95-F-6

International Public Goods and Contribution Productivity Differentials

by

Toshihiro Ihori
University of Tokyo

March 1995

International Public Goods and Contribution Productivity Differentials*

Toshihiro Ihori

Department of Economics

University of Tokyo

7-3-1 Hongo, Bunkyo-ku

Tokyo 113, JAPAN

September 1994

Revised January 1995

Abstract

This paper investigates the impacts of productivity differentials in contributing to the public good on national welfare in the context of non-cooperative voluntary provision of a pure public good. A country with high productivity does not necessarily enjoy high welfare. Subsidizing the other country in the contribution game may lead to a strong paradoxical result, while the lump sum transfer will lead to a weak paradoxical result. We also explore the relation between the price of the public good and the size of an open club in a multi-club model.

Keywords: international public goods, contribution productivity, national welfare

JEL classification numbers: H41, F13

Phone: 03-3812-2111, **Fax**: 03-3818-7082

Journal of Public Economics, forthcoming

1. Introduction

This paper investigates the impact of productivity differentials in contributing to the public good on national welfare in the context of non-cooperative voluntary provision of a pure public good. Examples of such public goods in an international context are cross border pollution, other global environmental issues, or national defense. The marginal cost of providing the public good depends on technology, which may well be different between countries.[1]

Most of models of public goods are highly abstract and usually assume a very simple cost structure for providing the public good. In fact the public good is a set of institutional arrangements, some of which involve procurement practices. If the government must impose an austerity program, one potential source of savings is the procurement process itself. Thus, it is important to analyze the economic effect of an institutional reform which can alter the cost of providing the public good.

Since Warr (1983)'s provocative paper, a considerable body of literature has grown up on the neutrality result that real equilibrium is unaffected by a redistribution of income when public goods are privately provided. It is, however, now well known that fiscal transfer policy will not be completely neutral in some important cases. First of all, as Bergstrom et al. (1986) showed, the neutrality result will not apply if some agents are making no contribution. Second, as Andreoni (1989) showed, if altruism is not 'pure' in the sense that agents get some benefits from their gift per se, the neutrality result will not hold. Third, Ihori (1992) showed that in the case of an impure public good the consequences of transfers on utility are paradoxical under certain conditions. Finally, the recent several papers (Buchholz and Konrad (1993) and Batina and Dion (1994)) incorporate productivity differentials into the model to have some interesting results. Namely, it is shown that a low productivity country will gain by giving a transfer.

We will explore the impact of changes in contribution productivity more fully in this paper. Namely, we investigate the welfare effects of changes in productivity differentials themselves on national welfare. It is shown that a country with high productivity (i.e., low cost of producing public goods) does not necessarily enjoy high welfare. A country with low productivity (i.e., high cost of producing public goods) can enjoy high welfare, which is a seemingly paradoxical result. A country may not have a strong incentive to reduce the marginal cost of providing the public good as a decrease in the cost may reduce the welfare of the country in the two-country framework. Subsidizing the other country in the contribution game may lead to a strong paradoxical result, while the lump sum transfer will lead to a weak paradoxical result. We also explore the relation between an increase in the price of the public good and the size of an open club in a multi-club model.

The paper is organized as follows. Section 2 outlines the model. Section 3 considers the non-cooperative Nash equilibrium of voluntary contributions to a pure public good when transfers are made. Section 4 considers the impacts of changes in contribution productivity on welfare. Section 5 extends the model into multi-club situations. Finally, section 6 concludes the paper.

2. Model

Assume that there are two countries in the world, country 1 and country 2. Country i's utility is given by

$$U^i = U^i(c_i, G) \tag{1}$$

where U^i is welfare of country i, c_i is private consumption of country i, and G is the benefit of a pure public good for country i. (i = 1,2). We assume that both c_i and G are normal goods.

G is given by

$$G = g_i + \sum_{j \neq i} g_j \tag{2}$$

where g_i is the international pure public good provided by country i.

Country i's budget constraint is given by

$$c_i + p_i g_i = Y_i \tag{3}$$

where Y_i is exogenously given national income of country i and p_i is the relative price of public goods in terms of private consumption in country i. Low (high) p_i means high (low) productivity of providing the public good.

Substituting (2) into (3), we have

$$c_i + p_i G = Y_i + p_i \sum_{j \neq i} g_j \tag{3)'$$

We assume that each government determines its public good, treating the other's public spending, prices p_i and Y_i as given. As in the standard model of voluntary provision of a pure public good, we will exclude binding contracts or cooperative behavior between the two agents (countries) and will explore the outcome of non-cooperative Nash behavior.

In this Cournot-Nash model, define the expenditure function:

Minimize $E^i = c_i + p_i G$ subject to $U^i = U$.

Then, the following equation will determine U^i as a function of real income, $Y_i + p_i \sum_{j \neq i} g_j$, which contains actual income and the externalities from the other country's provision of the public good.

$$E^i(U^i, p_i) = Y_i + p_i \sum_{j \neq i} g_j \tag{4}$$

By a variant of Shephard's Lemma we know

$$G = G^i(U^i, p_i) \tag{5}$$

where G^i ($\equiv \partial E^i / \partial p_i$) is the compensated demand function for the public good of country i.

By definition we have

$$E^j = Y_j + p_j(G - g_j) \tag{6}$$

From (4)(5) and (6) the two-country model may be summarized by the following three equations.

$$p_2 E^1(U^1, p_1) + p_1 E^2(U^2, p_2) = p_2 Y_1 + p_1 Y_2 + p_1 p_2 G \tag{7}$$

$$G = G^1(U^1, p_1) = G^2(U^2, p_2) \tag{8}$$

(7) comes from (6). Namely, multiply (6) for E^1 by p_2, multiply for E^2 by p_1 and add. Then we will get (7). (8) comes from (5). These three equations determine U^1 and U^2 (and G) as a function of Y_1, Y_2, p_1, and p_2. We assume the existence of a Nash equilibrium with $g_i > 0$.[2]

3. International Transfers

If $p_1 = p_2$, from (7) and (8) it is easy to see that the Nash solution depends on total income in the world, $Y_1 + Y_2$ and is independent of the distribution of Y_i. We have the well known neutrality result of transferring income among countries. Namely, when a single pure public good is provided at positive levels by countries, its provision and national welfare are unaffected by a redistribution of income. This holds regardless of differences in countries' preferences and despite differences in exogenous parameters Y_i. See Shibata (1971) and Warr (1983).[3]

If p_1 is not equal to p_2, we do not have the neutrality result even in the case of pure public goods. From (7) and (8), we have

$$\begin{bmatrix} p_2 c_U^1, & p_1 E_U^2 \\ G_U^1, & -G_U^2 \end{bmatrix} \begin{bmatrix} dU^1 \\ dU^2 \end{bmatrix} = -\begin{bmatrix} p_1 - p_2 \\ 0 \end{bmatrix} dY_1 \qquad (9)$$

where $dY_1 = -dY_2$ is used.

$E_U = \partial E / \partial U > 0, G_U = \partial G / \partial U > 0,\ E_U^i = p_i G_U^i + c_U^i.\ c_U^i \equiv \partial c^i / \partial U > 0.$ c^i is the compensated demand function for private consumption in country i. From (9) we have

$$\frac{dU^1}{dY_1} = \frac{G_U^2 (p_1 - p_2)}{\Delta} \qquad (10)$$

where $\Delta = -p_2 c_U^1 G_U^2 - p_1 E_U^2 G_U^1 < 0$.

Hence, if $p_1 > p_2$, (10) is negative; a transfer from country 2 to country 1 will always hurt country 1 and vice versa. We have the (weak) transfer paradox even in the case of two agents. From (9) we also have

$$\frac{dU^2}{dY_1} = \frac{(p_1 - p_2) G_U^1}{\Delta} \qquad (11)$$

If $p_1 > p_2$, a transfer from country 2 to country 1 will also hurt country 2 and vice versa.

These two equations (10) and (11) show that comparative statics results depend on the productivity differential p_1-p_2 and the marginal propensity to consume the public good G_U / E_U. Intuition is as follows. Since country 1 is less efficient to supply the public good, a transfer from country 2 to country 1 will reduce the total level of the public good, which hurts both countries. From (7) it is easy to see that both U^1 and U^2 increase with total income, $Y = p_2 Y_1 + p_1 Y_2$. A transfer from country 2 to country 1 reduces total income if $p_1 > p_2$. Thus, the effect may be called the total income effect.

Buchholz and Konrad (1993) and Batina and Dion (1994) showed that if countries differ in their contribution productivity, the less productive country gains by making unconditional transfers to the more productive country. However, they did not analyze the welfare effect of changes in productivity differentials. We are ready to investigate this point in the following sections.

4. Changes in Productivity Differentials

4.1 an increase in p_1

Let us investigate the welfare effect of changes in productivity of providing the public good, p_i. From (7) and (8), we also have

$$\begin{bmatrix} p_2 c_U^1, & p_1 E_U^2 \\ G_U^1, & -G_U^2 \end{bmatrix} \begin{bmatrix} dU^1 \\ dU^2 \end{bmatrix} = \begin{bmatrix} -E^2 + Y_2 + p_1 p_2 G_p^1 \\ -G_p^1 \end{bmatrix} dp_1 \tag{12}$$

Hence, we have

$$\frac{dU^1}{dp_1} = \frac{p_2 g_1 G_U^2 + G_p^1 p_1 c_U^2}{\Delta} \tag{13}$$

$$\frac{dU^2}{dp_1} = \frac{-p_2 E_U^1 G_p^1 + g_1 p_2 G_U^1}{\Delta} \tag{14}$$

Since $\Delta < 0$, $E_U > 0$, $c_U > 0$, $G_p < 0$, $G_U > 0$, and $-E^i + Y_i = -p_i g_j < 0$ $(i \neq j)$, (14) is always negative. An increase in the price of the public good in country 1 will hurt country 2. From (4) the sign of (14) is negative if and only if the spillover effect of

p_1 on g_1 is negative; i.e., $dg_1/dp_1 < 0$. An increase in p_1 will reduce real income of country 1 and hence reduce g_1. $p_2 g_1 G_U^1$ represents this income effect. An increase in p_1 will raise the price of G and hence reduce g_1. $-p_2 E_U^1 G_p^1$ represents this substitution effect. Since both the income effect and substitution effect reduce g_1, (14) is always negative. This overall price effect induces a decrease in g_1 and a reduction in the effective income of country 2; it harms country 2.

On the other hand, the sign of (13) is ambiguous. From (4) the sign of (13) is positive if and only if $-g_1+p_1 dg_2/dp_1 > 0$. An increase in p_1 will directly reduce income of country 1. $-g_1$ represents this direct income effect. An increase in p_1 will induce an increase in g_2. $p_1 \frac{dg_2}{dp_1} = -p_1 \frac{G_p^1 c_U^2}{G_U^2 p_2}$ represents this spillover price effect. It is possible to have $dU^1/dp_1 > 0$ if the spillover price effect is large, which is a seemingly paradoxical result.

In a model without public goods ($G = g_i$) an increase in the price level (p_i) or a decrease in the productivity of providing the good always reduces welfare. $dU^1/dp_1 > 0$ can occur only if each country provides the public goods (not the private goods). Intuition is as follows. An increase in p_1 will reduce g_1, which will hurt country 2. Then country 2 will react to increase her supply of the public good, g_2, which is beneficial to country 1. If this positive spillover price effect from country 2 is greater than the direct negative income effect of an increase in p_1, country 1 will gain.

The numerator of (13) A is reduced to

$$A = G_U^2 p_2 g_1 + G_p^1 c_U^2 p_1 = G(\frac{1-\alpha^2}{2} - \varepsilon^1 \alpha^2)/E_U^2$$

where $\alpha^2 = c_U^2/E_U^2, \varepsilon^1 = -G_p^1 p_1/G$. α^i is the marginal propensity to consume the private good and ε^i is the elasticity of the public good with respect to the price in country i.[4] It is likely to have the paradoxical result when the marginal propensity to consume the private good is high and the price elasticity of the public good is high. If $\varepsilon = 1$, then $\alpha > 1/3$ is sufficient to have A < 0 and hence (13) > 0. In the case of Cobb-Douglas utility function such as $U = c^a G^{1-a}$ we have $\alpha = \varepsilon = a$. Thus,

$a > 1/2$ is necessary and sufficient to have (13) > 0. This suggests that the paradoxical result is not regarded as an unusual case.[5]

Most models of public goods are highly abstract and usually assume a very simple cost structure for providing the public good. Embedded in the cost structure of providing, the public good is in fact a set of institutional arrangements, some of which involve procurement practices. If the government must impose an austerity program, one potential source of savings is the procurement process itself. Indeed, it may actually take a severe crisis to convince government bureaucrats and elected officials of the need for reforming the procurement process. An institutional reform which can lower the marginal cost of providing the public good is normally regarded as a part of the desirable austerity program.

In the closed model a decrease in p (an increase in the productivity of providing the public good) is normally beneficial to its own country. We have shown, however, that this result is not necessarily valid in the two country open model with international public goods. Our analysis suggests that a country may not have a strong incentive to reduce the marginal cost of providing the public good as a decrease in p may reduce (not raise) the welfare of the country in the two-country model.

If the preferences are the same between countries, then $G^i(\,)$ is the same. From (8) we know that $U^1 > U^2$ if and only if $p_1 > p_2$. Every country must have the same demand for G in the equilibrium. Since G_p is negative, a country with high p needs high welfare (i.e. high effective income) to demand the same level of G. Intuition is as follows. Suppose initially $p_1 = p_2$ and $U^1 = U^2$. An increase in p_1 reduces g_1, which will hurt country 2. Country 2 will react to raise g_2, which will benefit country 1. A country with high productivity does not necessarily enjoy high welfare. A country with high p (low productivity) can enjoy high welfare, which is a seemingly paradoxical result.[6]

4.2 an increase in p_1 when n > 2

We now consider the case where the number of countries n is more than 2. Denote by p_1 the productivity of providing the public good in country 1 and p_2 the productivity of providing the public good in the rest of countries from 2 to n. For simplicity assume that the rest of countries is homogenous and hence represented by country 2. Then the system will be summarized as

$$p_2E^1(p_1,U^1)+(n-1)p_1E^2(p_2,U^2)=p_2Y_1+(n-1)p_1Y_2+(n-1)p_1p_2G \tag{15}$$

$$G=G^1(p_1,U^1)=G^2(p_2,U^2) \tag{16}$$

Hence, we have

$$\frac{dU^1}{dp_1}=\frac{[p_2g_1G_U^2+G_p^1c_U^2p_1(n-1)]}{\Delta}=\frac{G}{\Delta}[\frac{1-\alpha^2}{n}-\varepsilon^1\alpha^2(n-1)]/E_U^2 \tag{17}$$

(17) implies that the (weak) paradoxical result of $dU^1/dp_1>0$ is more likely to occur if the number of countries n is large. The positive spillover effect of the rest of countries on country 1 becomes large if the number of countries is large. In other words, in the multi-country model where the number of countries is greater than 2, an increase in p_1 is more likely to raise welfare of country 1 since the positive spillover effect from the rest of world increases with the number of countries.

4.3 subsidizing the other country

One country is able to commit to a contribution to the public good before the other country makes its choice. The contribution by the first country affects the benefits that the second country receives from its contribution. In this subsection, following Varian (1994), we examine public goods games in which the agents can influence the cost to other countries of their contributions. In particular, we examine what will happen if one country has the opportunity to subsidize the other country's contributions.

Suppose that country 1 offers to subsidize country 2's contributions at rate s_2. p_2 and Y_1 will be rewritten as

$$p_2 = \bar{p}_2(1-s_2)$$

$$Y_1 = \bar{Y}_1 - s_2 g_2 \bar{p}_2$$

where $\bar{p}_2, \bar{Y}_1$ are before-subsidy price and income respectively. Now p_2, Y_1 are after-subsidy price and income respectively. Hence, we have

$$dp_2 = -\bar{p}_2\, ds_2$$

$$dY_1 = -\bar{p}_2\, g_2 ds_2$$

From these equations we have

$$g_2 dp_2 = dY_1 \tag{18}$$

By subsidizing country 2, Y_1 and p_2 are reduced. For giving country 1 (after-subsidy) income is reduced, while for receiving country 2 the (after-subsidy) relative price of the public good is reduced.

We have[7]

$$\begin{bmatrix} p_2 c_U^1, & p_1 E_U^2 \\ G_U^1, & -G_U^2 \end{bmatrix} \begin{bmatrix} dU^1 \\ dU^2 \end{bmatrix} = \begin{bmatrix} -E^1 + Y_1 \\ G_p^2 \end{bmatrix} dp_2 + \begin{bmatrix} p_2 \\ 0 \end{bmatrix} dY_1 \tag{19}$$

Considering (18), from the above equation (19) we have

$$\frac{dU^1}{dp_2} = \frac{[-g_2(p_2 - p_1)G_U^2 - p_1 E_U^2 G_p^2]}{\Delta} \tag{20}$$

$$= -\frac{p_2 g_2 G_U^2}{\Delta} + \frac{p_1 g_2 G_U^2 - p_1 G_p^2 E_U^2}{\Delta}$$

$$\frac{dU^2}{dp_2} = \frac{[p_2(E_U^1 - p_1 G_U^1)G_p^2 - G_U^1(p_2 - p_1)g_2]}{\Delta} \tag{21}$$

$$= -\frac{p_2 g_2 G_U^1}{\Delta} + \frac{p_1 g_2 G_U^1 + p_2 G_p^2 c_U^1}{\Delta}$$

The first term of (20) represents the effect of a decrease in Y_1. The second term represents the effect of a decrease in p_2 (an increase in g_2). As explained in section 4.1 the second term includes the income effect and the substitution effect (see (14)). If $p_1 \geq p_2$, the first term is dominated by the income effect of the second term, so that (20) is negative. $g_2(p_2 - p_1)G_U^2$ corresponds to the total income effect. $p_1 E_U^2 G_p^2$ corresponds to the substitution price effect. Both effects are beneficial to country 1. The less productive country gains by making subsidies to the more productive country.[8] This is qualitatively the same result as in the lump sum transfer case in section 3.

But the sign of (21) is ambiguous when $p_1 \geq p_2$. The first term of (21) represents the effect of a decrease in Y_1. The second term represents the effect of a decrease in p_2. The second effect includes the direct income effect and the spillover price effect as explained in section 4.1 (see (13)). The total income effect is beneficial to country 2 when $p_1 \geq p_2$, while the spillover price effect always hurts country 2. (21) may be positive if the absolute value of G_p^2 (the spillover price effect) is large. In such a case the more productive country loses by receiving subsidies; we have a strong paradoxical result. Remember that in the lump sum transfer case in section 3, we always have a weak paradoxical result; both countries gain. In the subsidizing transfer case considered here we may have a strong paradoxical result.

Intuition is as follows. A decrease in income of (less-productive) country 1 hurts both countries, while a decrease in the relative price of the public good in country 2 benefits country 1. It benefits country 2 due to the direct income effect but hurts country 2 due to the spillover price effect. Suppose $p_1 > p_2$. Then, the income effect due to a decrease in p_2 dominates the income effect due to a decrease in Y_1. Thus, country 1 gains. If, in addition, the spillover price effect dominates the direct income effect in country 2, then country 2 loses and we have the strong paradoxical result.

When $p_1 = p_2$, we always have the strong paradoxical result since in this case the income effect due to a decrease in Y_1 perfectly offsets the income effect due to a decrease in p_2, so that we only have the spillover price effect.

Suppose $p_1 < p_2$. Then, the sign of (20) becomes ambiguous, while the sign of (21) becomes positive. When the more productive country makes subsidies to the less productive country, the income effect hurts both countries. The spillover price effect benefits the giving country (the more productive country) and hurts the receiving country (the less productive country). Thus, if the spillover and substitution price effects dominate the income effects in both countries, we have the strong paradoxical result as well.

We have seen that each country may well prefer to subsidize the other country in our contribution game. We now consider the case where both countries simultaneously subsidize each other. In place of (18) we have

$$g_2 dp_2 = dY_1 = dY_2 = g_1 dp_1 \tag{18)'}$$

Considering (18)', we have

$$\frac{dU^1}{dY^1} = \frac{-p_1 p_2 G_U^2 G_p^1 + (p_1 - p_2)(g_2 - g_1)G_U^2 + p_1 E_U^2 (G_p^1 - G_p^2)}{\Delta} \tag{22-1}$$

$$\frac{dU^2}{dY^2} = \frac{-p_1 p_2 G_U^1 G_p^2 + (p_1 - p_2)(g_2 - g_1)G_U^1 + p_2 E_U^1 (G_p^2 - G_p^1)}{\Delta} \tag{22-2}$$

From (22) it is easy to see that if $p_1 = p_2$ and both countries are identical, then $G_p^1 = G_p^2$ and (22-1) (22-2) are negative; both countries gain by making subsidies to each other. If $(p_1-p_2)(g_2-g_1) > 0$, it is more likely to have (22) < 0. Since an increase in p will depress contributions of the public good, the above condition is likely to be satisfied.

Finally, if $p_1 = p_2$ and $G_p^1 < G_p^2$ $(G_p^1 > G_p^2)$, it is possible that the sign of (22-1) is positive (the sign of (22-2) is positive). Intuition is as follows. If $|G_p^1|$ is small, a decrease in p_1 will not raise g_1 much, making the spillover price effect on country 2 small. Hence, country 2 may be hurt by giving subsidies; (22-2) becomes positive (and vice versa).

5. Multi-Club Model

5.1 open-club model

In this section we consider an open club framework where the size of the club n is endogenously determined. We investigate a so-called multi-club model where the number of clubs is exogenously given and all countries must belong to one of the clubs. We assume that preferences and the contribution productivity are the same among countries within a club. For simplicity we assume that the productivity of technology is club-specific. Each country that belongs to a club can enjoy the benefits of its own international public goods but not those of other clubs. The international public good provides benefits only to those countries of a particular club.

When n is endogenous, it is useful to rewrite the utility function (1) as

$$U^i = U^i(c_i, \frac{G}{n}) \qquad (1)'$$

We now assume that an increase in the number of countries in the club n will reduce the benefit of the public good G.[9] For simplicity the benefit is assumed to be an increasing function of per country level of the public good, G* = G/n. There are a number of ways in which congestion might be modeled. An alternative is to account for congestion in the cost of providing the public good. The qualitative result would be the same as in the text.

The budget constraint (3)' will be rewritten as

$$c_i + np_i G^* = Y_i + p_i \sum_{j \neq i} g_j \qquad (3)''$$

The effective price of providing the public good G* is now np_i. Hence, the model will be reduced to

$$E(np_i, U) = y + p_i G^*(np_i, U)(n-1) \qquad (23)$$

where y is per capita income and G*( ) now denotes the compensated demand function for G*. From (23) we have

$$\frac{dU}{dn} = \frac{(n-1)p_i^2 G_p^*}{E_U - p_i(n-1)G_U^*} \qquad (24)$$

(24) is always negative. An increase in the number of countries in the club reduces national welfare in the club, which is a stability condition of the system.[10]

Suppose there are two clubs in the world and every country must belong to one of them, club A or club B. If $U_A < U_B$, an old country within club A will get out of the club and join club B. When U is increasing with n (dU/dn > 0), the equilibrium is not stable. If the club is disturbed from the equilibrium, it will tend to diverge. In such a case, if $U_A < U_B$, an increase in n will lower U_A, so that $U_A = U_B$ will not be realized. If dU/dn < 0, then $U_A = U_B$ will be realized by changes in n, which corresponds to the stability of the system.

Suppose club A has n countries and club B has m countries.

$$n + m = N \tag{25}$$

where N is the total number of countries in the world. In the long run equilibrium national welfare is equalized between the two clubs. Thus, we have as the multi-club model

$$E(np_A, U) = y_A + p_A G^*(np_A, U)(n-1) \tag{26-1}$$

$$E(mp_B, U) = y_B + p_B G^*(mp_B, U)(m-1) \tag{26-2}$$

where y_A and y_B are per-capita income of club A and club B respectively. Three equations (25)(26-1) and (26-2) determine U, n, and m. In the long run the size of the club is endogenous and changes in the exogenous parameters would affect it.

5.2 an increase in p_A

We now investigate how the multi-club model would be affected by changes in exogenous factors such as the price of the public good.

When p increases in club A only, considering (25) and (26) we have

$$\begin{bmatrix} E_U^A - p_A G_U^{*A}(n-1), & -p_A^2 G_p^{*A}(n-1) \\ E_U^B - p_B(m-1)G_U^{*B}, & p_B^2 G_p^{*B}(m-1) \end{bmatrix} \begin{bmatrix} dU \\ dn \end{bmatrix} = \begin{bmatrix} -G^{*A} + p_A G_p^{*A}(n-1)n \\ 0 \end{bmatrix} dp_A \tag{27}$$

Thus, we have

$$\frac{dU}{dp_A} = \frac{[-G^{*A} + p_A G_p^{*A}(n-1)n] p_B^2 G^{*B}(m-1)}{\Phi} \tag{28}$$

$$\frac{dn}{dp_A} = \frac{[G^{*A} - p_A G_p^{*A}(n-1)n][E_U^B - p_B(m-1)G_U^{*B}]}{\Phi} \tag{29}$$

where $\Phi = [E_U^A - p_A G_U^* A(n-1)] p_B^2 G_p^{*B}(m-1) + [E_U^B - p_B G_U^{*B}(m-1)] p_A^2 G_p^{*A}(n-1) < 0$.

Hence, from (28) and (29) we know that $dU/dp_A < 0$ and $dn/dp_A < 0$. An increase in the price of the public good in club A p_A will always reduce the size of the club and welfare. This is a plausible result. Intuition is as follows. If there is no migration we know from section 4 that a general productivity decrease in public provision harms the whole club. Hence, every country in that club is worse off. In particular every country in the club is worse off than when migrating to the other clubs. So, some countries will outmigrate if the productivity decreases in a club, until the overall utility level in any club is again the same. Hence, migration only partially offsets the effect of a productivity decrease for the club members.[11]

This may explain the advantage of NATO against COMECON in the second half of 1980s since p reflects the efficiency of government bureaucrat system.

So far it is assumed that each club has its own production technology of providing the public good. p is club-specific and hence the same within the club. If p is assumed to be specific to a country, how would the result be altered? It is easy to see that when less productive countries join the club, namely, when countries with higher p join the club, the negative spillover price effect of higher p will hurt the existing member countries.

5.3 transfer between two clubs

We now consider the transfer between two clubs. We have considering

$dy_A + dy_B = 0$

$$\frac{dU}{dy_A} = \frac{p_A G^{*A} \varepsilon^A (n-1)/n - p_B G^{*B} \varepsilon^B (m-1)/m}{\Phi} \qquad (30)$$

If $\varepsilon^A G^{*A}(n-1)/n = \varepsilon^B G^{*B}(m-1)/m$ and $p_B > p_A$, a transfer from club B to club A will raise national welfare of a country in club B as well as in club A. This result is qualitatively the same as the paradoxical result of section 3. (30) is similar to (10). However, in this section there is no link between two clubs as to the benefit of the public good. The public good is club specific and hence we do not expect the positive spillover effect of the public good from the other club. Still, the less productive club can improve national welfare by transferring income to the more productive club.

Intuition is as follows. An increase in Y_A will directly raise U_A, while a decrease in Y_B will directly reduce U_B. These two effects offset each other. However, an increase in Y_A will indirectly raise U_B by inducing lower m. This effect may be called another spillover price effect as a decrease in m will reduce the effective price of providing the public good mp_B. Thus, the magnitude of this effect is given by p_B. A decrease in Y_B will indirectly reduce U_A by inducing higher n, the spillover price effect. If $p_B > p_A$ and $\varepsilon^A G^{*A}(n-1)/n = \varepsilon^B G^{*B}(m-1)/m$, the positive spillover effect of an increase in Y_A dominates the negative spillover effect of a decrease in Y_B, so that the equilibrium level of national welfare is raised.

6. Conclusion

This paper has developed a general equilibrium model of multi countries that provide an international public good. It has been shown that an increase in the price of providing the public good or a decrease in productivity may raise national welfare of its own country, while this will hurt the rest of the world. A country with high productivity does not necessarily enjoy high welfare. A country with low productivity can enjoy high welfare in the closed club model.

A country may not have a strong incentive to reduce the marginal cost of providing the public good as a decrease in the cost may reduce (not raise) the welfare of the country in the two-country model. The less productive country benefits by making transfers to the more productive country. We have shown that subsidizing the other country in the contribution game may lead to a strong paradoxical result due to the spillover price effect, while the lump sum transfer will lead to a weak paradoxical result due to the total income effect. We have explored some paradoxical results due to the response of foreign countries' provision of international public goods by incorporating contribution productivity differentials.

We have also explored the relation between an increase in the price of the public good and the size of an open club in a multi-club model and have shown that the less productive club can improve national welfare by transferring income to the more productive club due to the spillover price effect. Transfers between clubs can be Pareto improving although public goods have no direct spillovers to other clubs. Simply the migration externality makes transfers mutually advantageous.

References

Andreoni, J., 1989, Giving with impure altruism: Applications to charity and Ricardian equivalence, Journal of Political Economy 97, 1447-1459.

Batina, R.G. and P. Dion, 1994, On the transfer problem with international public goods, mimeo.

Bergstrom,T., L. Blume and H. Varian, 1986, On the private provision of public good, Journal of Public Economics 29, 25-49.

Boadway, R., Pestieau, P., and D. Wildasin, 1989, Tax-transfer policies and the voluntary provision of public goods, Journal of Public Economics 39, 157-176.

Buchholz, W. and K.A. Konrad, 1993, Strategic international transfers and private provision of public goods, mimeo.

Cornes, R., and Sandler, T., 1986, Externalities, Public Goods and Club Goods, Cambridge University Press.

Cornes, R., and Sandler, T., 1994, The comparative static properties of the impure public good model, Journal of Public Economics 54, 403-421.

Gradstein, M., 1993, Rent seeking and the provision of public goods, Economic Journal, 1236-1243.

Ihori, T., 1992, Impure public goods and transfers in a three agent model, Journal of Public Economics 48, 385-401.

Ihori, T., 1993, Immiserizing growth with interregional externalities of public goods, Regional Science and Urban Economics, forthcoming.

Murdoch J.C. and T. Sandler, 1984, Complementarity, free riding, and the military expenditures of NATO allies, Journal of Public Economics 25, 83-101.

Reece, W.S. and K. Zieschang, 1985, Consistent estimation of the impact of tax deductibility on the level of charitable contributions, Econometrica 53, 271-294.

Shibata, H., 1971, A bargaining model of the pure theory of public expenditures, Journal of Political Economy 79, 1-29.

Varian, H.R., 1994, Sequential contributions to public goods, Journal of Public Economics 53, 165-186.

Warr, P.G., 1983, The private provision of a public good is independent of the distribution of income, Economics Letters 13, 207-211.

Notes

[*]. The author is grateful for the comments by two anonymous referees.

[1]. Consider the case of national defense as the public good. The cost of military hardware, for example, has increased dramatically in the last twenty years as research and development costs escalate. The cost depends on technology available for the country. Murdoch and Sandler (1984) estimate NATO nations' demand for military expenditures by assuming the price of military activity changes in the same proportions as the CPI for each ally. It is well known that changes in the CPI are different among countries. If income increases with such research and development costs, the relative cost of defense spending has not increased much. Since many development countries are attempting to acquire state-of-art weapons systems, they will face an increase in the cost of defense as a result.

[2]. In order to present the results in the simplest way and in their strongest form, we assume that non-negativity constraints on providing public goods are non-binding in equilibrium. As remarked by Boadway et al. (1989), this assumption is relatively weak in some of the situations we analyze. The number of agents is small in the context of providing international public goods.

[3]. When the public good is impure and its externalities are divergent, we would have the paradoxical results of transferring income and immiserizing growth. See Ihori (1992, 1993) and Cornes and Sandler (1994).

[4]. For simplicity it is assumed that $g_1 = g_2 = G/2$ initially.

[5]. Reece and Zieschang (1985) estimate linear donations functions for a cross section of individuals and estimate the income effect of donations to be 0.0342, which means $\alpha = 0.9658 > 1/2$.

[6]. If both p_1 and p_2 increase at the same time, we would expect that the welfare of country 1 will normally be reduced. Note that in this case both countries cannot be improved by the price change. At least one country must be hurt by the increase in p_1 and

p_2. When preferences are identical, both countries must be hurt. However, it is still possible that an increase in p_i in both countries may be beneficial to country 1 if ε^i is different. For example, if $\varepsilon^1 = 2$ and $\varepsilon^2 = 0.5$, then $\alpha > 1/2$ is necessary and sufficient to have $dU^1 / dp_1 > 0$.

[7]. Initially we assume $s_2 = 0$.

[8].Varian (1994) derived the similar result for quasilinear preferences. However, our result holds for general preferences with productivity differentials.

[9]. This assumption does not affect the analytical results in the previous sections as n has been fixed there.

[10]. All these assumptions are quite standard. See for example Cornes and Sandler (1986) and Gradstein (1993).

[11]. I am indebted to a referee for this intuition.