95-F-15

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April 1995
(November 1994)

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This version: November, 1994

Abstract

This paper investigates an overlapping generations model in which fiat money is used as the riskless asset. In this model, monetary equilibria exist for any combination of parameters. Moreover, this model indicates the possibilities that the non-monetary steady state Pareto-dominates the monetary one; and that the Mundell-Tobin effect is reversed, i.e., the intensification of inflationary finance reduces the capital stock in the monetary steady state.

Keywords: Fiat Money, Moral Hazard, The Mundell-Tobin Effect.
JEL classification numbers: D82, E31, E41.

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* I am indebted to Hideaki Murase, Noriyuki Yanagawa, Masaya Sakuragawa, Toshihiro Matsumura, Keiichi Koda, Tadashi Minagawa, Kiyohiko G. Nishimura, Katsuhito Iwai, Masahiro Okuno-Fujiwara, and seminar participants at University of Tokyo, Yokohama National University, and Nagoya City University for helpful comments.
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1. Introduction

More than three decades have passed since Tobin(1958) presented his celebrated idea: money is held because it is the safest asset. Now, his idea is fully exploited in the literature of finance and portfolio selection, but rarely mentioned in the literature of money. The purpose of this paper is to construct a model of money in which it is demanded as a riskless asset, and to compare its results with those of the preceding models.

The model presented here is a modified version of Tirole's(1985). As in Tirole's model, capital and fiat money, an intrinsically useless asset, are available as store of value. In addition, it employs the following assumptions: capital investment is attended by an idiosyncratic risk, i.e., it brings no return with a positive probability; the result of investment is not verifiable; and agents are endowed with a utility function of constant relative risk aversion type. These assumptions make it impossible to provide investment insurance, though agents desire that insurance. Thus, agents have to bear their own risks.

This model exhibits monetary equilibria in which aggregate variables, including the price level, are non-stochastically determined. In those equilibria, agents demand money, though its expected return being lower than that of capital. This is because money earns a positive return for certain. As Tobin pointed out, risk-averse agents often choose to invest part of their wealth in the 'riskless' asset, i.e., the asset with certain (but possibly lower) return, because they are concerned not only with the expected return of an asset but also with its riskiness. From their viewpoint, it is unwise to hold the entire portfolio in a single risky asset, such as capital in this model. The price level being deterministic, money can serve as the 'riskless' asset in these equilibria.

It is natural to ask why money can be the riskless asset in the economy with uninsured risk. The reason is that the risk present here is not 'aggregate' but 'idiosyncratic.' This property of the risk makes self-fulfilling the expectation that money earns a positive return for certain: thanks to the law of large numbers, the behaviors of agents having such an expectation are made into a deterministic sequence of aggregate variables consistent with that expectation. So to speak, the idiosyncratic risk and agents' expectation jointly enthrone money as the riskless asset. In some cases, the valuation of money is indispensable for agents to live a decent life. As will be shown, highly risk-averse agents are worst-off if money is not valued.

This model yields some 'non-standard' results which are never obtained in the standard models of fiat money [e.g. Samuelson(1958), Wallace(1980), Tirole(1985)]. First, this model shows that monetary equilibria exist for any combination of parameters. On the other hand, the standard models showed that monetary equilibria exist only when the non-monetary equilibrium is dynamically inefficient. This contrast is due to the additional assumptions mentioned above. They alter the role of money from a perfect substitute for capital to a surrogate for insurance. As a result, even when the economy can be regarded as dynamically efficient, agents of this model hold money for the purpose of protecting themselves against their own risks.

Second, the non-monetary steady state of this model may Pareto-dominate the monetary one. In the standard models, a stationary monetary equilibrium, or a monetary equilibrium converging to the monetary steady state, achieves an efficient allocation: the long-run consumption of such an equilibrium is larger than that of any other one. In this model, on the other hand, the expected utility agents enjoy in the monetary steady state can be lower than the counterpart in the non-monetary one. When the economy is dynamically efficient, the valuation of money has a redistribution effect unfavorable to future generations, together with the effect providing them with a self-insurance device. In some cases, the former 'unfavorable' effect dominates the latter 'favorable' one. This result implies that the valuation of money does not necessarily make all agents better off.

And third, the Mundell-Tobin effect may be reversed in this model. As is widely known, Tirole's model is a typical environment where the Mundell-Tobin effect obtains: in his model, inflationary finance always increases the capital stock of the monetary steady state. [1] The reason is that inflation makes money less attractive as a store of value by lowering its return. On the other hand, this model shows that the intensification of inflationary finance may reduce the long-run capital stock if agents are sufficiently risk-averse. In a highly inflationary environment, such agents

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[1] This effect is originally pointed out by Mundell(1965) and Tobin(1965).

reduce capital investment in response to a small acceleration of inflation, which ultimately leads to the reduction of the long-run capital stock.

There are some works closely related to this paper. Smith(1986) also presented a model in which the valuation of money is essential to the existence of equilibria. In his model, adverse selection makes it impossible for either pooling or separating equilibrium to exist if money is valueless; but some equilibria emerge if money has positive value. Azariadis and Smith(1993) reconsidered the Mundell-Tobin effect in an overlapping generations model with an informational friction. They also pointed out the possibility that an informational friction can reverse the Mundell-Tobin effect, but their logic is quite different from the one working here.[2] Grossman and Yanagawa(1993) pointed out that the valuation of money may be malign to future generations. They obtained this result from an overlapping generations model with a Romer-type externality. One problem with their model is that they assumed, rather than derived, the externality. However, the model presented here suggests that such an externality can be derived from an informational friction.

The rest of this paper is organized as follows. The model is constructed in the next section. Then, the case without informational friction is investigated in Section 3, and the case with informational friction in Section 4. Section 5 is conclusion.

2. The Model

An overlapping generations economy a la Tirole(1985) is considered. Time is indexed by t=1,2,... and the economy consists of an infinite sequence of two-periods-lived overlapping generations. In the initial period (t=1) there is a unit mass of old consumers, each of whom is endowed with M units of fiat money[3] and K_1 units of capital goods[4]. They exchange their money

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[2] They derived the reversal of the Mundell-Tobin effect from an incentive constraint preventing non-investors from pretending to be genuine investors.

[3] For the time being, monetary authority is assumed to inject no additional money into the economy. Thus, the stock of fiat money equals M in any period.

[4] Capital goods are assumed to be not consumable.

and capital for consumption goods, consume them, and die at the end of that period. All of them have the same utility function $v(c_1)$, where v is an increasing function of consumption c_1.

In addition, a new generation containing a continuum of agents of measure one is born at the beginning of each period. The life of such an agent is as follows: when young, she supplies one unit of labor[5] and saves all of her wage for her old age; when old, she does not work at all but enjoys consumption. This implies that the labor force in period t, L_t, equals the population of young generation in that period; and that aggregate savings in period t, s_t, equals aggregate labor income in that period. That is:

$$L_t = 1, \quad \forall t \geq 1; \tag{1}$$

$$s_t = w_t L_t = w_t, \quad \forall t \geq 1. \tag{2}$$

where w_t is wage in terms of consumption goods in period t.

There are two ways of savings: one is to hold intrinsically useless money; the other is to produce capital goods.[6] Needless to say, the former is available only when fiat money has positive value. On the other hand, the latter is always available but subject to some technical constraints:

Assumption 1: *There is a technology transforming the consumption goods into the capital goods on one-to-one basis. This transformation is irreversible and takes one period gestation. Moreover, it is attended by the following risk. Let r_{t+1} denote rental price of capital in terms of consumption goods in period t+1. Then, $\tilde{I}_t$ units of investments results in $r_{t+1}\tilde{I}_t$ units of consumption in the next period with probability θ; and 0 with probability $1-\theta$, where θ is a constant satisfying $\theta \in (0,1)$. This risk is not aggregate but idiosyncratic.*

The law of large numbers implies the next aggregate relations:

$$K_{t+1} = \theta I_t L_t = \theta I_t; \tag{3}$$

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5 It is assumed that agents can supply labor without incurring disutility.

6 Consumption goods are assumed to be perishable, so that their storage is impossible.

$$m_t = w_t - I_t = \frac{M}{p_t}. \tag{4}$$

where K_{t+1} denotes aggregate level of capital stock in period t+1; m_t money demand per capita in period t; I_t investment per capita in period t; and p_t money price of consumption goods in period t.

Newly born agents are endowed with the same expected utility function:

$$\theta u(c^G) + (1-\theta)u(c^B)$$

where $c^G (c^B)$ denotes consumption when investment turns out to be successful (that when not successful, respectively). We even assume that:

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}, \qquad \sigma\text{: a positive constant.}$$

That is, agents are of constant relative risk aversion (CRRA). This assumption does not affect the analysis of Section 3 at-all, but makes that of Section 4 considerably easy.

Consumption goods are produced from capital and labor by competitive producers with a constant-return-to-scale technology of Cobb-Douglas type:

$$F(K_t, L_t) \equiv K_t^{\rho} L_t^{1-\rho}, \quad \rho \in (0,1).$$

Competitive production makes the following relations to hold:

$$r_t = \rho K_t^{\rho-1}, \quad w_t = (1-\rho)K_t^{\rho}, \quad \forall t \geq 1. \tag{5}$$

Moreover, capital depreciates completely, once input into the production process.

It is obvious that agents of this economy potentially demand investment insurance: being risk-averse, they prefer receiving expected return for certain. Such an insurance seems feasible, because the risk is not aggregate but idiosyncratic. However, agents are not necessarily insured. Some informational frictions can be an obstacle to provision of insurance. In what follows, we will see how equilibrium outcomes are changed by the verifiability of the results of investments.

3. The Case without Informational Friction

Let us first consider the case without any kind of informational friction. In this case, insurance companies can offer insurance contracts to young agents, each of whom is allowed to accept only one of these offers. An insurance contract consists of an insurance premium α_{t+1} and an insurance money β_{t+1}: if an insured agent succeeds in investment, she must transfer α_{t+1} units of capital goods to the insurance company; if fails, she is transferred β_{t+1} units from the company.

Competition among insurance companies gives the following features to equilibrium contracts: equilibrium contracts (a) are actuarially fair, i.e.,

$$\theta\alpha_{t+1} - (1-\theta)\beta_{t+1} = 0;$$

and (b) maximize each agent's utility, given her size of investment. Consider an agent having an intention to transform $\hat{I}_t$ units of the consumption goods into the capital goods. If insured, her expected utility can be written as:

$$\theta u[r_{t+1}(\hat{I}_t - \alpha_{t+1}) + \frac{p_t}{p_{t+1}}(w_t - \hat{I}_t)] + (1-\theta)u[\frac{\theta}{1-\theta}r_{t+1}\alpha_{t+1} + \frac{p_t}{p_{t+1}}(w_t - \hat{I}_t)].^{7}$$

As is easily established, the maximizing α_{t+1} satisfies the next relation:

$$r_{t+1}(\hat{I}_t - \alpha_{t+1}) + \frac{p_t}{p_{t+1}}(w_t - \hat{I}_t) = \frac{\theta}{1-\theta}r_{t+1}\alpha_{t+1} + \frac{p_t}{p_{t+1}}(w_t - \hat{I}_t);$$

or

$$\alpha_{t+1} = (1-\theta)\hat{I}_t.$$

That is, the equilibrium contract protects the agent completely from her individual risk. Since it makes the return on investment certain and equal to θr_{t+1}, the following must hold if agents demand money:

$$\theta r_{t+1} = p_t / p_{t+1}. \qquad (6)$$

Now, we are in a position to define the equilibrium for this case:

Definition: *An equilibrium for the case without informational friction is a sequence* $\{(K_t, L_t, I_t, r_t, w_t, p_t, m_t)\}_{t=1}^{\infty}$ *satisfying (1)-(5), either (6) or*

$$m_t = 0, \quad \forall t \geq 1; \qquad (7)$$

and the feasibility condition:

$$M/p_t < w_t, \quad \forall t \geq 1. \qquad (8)$$

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[7] Eq.(5) and the budget constraint, i.e., $w_t = \hat{I}_t + \hat{m}_t$, jointly yield this expression. If money is valueless, $\hat{I}_t = w_t$.

An equilibrium is called 'non-monetary' if (7) holds; and 'monetary,' otherwise.

Using money-savings ratio, i.e., $a_t \equiv \dfrac{M/p_t}{w_t}$, we can summarize (1)-(8) as follows:

$$a_{t+1} = \theta r^* \frac{a_t}{1-a_t}, \quad a_t \in [0,1), \quad \forall t \geq 1 \tag{9}$$

$$K_{t+1} = \theta(1-\rho)(1-a_t)K_t^{\rho}. \tag{10}$$

where $r^* \equiv \rho/\theta(1-\rho)$. Given K_1, a path of money-savings ratio satisfying (9) determines a unique path of capital accumulation, which in turn determines paths of the other endogenous variables. Thus, each sequence satisfying (9) corresponds to an equilibrium defined above.

The dynamics of money-savings ratio are depicted in Figure 1. As shown in Figure 1, there are two possible cases.[8] If $\theta r^* > 1$, no sequence but $a_t = 0, \forall t \geq 1$ satisfies (9). That is, there is a unique equilibrium in this case. If $\theta r^* < 1$, there are such other sequences, in addition to that zero sequence. These sequences are divided into two types: the first type is such that

$$a_t = 1 - \theta r^*, \quad \forall t \geq 1; \tag{11}$$

and the second one is:

$$1 - \theta r^* > a_1 > a_2 > \cdots > a_t > \cdots > 0, \quad \lim_{t \to +\infty} a_t = 0. \tag{12}$$

The equilibrium corresponding to the zero sequence is 'non-monetary.' In that equilibrium, capital accumulates according to the following rule:

$$K_{t+1} = \theta(1-\rho)K_t^{\rho} \tag{10'}$$

Since (10') implies that savings, the denominator of a_t, are positively finite in any period, money cannot have any positive value. This equilibrium exhibits that:

$$\lim_{t \to +\infty} (K_t, \theta r_t, w_t) = (K^N, \theta r^*, w^N),$$

where $K^N \equiv [\theta(1-\rho)]^{1/(1-\rho)}$ and $w^N \equiv (1-\rho)(K^N)^{\rho}$. In this section, we call the long-run state of this equilibrium 'non-monetary steady state (NMSS).'

Obviously, any equilibrium corresponding to another sequence is 'monetary.' Monetary

[8] The analysis of the border case, $\theta r^* = 1$, is omitted here.

equilibria corresponding to sequences such as (12) exhibit that:

$$\lim_{t \to +\infty} (m_t, K_t, \theta r_t, w_t) = (0, K^N, \theta r^*, w^N).$$

That is, these equilibria converge to the NMSS. On the other hand, the equilibrium corresponding to the sequence (11) exhibits that:

$$\lim_{t \to +\infty} (m_t, K_t, \theta r_t, w_t) = (\overline{m}^*, \overline{K}^M, 1, \overline{w}^M),$$

where $\overline{K}^M \equiv (\theta\rho)^{1/(1-\rho)}$, $\overline{m}^* \equiv (1-2\rho)(\overline{K}^M)^\rho$, and $\overline{w}^M \equiv (1-\rho)(\overline{K}^M)^\rho$. In this section, we call the long-run state of this equilibrium 'monetary steady state (MSS).'

Notice that monetary equilibria exist if and only if the long-run interest rate, θr^*, is lower than the growth rate, 1. The condition, $\theta r^* < 1$, implies that the stock of capital in the NMSS is inefficiently large, or equivalently that capital accumulates too much both in the non-monetary equilibrium and in the monetary ones converging to the NMSS. Such an inefficiency is often called 'dynamic inefficiency.' Tirole(1985) showed that dynamic inefficiency is a sufficient condition for the existence of monetary equilibria in such an overlapping generations economy as the present one.

In fact, the model analyzed here is essentially the same as Tirole's. In addition to the above result, Tirole also showed that the monetary equilibrium converging to the monetary steady state achieves an efficient allocation; and that the monetary steady state is the 'golden rule,' i.e., the steady state in which consumption per capita is maximized. These results are also our results: we can show that the monetary equilibrium converging to the MSS is efficient; and that the MSS is the golden rule.[9]

In this case, money is demanded as a perfect substitute for insured investment: agents see no difference between money and insured investment, because both earn the same return for certain. Note that this result depends considerably on investment insurance: no such result is obtained, if the insurance is not available for some reason.

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[9] It is a well-known fact that an allocation is efficient if and only if the infinite sum of t-period gross interest rates diverges. Using this criterion, we can prove the first fact. To prove the second one, we only have to point out that K^M maximizes $K^\rho - (K/\theta)$.

4. The Case with Informational Friction

Next, consider the case in which the results of investments are not verifiable. In this case no insurance is provided, which can be explained as follows. If insurance were provided, all 'successful' investors would evade payment of insurance premia, or rather require payment of insurance money, insisting that they are 'unsuccessful.' The law cannot order them to pay premia, because it is unable to prove that they are telling a lie. Since premia are hardly collected, insurance contracts always yield loss. Thus, no insurance companies have any incentive to provide investment insurance and, as a result, investors have to bear their own risks. [10]

When money has positive value, agents born in period t have to solve the following maximization problem:

$$Max_{(m_t, I_t)} \theta u[r_{t+1} I_t + \frac{p_t}{p_{t+1}} m_t] + (1-\theta) u[\frac{p_t}{p_{t+1}} m_t] \quad s.t. \, w_t = I_t + m_t. \tag{13}$$

Note that agents foresee future prices such as r_{t+1} and p_{t+1}. They know that endogenous variables are non-stochastically determined, though uninsured risks are present in this economy. The main task of this section is to show the existence of equilibria consistent with such expectations.

Having assumed that u is of CRRA type, we can obtain explicit solutions for (13):

$$m_t = \frac{R_{t+1}}{R_{t+1} - 1 + [\frac{\theta}{1-\theta}(R_{t+1} - 1)]^{1/\sigma}} w_t;$$

$$I_t = \frac{[\frac{\theta}{1-\theta}(R_{t+1} - 1)]^{1/\sigma} - 1}{R_{t+1} - 1 + [\frac{\theta}{1-\theta}(R_{t+1} - 1)]^{1/\sigma}} w_t, \tag{14}$$

where $R_{t+1} \equiv (p_{t+1}/p_t) r_{t+1}$. Notice that agents produce capital goods if and only if $[\frac{\theta}{1-\theta}(R_{t+1} - 1)]^{1/\sigma} > 1$, i.e.,

$$\theta r_{t+1} > p_t / p_{t+1}, \tag{15}$$

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[10] So-called 'self-enforcing contract' cannot be written in our model. Such a contract prevent betrayals by punishing the betraying party in subsequent periods. This kind of punishments are not available here.

while they demand money if only it has positive value.[11] In this economy agents demand money, even when its expected return is lower than that on investment. This is because the uninsured idiosyncratic risk turns money into a imperfect substitute for investment: while investment yields a positive return only stochastically, money earns a positive return for certain. That is, money is the 'riskless' asset, whereas investment the 'risky' one. Tobin(1958) pointed out that risk-averse agents in such a situation tend to invest part of wealth in the 'riskless' asset, even if its expected return is dominated by that on the 'risky' one. The reason is that they appreciate not only the expected return but also the safeness of each asset. Especially if endowed with a utility function satisfying the Inada condition, agents always demand the 'riskless' asset. CRRA is one of such utility functions.

When money is valueless, all agents can do is to produce capital goods. However, this does not always lead to their investments. In order to motivate them to invest, the following condition must hold:

$$\theta u[r_{t+1}w_t] + (1-\theta)u(0) > -\infty. \tag{16}$$

Suppose that agents are highly risk-averse, i.e., $\sigma \geq 1$. Valueless money makes their expected utilities fixed to be negatively infinite, independent of their behaviors.[12] In this case, all the model can predict is that no agents demand money. It cannot predict whether they make investments or not, because such actions cannot either raise or lower their expected utilities. That is, the non-valuation of money leads directly to the economic disorder. It is sensible to exclude such an abnormal situation from the set of equilibria. Thus, we adopt (16) as an equilibrium condition.

In this case we define monetary and non-monetary equilibrium as follows:

Definition: *(a) A monetary equilibrium for the case with informational friction is a sequence*

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[11] If $\theta r_{t+1} \leq p_t / p_{t+1}$, agents demand money exclusively. But such a thing does not happen in equilibrium. If so, no production of consumption goods occurs in period t+1. Thus money has no purchasing power in that period, which contradicts the assumption that money earns higher return than capital. In other words, (16) is one of the equilibrium conditions.

[12] On the other hand, such a problem is not present if $0 < \sigma < 1$.

$\{(K_t, L_t, I_t, r_t, w_t, p_t, m_t)\}_{t=1}^{\infty}$ *satisfying (1)-(5),(8),(14),(15); (b) A non-monetary equilibrium for the case with informational friction is a sequence* $\{(K_t, L_t, I_t, r_t, w_t, p_t, m_t)\}_{t=1}^{\infty}$ *satisfying (1)-(5),(16) and*

$$m_t = M/p_t = 0, \quad \forall t \geq 1. \tag{17}$$

Using money-savings ratio, we can reduce equilibrium conditions (1)-(5),(8),(14)-(16) to (10) and the dynamic relation:

$$\theta r^* \frac{a_t}{1-a_t} = a_{t+1}[(1-\theta)(1+\frac{r^*}{a_{t+1}})^{\sigma} + \theta], \quad a_t \in [0,1), \quad \forall t \geq 1. \tag{18}$$

As in the preceding section, a sequence of money-savings ratio corresponds to an equilibrium defined above.[13] Note that eq.(18) is reduced to (9) if agents are risk-neutral, i.e., $\sigma = 0$.

By setting $a_t = a_{t+1} = a^*$ in (18), we obtain the following equation:

$$\frac{\theta r^*}{1-a^*} = (1-\theta)(1+\frac{r^*}{a^*})^{\sigma} + \theta. \tag{19}$$

This equation has a unique root satisfying $a^* \in (0,1)$[14], which implies that there is at least one monetary equilibrium for any combination of parameters of technology (ρ), risk (θ), and risk aversion (σ).

The dynamics of money-savings ratio are depicted in Figure 2.

Case 1: $0 < \sigma < 1$ (Figure 2.a)

There are three types of sequences satisfying (18). The first type is such that:

$$a_t = 0, \quad \forall t \geq 1;$$

which corresponds to the non-monetary equilibrium. This equilibrium exhibits that:

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[13] Again in this case, both money-savings ratio and capital satisfy (10). Thus, the path of capital is uniquely determined by a sequence of money-savings ratio. Paths of other endogenous variables are uniquely determined by that path of capital.

[14] The LHS of (19) is a monotone increasing function of a^*, while the RHS a monotone decreasing function. Moreover, $\lim_{a^* \to 1} LHS = +\infty$ and $\lim_{a^* \to 0} RHS = +\infty$. Therefore, we can conclude that eq.(19) has a unique root satisfying $a^* \in (0,1)$.

$$\lim_{t\to+\infty}(K_t, r_t, w_t) = (K^N, r^*, w^N),$$

where K^N, r^*, w^N were defined in Section 3. Again in this section, we call the long-run state of this equilibrium 'non-monetary steady state (NMSS).' The second type is such that:

$$a^* > a_1 > a_2 > \cdots > a_t > \cdots > 0, \quad \lim_{t\to+\infty} a_t = 0.$$

Such a sequence corresponds to a monetary equilibrium converging to the NMSS. That is:

$$\lim_{t\to+\infty}(m_t, K_t, r_t, w_t) = (0, K^N, r^*, w^N).$$

And the third type is such that:

$$a_t = a^*, \quad \forall t \geq 1;$$

which corresponds to a monetary equilibrium exhibiting that:

$$\lim_{t\to+\infty}(m_t, K_t, r_t, w_t) = (m^*, K^M, r^M, w^M),$$

where $K^M \equiv [\theta(1-\rho)(1-a^*)]^{1/(1-\rho)}$, $m^* \equiv a^*(1-\rho)(K^M)^\rho$, $r^M = \rho(K^M)^{\rho-1}$, and $w^M \equiv (1-\rho)(K^M)^\rho$. In this section, we call the long-run state of this equilibrium 'monetary steady state (MSS).'

Case 2: $\sigma = 1$ (Figure 2.b)

No sequence but $\{a^*\}_{t=1}^\infty$ satisfies (18). That is, there is a unique equilibrium in this case. This equilibrium is monetary and converges to the MSS.

Case 3: $\sigma > 1$ (Figures 2.c and 2.d)

The dynamics is of Figure 2.c if σ is close to 1; and of Figure 2.d if σ is sufficiently large.[15] As shown in Figure 2.c, no sequence but $\{a^*\}_{t=1}^\infty$ satisfies (18) when σ is close to 1. This sequence corresponds to a monetary equilibrium converging to the MSS. When σ is sufficiently large, there may be other positive sequences satisfying (18), in addition to $\{a^*\}_{t=1}^\infty$. If exist, they correspond to

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[15] The gradient of the RHS of (18) at $a_{t+1} = a^*$ is as follows:

$$dRHS/da_{t+1} = [\theta r^*/(1-a^*)][1-\sigma(r^*+a^*-1)/(r^*+a^*)].$$

Since (19) ensures $0 < (r^*+a^*-1)/(r^*+a^*) < 1$, the sign of the above derivative is positive if σ is close to 1; negative if σ is sufficiently large.

monetary equilibria exhibiting erratic behaviors. As Figure 2.d suggests, none of such monetary equilibria would never converge to the NMSS.

Notice that there is not any non-monetary equilibrium in either Case 2 or Case 3. In those cases, every candidate for non-monetary equilibrium violates the condition (18). Also notice that no monetary equilibrium converges to the NMSS in these cases.

These results contrast with the counterparts of the preceding section. In the model of Section 3, non-monetary equilibrium exists in any case, whereas monetary equilibria exist only when the non-monetary equilibrium is inefficient. In this model, by contrast, monetary equilibrium exists in any case, whereas non-monetary equilibrium exists only when agents are not so risk-averse. This contrast is caused by the informational friction which make the provision of investment insurance impossible: money is demanded as a perfect substitute for insured investment when insurance is available; on the other hand, it is demanded as a surrogate for insurance when insurance is not available. We must also notice that the belief that money is the 'riskless' asset that lets money be the 'riskless' asset. If such a belief is absent, money can no longer serve as a surrogate for insurance. Especially when agents are highly risk-averse, there is no 'well-behaving' deterministic equilibrium, as shown above.

In addition, our model yields two more 'non-standard' results, both of which are related to the monetary steady state. First, our model suggests that the monetary steady state is no longer the 'golden rule' in the presence of informational friction.

Proposition 1: *The non-monetary steady state Pareto-dominates the monetary one if* $0 < \sigma < 1$ *and*

$$(1-a^*)^{0r^*(1-\sigma)-1}\left(\frac{r^*}{r^*+a^*}\right)^{\sigma} < 1. \tag{20}$$

(Proof) See Appendix.

Note that the condition (20) is satisfied if $\theta r^*(1-\sigma) > 1$. Define $W^M, W^N, \overline{W}^M$ as:

$$W^M \equiv \theta u[r^M(w^M - m^*) + m^*] + (1-\theta)u[m^*];$$
$$W^N \equiv \theta u[r^* w^N] + (1-\theta)u[0];$$
$$\overline{W}^M \equiv \theta u[r^M w^M] + (1-\theta)u[0].$$

Obviously, $W^M (W^N)$ is the level of expected utility in the MSS (that in the NMSS, respectively). On the other hand, $\overline{W}^M$ is the level of expected utility in an imaginary state in which factor prices are at the same levels as in the MSS but money is valueless. Using these notations, we can divide the change of expected utility due to the valuation of money, $W^M - W^N$, into the two effects:

$$W^M - W^N = (W^M - \overline{W}^M) + (\overline{W}^M - W^N).$$

The first term of the RHS represents the insurance effect, i.e., welfare gain from that the valuation makes it possible for agents to self-insure themselves; the second the redistribution effect, i.e., welfare gain or loss from that the valuation changes intergenerational distribution by reducing capital stock. While the first term is trivially positive-valued, the sign of the second term is positive if $\theta r^* < 1$; and negative if $\theta r^* > 1$.[16] When $\theta r^* > 1$, the valuation of money is accompanied by both the 'favorable' insurance effect and the 'unfavorable' redistribution effect. Proposition 1 insists that there are some cases where the 'unfavorable' effect dominates the 'favorable' one. This proposition also implies the next corollary:

Corollary: *Suppose that* $0 < \sigma < 1$, *and that (20) holds. Then, the monetary equilibrium converging to the monetary steady state and the non-monetary equilibrium cannot be Pareto-ranked.*

(Proof) See Appendix.

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16 The sign of the second term coincides with that of

$$r^M w^M - r^* w^N = \rho(1-\rho)[(K^M)^{2\rho-1} - (K^N)^{2\rho-1}].$$

Since $K^M < K^N$, the above is positive if $\rho < 1/2$; and negative if $\rho > 1/2$. Note that:

$$\rho > (<) 1/2 \Leftrightarrow \theta r^* > (<) 1;$$

and that $\theta r^* > 1$ implies dynamic efficiency.

That is, the valuation of money harms future generations by impeding capital accumulation, though making the initial old better off.

And next, the Mundell-Tobin effect can be reversed in our model. To see this, consider the following monetary policy:

$$M_{t+1} = \lambda M_t, \tag{21}$$

where λ is a constant larger than 1. The injection is through money-financed purchase of consumption goods.[17] In the absence of informational friction, this policy makes the next relation to hold in the long run:[18]

$$\theta \rho K^{\rho-1} = 1/\lambda. \tag{22}$$

That is, the long-run capital stock becomes larger, as the government accelerates money injection. The reason is obvious: if inflation accelerates, money becomes less attractive as a store of value, which motivates agents to make more capital investments. This correlation between inflation and capital accumulation is called the Mundell-Tobin effect. In the presence of informational friction, however, such a correlation is not necessarily observed.

Proposition 2: *The small increase in λ reduces the capital stock in the monetary steady state if*

$$1 - \sigma \frac{\lambda r^* + a^* - 1}{\lambda r^* + a^*} \leq 0. \tag{23}$$

Otherwise, it enhances the capital stock in the monetary steady state.

(Proof) The policy concerned modifies (18) as follows:

$$\theta r^* \frac{a_t}{1-a_t} = \frac{a_{t+1}}{\lambda}[(1-\theta)(1+\frac{r^*}{a_{t+1}/\lambda})^{\sigma} + \theta], \quad a_t \in [0,1), \quad \forall t \geq 1. \tag{18'}$$

The dynamics is like Figure 2.d if (23) holds; and like one of Figures 2.a, 2.b, and 2.c, otherwise. The small increase in λ raises a^* if (23) holds; and lowers in the other cases. This is apparent from the fact that it enlarges the graph of the RHS of (18') rightward, leaving that of the LHS

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[17] The government is assumed to consume purchased goods.

[18] This relation can be obtained from (5),(6), and (21).

unchanged. In the monetary steady state, eq.(10) is reduced to:

$$K^{1-\rho} = \theta(1-\rho)(1-a^*).$$

Thus, we have obtained the desired result. Q.E.D.

Note that the Mundell-Tobin effect is true if agents are not so risk-averse, i.e., $0 < \sigma \leq 1$. This is because the condition (23) never holds in such cases. The Mundell-Tobin effect is reversed in a highly inflationary economy with sufficiently risk-averse agents. Consider the portfolio selection of such a risk-averse agent. From (14), we can obtain the derivative:

$$\frac{d(I_t/w_t)}{dR_{t+1}} = \frac{\frac{1}{\sigma}\left[1-\sigma+\frac{1}{R_{t+1}-1}\right]\left[\frac{\theta}{1-\theta}(R_{t+1}-1)\right]^{\frac{1}{\sigma}}+1}{\left\{R_{t+1}-1+\left[\frac{\theta}{1-\theta}(R_{t+1}-1)\right]^{\frac{1}{\sigma}}\right\}^2}.$$

When $\sigma > 1$, the sign of this derivative is positive if the nominal return on capital investment, $R_{t+1} = (p_{t+1}/p_t)r_{t+1}$, is close to 1; negative if sufficiently large. That is, in response to a small increase in R_{t+1}, agents reduce investment when R_{t+1} is sufficiently high, and increase when not so high. As is easily confirmed, the condition (23) means that the sign of the above derivative is negative in the MSS. Thus, if (23) is true, agents reduce investment in response to a small increase in λ , which induces further reduction of investment by raising rental price and lowering wage, and ultimately the reduction of the long-run capital stock. Also notice that the reversal of the Mundell-Tobin effect may be caused by a monetary policy: a situation allowing the reversal emerges if the government sets λ to a sufficiently high level.

5. Conclusion

We have seen that an informational friction can create a situation in which no insurance is available and money is demanded as a surrogate for insurance. In such a situation, inflationary finance may not promote but impede capital accumulation; and the valuation of money may make future generations worse off.

Suggestive as it is, our model has some problems. For example, it is based on specific utility and production functions. We should have examined whether the same results are obtained in more general settings. However, that is a formidable task, because a slight generalization makes the model considerably intractable. The author examined some generalized models, only to find it difficult to even confirm the existence of monetary steady state in such a setting. Generalization is indeed desirable, but not assured to be rewarding.

Some readers may wonder what results are obtained if another type of informational friction is present. For example, we can imagine a less restrictive type of informational friction allowing only incomplete insurance to be provided. Kitagawa(1994) investigated an overlapping generations economy with such a friction.

Appendix

Proof of Proposition 2: Throughout this proof, we use the following notations:

W^M: the level of expected utility at the monetary steady state;

W^N: the level of expected utility at the non-monetary steady state;

K^M: the level of aggregate capital at the monetary steady state;

K^N: the level of aggregate capital at the non-monetary steady state;

I^M: the level of individual investment at the monetary steady state;

I^N: the level of individual investment at the non-monetary steady state;

r^M: the rental price of capital at the monetary steady state;

r^N: the rental price of capital at the non-monetary steady state;

m^*: the value of money at the monetary steady state.

Note that:

$$\theta I^M = K^M = [\theta(1-\rho)(1-a^*)]^{1/(1-\rho)}; \tag{A1}$$

$$\theta I^N = K^N = [\theta(1-\rho)]^{1/(1-\rho)}; \tag{A2}$$

$$r^M = \rho(K^M)^{\rho-1}; \tag{A3}$$

$$r^N = \rho(K^N)^{\rho-1} = r^*; \tag{A4}$$

$$m^* = a^*(1-\rho)(K^M)^{\rho}. \tag{A5}$$

Using (A1)-(A5) and (19), we can obtain:

$$W^N = \frac{\theta(r^N I^N)^{1-\sigma} - 1}{1-\sigma} = \frac{\theta[r^*(1-\rho)(K^N)^{\rho}]^{1-\sigma} - 1}{1-\sigma}; \tag{A6}$$

$$W^M = \frac{\theta(r^M I^M + m^*)^{1-\sigma} + (1-\theta)(m^*)^{1-\sigma} - 1}{1-\sigma}$$
$$= \frac{\dfrac{\theta[r^*(1-\rho)(K^M)^{\rho}]^{1-\sigma}}{1-a^*}\left[\dfrac{r^*}{r^*+a^*}\right]^{\sigma} - 1}{1-\sigma}. \tag{A7}$$

Eqs.(A6) and (A7) jointly yield the next equation:

$$W^N - W^M = \frac{\theta}{1-\sigma}[r^*(1-\rho)(K^N)^{\rho}]^{1-\sigma}\{1 - \frac{\left(K^M/K^N\right)^{\rho(1-\sigma)}[r^*/(r^*+a^*)]^{\sigma}}{1-a^*}\}$$
$$= \frac{\theta}{1-\sigma}[r^*(1-\rho)(K^N)^{\rho}]^{1-\sigma}\{1-(1-a^*)^{\theta r^*(1-\sigma)-1}\left(\frac{r^*}{r^*+a^*}\right)^{\sigma}\}$$

which implies that $W^N > W^M$ if (23) holds. Q.E.D.

Proof of Corollary: In addition to W^N and W^M, we use the following notations:

W_t^M: the level of expected utility of generation t in the monetary equilibrium;

W_t^N: the level of expected utility of generation t in the non-monetary equilibrium;

W_0^M: the level of utility of the initial old in the monetary equilibrium;

W_0^N: the level of utility of the initial old in the non-monetary equilibrium;

K_t^M: the level of aggregate capital in period t in the monetary equilibrium;

K_t^N: the level of aggregate capital in period t in the non-monetary equilibrium;

r_t^M: the rental price of capital in period t in the monetary equilibrium;

r_t^N: the rental price of capital in period t in the non-monetary equilibrium;

$m_t{}^*$: the value of money in period t in the monetary equilibrium.

W_0^M and W_0^N are as follows:

$$W_0^M = v(r_1^M K_1^M + m_1{}^*);$$

$$W_0^N = v(r_1^N K_1^N).$$

Since $K_1^M = K_1^N = K_1$ and $r_1^M = r_1^N = \rho(K_1)^{\rho-1}$, we can conclude that $W_0^M > W_0^N$. Thus, if one equilibrium Pareto-dominates the other, the dominating equilibrium is the monetary one. If so, the following relations must hold:

$$W_t^M \geq W_t^N, \quad \forall t \geq 1$$

or

$$\lim_{t \to +\infty} (W_t^M - W_t^N) \geq 0. \tag{A8}$$

Since $\lim_{t \to +\infty} W_t^i = W^i$ $(i = M, N)$, (A8) means that:

$$W^M - W^N \geq 0;$$

which contradicts Proposition 2. Q.E.D.

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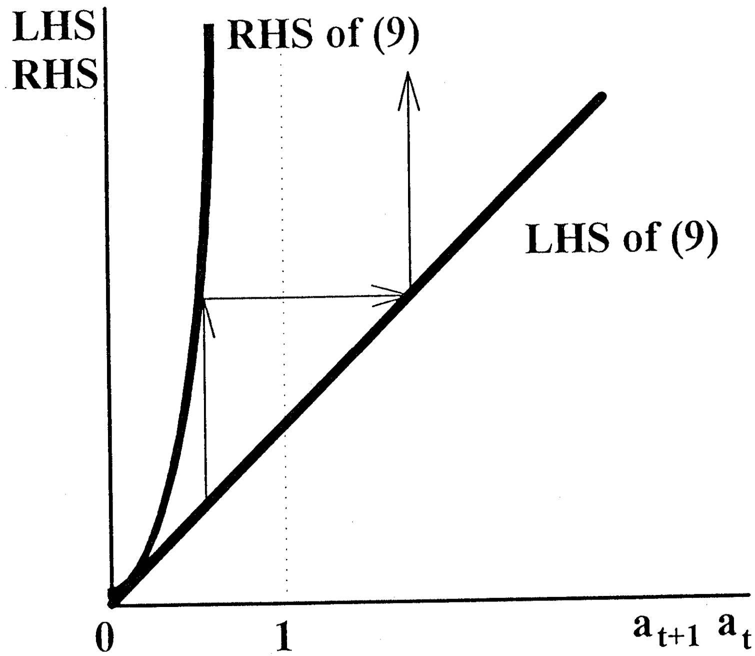

Figure 1.a: the case of $\theta r^{*}>1$

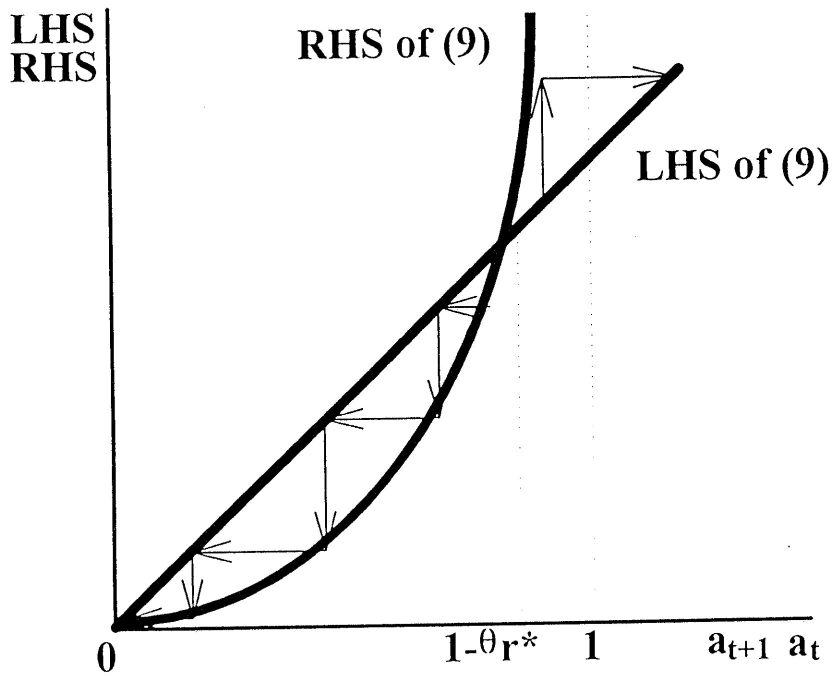

Figure 1.b: the case of $\theta r^{*}<1$

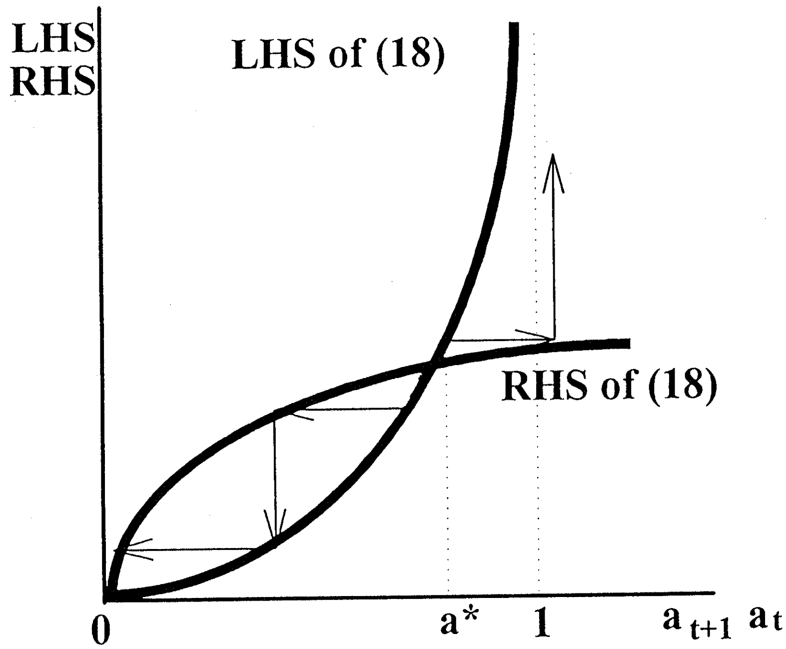

Figure 2.a: the case of $0<\sigma<1$

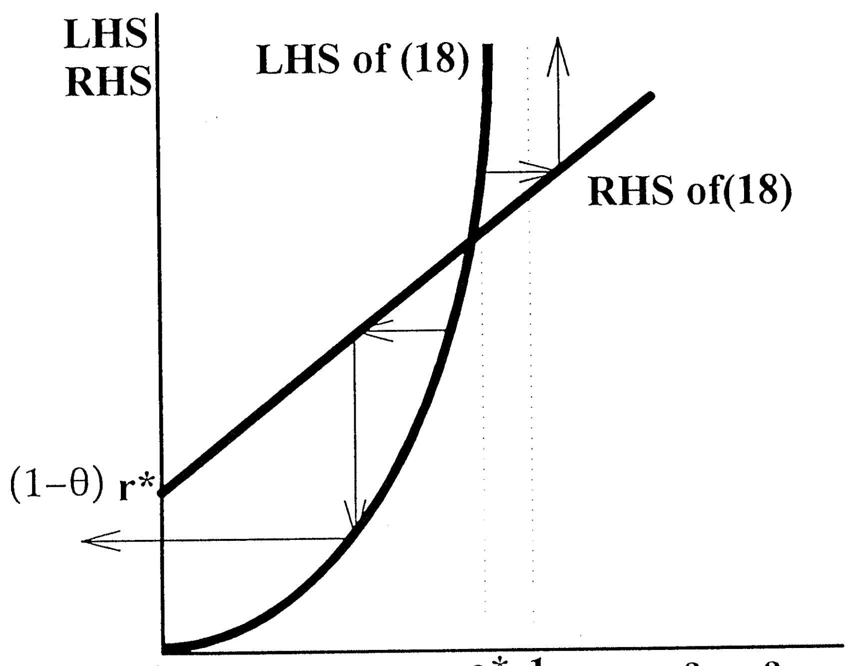

Figure 2.b: the case of $\sigma=1$

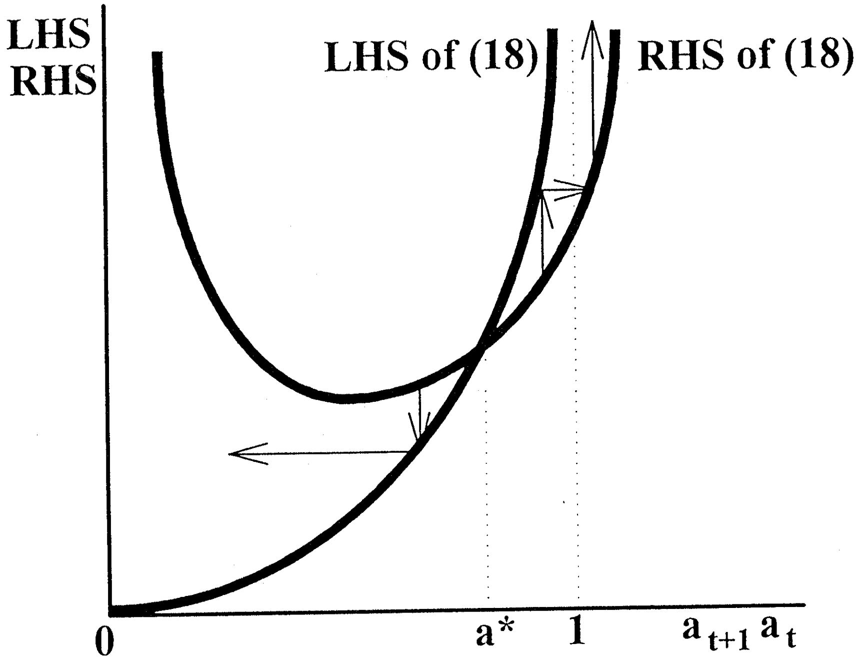

Figure 2.c: the case of $\sigma>1$ (1)

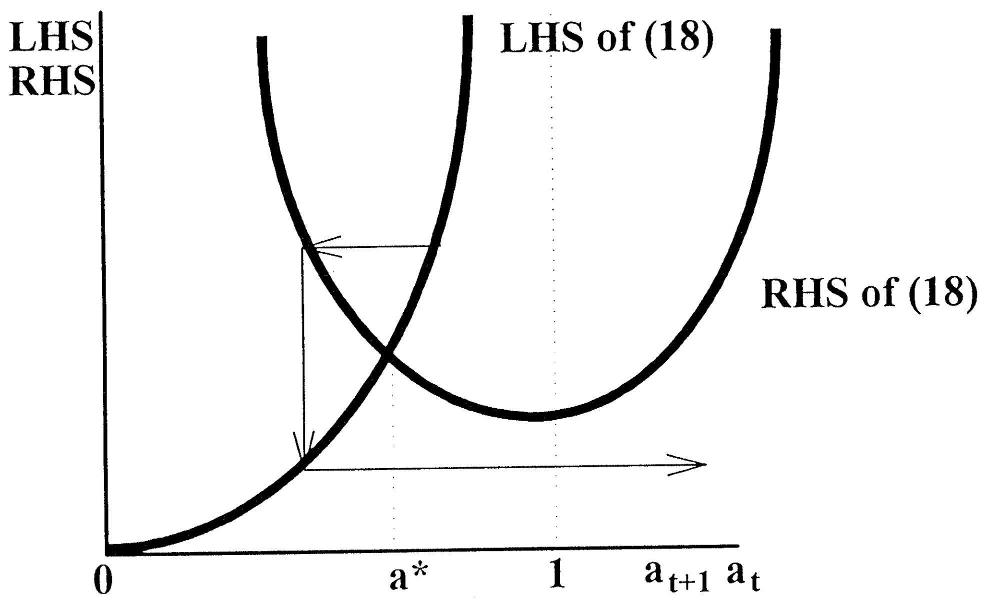

Figure 2.d: the case of $\sigma>1$ (2)