95-F-22

Multimarket Contact, Imperfect Monitoring, and Implicit Collusion

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October 1995

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February 1993

This version, October 1995

* I am grateful to Michihiro Kandori and Noriyuki Yanagawa for their helpful comments.

ABSTRACT

It is well known in the industrial organization literature that when the demand is random and the rivals' choices of supply are unobservable, it is difficult for oligopolistic firms to establish implicit collusion. We will show in an infinitely repeated game with imperfect monitoring that multimarket contact enhances implicit collusion: As the number of distinct markets in which firms contact with each other increases, the optimal supergame equilibria converge to full collusion. This folk theorem property holds on almost the same condition as the perfect monitoring case with respect to the discount factor, that is, even in the low discount factor case.

KEYWORDS: infinitely repeated games, implicit collusion, imperfect monitoring, multimarket contact, the folk theorem, low discount factor.

1. INTRODUCTION

In a real economic environment, large enterprises produce a group of products, and even single-product firms operate in many distinct geographic markets. As a result, firms may contact with each other in multiple markets. In this paper, we will explain that multimarket contact enhances the possibility of implicit collusion among the oligopolistic firms.

In an oligopolistic industry, the rival firms sometimes make the cartel agreement, without explicitly contracting, to earn the excessive profits by keeping the market price higher than the competitive price. The possibility of implicit collusion in a self-enforcing way is well-described by perfect equilibria in a model of infinitely repeated games, provided that firms are patient enough and compete each other in infinitely many periods (see Friedman (1971)).[1] It, however, is well known in the industrial organization literature that when the market demand fluctuates randomly and the rivals' choices of supply are unobservable, it is difficult for firms to detect secret price cuts by the opponents, which seriously hinders implicit collusion (see, for example, Stigler (1964)). Green and Porter (1984) modelled the long-term quantity-setting duopoly as an infinitely repeated game with imperfect monitoring, where single-product firms cannot observe the opponents' choices of supply but can observe the realized market-clearing price which is a random variable depending on their choices of supply as well as some unobservable noisy factor. In this paper, it will be shown that multimarket contact dramatically eliminates the incentive for firms to make secret price cuts.

The possibility that multimarket contact fosters implicit collusion and several related empirical evidences were first raised by Edwards (1955).

Bernheim and Whinston (1990) presented a systematic analysis in infinitely repeated games with perfect monitoring where firms can observe the rivals' choices of supply directly. They showed that when firms differ in their costs of production across markets, multimarket contact develops the spheres of influence fostering implicit collusion. Multimarket contact serves to pool the incentive constraints and relax binding incentive constraints by shifting the slack enforcement power in the collusive markets to the competitive markets. However, multimarket contact never enhances implicit collusion with perfect monitoring, when markets and firms are identical.

In this paper, we will give an alternative reason why multimarket contact fosters implicit collusion. We assume imperfect monitoring and assume that the number of distinct markets is large enough. A supergame strategy profile will be constructed, which approximately sustains the fully collusive payoff vector. According to this strategy profile, firms continue to collude in all markets as long as the number of markets in which the competitive price occurs is less than some threshold level. Once this number is more than or equal to this threshold level, they stop to collude and start to behave competitively in all markets forever from the next period.

It will be shown that this strategy profile can be a perfect equilibrium provided that the number of distinct markets is large enough: As the number of distinct markets increases, the optimal perfect equilibria converge to full collusion. Surprisingly, this permissive result holds on almost the same condition as the perfect monitoring case with respect to the discount factor. Hence, we can conclude that multimarket contact totally eliminates the obstacles of imperfect monitoring, and full collusion can be approximated by perfect equilibrium outcomes even in the low discount factor case. This permissive result needs no technological asymmetry among firms like the one in Bernheim and Whinston (1990).

This paper might be the first attempt to derive the folk theorem with low discount factor in the proper imperfect monitoring case with diffuse information. Several authors have attempted to prove the folk theorem with imperfect monitoring. All of them, however, have commonly assumed that the discount factor is close to unity. (see Radner (1986), Matsushima (1989), Fudenberg, Levine and Maskin (1994), Abreu, Milgrom and Pearce (1991), Kandori and Matsushima (1994), and others).

In order to detect firms' global deviation, the law of large numbers plays the central role: A firm hesitates to deviate at a time in many markets, because it would be almost surely detected through the statistical test mentioned above. The use of the law of large numbers, however, is not sufficient to derive the equilibrium property. A firm may still have incentive to deviate locally in a small number of markets, because it would be difficult to detect it in the imperfect monitoring case. We will present an idea how to specify the threshold level so as to eliminate this incentive to deviate locally, which is the central issue in the proof of the main theorem from the technical view-point. By choosing the threshold level in an appropriate way, a firm hesitates to deviate in a small number of markets, for fear of getting the global punishment in all distinct markets from the next period.

For simplicity of arguments, we will consider a symmetric prisoner-dilemma game only. This is essentially the same game as the one presented by Radner, Myerson and Maskin (1986), where all of perfect equilibria are bounded away from full collusion uniformly in the discount factor.

The organization of this paper is as follows. In Section 2, the symmetric prisoner-dilemma game is introduced. The strategic aspects in all distinct markets are commonly described by this same game. In Section 3, it will be shown that full collusion is approximately attainable, if the discount factor is <u>close</u> to unity and the number of distinct markets is large enough. In Section 4, we present the main theorem and give the sketch of its proof, by using two numerical examples. The complete proof will be given in the Appendix. Section 5 gives further discussions.

2. THE MODEL

2.1. PRISONER-DILEMMA GAME

A symmetric prisoner-dilemma game $G = (N, A_1, A_2, u_1, u_2)$ is introduced, where $N = \{1,2\}$, $A_1 = A_2 = \{c,d\}$, and

$$u_1(c,c) = u_2(c,c) = 1,\ u_1(d,d) = u_2(d,d) = 0,$$

$$u_1(d,c) = u_2(c,d) = 1 + M,\ u_1(c,d) = u_2(d,c) = -L,$$

$$M > 0,\ L > 0, \text{ and } 1 > M - L.$$

Let $u(a) = (u_1(a), u_2(a))$. Action profile (d,d) is the unique Nash equilibrium, and the associated payoff vector $u(d,d) = (0,0)$ is Pareto-inferior to the efficient payoff vector $u(c,c) = (1,1)$. One interpretation is that in a quantity-setting duopoly, each player i (firm i), $i = 1,2$, simultaneously chooses either a small supply $a_i = c$ (cooperation) or a large supply $a_i = d$ (defection).

We assume <u>imperfect public monitoring</u>. That is, each player i cannot observe the opponent's choice of action a_j, $j \neq i$, and players commonly observe a public random signal ω which depends on the action profile $a \in A$. Let $\Omega = \{L,H\}$ denote the set of possible ω. Players observe $\omega = L$ with probability $p(a)$ if they choose $a \in A$, where $0 \leqq p(a) \leqq 1$, $p(d,c) = p(c,d)$, and $p(c,c) < p(d,c)$. We must note that $u_i(a)$ is the expected payoff.

In a quantity-setting duopoly, public signal ω is the market-clearing price, where $\omega = L$ is the <u>low</u> (competitive) price and $\omega = H$ is the <u>high</u> (collusive) price. Because of the demand uncertainty, the market-clearing price fluctuates randomly according to the probability function $p(a)$. Hence, the realized market price gives a noisy information about whether the rival firm behaves competitively or cooperatively.

2.2. MULTIMARKET CONTACT AND REPEATED PLAY

Multimarket contact is introduced as follows. Two firms are active at a time in the same multiple duopolistic markets $h = 1,\dots,m$. Each market is modelled by the identical prisoner-dilemma game defined in the preceding subsection. In the h-th market, firm i chooses $a_i^h \varepsilon \{c,d\}$, and observes public signal $\omega^h \varepsilon \{L,H\}$.

We will introduce the infinitely repeated game with multimarket contact denoted by $G^\infty(m,\delta)$, where m is the number of distinct markets, and $\delta \varepsilon (0,1)$ is the discount factor. In each period t, firm i chooses $(a_i^1(t),\dots,a_i^m(t)) \varepsilon A_i^m$ and observes $(\omega^1(t),\dots,\omega^m(t)) \varepsilon \Omega^m$. Formally, let $S_i = \{c,d\}^m$ denote the set of firm i's choices, and $\Phi = \Omega^m$ denote the set of signal profiles. A strategy for firm i in $G^\infty(m,\delta)$ is defined by $\sigma_i : \bigcup_{t=0}^{\infty} \Phi^t \to S_i$, where $\Phi^0 = \{\phi^0\}$, ϕ^0 is the null history,

$$\phi^t = (\Phi(1),\dots,\Phi(t)) \varepsilon \Phi^t, \; \Phi(\tau) = (\omega^1(\tau),\dots,\omega^m(\tau)) \varepsilon \Phi,$$

$$\sigma_i(\phi^t) = (\sigma_i^1(\phi^t),\dots,\sigma_i^m(\phi^t)),$$

and $\sigma_i^h(\phi^t) \varepsilon A_i$ is firm i's choice of supply in period t + 1 in the h-th market given history ϕ^t up to period t. It must be noted that a firm's choice in a market depends on histories in all markets. Let $\sigma = (\sigma_1, \sigma_2)$,

$v_i(\sigma,\delta)$ denote the normalized expected payoff for firm i induced by strategy profile σ, and $v(\sigma,\delta) = (v_1(\sigma,\delta), v_2(\sigma,\delta))$.

For every ϕ^t, let $\sigma_i|_{\phi^t}$ denote the strategy for firm i in period t + 1 induced by σ_i given history ϕ^t, and let $\sigma|_{\phi^t} = (\sigma_1|_{\phi^t}, \sigma_2|_{\phi^t})$. A strategy profile σ is a perfect equilibrium in $G^{\infty}(m,\delta)$ if for every t and every ϕ^t, every i = 1,2, and every player i's strategy σ_i',

$$v_i(\sigma|_{\phi^t},\delta) \geqq v_i(\sigma_i', \sigma_j|_{\phi^t}, \delta).$$

(In a general formulation, a strategy for player i may depend on private histories of her own actions as well as public histories of signal profiles, and may be a mixed strategy also. The perfect equilibrium property in our definition is robust in the sense that there is no mixed strategies depending on private histories as well as public histories which violate the incentive constraints. Moreover, in Section 3, we will construct strategy profiles with public randomization, and use the similarly defined perfect equilibrium concept.)

2.3. BASIC RESULT: PERFECT MONITORING

Before starting the analysis of the imperfect monitoring case, it might be helpful to consider the perfect monitoring case at the outset, in which each firm directly observes the opponent's choice of action. We define a strategy for player i, so-called a trigger strategy, as follows:

$$\text{Choose } (a_i^1(1),\ldots,a_i^m(1)) = (c,\ldots,c) \text{ in period } 1.$$

For every t > 1,

choose $(a_i^1(t),\ldots,a_i^m(t)) = (c,\ldots,c)$ in period t if player i observed $((c,\ldots,c),(c,\ldots,c))$ in all previous periods,

and

choose $(a_i^1(t),\ldots,a_i^m(t)) = (d,\ldots,d)$ in period t, otherwise.

If both players conform to this trigger strategies, they obtain the efficient normalized payoff vector (m,m) in every period. Moreover, the trigger strategy profile is a perfect equilibrium, if and only if players are patient enough to satisfy

(1) $$\delta \geqq \frac{M}{M + 1}.$$

(If a firm deviates in a market, then the opponent firm, according to the trigger strategy, will choose d forever in all markets from the next period. Hence, choosing d in all markets in all periods is the best strategy among all deviating strategies, which gives this deviant firm the normalized payoff $(1 - \delta)m(1 + M)$. This value is less than or equal to m if and only if inequality (1) holds.) Since the one-shot Nash equilibrium payoff vector (0,0) is the minimax point in the prisoner-dilemma game, the repetition of (d,d) in all markets is regarded as the severest equilibrium punishment for both players. These observations say that inequality (1) is not only sufficient but also necessary for the attainability of full collusion, even though all strategies other than trigger strategies are taken into account (see, for example, Abreu (1988)).

We must note that the necessary and sufficient condition (1) is independent of the number of markets m. This corresponds to the irrelevance result presented in Bernheim and Whinston (1990, section 3), which says that in a symmetric model multimarket contact gives no influence on the possibility of implicit collusion.

3. IMPERFECT MONITORING AND LEAST UPPER BOUNDS

From now on, we will investigate the <u>imperfect</u> monitoring case. We denote by x(m) the maximum of $2\{m - zp(c,c)^m\}$ with respect to $z \geqq 0$ subject to

$$z\{p(d,c)^k p(c,c)^{m-k} - p(c,c)^m\} \geqq kM \text{ for all } k = 1,\ldots,m.$$

That is,

$$x(m) = 2\{m - \frac{Mp(c,c)}{p(d,c) - p(c,c)}\}.$$

In this section we assume that $p(c,c) > 0$, and

(2) $$x(m) \geqq m(1 + M - L), \text{ and } x(m) \geqq 0.$$

Since $\lim_{m\uparrow\infty} \frac{x(m)}{m} = 2$ and $1 > M - L$, there exists $\bar{m}$ such that for every $m \geqq \bar{m}$, inequalities (2) hold.

In the same way as Radner, Myerson and Maskin (1986), we can check that x(m) is an upper bound of all perfect equilibrium payoff vectors, in the sense that for every $\delta\varepsilon(0,1)$ and every perfect equilibrium σ in $G^\infty(m,\delta)$,

$$x(m) \geqq v_1(\sigma,\delta) + v_2(\sigma,\delta).$$

(Similarly, we can check that x(m) is an upper bound of all perfect equilibria with and without public randomization also.) Since $x(m) < 2m$, all perfect equilibria has an <u>uniformly</u> inefficient upper bound.

We will show below that x(m) is the <u>least</u> upper bound also. Fix a real number $q\varepsilon[0,1]$ arbitrarily, which will be specified later. We define a strategy profile with <u>public randomization</u> as follows:

Choose ((c,...,c),(c,...,c)) with probability 1 in period 1.

If the strategy profile continued to choose ((c,...,c),(c,...,c)) from period 1 through period t - 1 and players observed $\Phi(t-1) \neq (L,\ldots,L)$ in period t - 1, then

choose ((c,...,c),(c,...,c)) with probability 1 in period t.

If it continued to choose ((c,...,c),(c,...,c)) from period 1 through period t - 1 and players observed $\Phi(t-1) = (L,\ldots,L)$ in period t - 1, then

choose ((d,...,d),(d,...,d)) with probability q,

and

choose ((c,...,c),(c,...,c)) with probability 1 - q in period t.

If it chose ((d,...,d),(d,...,d)) in period t - 1, then

choose ((d,...,d),(d,...,d)) with probability 1 in period t.

According to this strategy profile, both firms continue to supply the small amounts in all markets as long as they observed the high price in at least one market in all previous periods. On the other hand, if the low price occured at a time in <u>all</u> markets, then, with probability q, both firms stop to behave collusively and continue to supply large amounts in all markets forever from the next period.[2,3]

According to this strategy profile, firm i obtains the normalized expected payoff

$$v_i = (1 - \delta)m + \delta\{1 - qp(c,c)^m\}v_i, \text{ that is,}$$

(3) $$v_i = \frac{(1 - \delta)m}{1 - \delta + \delta qp(c,c)^m}.$$

If a firm chooses d in k markets and chooses c in m - k markets in period 1, and conforms to this strategy profile from period 2, then it obtains the normalized expected payoff

$$(1 - \delta)(m + kM) + \delta\{1 - qp(d,c)^k p(c,c)^{m-k}\}v_i$$

$$= v_i + (1 - \delta)kM - \delta q p(c,c)^m\{(\frac{p(d,c)}{p(c,c)})^k - 1\}v_i$$

$$= v_i + \sum_{h=1}^{k}[(1 - \delta)M - \delta q p(c,c)^m\{(\frac{p(d,c)}{p(c,c)})^k - (\frac{p(d,c)}{p(c,c)})^{k-1}\}v_i]$$

$$\leqq v_i + k\{(1 - \delta)M - \delta q p(c,c)^m(\frac{p(d,c)}{p(c,c)} - 1)v_i\},$$

where the last inequality is derived from the fact that $p(c,c) < p(d,c)$. Hence, this strategy profile is a perfect equilibrium if inequality

$$(4) \qquad (1 - \delta)M - \delta q p(c,c)^m(\frac{p(d,c)}{p(c,c)} - 1)v_i \leqq 0$$

is satisfied. On the other hand, if this strategy profile is a perfect equilibrium, then the deviant's payoff for $k = 1$ must be less than or equal to v_i, that is, inequality

$$v_i + (1 - \delta)M - \delta q p(c,c)^m(\frac{p(d,c)}{p(c,c)} - 1)v_i \leqq v_i$$

must hold. Since this inequality is equivalent to inequality (4), one gets that inequality (4) is not only sufficient but also necessary for the perfect equilibrium property. Equality (3) and inequality (4) imply that

$$(5) \qquad q \geqq \frac{(1 - \delta)M}{\delta B(m)},$$

where

$$B(m) = p(c,c)^m[\frac{\{p(d,c) - p(c,c)\}m}{p(c,c)} - M]$$

$$= p(c,c)^m(\frac{p(d,c) - p(c,c)}{p(c,c)})\cdot\frac{x(m)}{2} \geqq 0.$$

(The last inequality is derived from the fact that $x(m) \geqq 0$.) There exists q in the interval [0,1] which satisfies inequality (5), if and only if players are patient enough to satisfy

$$(6) \qquad \delta \geqq \frac{M}{M + B(m)},$$

where the right hand side of (6) is in the interval [0,1], because $B(m) > 0$. Let $q \in [0,1]$ be specified so as to satisfy inequality (5) with <u>equality</u>, that is, $q = \frac{(1-\delta)M}{\delta B(m)}$. Then, one gets from equality (3) that

$$v_i = m - \frac{Mp(c,c)}{p(d,c) - p(c,c)}, \text{ i.e., } v_1 + v_2 = x(m).$$

This implies that if players are patient enough to satisfy inequality (6), then this strategy profile is a perfect equlibrium and the sum of the normalized expected payoffs induced by it is equivalent to the upper bound $x(m)$. Hence, $x(m)$ is regarded as the least upper bound.

As the number of markets m increases, the average payoff vector <u>per</u> market

$$\left(1 - \frac{Mp(c,c)}{m\{p(d,c) - p(c,c)\}}, 1 - \frac{Mp(c,c)}{m\{p(d,c) - p(c,c)\}}\right)$$

converges to the efficient payoff vector (1,1). Hence, full collusion can be approximately attained by a perfect equilibrium if and only if the number of markets m is sufficiently large and players are patient enough to satisfy inequality (6).

However, we would like to say that the positive result presented above is <u>not</u> satisfactory: First of all, inequality (6) is much more restrictive than inequality (1), the necessary and sufficient condition in the perfect monitoring case. Moreover, the right hand side of inequality (6) converges to unity as m increases, which implies that inequality (6) requires the discount factor δ to be near <u>unity</u> in the case of large m. (Since $\lim_{m\uparrow\infty} mp(c,c)^m = 0$, one gets

$$\lim_{m\uparrow\infty} B(m) = \frac{p(d,c) - p(c,c)}{p(c,c)} \left[\lim_{m\uparrow\infty} \{mp(c,c)^m - Mp(c,c)^m\}\right] = 0,$$

which implies that the right hand side of inequality (6) converges to unity as m increases.)

4. FOLK THEOREM WITH LOW DISCOUNT FACTOR

We will present the main theorem as follows:

THEOREM 2: Suppose that inequality (1) holds strictly, that is,

$$\delta > \frac{M}{M+1}.$$

Then, for every large enough m, there exists a perfect equilibrium $\sigma^{[m]}$ in $G^{\infty}(m,\delta)$ such that

$$\lim_{m\uparrow\infty} \frac{v(\sigma^{[m]},\delta)}{m} = (1,1).$$

Theorem 2 says that full collusion can be approximately attained through multimarket contact on almost the same condition as the perfect monitoring case.

The proof of Theorem 2 is constructive: Fix a positive integer $r(m) \in \{1,\ldots,m\}$ arbitrarily, where the function $r(m)$ will be called the threshold function. We define a strategy for player i $\sigma_i = \sigma_i^{[m]}$ by

$$\sigma_i(\phi^0) = (c,\ldots,c),$$

for every $t \geqq 1$,

$$\sigma_i(\phi^t) = (c,\ldots,c) \text{ if } \#\{h\colon \omega^h(\tau) = L,\ h = 1,\ldots,m\} < r(m) \text{ for all } \tau = 1,\ldots,t,$$

and

$\sigma_i(\phi^t) = (d,\ldots,d)$ if $\#\{h: \omega^h(\tau) = L,\ h = 1,\ldots,m\} \geqq r(m)$ for some $\tau = 1,\ldots,t$.

According to $\sigma = \sigma^{[m]}$, each firm continues to choose c in all markets as long as the number of markets h in which $\omega^h = L$ occurs is less than the threshold r(m). Once this number is more than or equal to r(m), both firms immediately stop to behave collusively and continue to choose d in all markets from the next period.[4]

All we have to do is to specify the threshold function r(m) so as to satisfy that the above defined $\sigma^{[m]}$ is a perfect equilibrium and $v(\sigma^{[m]},\delta)$ is approximately efficient for every large enough m. In order to avoid a long struggle with the difficulty of this specification, we will start with the following two numerical examples with moving support, which might be helpful to understand the logical core of the proof. The complete proof of Theorem 2 will be presented in the Appendix.

Consider the first example, Example I, which may be the easiest case to check the properties in Theorem 2. We assume

$$p(c,c) = 0 \text{ and } p(d,c) = \frac{1}{2}.$$

Players never observe signal L whenever they chose (c,c).

In this example, we shall specify

$$r(m) = 1.$$

Firms stop to collude immediately, once they observed the small price in one or more markets. According to $\sigma = \sigma^{[m]}$, firm i obtains the efficient payoff vector

$$v_i(\sigma,\delta) = m,$$

because signal L is never observed on the induced path.

If firm i deviates from σ_i by choosing d in k market and c in m - k markets in period 1 and conforming to σ_i from the next period, then it is punished with probability $1 - (\frac{1}{2})^k$ and obtains the expected normalized payoff

$$(1 - \delta)(m + kM) + \delta(\tfrac{1}{2})^k v_i(\sigma,\delta)$$

$$= (1 - \delta)(m + kM) + \delta m(\tfrac{1}{2})^k = m + (1 - \delta)kM - \delta m\{1 - (\tfrac{1}{2})^k\}$$

$$= m + \sum_{h=1}^{k} [(1 - \delta)M - \delta m\{(\tfrac{1}{2})^{h-1} - (\tfrac{1}{2})^h\}].$$

This value must be less than or equal to $v_i(\sigma,\delta) = m$ for every $k = 1,\ldots,m$ if σ is a perfect equilibrium. That is, it must be satisfied that for every $k = 1,\ldots,m$,

$$\sum_{h=1}^{k} [(1 - \delta)M - \delta m\{(\tfrac{1}{2})^{h-1} - (\tfrac{1}{2})^h\}] \leqq 0. \tag{7}$$

Since $(\frac{1}{2})^{h-1} - (\frac{1}{2})^h$ is decreasing with respect to h, one gets that inequality (7) holds for all $k = 1,\ldots,m$, if and only if inequality (7) holds for $k = m$, that is, if and only if

$$\sum_{h=1}^{m} [(1 - \delta)M - \delta m\{(\tfrac{1}{2})^{h-1} - (\tfrac{1}{2})^h\}] = (1 - \delta)mM - \delta m\{1 - (\tfrac{1}{2})^m\}$$

$\leqq 0$, or

$$\delta \geqq \frac{M}{M + 1 - (\frac{1}{2})^m}. \tag{8}$$

Since the right hand side of inequality (8) approaches $\frac{M}{M+1}$ as m increases, we conclude that the properties of Theorem 2 hold true in Example I.

Next, consider the second example, Example II, in which we assume

$$p(c,c) = \frac{1}{2} \text{ and } p(d,c) = 1.$$

Players can not observe signal H when they choose either (d,c) or (c,d).

Fix r(m) arbitrarily, which will be specified later. Let f(r:m,k) denote the probability that r is the number of markets in which signal L is observed, i.e., $\#\{h: \omega^h = L, h = 1,\ldots,m\} = r$, provided that a firm chooses d in k markets, chooses c in m - k markets, and the opponent chooses c in all markets. In Example II,

$$f(r:m,k) = \frac{(m-k)!}{(m-r)!(r-k)!}\cdot(\frac{1}{2})^{m-k} \text{ for } k \leqq r, \text{ and}$$

$$f(r:m,k) = 0 \text{ for } k > r.$$

According to $\sigma = \sigma^{[m]}$, firm i obtains

$$v_i(\sigma,\delta) = (1-\delta)m + \delta \sum_{r<r(m)} f(r:m,0)v_i(\delta,\sigma), \text{ that is,}$$

$$v_i(\sigma,\delta) = \frac{(1-\delta)m}{1-\delta \sum_{r<r(m)} f(r:m,0)}.$$

As the first step of the specification of r(m), we will require that

(9) $$\lim_{m\uparrow\infty} \{ \sum_{r<r(m)} f(r:m,0)\} = 1, \text{ and}$$

(10) $$\lim_{m\uparrow\infty} \frac{r(m)}{m} = p(c,c).$$

We must note that such a function r(m) exists. (Bernoulli's law of large numbers says that for every $\varepsilon > 0$ and every $\eta > 0$, there exists $\bar{m}$ such that for every $m \geqq \bar{m}$,

$$\sum_{r<m\{p(c,c)+\varepsilon\}} f(r:m,0) > \sum_{|mp(c,c)-r|<\varepsilon} f(r:m,0) > 1 - \eta.$$

Hence, there exists a function r(m) which satisfies equalities (9) and (10).)

Equality (9) says that for every large enough m, $\sigma = \sigma^{[m]}$ realizes the approximately efficient payoff vector, that is,

(11) $$\lim_{m\uparrow\infty} \frac{v_i(\sigma^{[m]},\delta)}{m} = \lim_{m\uparrow\infty} \frac{1-\delta}{1-\delta \sum_{r<r(m)} f(r:m,0)} = 1.$$

If firm i deviates from σ_i by choosing d in k markets and c in m - k markets in period 1 and conforming to σ_i from the next period, then it is punished with probability $\sum_{r\geqq r(m)} f(r:m,k)$ and obtains the expected normalized payoff

(12) $$w_i(k,m) = (1-\delta)(m + kM) + \delta \sum_{r<r(m)} f(r:m,k)v_i(\sigma,\delta).$$

Firm i has no incentive to choose d in k markets if $w_i(k,m) \leqq v_i$, that is,

(13) $$(1-\delta)(1 + \frac{kM}{m}) \leqq \{1 - \delta \sum_{r<r(m)} f(r:m,k)\}\cdot\frac{v_i(\sigma^{[m]},\delta)}{m}.$$

We must note that firms have no incentive to choose d at a time in a nonnegligible number of markets if m is large enough. That is, for every $\varepsilon >$ 0, there exists $\bar{m}$ such that $w_i(k,m) \leqq v_i(\sigma^{[m]},\delta)$ for every $m \geqq \bar{m}$ and for every k satisfying $k \geqq \varepsilon m$. (The law of large numbers says that for every ε' > 0 and every $\eta > 0$, there exists $\bar{m}$ such that for every $m \geqq \bar{m}$,

(14) $$\sum_{r\geqq m\{(1-\varepsilon)p(c,c)+\varepsilon p(d,c)-\varepsilon'\}} f(r:m,k)$$
$$\geqq \sum_{r\geqq(m-k)p(c,c)+kp(d,c)-\varepsilon' m} f(r:m,k) > 1 - \eta.$$

From equality (10) and by choosing $\varepsilon' < \varepsilon\{1 - p(c,c)\}$, one gets that for every large enough m,

$$(1-\varepsilon)p(c,c) + \varepsilon p(d,c) - \varepsilon' = p(c,c) + \varepsilon\{1 - p(c,c)\} - \varepsilon'$$

$$> \frac{r(m)}{m}.$$

Hence, the left hand side of (14) is less than or equal to $\sum_{r \geqq r(m)} f(r:m,k)$, and therefore, $\sum_{r<r(m)} f(r:m,k)$ is near zero for every large enough m. This, together with equality (11), implies that the right hand side of inequality (13) is approximated by $1 - \delta$ for every large enough m. On the other hand, the left hand side of (13) is more than or equal to $(1 - \delta)(1 - \varepsilon M)$, which is less than $1 - \delta$. Hence, we have proven that $w_i(k,m) \leqq v_i(\sigma,\delta)$ for every large enough m.)

However, it is important to note that the requirements of (9) and (10) are not sufficient to guarantee the perfect equilibrium property. Firms may still have incentive to deviate in a small number of markets. The requirements of (9) and (10) only implies that for every large enough m there exists an integer k(m) such that firms have no incentive to choose d at a time in k(m) or more markets. Surely $\frac{k(m)}{m}$ approaches zero as m increases, but for any large m, k(m) may still be more than 1. Hence, we have not yet eliminated the incentive for a firm to choose d in k(m) - 1 or less markets.

Based on these observations, we will require, in addition to (9) and (10), that for every large enough m,

(15) $$\delta m\{p(d,c) - p(c,c)\}f(r(m)-1:m-1,0) > (1 - \delta)M.$$

In Example II, inequality (15) is equivalent to

$$\delta m f(r(m)-1:m-1,0) > 2(1 - \delta)M.$$

As it will be shown in Lemma A-3 in the Appendix, there exists a threshold function r(m) which satisfies inequality (15) as well as equalities (9) and

(10). We will show below that for every large enough m, firm i has no incentive to choose d in a <u>single</u> market: By definition,

$$\sum_{r\geq r(m)} f(r:m,k) = \sum_{r\geq r(m)} f(r:m-1,k-1) + f(r(m)-1:m-1,k-1), \text{ and}$$

$$\sum_{r\geq r(m)} f(r:m,k-1) = \sum_{r\geq r(m)} f(r:m-1,k-1) + \frac{1}{2} f(r(m)-1:m-1,k-1),$$

and therefore,

$$\sum_{r\geq r(m)} f(r:m,k) - \sum_{r\geq r(m)} f(r:m,k-1) = \frac{1}{2} f(r(m)-1:m-1,k-1).$$

Hence, one gets

$$w_i(1,m) - v_i(\sigma^{[m]},\delta)$$

$$= (1 - \delta)M - \delta\{\sum_{r\geq r(m)} f(r:m,1) - \sum_{r\geq r(m)} f(r:m,0)\}v_i(\sigma^{[m]},\delta)$$

$$= (1 - \delta)M - \frac{\delta}{2} f(r(m)-1:m-1,0)v_i(\sigma^{[m]},\delta),$$

which is approximated by $(1 - \delta)M - \frac{\delta m}{2} f(r(m)-1:m-1,0)$ for every large enough m, because of equality (11). Here, $(1 - \delta)M$ is the instanteneous gain from deviation in a single market, and $\frac{\delta m}{2} f(r(m)-1:m-1,0)$ approximates the associated increase of the expected value of future penalty. Inequality (15) says that $(1 - \delta)M - \frac{\delta m}{2} f(r(m)-1:m-1,0)$ is negative, and therefore, $w_i(1,m) < v_i(\sigma^{[m]},\delta)$, i.e., firms have no incentive to choose d in a single market for every large enough m.

We can also check that if a firm i has incentive to deviate from $\sigma_i^{[m]}$, it is beneficial for it to choose d either in a single market or in all markets: As it will be shown in Lemma A-2 in the Appendix, for every m and every $r\varepsilon\{0,\ldots,m\}$, $f(r:m,k)$ is single-peaked with respect to $k\varepsilon\{0,\ldots,m\}$.

Hence, $f(r(m)-1:m-1,k-1)$ is single-peaked with respect to $k \in \{1,...,m\}$, and therefore, there exists $k^* \in \{1,...,m\}$ such that $f(r(m)-1:m-1,k-1)$ is nondecreasing with respect to k in $\{1,...,k^*\}$, and is nonincreasing with respect to k in $\{k^*,...,m\}$. We must note

$$w_i(k,m) - w_i(k-1,m)$$

$$= (1 - \delta)M - \delta\{ \sum_{r \geqq r(m)} f(r:m,k) - \sum_{r \geqq r(m)} f(r:m,k-1)\}v_i(\sigma^{[m]},\delta)$$

$$= (1 - \delta)M - \frac{\delta}{2} f(r(m)-1:m-1,k-1)v_i(\sigma^{[m]},\delta),$$

which is nonincreasing with respect to k in $\{1,...,k^*\}$, and is nondecreasing with respect to k in $\{k^*,...,m\}$. This implies that all we have to check is whether a firm has incentive to choose d either in a single market or in all markets. From the above arguments, we have proven Theorem 2 in the case of Example II.

5. FURTHER DISCUSSIONS

Throughout this paper we have assumed that all firms operate in the same multiple markets. This assumption is irrelevant to our arguments: In the same way as Section 4, we can also check, in the case of three or more firms operating in their respective multiple markets, that whenever each firm can observe the realized prices in all markets including the ones in which it does not operate, then the similar folk theorem properties still hold, even though two firms may contact with each other in a single market only.

We have also assumed that there exists no macro random shock which commonly influences all markets. This assumption is crucial for the proof of the main theorem. I think that it would be quite meaningful to take the existence of macro shocks into account, from the view-point of applicability: In a real economic environment, several distinct markets are actually influenced by common random factors and these market demands fluctuate in a correlated way. This consideration is beyond the purpose of this paper. See, however, my companion papers (1994, 1995), in which several related problems have been intensively studied.

FOOTNOTES

1 For readers unfamiliar with the study of repeated games, see Pearce (1992).

2 At the expense of irrelevant difficulty, we can derive the similar result by constructing a pure strategy equilibrium with finite-period penalty phases.

3 The readers familiar with the literature of repeated games may find that our construction in this section is similar to the one by Abreu, Milgrom and Pearce (1991) presented in a different context.

4 In the construction of review strategies in repeated partnerships, Radner (1986) organized similar statistical tests through multiple successive periods. Radner assumed no discounting and used the limit-of-average criterion. These assumptions drastically simplify the incentive aspects of the Folk theorem, by avoiding the troublesome restriction of deviation in a small number of periods. For the criticism of the limit-of-average criterion, see Pearce (1992).

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APPENDIX: PROOF OF THEOREM 2

Similarly to Example II in Section 4, let $f(r:m,k)$ denote the probability that r is the number of markets in which signal L is observed, i.e., $\#\{h: \omega^h = L, h = 1,\dots,m\} = r$, provided that a firm chooses d in k markets, chooses c in m - k markets, and the opponent chooses c in all markets. We must note

$$f(r:m,k) = \sum_{h=\max[0,r-k]}^{\min[r,m-k]} D(h,r,m,k),$$

where

$$D(h,r,m,k) = \frac{k!(m-k)!}{h!(m-k-h)!(r-h)!(k-r+h)!}$$

$$\cdot p(c,c)^h\{1 - p(c,c)\}^{m-k-h}p(d,c)^{r-h}\{1 - p(d,c)\}^{k-r+h}$$

is the product of probabilities that h is the number of markets in which the deviant firm chooses c and observes signal L and that r - h is the number of markets in which the deviant firm chooses d and observes signal L. We must note that

(A1) $$f(r:m,k) = p(d,c)f(r-1:m-1,k-1) + \{1 - p(d,c)\}f(r:m-1,k-1),$$

and

(A2) $$f(r:m,k-1) = p(c,c)f(r-1:m-1,k-1) + \{1 - p(c,c)\}f(r:m-1,k-1).$$

Before starting the proof of Theorem 2, it might be helpful to present the following three lemmata.

LEMMA A-1: $f(r:m,k)$ _is single-peaked with respect to_ r, _that is, there exists_ $r^* = r^*(m,k)$ _such that_

$$f(r:m,k) \geqq f(r-1:m,k) \text{ for } r \leqq r^*, \text{ and}$$

$$f(r:m,k) \geqq f(r+1:m,k) \text{ for } r \geqq r^*,$$

where $r^*(m,k)$ is nondecreasing with respect to k, that is,

$$r^*(m,k) \geqq r^*(m,k-1) \text{ for all } k \geqq 1.$$

PROOF: Let $k = 0$. Then, by definition,

$$f(r:m,0) = \frac{m!}{r!(m-r)!} \cdot p(c,c)^r \{1 - p(c,c)\}^{m-r},$$

and therefore,

$$\frac{f(r+1:m,0)}{f(r:m,0)} = \frac{(m - r)p(c,c)}{(r + 1)\{1 - p(c,c)\}},$$

which is less than or equal to 1 if and only if $r \leqq (m + 1)p(c,c) - 1$. Hence, $f(r:m,0)$ is single-peaked with respect to r.

Let $k \geqq 1$. Suppose that $f(r:m-1,k-1)$ is single-peaked with respect to r. For every $r \varepsilon \{1,\ldots,m-1\}$, if $1 \leqq r \leqq r^*(m-1,k-1)$, then, by using equality (A1),

$$\begin{aligned} f(r:m,k) &= p(d,c)f(r-1:m-1,k-1) + \{1 - p(d,c)\}f(r:m-1,k-1) \\ &\geqq p(d,c)f(r-2:m-1,k-1) + \{1 - p(d,c)\}f(r-1:m-1,k-1) \\ &= f(r-1:m,k), \end{aligned}$$

where $f(-1:m-1,k-1) = 0$. If $r^*(m-1,k-1) + 1 \leqq r \leqq m-1$, then, by using equality (A1),

$$\begin{aligned} f(r:m,k) &= p(d,c)f(r-1:m-1,k-1) + \{1 - p(d,c)\}f(r:m-1,k-1) \\ &\geqq p(d,c)f(r:m-1,k-1) + \{1 - p(d,c)\}f(r+1:m-1,k-1) \\ &= f(r+1:m,k), \end{aligned}$$

where $f(m:m-1,k-1) = 0$. Hence, $f(r:m,k)$ is single-peaked with respect to r, and therefore, the first part of this lemma has been proven.

Consider the second part. Suppose that $f(r+1:m,k-1) \geqq f(r:m,k-1)$. By using equality (A2)

$$f(r+1:m,k-1) - f(r:m,k-1)$$
$$= p(c,c)\{f(r:m-1,k-1) - f(r-1:m-1,k-1)\}$$
$$+ \{1 - p(c,c)\}\{f(r+1:m-1,k-1) - f(r:m-1,k-1)\}$$
$$\geqq 0.$$

This inequality, together with the single-peakedness with respect to r, implies that

$$f(r:m-1,k-1) \geqq f(r-1:m-1,k-1)$$

must hold. Suppose that $f(r+1:m-1,k-1) \geqq f(r:m-1,k-1)$. Then, it is obvious that $f(r+1:m,k) \geqq f(r:m,k)$. (By using equality (A1),

$$f(r+1:m,k) - f(r:m,k)$$
$$= p(d,c)\{f(r:m-1,k-1) - f(r-1:m-1,k-1)\}$$
$$+ \{1 - p(d,c)\}\{f(r+1:m-1,k-1) - f(r:m-1,k-1)\}$$
$$\geqq 0.)$$

Next, suppose that $f(r+1:m-1,k-1) < f(r:m-1,k-1)$. Then, obviously,

$$f(r:m-1,k-1) - f(r-1:m-1,k-1) \geqq 0 > f(r+1:m-1,k-1) - f(r:m-1,k-1).$$

This, together with equalities (A1), (A2), and inequalities $p(c,c) < p(d,c)$, implies

$$f(r+1:m,k) - f(r:m,k)$$
$$= p(d,c)\{f(r:m-1,k-1) - f(r-1:m-1,k-1)\}$$
$$+ \{1 - p(d,c)\}\{f(r+1:m-1,k-1) - f(r:m-1,k-1)\}$$
$$> p(c,c)\{f(r:m-1,k-1) - f(r-1:m-1,k-1)\}$$
$$+ \{1 - p(c,c)\}\{f(r+1:m-1,k-1) - f(r:m-1,k-1)\}$$
$$= f(r+1:m,k-1) - f(r:m,k-1)$$
$$\geqq 0.$$

Hence, we have proven that if $f(r+1:m,k-1) \geqq f(r:m,k-1)$, then $f(r+1:m,k) \geqq f(r:m,k)$. This implies that $r^*(m,k) \geqq r^*(m,k-1)$. **Q.E.D.**

LEMMA A-2: $f(r{:}m,k)$ <u>is single-peaked with respect to</u> k, <u>that is, there exists</u> $k^* = k^*(r,m)$ <u>such that</u>

$$f(r,m,k) \geqq f(r,m,k-1) \text{ for } k \leqq k^*, \text{ and}$$

$$f(r,m,k) \leqq f(r,m,k-1) \text{ for } k > k^*.$$

PROOF: Fix (r,m,k) arbitrarily. Suppose that $0 \leqq r \leqq r^*(m-1,k-1)$. Lemma A-1 says that $r^*(m-1,k) \geqq r^*(m-1,k-1)$, and therefore,

$$0 \leqq r \leqq r^*(m-1,k), \text{ or, } f(r-1{:}m-1,k) \leqq f(r{:}m-1,k).$$

By using equalities (A1), (A2), and inequalities $p(c,c) < p(d,c)$,

$$f(r{:}m,k) - f(r{:}m,k+1)$$
$$= \{p(d,c) - p(c,c)\}\{f(r{:}m-1,k) - f(r-1{:}m-1,k)\}$$
$$\geqq 0.$$

Next, suppose that $r^*(m-1,k-1) < r \leqq m$. Hence,

$$f(r-1{:}m-1,k-1) \geqq f(r{:}m-1,k-1).$$

By using equalities (A1), (A2), and inequalities $p(c,c) < p(d,c)$,

$$f(r{:}m,k-1) - f(r{:}m,k)$$
$$= \{p(d,c) - p(c,c)\}\{f(r{:}m-1,k-1) - f(r-1{:}m-1,k-1)\}$$
$$\leqq 0.$$

Hence, we have shown that for every (r,m,k),

$$\text{either } f(r{:}m,k) \geqq f(r{:}m,k-1) \text{ or } f(r{:}m,k) \geqq f(r{:}m,k+1).$$

This implies the single-peakedness with respect to k. **Q.E.D.**

LEMMA A-3: <u>There exists a threshold function</u> $r(m)$ <u>which satisfies equalities</u> (9) <u>and</u> (10) <u>and inequalities</u> (15), <u>that is,</u>

$$\lim_{m\uparrow\infty} \{ \sum_{r<r(m)} f(r{:}m,0)\} = 1,$$

$$\lim_{m\uparrow\infty} \frac{r(m)}{m} = p(c,c),$$

<u>and for every large enough</u> m,

$$\delta m\{p(d,c) - p(c,c)\}f(r(m)-1:m-1,0) > (1 - \delta)M.$$

PROOF: The law of large numbers says that for every $\varepsilon > 0$ and every $\eta > 0$, there exists $\bar{m}$ such that for every $m \geqq \bar{m}$,

$$\sum_{|mp(c,c)-r|<m\varepsilon} f(r:m,0) > 1 - \eta.$$

Hence, there exists a function $\varepsilon(m)$ such that for every m, $\varepsilon(m) > 0$, and

$$\lim_{m\uparrow\infty} \varepsilon(m) = 0, \text{ and}$$

$$\lim_{m\uparrow\infty} \{\sum_{|mp(c,c)-r|<m\varepsilon(m)} f(r:m,0)\} = 1.$$

For every large enough m, we will specify r(m) as the maximum among integers r which satisfy inequalities (15) and

$$r \leqq m\{p(c,c) + \varepsilon(m)\}.$$

(Such a function r(m) exists: If not, then, for every large enough m and every r satisfying $r \leqq m\{p(c,c) + \varepsilon(m)\}$,

$$f(r-1:m-1,0) \leqq \frac{(1 - \delta)M}{\delta m\{p(d,c) - p(c,c)\}}, \text{ that is,}$$

$$f(r:m,0) = \frac{m!}{r!(m-r)!}\cdot p(c,c)^r\{1 - p(c,c)\}^{m-r}$$

$$= \frac{mp(c,c)}{r}\cdot f(r-1:m-1,0)$$

$$\leqq \frac{(1 - \delta)Mp(c,c)}{\delta r\{p(d,c) - p(c,c)\}}.$$

Hence,

$$\sum_{|mp(c,c)-r|<m\varepsilon(m)} f(r:m,0)$$

$$\leqq \sum_{m\{p(c,c)-\varepsilon(m)\}<r<m\{p(c,c)+\varepsilon(m)\}} \frac{(1-\delta)Mp(c,c)}{\delta r\{p(d,c)-p(c,c)\}}$$

$$\leqq \frac{2\varepsilon(m)(1-\delta)Mp(c,c)}{\delta\{p(c,c)-\varepsilon(m)\}\{p(d,c)-p(c,c)\}},$$

the left hand side of which approaches to 0 as m increases. But, this is a contradiction.)

We will show below that the specified r(m) satisfies equalities (9) and (10) also. We must note

$$\lim_{m\uparrow\infty} \{\sum_{r<r(m)} f(r:m:0)\}$$

$$\geqq \lim_{m\uparrow\infty} [\sum_{|mp(c,c)-r|<m\varepsilon(m)} f(r:m,0) - \sum_{r(m)\leqq r<m\{p(c,c)+\varepsilon(m)\}} f(r:m,0)]$$

$$\geqq \lim_{m\uparrow\infty} [\sum_{|mp(c,c)-r|<m\varepsilon(m)} f(r:m,0) - \frac{2\varepsilon(m)m(1-\delta)Mp(c,c)}{\delta r(m)\{p(d,c)-p(c,c)\}}]$$

$$= 1 - 0 = 1,$$

which implies equality (9). Suppose that equality (10) does not hold. Since $r(m) \leqq m\{p(c,c)+\varepsilon(m)\}$, there must exist an increasing subsequence $(m_s)_{s=1}^{\infty}$ and $\eta > 0$ such that

$$r(m_s) \leqq m_s\{p(c,c)-\eta\} \text{ for all s.}$$

Notice

$$\sum_{r<r(m_s)} f(r:m_s,0) \leqq \sum_{r<m_s\{p(c,c)-\eta\}} f(r:m_s,0)$$

$$= 1 - \sum_{r\geqq m_s\{p(c,c)-\eta\}} f(r:m_s,0),$$

which approaches to zero as s increases, because of the law of large numbers. But, this is a contradiction of equality (9). Hence equality (10) holds.

Q.E.D.

We will present below the proof of Theorem 2, which is essentially the same as that in the case of Example II in Section 4. From Lemma A-3, we can choose r(m) which satisfies equalities (9), (10), and inequalities (15). According to $\sigma = \sigma^{[m]}$, firm i gets

$$v_i(\sigma,\delta) = \frac{(1-\delta)m}{1 - \delta \sum_{r<r(m)} f(r:m,0)}.$$

Equality (9) says that for every large enough m, $\sigma = \sigma^{[m]}$ realizes the approximately efficient payoff vector, that is, equality (11) holds.

If firm i deviates from σ_i by choosing d in k markets and c in m - k markets in period 1 and conforming to σ_i from the next period, then it is punished with probability $\sum_{r \geqq r(m)} f(r:m,k)$ and obtains the expected normalized payoff $w_i(k,m)$, which is defined by equality (12). Firm i has no incentive to choose d in k markets if $w_i(k,m) \leqq v_i$, i.e., if inequality (13) holds.

First, we can check that for every large enough m, firms have no incentive to choose d in <u>all</u> markets, i.e.,

$$w_i(m,m) \leqq v_i(\sigma^{[m]},\delta).$$

(The law of large numbers says that

$$\lim_{m\uparrow\infty} \{ \sum_{r \geqq m\{p(d,c)-\varepsilon\}} f(r:m,m)\} = 1 \text{ for every } \varepsilon > 0.$$

By choosing $\varepsilon < p(d,c) - \frac{r(m)}{m}$, one gets

$$\lim_{m\uparrow\infty} \{ \sum_{r<r(m)} f(r:m,m)\} \leqq 1 - \lim_{m\uparrow\infty} \{ \sum_{r \geqq m\{p(d,c)-\varepsilon\}} f(r:m,m)\} = 0.$$

This, together with equalities (11) and (13), implies that

$$\lim_{m\uparrow\infty} w_i(m,m) = (1-\delta)m(1+M) + \lim_{m\uparrow\infty} \{ \sum_{r<r(m)} f(r:m,m) v_i(\sigma^{\{m]},\delta)\}$$

$$= (1 - \delta)m(1 + M),$$

which is less than $v_i(\sigma^{[m]},\delta)$ for every large enough m, because of strict inequality (4) and equality (11).)

Moreover, we can check that for every large enough m, firm i has no incentive to choose d in a <u>single</u> market. (By definition,

$$\sum_{r \geqq r(m)} f(r:m,k) = \sum_{r \geqq r(m)} f(r:m-1,k-1) + p(d,c)f(r(m)-1:m-1,k-1),$$

and

$$\sum_{r \geqq r(m)} f(r:m,k-1) = \sum_{r \geqq r(m)} f(r:m-1,k-1) + p(c,c)f(r(m)-1:m-1,k-1),$$

and therefore,

$$\sum_{r \geqq r(m)} f(r:m,k) - \sum_{r \geqq r(m)} f(r:m,k-1)$$

$$= \{p(d,c) - p(c,c)\}f(r(m)-1:m-1,k-1).$$

Hence,

$$w_i(1,m) - v_i(\sigma^{[m]},\delta)$$

$$= (1 - \delta)M - \delta\{\sum_{r \geqq r(m)} f(r:m,1) - \sum_{r \geqq r(m)} f(r:m,0)\}v_i(\sigma^{[m]},\delta)$$

$$= (1 - \delta)M - \delta\{p(d,c) - p(c,c)\}f(r(m)-1:m-1,0)v_i(\sigma^{[m]},\delta),$$

which is approximated by $(1 - \delta)M - \delta m\{p(d,c) - p(c,c)\}f(r(m)-1:m-1,0)$ for every large enough m, because of equality (11). Inequalities (15) say that this value is negative, and therefore, $w_i(1,m) < v_i(\sigma^{[m]},\delta)$, i.e., firms have no incentive to choose d in a single market for every large enough m.)

Finally, We can check that if a firm i has incentive to deviate from $\sigma_i^{[m]}$, it is beneficial for it to choose d either in a single market or in all markets. (By definition,

$$w_i(k,m) - w_i(k-1,m)$$

$$= (1 - \delta)M - \delta\{\sum_{r \geqq r(m)} f(r:m,k) - \sum_{r \geqq r(m)} f(r:m,k-1)\}v_i(\sigma^{[m]},\delta)$$

$$= (1 - \delta)M - \delta\{p(d,c) - p(c,c)\}f(r(m)-1:m-1,k-1)v_i(\sigma^{[m]},\delta).$$

From Lemma A-2 and inequality $p(c,c) < p(d,c)$, this value is nonincreasing with respect to k in $\{1,\ldots,k^*\}$, and is nondecreasing with respect to k in $\{k^*,\ldots,m\}$. This implies that all we have to check is whether a firm has incentive to choose d either in a single market or in all markets.)

From the above arguments, we have proven Theorem 2.