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Protection against National Emergency: International Public Goods and Insurance

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Abstract

This paper develops a model of economic protection against random emergency costs. To mitigate the effects of these disruptions, each country creates a private mutual insurance market and provides voluntarily an international public good. We will explore how protection through voluntary provision of an international public good as well as mutual insurance would affect welfare. The existence of both mutual insurance and an international public good is crucial to obtain welfare equalization and the weak paradox of international transfer.

Keywords: risk of emergency, insurance, public goods

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1. Introduction

Governments need not passively accept risks that production and hence consumption would reduce in emergency situations. There are several measures to protect against national emergency. Typically, as McGuire (1994) stressed, an insurance motive to form alliances implies connections between countries to share effective income against production uncertainty. The mutual insurance will minimize the expected welfare loss of emergency costs, and players are competitive consumers in an international private market. On the other hand, as Olson and Zeckhauser (1966) propounded, countries may also allocate some fraction of national income to international (or regional) public goods (for example, special trading agreements, formation of international organizations, military preparedness, active international diplomacy, foreign aid) to reduce regional and international tension and to avoid random emergency costs. The public good is voluntarily provided by countries' governments. [1] These two measures are similar in the sense that they may capture the benefits of risk sharing by confederating economically with each other and hence by creating alliances. Each country that belongs to the alliances can enjoy the benefits of its mutual insurance and international public good. This feature may reflect realities in international defense alliances involving Japan, the European Community, and the United States.

McGuire (1994) incorporated both an insurance motive and voluntary provision of public goods into a rigorous general equilibrium model of economic protection under uncertainty where military preparedness can reduce the risks of war. He then showed that where security expenditures complement each other,

1 . As explained in Sandler (1993), the notion of public good among and within allies is a useful theoretical idea that is a reasonable approximation under some circumstances such as environmental pacts, the United Nations, and common markets. Ihori (1994) presented a model of economic integration with international public goods. Alesina and Perotti (1995) investigated economic risk and political risk in fiscal unions, and Alesina and Spolaore (1995) discussed the endogenous formation of countries in the case of public good provision.

defense produces such great common benefit that bilateral insurance and risk sharing is reduced. We will instead incorporate pure publicness of defense within allies into the utility function and clarify the effect of protection through insurance and public good spending on national welfare. We will then analyze the effects of changes in emergency costs and international transfers on welfare[2]. By doing so, we can explore how voluntary provision of an international public good and mutual insurance between countries would protect against national emergency.

Section 2 describes each country's optimizing behavior in the international private insurance market, in which insurance motives exist but voluntary provision of international public goods does not. Section 3 investigates some comparative statics results including changes in emergency costs and national income. We will explore how changes in such exogenous parameters may lead to different spillover welfare effects between countries. Section 4 develops a model of voluntary provision of international public goods and investigates how protection through international public goods would affect welfare of each country. Section 5 introduces both the mutual insurance market and the voluntary provision of the international public good into the model. It will be shown that expected welfare is equalized, irrespective of differences in emergency costs. Furthermore, if emergency costs are the same between countries, the public good can essentially crowd out mutual insurance since the private insurance market plays a role of sharing effective income against production uncertainty. We also investigate some comparative statics results. Section 6 finally concludes the paper.

2. Insurance Model

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2 . International transfers between allies may well occur in the real world. For example, Japan contributed an additional $9 billion, on top of the $4 billion it offered in 1990, to the allied effort on the Gulf war in 1991. The $9 billion may be regarded as a transfer from Japan to the United States.

There are two countries in the alliances, country 1 and country 2. They are identical in preferences but may be heterogeneous in income and emergency costs. They do not provide an international public good. There is an international private insurance market.

Assuming the additively separable utility function, country i's expected utility is given by

$$W_i = (1-\alpha)V(c_i^A) + \alpha V(c_i^B) \quad (1)$$

where W_i is expected welfare of country i, c_i is private consumption of country i (i=1,2). c_i is subject to uncertainty. In state A, which occurs at the probability of 1-α, country i enjoys c_i^A. In state B, which occurs at the probability of α, country i cannot enjoy c_i^A but can enjoy c_i^B. α indicates the probability of an economically disruptive production emergency or a 'war'. Note that subscript i refers to the country.

Country i's budget constraint in each state is given by

$$c_i^A = Y_i - ps_i \quad (2\text{-}1)$$

$$c_i^B = (1-\pi_i)Y_i - ps_i + s_i = c_i^A - \pi_i Y_i + s_i \quad (2\text{-}2)$$

where Y_i is exogenously given (identical) national income of country i. π_i (>0) indicates the net quantity of resources lost to each dollar of private production during the period of contingency when a war or natural disaster actually occurs -- resources lost by reason of being diverted to a war effort or being cut off because of disruptions in production activities. Thus, π is called the penalty ratio.

s is insurance return in event of emergency with p the price of insurance. In other words, ps indicates a demand country's premium paid to a supply country during the state of A and s represents the demand country's return in event of state B with p the price of insurance, that is, the premium per dollar of insurance coverage. Country 1's return s_1 may be positive or negative with country 2's return, s_2, necessarily opposite in sign. It is also assumed that uncertainty is restricted to production; the insurance premium paid from a demand country to a supply country in state B is risk free and is not subject to the penalty ratio.

We assume that each government (or consumer) determines its insurance demand or supply, treating exogenous parameters α, π and insurance price p as given[3,4].

(2) may be rewritten as

$$pc_i^B + \rho c_i^A = (1 - p\pi_i)Y_i \tag{2'}$$

where $\rho \equiv 1 - p$. We assume $p < 1$ $(\rho > 0)$ to investigate a meaningful solution. (2-2) means that the effective rate of return on insurance premium ps is (1-p)/p. The price of insurance, p, also means the price of consumption in state B, while 1-p means the price of consumption in state A. Effective income in the left hand side of (2)' evaluates emergency costs $\pi_i Y_i$ using p, the price of consumption in state B.

Let us define the following expenditure function:

$$Min\, E_i \equiv pc_i^B + \rho c_i^A \quad subject\ to\ W_i \geq \overline{W_i}$$

Since preferences are identical between countries, the functional form is the same between countries. Considering (2)', we have

$$E(W_i, \alpha, 1-p, p) = \tilde{E}(W_i, \alpha, \rho, p) = (1 - p\pi_i)Y_i \tag{3}$$

From (3) the expenditure function E will determine W_i as a function of income Y_i, the probability of "war" α, the penalty ratio π_i, and the price of insurance p.

By a variant of Shephard's Lemma we know

$$\tilde{E}_{ip} \equiv \partial\tilde{E}(W_i, \alpha, \rho, p) / \partial p = c^B(W_i, \alpha, \rho, p) \tag{4}$$

$$\tilde{E}_{i\rho} = \partial\tilde{E}(W_i, \alpha, \rho, p) / \partial \rho = c^A(W_i, \alpha, \rho, p) \tag{5}$$

$$E_{ip} = \partial E(W_i, \alpha, 1-p, p) / \partial p = c_i^B - c_i^A = s(W_i, \alpha, 1-p, p, \pi_i Y_i) - \pi_i Y_i \tag{6}$$

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3 . Although p might be determined in bilateral negotiation in the real world, we focus on the case in which p is expressed as the outcome of non-strategic, competitive equalization of insurance supply and demand in the private insurance market.

4 . α could be a decreasing function of security spending as in Shibata (1986, 1987) and McGuire (1994). Shibata analyzed relative merits of stockpiling and protection of domestic industries as a way of defending a nation against possible disruptions in international trade.

where s( ) is the compensated demand (or supply) function for insurance and $c_i^j()$ is the compensated demand function for private consumption in state j (j=A,B) for country i. (6) means that $s_\pi \equiv \partial s / \partial \pi Y = 1$.

The two country model will then be summarized by

$$E(W_1, p) = (1 - p\pi_1)Y_1 \tag{7}$$

$$E(W_2, p) = (1 - p\pi_2)Y_2 \tag{8}$$

$$s(W_1, p, \pi_1 Y_1) + s(W_2, p, \pi_2 Y_2) = 0 \tag{9}$$

(7) gives expected welfare of country 1 as a function of the insurance price, p, and its own effective income, $(1 - p\pi_1)Y_1$. Similarly, (8) gives expected welfare of country 2. (9) is the equilibrium condition of the private mutual insurance market and gives the equilibrium level of insurance price, p. Since there are no externalities, the real equilibrium is Pareto efficient.

The benefits of insurance-alliance formation depend crucially on the nature of relative risks of emergency[5]. (9) implies that s_1 and s_2 must have opposite signs. Without loss of generality, assume $s_1 > 0$ and $s_2 = -s_1 < 0$. Country 1 is a demand country, while country 2 is a supply country. If $Y_1 = Y_2$ and $\pi_1 = \pi_2$, both countries are identical and hence $W_1 = W_2$ and $s_1 = s_2 = 0$. Heterogeneous income and/or penalty ratios are crucial for the mutual insurance. We now briefly explore how such a situation would occur.

Suppose that Y is the same between two countries. If π is high in country 1 and low in country 2, it is still possible to have mutual insurance. Since $s_\pi = 1$, s_i is increasing with π_i. Therefore, when $\pi_1 > \pi_2$, it is possible to have $s_1 = -s_2 > 0$. From (7) and (8) we know $W_1 < W_2$ as $\pi_1 > \pi_2$.

For example, consider the log-linear utility function

$$W = (1-\alpha)\log c_i^A + \alpha \log c_i^B \tag{10}$$

Then the ordinary demand functions for c_i^A, c_i^B, and s_i are respectively given by

$$c_i^A = \frac{1-\alpha}{1-p}(1 - p\pi_i)Y \tag{11-1}$$

5 . We assume that two countries face a common risk. McGuire (1994) investigated a general case of a joint distribution of risks.

$$c_i^B = \frac{\alpha}{p}(1 - p\pi_i)Y \tag{11-2}$$

$$s_i = \frac{1 - p - (1-\alpha)(1 - p\pi_i)}{p(1-p)} Y \tag{11-3}$$

(11-3) means that if

$$\frac{p(1-\pi_2)}{1 - p\pi_2} > \alpha > \frac{p(1-\pi_1)}{1 - p\pi_1} \tag{12}$$

then, $s_1 > 0 > s_2$; we can have mutual insurance. Since p <1, inequality (12) implies p> α. From (9) and (11-3) the equilibrium level of p is given by

$$p = \frac{2\alpha}{2 - (1-\alpha)(\pi_1 + \pi_2)}$$

which also means p> α.

From (11-1) and (11-2), we also have

$$c_i^B - c_i^A = \frac{(1 - p\pi_i)(\alpha - p)}{p(1-p)} Y \tag{13}$$

which is negative if p> α. As shown in Appendix 1, in such a case $s_W < 0$. We assume this.

In summary, if $\pi_1 Y_1 \geq \pi_2 Y_2$ and $Y_1 \leq Y_2$ (and if either condition does not hold in equality), then $W_1 < W_2$ and $s_1 > s_2$.

3. Comparative Statics Results in the Insurance Model

Let us investigate some comparative statics results in the insurance model. Totally differentiating (7)(8) and (9), we have

$$\begin{bmatrix} E_{1W} & 0 & s_1 \\ 0 & E_{2W} & s_2 \\ s_{1W} & s_{2W} & s_{1p} + s_{2p} \end{bmatrix} \begin{bmatrix} dW_1 \\ dW_2 \\ dp \end{bmatrix} = \begin{bmatrix} -pY_1 \\ 0 \\ -Y_1 \end{bmatrix} d\pi_1 \tag{14}$$

Hence, we have

$$\frac{dW_1}{d\pi_1} = -\frac{Y_1}{\Delta}\{p[E_{2W}(s_{1p} + s_{2p}) - s_2 s_{2W}] - s_1 E_{2W}\} \tag{15}$$

$$\frac{dW_2}{d\pi_1} = -\frac{Y_1}{\Delta} s_2 (p s_{1W} - E_{1W}) \tag{16}$$

$$\frac{dp}{d\pi_1} = \frac{Y}{\Delta} E_{2W}(p s_{1W} - E_{1W}) \tag{17}$$

where $\Delta \equiv E_{1W}E_{2W}(s_{1p} + s_{2p}) - s_1E_{2W}s_{1W} - s_2E_{1W}s_{2W}$ and the first subscript on the partial derivatives refers to the country. As shown in Appendix 1, $E_W > 0$, $s_p < 0$. Δ is negative from the stability condition.

It is easy to see that (17) is positive. Namely, an increase in π_1 raises the price of insurance, p; an increase in the penalty ratio will raise the price of insurance. Since $s_2 < 0$, (16) is positive. An increase in p is desirable for the supply country; an increase in π_1 has a positive spillover welfare effect on country 2. This is called the price effect. Since $s_1 > 0$, the sign of (15) is negative. An increase in π_1 directly reduces the expected income of country 1. This is called the income effect. As country 1 is a demand country, an increase in p also has an unfavorable price effect, which will strengthen the unfavorable income effect of the increase in the penalty ratio. Hence, an increase in π_1 hurts country 1, while it benefits country 2.

An increase in π_2 may be analyzed in the similar way. Namely, it also increases the price of insurance, hurting country 1 due to the price effect. On the other hand, it directly reduces country 2's own effective income but it indirectly raises its welfare due to the price effect. Since the latter price effect offsets the former income effect, the overall welfare effect on country 2 is ambiguous.

These results suggest that an increase in the emergency cost has different spillover effects, depending on where it occurs. If the penalty ratio rises in the demand country, it has a positive spillover effect on the supply country. On the other hand, if the penalty ratio rises in the supply country, it has a negative spillover effect on the demand country. This is because an increase in the penalty ratio in any country will raise the price of insurance. These results are intuitively plausible.

A decrease in π_i has the same qualitative effect as an increase in Y_i, national income of country i, reducing the price of insurance. This is because the penalty ratio does not directly affect the price of insurance. Changes in the penalty ratio will affect the economy only through changes in emergency costs.

Therefore, an increase in Y_1 will hurt country 2, while an increase in Y_2 benefits country 1. Each country directly gains by its own economic growth due to the income effect but country 2 gets smaller benefits of its own economic growth than country 1 due to a reduction in the price of insurance. World-wide economic growth is more beneficial to the demand country than to the supply country.

We now analyze the effect of transferring income between countries. This case may be regarded as a situation where the penalty ratio rises in one country but it decreases in another country. Considering the constraint of $dY_1 + dY_2 = 0$, from (7) (8) and (9) we have

$$\frac{dW_1}{dY_1} = \frac{1}{\Delta}[(1 - p\pi_1)E_{2W}(s_{1p} + s_{2p}) + (\pi_2 - \pi_1)s_1(ps_{2W} - E_{2W})] \qquad (18)$$

$$\frac{dW_2}{dY_1} = -\frac{1}{\Delta}[(1 - p\pi_2)E_{1W}(s_{1p} + s_{2p}) - (\pi_2 - \pi_1)s_2(ps_{1W} - E_{1W})] \qquad (19)$$

$$\frac{dp}{dY_1} = \frac{1}{\Delta}[-(1 - p\pi_1)s_{1W}E_{2W} + (1 - p\pi_2)s_{2W}E_{1W} + (\pi_2 - \pi_1)E_{1W}E_{2W}] \quad (20)$$

(20) implies that if $\pi_2 > \pi_1$, then p will decline and vice versa. Namely, if the penalty ratio is higher in country 1, a transfer from country 2 to country 1 will raise the price of insurance, hurting the demand country and benefiting the supply country. This is the price effect. As for the income effect, the receiving country gains and the giving country loses. Thus, (18) is positive if $\pi_2 > \pi_1$. In this case both the income and price effects benefit country 1. However, if $\pi_1 > \pi_2$, the price effect hurts country 1, while the income effect benefits country 1. (18) becomes positive when the income effect dominates the price effect. (19) is negative if $\pi_2 > \pi_1$. In such a case both the income and price effects hurt country 2. However, if $\pi_1 > \pi_2$, the sign of (19) becomes ambiguous since the income and price effects offset each other. To sum up, when $\pi_2 > \pi_1$, a country which receives a gift becomes better and a country which gives a gift becomes worse. The transfer paradox would not occur if $\pi_2 > \pi_1$. However, if

$\pi_2 < \pi_1$, we could have the strong transfer paradox that a giving country gains and a receiving country loses[6].

4. International public goods

We now develop a model of an international public good, G. Characterizing deterrence as a pure public good is a standard idea since Olson and Zeckhauser (1966). The international public good such as defense benefits may well be nonrival and nonexcludable among allies. For example, deterrence as provided by a nuclear triad is nonrival among allies if the retaliatory is automatic. The interdependence of economic activities among allies collectivizes the security of the allies and hence increases nonexcludability of defense benefits[7].

Then, the utility function (1) may be rewritten as

$$W_i = (1-\alpha)V(c_i^A) + \alpha V(c_i^B) + U(G) \qquad (1)'$$

It might be plausible to assume that the international public good is more beneficial when the emergency occurs than when it does not. In this paper we consider the case where the benefit of the international public good is independent of the state of nature. The qualitative results are almost the same even if G is assumed to be more beneficial when the emergency occurs.

Since G is a pure public good, G is given by

$$G = g_i + \sum_{j \neq i} g_j \qquad (21)$$

where g_i is the international public good voluntarily provided by country i.

Country i's budget constraint in each state is now given by

$$c_i^A = Y_i - g_i \qquad (2\text{-}1)'$$

$$c_i^B = (1-\pi_i)Y_i - g_i = c_i^A - \pi_i Y_i \qquad (2\text{-}2)'$$

where the relative price of public goods in terms of private consumption is assumed to be unity. Substituting (2-2)' into (1)', we have

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6. As for the transfer paradox in the standard framework of international trade see Bhagwati, Brecher and Hatta (1983).

7 . As stressed by McGuire (1990) and Sandler (1993) among others, some of defense benefits are not purely public.

$$W_i = (1-\alpha)V(c_i^A) + \alpha V(c_i^A - \pi_i Y_i) + U(G) \quad (1)''$$

We assume that each government in a noncooperative setting determines its public good provision, treating the other country's public spending as given[8]. Considering (21), (2-1)' may be rewritten as

$$c_i^A + G = Y_i + \sum_{i \neq j} g_j \quad (2\text{-}1)''$$

Let us now define the following expenditure function:

$$Min \;\; E_i \equiv c_i^A + G \quad subject\ to\ W_i \geq \overline{W_i}$$

Then in place of (3) we have

$$E(W_i, \pi_i Y_i) = Y_i + \sum_{i \neq j} g_j \quad (3)'$$

The two country model with voluntary provision of the public good will be summarized by

$$E(W_1, \pi_1 Y_1) + E(W_2, \pi_2 Y_2) = Y_1 + Y_2 + G(W_1, \pi_1 Y_1) \quad (23)$$

$$G(W_1, \pi_1 Y_1) = G(W_2, \pi_2 Y_2) \quad (24)$$

where G( ) is the compensated demand function for the public good. (23) comes from (3)' and (21). (24) means that each country demands the same amount of the public good in equilibrium.

From (24) if emergency costs are equal between countries, that is,

$$\pi_1 Y_1 = \pi_2 Y_2$$

then, expected welfare is also equalized.

$$W_1 = W_2$$

Even if income is different between countries, expected welfare would be equalized when emergency costs are the same. This is because the voluntary provision of pure public good can offset the difference with respect to income before uncertainty. Similarly, if emergency costs in country 1 are greater than in country 2 ($\pi_1 Y_1 > \pi_2 Y_2$), then welfare in country 1 is less than in country 2 ($W_1 < W_2$). This result comes from the property that the compensated demand

8 . This public good model is a standard private provision model. See Cornes and Sandler (1996) among others.

function for G is increasing with the emergency costs; $G_\pi \equiv \partial G / \partial \pi Y > 0$ (See Appendix 2).

Totally differentiating (23) and (24), we have

$$\begin{bmatrix} E_{1W} - G_{1W} & E_{2W} \\ G_{1W} & -G_{2W} \end{bmatrix} \begin{bmatrix} dW_1 \\ dW_2 \end{bmatrix} = \begin{bmatrix} -Y_1 c_{1\pi}^A \\ -Y_1 G_{1\pi} \end{bmatrix} d\pi_1 + \begin{bmatrix} -\pi_1 E_{1\pi} + \pi_2 E_{2\pi} + G_{1\pi}\pi_1 \\ -\pi_1 G_{1\pi} - \pi_2 G_{2\pi} \end{bmatrix} dY_1$$

We assume $dY_1 = -dY_2$ when we consider the income transfer from country 2 to country 1.

Or, we have

$$\frac{dW_1}{d\pi_1} = \frac{1}{\overline{\Delta}}[Y_1 c_{1\pi}^A G_{2W} + Y_1 G_{1\pi} E_{2W}] \tag{25-1}$$

$$\frac{dW_2}{d\pi_1} = \frac{1}{\overline{\Delta}}[(E_{1W} - G_{1W})(-Y_1 G_{1\pi}) + G_{1W} Y_1 c_{1\pi}^A] \tag{25-2}$$

$$\frac{dW_1}{dY_1} = \frac{1}{\overline{\Delta}}[(-\pi_1 c_{1\pi}^A + \pi_2 E_{2\pi})(-G_{2W}) + (\pi_1 G_{1\pi} + \pi_2 G_{2\pi})E_{2W}] \tag{25-3}$$

$$\frac{dW_2}{dY_1} = \frac{1}{\overline{\Delta}}[(-\pi_1 c_{1\pi}^A + \pi_2 E_{2\pi})(-G_{1W}) - (\pi_1 G_{1\pi} + \pi_2 G_{2\pi})(E_{1W} - G_{1W})] \tag{25-4}$$

where $\overline{\Delta} \equiv (E_{1W} - G_{1W})(-G_{2W}) - E_{2W}G_{1W} < 0$, $c_\pi^A \equiv \partial c^A / \partial \pi Y > 0$, and $E_\pi \equiv \partial E / \partial \pi Y > 0$.

First of all, let us examine the welfare effect of an increase in π_1. (25-1) is negative but the sign of (25-2) is ambiguous. An increase in π_1 will reduce the total effective income, hurting both countries. This is called the total income effect. An increase in π_1 raises the demand for the public good in country 1, inducing more provision of g_1 and less provision of g_2. This hurts country 1 but benefits country 2, which is called the relative burden effect. Thus, country 1 loses but the welfare effect on country 2 is ambiguous. Country 2 loses when the total income effect dominates the relative burden effect.

Let us then consider the income transfer policy. The first term in (25-3,4) is ambiguous. This is because an increase in the emergency costs $\pi_1 Y_1$ in country 1 will hurt both countries, while a decrease in the emergency costs $\pi_2 Y_2$ in country 2 will benefit both countries. The second term is positive in (25-3), while it is negative in (25-4). This is because an increase in the emergency costs in country 1 induces more provision of g_1, while a decrease in the emergency costs in country 2 induces less provision of g_2. This is the relative burden effect. Thus, (25-3) could be positive, while (25-4) could be negative. We could have the strong paradox if the relative burden effect dominates the sign[9].

5. Insurance Market and International Public Good

This section incorporates both the insurance market and the international public good. Country i's budget constraint is now rewritten as

$$c_i^A = Y_i - ps_i - g_i \tag{2-1)''}$$

$$c_i^B = (1-\pi_i)Y_i - ps_i + s_i - g_i \tag{2-2)''}$$

Hence, we have

$$pc_i^B + \rho c_i^A + g_i = (1-p\pi_i)Y_i \tag{2)''}$$

The game is a single-shot one stage game. Before the game the equilibrium price of insurance p is announced and binding commitments regarding insurance are determined in the private insurance market. Then two countries simultaneously choose the provision of the public good g_i and have Nash conjectures about the choice of the other's public good. Finally nature decides which state (A or B) occurs.

Considering (21), (2)" may be rewritten as

$$pc_i^B + \rho c_i^A + G = (1-p\pi_i)Y_i + \sum_{i\neq j} g_j \tag{22)'}$$

Let us now define the following expenditure function:

$$Min\, E_i \equiv pc_i^B + \rho c_i^A + G \quad subject\ to\ W_i \geq \overline{W_i}$$

Then in place of (3) we have

$$E(W_i, \alpha, 1-p, p) = (1-p\pi_i)Y_i + \sum_{i\neq j} g_j \tag{3)''}$$

Let us investigate properties of compensated demand and supply functions.

As shown in Appendix 3, the properties of c^A, c^B, E, and s functions are

9. As for the neutrality result see Shibata (1971) and Warr (1983). The nonneutrality result here comes from nation-specific aspects as in Cornes and Sandler (1984, 1994, 1996). See also Ihori

qualitatively the same as in section 2. We also have G(W, p); the compensated demand function for the public good as in section 4. It is intuitively plausible to see that $G_W \equiv \partial G / \partial W$ is positive. From (A23) and (A24) $G_p \equiv \partial G / \partial p$ is negative since we assume $s_W < 0$. G is substitutable with c^A and complementable with c^B.

By definition we have as in section 2

$$s(W_i, p, \pi_i Y_i) \equiv c^B(W_i, p) - c^A(W_i, p) + \pi_i Y_i$$

The two country model with mutual insurance motives and public goods will then be summarized by

$$E(W_1, p) + E(W_2, p) = Y_1 + Y_2 - p(Y_1\pi_1 + Y_2\pi_2) + G(W_1, p) \tag{26}$$

$$G(W_1, p) = G(W_2, p) \tag{27}$$

$$c^B(W_1, p) - c^A(W_1, p) + \pi_1 Y_1 + c^B(W_2, p) - c^A(W_2, p) + \pi_2 Y_2 = 0 \tag{28}$$

(26) and (27) correspond to (23) and (24), respectively. (28) corresponds to (9).

From (27) when preferences are identical between countries we always have

$$W_1 = W_2 \tag{29}$$

We also have

$$s_1 = s_2 = 0 \tag{30}$$

if emergency costs are equal between countries[10]:

$$\pi_1 Y_1 = \pi_2 Y_2 \tag{31}$$

(29) means that $c_1^A = c_2^A$, $c_1^B = c_2^B$. Hence, from (28) we know

$$s_1 > 0 > s_2 \quad \text{if and only if} \quad \pi_1 Y_1 > \pi_2 Y_2 \tag{32}$$

Suppose (31) holds but income is different between countries ($Y_1 \neq Y_2$). Then (30) holds when we allow for an international public good as well as the mutual insurance market.

(1992a,b) and Buchholz and Konrad (1995).

10. McGuire (1994) showed that defense effort reduces the scope for mutual insurance, which is consistent with out result.

These results show how the interdependence between allies will be affected by the existence of insurance motives and an international public good. It should be stressed that (29) does not hold without incorporating international public goods in addition to the mutual insurance market. Once we incorporate both the mutual insurance market and the international public good at the Nash solution, expected welfare is equalized, irrespective of differences in income, the penalty ratio, or the type of insurance. When the insurance market works and each country voluntarily provides an international public good, the divergence between the effective income ($(1-p\pi_i)Y_i$) does not matter. On the contrary, the total effective income $Y_1+Y_2-p(\pi_1Y_1+\pi_2Y_2)$ and total emergency costs $\pi_1Y_1+\pi_2Y_2$ do matter. Each country's expected welfare is affected in the same way. This is because both countries face the same price of insurance and different emergency costs have the income effect only[11]. Remember that in the insurance market model, welfare is equalized if the expected disposable income $(1-p\pi_i)Y_i$ is equal. In the public good provision model, welfare is equalized if the emergency cost π_iY_i is equal. Here in a two-ally model with both insurance and the public good expected welfare is always equalized so long as the preference is the same.

From (26)(27)(28) and (29), we have

$$2E(W,p)=(1-p\pi_1)Y_1+(1-p\pi_2)Y_2+G(W,p) \tag{33}$$

$$s(W,p,\pi_1Y_1)+s(W,p,\pi_2Y_2)=0 \tag{34}$$

Hence, assuming that an income transfer from country 2 to country 1 occurs before the true state (A or B) is known and hence considering $dY_1=-dY_2$, we have

$$\begin{bmatrix} 2E_W-G_W & -G_p \\ s_{1W}+s_{2W} & s_{1p}+s_{2p} \end{bmatrix} \begin{bmatrix} dW \\ dp \end{bmatrix} = \begin{bmatrix} -pY_1 \\ -Y_1 \end{bmatrix} d\pi_1 + \begin{bmatrix} -p(\pi_1-\pi_2) \\ -\pi_1+\pi_2 \end{bmatrix} dY_1 \tag{35}$$

Or

11 . This is an extension of the neutrality result.

$$\frac{dW}{d\pi_1} = \frac{-Y_1[p(s_{1p} + s_{2p}) + G_p]}{(2E_W - G_W)(s_{1p} + s_{2p}) + (s_{1W} + s_{2W})G_p} \tag{36-1}$$

$$\frac{dW}{dY_1} = \frac{-(\pi_1 - \pi_2)[p(s_{1p} + s_{2p}) + G_p]}{(2E_W - G_W)(s_{1p} + s_{2p}) + (s_{1W} + s_{2W})G_p} \tag{36-2}$$

We know $2E_W - G_W > 0$, $s_p < 0$, $G_p < 0$. The denominator is negative from the stability condition.

An increase in π_1 reduces the total expected income, hurting both countries. This is the total income effect as in section 4. An increase in π_1 raises the price of insurance, inducing less supply of the pure public good. This price effect also hurts both countries. Hence, (36-1) is always negative.

Let us then investigate the welfare effect of international transfer. Remember that the transfer policy usually has normal welfare effects in the insurance model of section 3; a receiving country becomes better and a giving country becomes worse when the direct income effect dominates the price effect. However, the transfer policy could have the strong paradoxical effects in the insurance model of section 3 and in the model of international public goods of section 4 when the price effect in section 3 and the relative burden effect in section 4 dominate the sign respectively. We now derive the weak transfer paradox in the model with both insurance and an international public good. Since expected welfare is equalized, both countries may either gain or lose from the international income transfer. We have the weak paradox here. We would like to investigate how both countries are affected by the transfer policy.

The numerator in (36-2) is positive if and only if $\pi_2 > \pi_1$. Thus, if $\pi_2 < \pi_1$, (36-2) is negative; a transfer from country 2 to country 1 will reduce welfare of each country. Both a receiving country and a giving country lose when income is transferred to the country which has a larger penalty ratio. Intuition is as follows. $dY_1 = -dY_2$ means that in state A country 1 gains 1\$ and country 2 loses this dollar. But in state B country 1 gains $(1-\pi_1)$\$ while country 2 loses

$(1-\pi_2)\$$. If $\pi_1 > \pi_2$, it is clear that aggregate income in this state decreases in at least one country. The existence of an insurance market and a public good means welfare equalization, which then implies that both countries' expected welfare decreases. In this sense, the existence of both the insurance market and the international public good is crucial to obtain the weak paradoxical result of international transfer.

6. Conclusion

We have investigated welfare implications of the interdependence between countries of allies through insurance motives and voluntary provision of international public goods toward country risk. We have considered the case where two countries are identical in preferences but they may be heterogeneous with respect to income and the penalty ratio on production.

The main results are as follows: When international public goods are not provided, mutual insurance could work in the private insurance market. In such a case an increase in the penalty ratio in country 1 (the demand country) will hurt country 1 to a large extent, while it benefits country 2 (the supply country). On the other hand, an increase in the penalty ratio in country 2 hurts country 2 to a small extent, while it hurts country 1. An increase in national income (or a decrease in the penalty ratio) in some countries will reduce the price of insurance. Hence it benefits the demand country and hurts the supply country due to the price effect. We have explored how the income and price effects may lead to different spillover welfare effects between countries.

With the international public good only, the total income effect and the relative burden effect are shown to be relevant. A country with larger emergency costs becomes worse than a country with smaller emergency costs. Each country loses from an increase in its own penalty ratio due to the total income effect, but it has a beneficial spillover effect on the other country due to the relative burden effect. Welfare equalization occurs only when emergency costs are the same between countries. Furthermore, we could have the strong paradox of transfers; a giving country gains and a receiving country loses when the relative burden effect dominates the sign.

With both the mutual insurance market and the voluntary provision of international public goods, expected welfare is always equalized, irrespective of differences in income, the penalty ratio, or the type of insurance. In such a case the divergence between the effective income does not matter since the total effective income and total emergency costs affect each country's welfare in the same way. As welfare is equalized, we have the weak version of transfer paradox; both a receiving country and a giving country lose when income is transferred to the country with a larger penalty ratio. International transfers between allies may well occur in the real world. We have shown that welfare implications of such transfers crucially depend on how protection against national emergency is used.

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Appendix 1

The first order condition for each country's optimization is given by

$$\alpha\rho V_c^B = p(1-\alpha)V_c^A \tag{A1}$$

where $V_c^A \equiv dV/dc^A, V_c^B \equiv dV/dc^B$. At the optimum, the marginal utility gain from allocation of an extra dollar to s (LHD of (A1)) just equals the expected marginal utility cost of that last dollar (RHD of (A1)). For simplicity subscript i, which is referring to the country, is omitted and c_i is rewritten as c.

From (1) and (A1), we solve for s, c^A and c^B respectively as functions of W and p, which give the compensated demand (or supply) functions for s, c^A and c^B; equations (4), (5) and (6), respectively.

Totally differentiating (1) and (A1), we have

$$\begin{bmatrix} \alpha\rho V_c^B/p, & \alpha V_c^B \\ -p(1-\alpha)V_{cc}^A, & \rho\alpha V_{cc}^B \end{bmatrix} \begin{bmatrix} dc^A \\ dc^B \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} dW + \begin{bmatrix} 0 \\ \alpha V_c^B/p \end{bmatrix} dp \tag{A2}$$

where $V_{cc}^A \equiv d^2V^A/dc^A dc^A, V_{cc}^B \equiv d^2V^B/d\,c^B dc^B$, $V^A \equiv V(c^A), V^B \equiv V(c^B)$.

Hence we have

$$c_W^A \equiv \partial c^A/\partial W = \rho\alpha V_{cc}^B/\gamma > 0 \tag{A3}$$

$$c_W^B \equiv \partial c^B/\partial W = p(1-\alpha)V_{cc}^A/\gamma > 0 \tag{A4}$$

$$s_W \equiv \partial s/\partial W = c_W^B - c_W^A = [p(1-\alpha)V_{cc}^A - \alpha\rho V_{cc}^B]/\gamma \tag{A5}$$

$$E_W \equiv \partial E/\partial W = p\partial c^B/\partial W + \rho\partial c^A/\partial W$$

$$= [p^2(1-\alpha)V_{cc}^A + \rho^2\alpha V_{cc}^B]/\gamma = \frac{p}{\alpha V_c^B} > 0 \tag{A6}$$

$$c_p^A \equiv \partial c^A/\partial p = -\alpha^2 V_c^B V_c^B/p\gamma > 0 \tag{A7}$$

$$c_p^B \equiv \alpha^2 V_c^B V_c^B \rho/p^2\gamma < 0 \tag{A8}$$

$$s_p \equiv \partial s/\partial p = c_p^B - c_p^A = \alpha^2 V_c^B V_c^B/p^2\gamma < 0 \tag{A9}$$

where $\gamma \equiv \alpha V_c^B[p^2(1-\alpha)V_{cc}^A + \rho^2\alpha V_{cc}^B]/p < 0$.

An increase in W raises c^A, c^B, and E. An increase in p raises c^A but reduces c^B and s. These results are qualitatively plausible. The sign of (A9) is ambiguous. Suppose the relative risk aversion is constant ($\frac{cV_{cc}}{V_c} = -\lambda$). Then considering the first-order condition (A1), (A9) may be rewritten as

$$s_W = \alpha\rho\lambda V_c^B(c^A - c^B)/c^A c^B \gamma \qquad \text{(A9)'}$$

Thus, if $c^A > c^B$, then $s_W < 0$ and $E_p \equiv \partial E/\partial p = c^B - c^A < 0$. We assume this.

Appendix 2

With the public good the first order condition (A1) is rewritten as

$$(1-\alpha)V_c^A + \alpha V_c^B = U_G \qquad \text{(A1)'}$$

where $U_G \equiv dU/dG$. Let us investigate properties of expenditure and compensated demand functions. Totally differentiating (1)' and (A1)', we have

$$\begin{bmatrix} (1-\alpha)V_{cc}^A + \alpha V_{cc}^B & -U_{GG} \\ (1-\alpha)V_c^A + \alpha V_c^B & U_G \end{bmatrix} \begin{bmatrix} dc^A \\ dG \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} dW + \begin{bmatrix} \alpha V_{cc}^B \\ \alpha V_c^B \end{bmatrix} \pi_i Y_i \qquad \text{(A10)}$$

Hence we have

$$c_W^A = \frac{1}{\bar{\gamma}} U_{GG} > 0 \qquad \text{(A11)}$$

$$G_W \equiv \frac{\partial G}{\partial W} = \frac{1}{\bar{\gamma}}[(1-\alpha)V_{cc}^A + \alpha V_{cc}^B] > 0 \qquad \text{(A12)}$$

$$c_\pi^A \equiv \frac{\partial c^A}{\partial \pi Y} = \frac{1}{\bar{\gamma}}[\alpha V_{cc}^B U_G + \alpha V_c^B U_{GG}] > 0 \qquad \text{(A13)}$$

$$G_\pi \equiv \frac{\partial G}{\partial \pi Y} = \frac{\alpha}{\bar{\gamma}}\{[(1-\alpha)V_{cc}^A + \alpha V_{cc}^B]V_c^B - [(1-\alpha)V_c^A + \alpha V_c^B]V_{cc}^B\} \qquad \text{(A14)}$$

$$E_\pi \equiv \frac{\partial E}{\partial \pi Y} = \frac{\alpha}{\bar{\gamma}}\{V_c^B U_{GG} + [(1-\alpha)V_{cc}^A + \alpha V_{cc}^B]V_c^B\} > 0 \qquad \text{(A15)}$$

$$E_W = c_W^A + G_W > 0 \qquad \text{(A16)}$$

where $\bar{\gamma} \equiv [(1-\alpha)V_{cc}^A + \alpha V_{cc}^B]U_G + [(1-\alpha)V_c^A + \alpha V_c^B]U_{GG} < 0$. Suppose the relative risk aversion is constant ($\frac{cV_{cc}}{V_c} = -\lambda$). Then (A14) may be rewritten as

$$\frac{\partial G}{\partial \pi Y} = \frac{\alpha\lambda V_c^A V_c^B \alpha(1-\alpha)}{c^A c^B}(c^A - c^B) > 0 \qquad \text{(A14)'}$$

Appendix 3

With both the public good and the insurance market the first order condition (A1) is rewritten as

$$\alpha\rho V_c^B = p(1-\alpha)V_c^A = p\rho U_G \qquad \text{(A1)''}$$

Totally differentiating (1)'' and (A1)'', we have

$$\begin{bmatrix} \alpha\rho V_c^B / p, & \alpha V_c^B, & \alpha V_c^B / p \\ -p(1-\alpha)V_{cc}^A, & \alpha\rho V_{cc}^B, & 0 \\ 0 & \alpha V_{cc}^B, & -pU_{GG} \end{bmatrix} \begin{bmatrix} dc^A \\ dc^B \\ dG \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} dW + \begin{bmatrix} 0 \\ \alpha V_c^B / p \\ \alpha V_c^B / p \end{bmatrix} dp \tag{A17}$$

Hence we have

$$c_W^A = \frac{1}{\gamma *}[-\alpha\rho p V_{cc}^B U_{GG}] > 0 \tag{A18}$$

$$c_W^B = \frac{1}{\gamma *}[-p^2(1-\alpha)V_{cc}^A U_{GG}] > 0 \tag{A19}$$

$$G_W = \frac{1}{\gamma *}[-p(1-\alpha)\alpha V_{cc}^A V_{cc}^B] > 0 \tag{A20}$$

$$c_p^A = \frac{1}{\gamma *}[\alpha^2 (V_c^B)^2 (U_{GG} + \frac{\alpha}{p} V_{cc}^B)] > 0 \tag{A21}$$

$$c_p^B = \frac{1}{\gamma *}[-\alpha^2 \rho (V_c^B)^2 U_{GG} / p - (1-\alpha)\alpha^2 V_{cc}^A V_c^B V_{cc}^B] < 0 \tag{A22}$$

$$G_p = \frac{1}{\gamma *}\frac{\alpha^2}{p}(V_c^B)^2[-\alpha\rho V_{cc}^B + p(1-\alpha)V_{cc}^A] \tag{A23}$$

where $\gamma * \equiv -V_c^B[p^2\alpha(1-\alpha)V_{cc}^A U_{GG} + \rho^2\alpha^2 V_{cc}^B U_{GG} + (1-\alpha)\alpha^2 V_{cc}^A V_{cc}^B] < 0.$

Since $s = c^B - c^A + \pi Y$, we have

$$s_W = \frac{1}{\gamma *}(-p)U_{GG}[p(1-\alpha)V_{cc}^A - \alpha\rho V_{cc}^B] \tag{A24}$$

$$s_p = c_p^B - c_p^A < 0 \tag{A25}$$

Finally, we have

$$E_W = pc_W^B + \rho c_W^A + G_W > 0 \tag{A26}$$

$$E_p = c^B - c^A \tag{A27}$$