) \\ T^* \text{ and } \infty & \text{when } p_0 = \xi(\tau) \\ T^* & \text{when } p_0 > \xi(\tau), \end{cases}$$

where

$$\xi(\tau) = \frac{C}{i-\theta}\{-\frac{1}{\tau}\theta + \theta + \frac{iv}{R-v}\} > C\frac{v}{R-v} \quad \text{for } \tau > \frac{R-v}{R}.$$

If $\tau = \frac{R-v}{R}$, then

$$T^{**}(\tau,p_0) = \begin{cases} \infty & \text{when } p_0 < C\frac{v}{R-v} \\ [T^*,\infty) & \text{when } p_0 = C\frac{v}{R-v}. \\ T^* & \text{when } p_0 > C\frac{v}{R-v} \end{cases}$$

PROOF: See the Appendix.

Comparing $P(0,T^*)$ and $P(T^{**},T^{**})$, we obtain the optimal solution.

PROPOSITION 5. If $R(t) = Re^{\theta t}$ and $v(t) = ve^{\theta t}$, then the optimal plan satisfies the following conditions.

(1) $\tau < \frac{R-v}{R}$

$$(S,T) = \begin{cases} (\hat{T},\hat{T}) & \text{if } 0 \leq p_0 < \psi(\tau) \\ (\hat{T},\hat{T}) \text{ and } (0,T^*) & \text{if } p_0 = \psi(\tau) \\ (0,T^*) & \text{if } p_0 > \psi(\tau) \end{cases}$$

(2) $\tau \geq \frac{R-v}{R}$

$$(S,T) = \begin{cases} (\infty,\infty) & \text{if } 0 \leq p_0 < \psi_\infty(\tau) \\ (\infty,\infty) \text{ and } (0,T^*) & \text{if } p_0 = \psi_\infty(\tau) \\ (0,T^*) & \text{if } p_0 > \psi_\infty(\tau) \end{cases},$$

where

$$\hat{T} \to \infty \text{ as } \tau \to \frac{R-v}{R}.$$

and

$$T^* = \frac{1}{\theta}\ln\left[\frac{C}{R-v}i\right].$$

$$\hat{T} = \hat{T}(\tau,p_0) = \frac{1}{\theta}\ln\left[\frac{(1-\tau)C-\tau p_0}{(1-\tau)R-v}i\right]$$

$$P(0,T^*,\tau,\psi(\tau)) \equiv P(\hat{T}(\tau,\psi(\tau)),\hat{T}(\tau,\psi(\tau)),\tau,\psi(\tau)),$$

$$\psi_\infty(\tau) = \frac{v}{i-\theta} + (1-\frac{1}{\tau})\frac{\theta}{i-\theta}\left[\frac{i\,C}{R-v}\right]^{-i/\theta}.$$

Proof: See the Appendix.

This result is illustrated in Fig. 1. If p_0 is higher than $\psi(\tau)$ or $\psi_\infty(\tau)$, then the owner sells the land at the initial time and development occurs at the first best T^*. If p_0 is lower than $\psi(\tau)$ and if τ is lower than $(R-v)/R$, then the owner sells the land at $\hat{T}$ when development occurs. If p_0 is lower than $\psi_\infty(\tau)$ and if τ is higher than $(R-v)/R$, then the owner keeps the land forever and development never occurs. In the latter two cases the capital gains tax has the lock–in effect but in the first case it has the effect of speeding up the sale of land.

7. Concluding Discussions

In our model of land development, taxation of realized capital gains may or may not produce the lock–in effect. The lock–in effect occurs when the purchase price is low, but if the purchase price is high, the tax induces the owner to sell her land immediately. This reflects the fact that the owner who cannot develop the land by herself is equivalent to being a speculator. Because the capital gains tax represents an additional cost for speculative trading, it discourages speculative purchase of land and encourages sale of land by a speculator.

The result that the lock–in effect may or may not appear depending on the purchase price is not limited to land development. The same result obviously holds for securities. Distortions in real resource allocation are however different in the case of securities. In a public corporation where equity owners are separate from the management, whether or not equity owners are locked in does not have any direct effects on the management of the corporation. Distortionary effects arise only through distortion in the portfolio of locked–in owners.

References

Auerbach, A.J., 1991, "Retrospective Capital Gains Taxation," *American Economic Review* 81, 167–178.

Becker, G., 1962, "Investment in Human Capital: a Theoretical Analysis," *Journal of Political Economy* 70, Suppl., 9–49.

Bentick, B. L., 1979, "The Impact of Taxation and Valuation Practices on the Timing and Efficiency of Land Use," *Journal of Political Economy* 87, 859–874.

Constantinides, G.M., 1983, "Capital Market Equilibrium with Personal Tax," *Econometrica* 51, 611–636.

Fujita, M., 1986, "Urban Land Use Theory," in *Location Theory* (Fundamentals of Pure and Applied Economics, Vol.5), 73–149.

Kanemoto, Y., 1985, "Housing as an Asset and the Effects of Property Taxation on the Residential Development Process," *Journal of Urban Economics* 17, 145–166.

Kanemoto, Y., 1990, "Contract Types in the Property Market," *Regional Science and Urban Economics* 20, 5–22.

Sinn, H–W., 1986, "Vacant Land and the Role of Government Intervention," *Regional Science and Urban Economics* 16, 353–385.

Skouras, A., 1978, "The Non–Neutrality of Land Taxation," *Public Finance* 33, 113–134.

Wildasin, D., 1982, "More on the Neutrality of Land Taxation," *National Tax Journal* 35, 105–111.

Appendix

Proof of Lemma 9:

The second order derivative of $\Pi(T)$ at $\hat{T}$ is

$$\Pi''(\hat{T}) = [\tau - \frac{R-v}{R}]R\theta e^{-(i-\theta)\hat{T}}.$$

Hence, $\hat{T}$ maximizes $\Pi(T)$ if $\tau < (R-v)/R$, but it minimizes $\Pi(T)$ if $\tau > (R-v)/R$. We examine these cases separately.

(1) $\tau < \frac{R-v}{R}$

In this case, $\hat{T}$ maximizes $\Pi(T)$ by uniqueness of $\hat{T}$. However, if $p_0 \geq C\frac{v}{R-v}$, then $\hat{T} \leq T^*$. In this case, $\Pi'(T) \leq 0$ for $T \in [T^*,\infty)$ and $\Pi(T)$ attains its maximum at T^*. If $p_0 < C\frac{v}{R-v}$, then $\hat{T}$ is optimal.

(2) $\tau > \frac{R-v}{R}$

In this case $\hat{T}$ minimizes $\Pi(T)$. We have to distinguish between the following two cases.

(a) $p_0 > C\frac{v}{R-v}$

In this case we have $\hat{T} > T^*$ and hence $\Pi(T)$ is maximized either at $T = T^*$ or $T = \infty$. From Lemma 6, $P(T,T)$ is maximized at $T=T^*$ if $p_0 > \xi(\tau)$; and at $T=\infty$ if $p_0 < \xi(\tau)$, where

$$\xi(\tau) = \frac{C}{i-\theta}\{-\frac{1}{\tau}\theta + \theta + \frac{iv}{R-v}\}$$

If $p_0 = \xi(\tau)$, then the maximum is attained at both $T=T^*$ and $T=\infty$.

From the definition of $\xi(\tau)$, we have

$$\xi(\frac{R-v}{R}) = C\frac{v}{R-v}.$$

Since $\xi(\tau)$ is increasing in τ, this shows that $\xi(\tau) > C\frac{v}{R-v}$ for $\tau > \frac{R-v}{R}$.

(b) $p_0 \leq C\frac{v}{R-v}$

In this case, $\hat{T} \leq T^*$. From $\Pi'(T) \geq 0$ for $T \in [T^*,\infty)$, the maximum occurs at $T = \infty$.

(3) $\tau = \frac{R-v}{R}$

In this case we have

$$\Pi'(T) = i[(1-\tau)C-\tau p_0]e^{-iT} \gtreqless 0 \text{ as } p_0 \lesseqgtr C\frac{v}{R-v}.$$

Hence, if $p_0 < C\frac{v}{R-v}$, then $\Pi(T)$ is maximized at $T = \hat{T} = \infty$; if $p_0 > C\frac{v}{R-v}$, then $\Pi(T)$ is maximized at $T = T^*$; and if $p_0 = C\frac{v}{R-v}$, then $\Pi(T)$ is constant.

Q.E.D.

Proof of Proposition 5:

(1) $\tau < \frac{R-v}{R}$

First, we show that $C\frac{v}{R-v} > \phi(T^*)$. From the definition of $\phi(T^*)$,

$$C\frac{v}{R-v} - \phi(T^*) = \frac{v}{i(i-\theta)(1-X^{-i/\theta})}\{-i+(i-\theta)X+\theta X^{-(i-\theta)/\theta}\}.$$

Let $g(X) \equiv -i+(i-\theta)X+\theta X^{-(i-\theta)/\theta}$. Then, $g(1) = 0$ and

$$g'(X) = (i-\theta)($$

$$P(0,T^*) > P(T^*,T^*) < P(\hat{T},\hat{T}).$$

From Lemma 4, we obtain

$$P(0,T^*) \gtreqless P(\hat{T},\hat{T}) \text{ as } p_0 \gtreqless \psi(\tau).$$

(c) $p_0 \geq C\frac{v}{R-v}$

From Lemmas 1 and 10, we obtain

$$P(0,T^*) > P(T^*,T^*) \geq P(\hat{T},\hat{T}).$$

Hence, $(0,T^*)$ is optimal. In this case, inequality $P(0,T^*) > P(\hat{T},\hat{T})$ is satisfied. Hence, we must have $p_0 > \psi(\tau)$ for any τ in $[0,\frac{R-v}{R})$. This implies that $C\frac{v}{R-v} > \psi(\tau)$ for any τ in $[0,\frac{R-v}{R})$.

(2) $\tau > \frac{R-v}{R}$

In this case $P(T,T)$ is maximized at $T=T^*$ if $p_0 > \xi(\tau)$; and at $T=\infty$ if $p_0 < \xi(\tau)$. From Lemma 10 we have

$$\xi(T^*,\frac{R-v}{v}) > \frac{vC}{R-v} > \phi(T^*).$$

Hence, we obtain the following results.

(a) $p_0 \leq \phi(T^*)$

Lemmas 1 and 10 yield

$$P(0,T^*) \leq P(T^*,T^*) < P(\infty,\infty)$$

Hence (∞,∞) is optimal.

(b) $\phi(T^*) < p_0 < \xi(T^*,\tau)$

From Lemmas 1 and 10, we obtain

$$P(0,T^*) > P(T^*,T^*) < P(\infty,\infty).$$

Lemma 5 yields

$$S = 0 \text{ and } T = T^* \text{ if } p_0 > \psi_\infty(\tau)$$
$$S = T = \infty \text{ if } p_0 < \psi_\infty(\tau),$$

where

$$\psi_\infty(\tau) = \frac{v}{1-\theta} + (1-\frac{1}{\tau})\frac{\theta}{1-\theta}X^{-i/\theta}.$$

(c) $p_0 \geq \xi(T^*,\tau)$

From Lemmas 1 and 10, we have

$$P(0,T^*) > P(T^*,T^*) \geq P(\infty,\infty)$$

Hence, $(0,T^*)$ is optimal.

(3) $\tau = \frac{R-v}{R}$

In the same way as in (2), Lemmas 1 and 10 yield the following results.

(a) $p_0 \leq \phi(T^*)$: (∞,∞) is optimal.

(b) $\phi(T^*) < p_0 < C\frac{v}{R-v}$:

$$S = 0 \text{ and } T = T^* \text{ if } p_0 > \psi_\infty(\frac{R-v}{R})$$
$$S = T = \infty \text{ if } p_0 < \psi_\infty(\frac{R-v}{R}),$$

where

$$\psi_\infty(\frac{R-v}{R}) = \psi(\frac{R-v}{R}).$$

(c) $p_0 \geq C\frac{v}{R-v}$: $(0,T^*)$ is optimal.

Q.E.D.

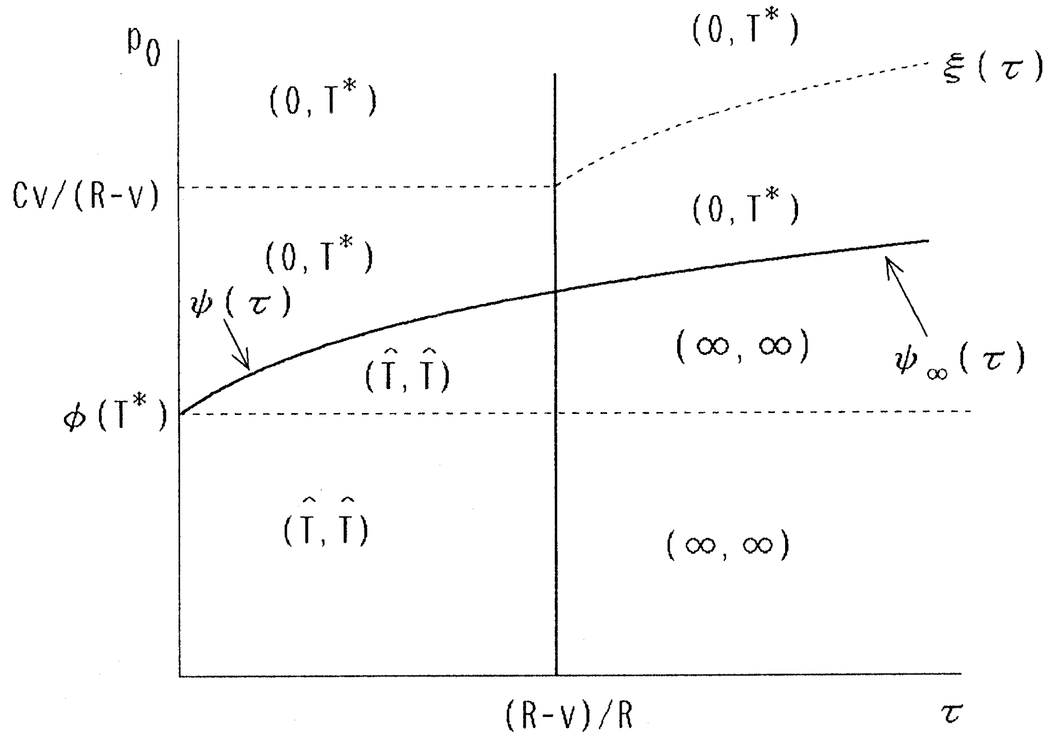

Fig.1. Optimal Timing of Sale and Development: An Exponential Example