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Abstract

This paper investigates dynamic properties of environmental externalities with a framework of voluntary provision of a public good by analyzing the infinite duration dynamic game. We compare the first best solution, the open-loop solution under enforceable commitments, and the closed-loop solution without commitment. We explore the free riding problem and consider the normative role of Pigovian consumption taxes to internalize the free riding problem. We also investigate the adjustment speeds of environmental quality under alternative solutions and examine the impact of Pigovian consumption taxes on the adjustment speed.

Key words: Environmental externalities, voluntary provision, dynamic game
JEL classification numbers: H41, F13, D62

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1. Introduction

There has been much attention on the long-run externality effects of economic activities on the world environment. In the analysis of environmental problems we could consider two types of conflicts; intergenerational and intragenerational conflicts within and between generations due to free riding. Recently John et al (1995), Farzin (1996), Mohtadi (1996), Bovenberg and de Mooij (1997), and Yoshida (1996) investigated dynamic implications of intergenerational conflict. However, there have been few papers on the dynamic analysis of intragenerational conflict, although the free-rider problem within a generation has been much investigated in a static framework[1]. Since environmental quality would gradually change over time, it is important to explore dynamic free-rider properties of collective and continuous adjustments in the stock of world-wide environment and to investigate mechanisms under which a decentralized economy might successfully internalize environmental externalities. Answers to these questions can be important in investigating how best to assist the developing countries to grow with better environmental quality.

Fershtman and Nitzan (1991) considered a dynamic public good contribution game in which individuals' contributions are accumulated over time[2]. They showed that the free riding problem is aggravated when players' contributions are conditional on the observable collective contributions. We apply the similar theoretical framework to the world-wide environmental problem[3]. Our analysis is an extension of their model in the following sense.

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[1] . See Bergstrom et. al. (1986), Cornes and Sandler (1994), and Itaya and Dasgupta (1995) among others.

[2] . See also Fershtman and Kamien (1987).

[3] . We may develop a similar dynamic model of fiscal reconstruction where a number of interest groups voluntarily pay taxes to obtain more public spending. See Ihori and Itaya

First, instead of introducing the ad-hoc production function with ad-hoc adjustment costs into the objective function, we use a second-order Taylor-series approximation of the plausible utility function. The second-order term on consumption appearing in the implied quadratic utility function ensures that the level of environmental stock is a state variable that does not jump but evolve gradually. This formulation not only enhances the reality of the model but also enables us to analyze the intrinsic dynamics of environmental stock without appealing to a function of ad-hoc adjustment costs. Second, we introduce two types of contributions to public goods; aggregate consumption accumulates pollution, while aggregate environmental expenditures improve environmental quality. Third, we explore the normative role of Pigovian consumption taxes to internalize the free riding problem in an intertemporal setting. Although most of the literature on environmental problems has paid much attention on the emission tax or the pollution tax in correcting specific stock externalities based on a partial-equilibrium framework, our analytical focus on Pigovian taxes on consumption would serve to highlight the potential general-equilibrium effectiveness of this tax in the presence of widespread environmental disexternalities of consumption. We derive the optimal rates of consumption taxes at non-cooperative solutions in a decentralized world. Forth, we explicitly derive the adjustment speeds of environmental quality under alternative solutions of dynamic game and show that the adjustment speed of the Pareto efficient path is greater than either that of the open- or closed-loop equilibrium path. Finally, we examine the effect of consumption taxes on the adjustment speed. By doing so, we clarify welfare implications of Pigovian tax policy on cumulative environmental externalities during the transition as well as in a steady state.

(1996).

Section 2 presents the model. Section 3 investigates the Pareto efficient outcome. Section 4 considers the open-loop solution under enforceable commitments. Section 5 considers the closed-loop solution without commitment. Section 6 introduces Pigovian taxes on consumption and derives the optimal consumption tax rate. Section 7 investigates the adjustment speed under alternative solutions. Finally, section 8 concludes the paper. Mathematical derivations are found in the Appendix.

2. The model

The instantaneous utility of country i (or agent i) is assumed to be separable in consumption and the quality of environment for country i, which is common to all countries and may be viewed as a pure public good, i.e.,

$$U^i(c_i, G) = U_1^i(c_i) + U_2^i(G) \quad (1)$$

which ensures that c_i and G are normal goods. Moreover, in order to be able to obtain analytical solutions, a second-order Taylor-series approximation of the above utility function is employed. This enables a comparison of different game equilibria as well as a sensitivity (or transitional) analysis with respect to the underlying parameters of the model, i.e.,

$$U_1^i(c) + U_2^i(G) = \alpha^i + \beta_1^i c_i - \frac{\gamma_1^i}{2} c_i^2 + \beta_2^i G - \frac{\gamma_2^i}{2} G^2 \quad (2)$$

$$\beta_1, \beta_2, \gamma_1, \gamma_2 > 0$$

The intertemporal utility function of country i for the infinite-horizon problem starting at time 0 is given by

$$\int_0^\infty [U_1^i(c_i) + U_2^i(G)] e^{-\rho t} dt \quad (3)$$

where ρ (>0) is the constant discount rate.

The environmental quality (or environmental stock), G, will change over time. The dynamic evolution of G is given by

$$\dot{G} = -\sum_{i=1}^{n} \varepsilon_i c_i + \sum_{i=1}^{n} \sigma_i g_i - \delta G \tag{4}$$

where g_i is payment of an environmental maintenance and improvement provided by country i at time t, δ measures the depreciation of environmental quality[4]. ε_i indicates the degree of environmental degradation caused by country i's consumption and σ_i indicates the degree of environmental improvement from expenditures by country i.

Country i's flow budget constraint at each point in time is written as

$$c_i + g_i = Y_i \tag{5}$$

where Y_i is exogenously given income of country i and the relative price of two goods is set to be unity. For analytical simplicity these variables are assumed to be fixed through time. Since each country is a small country in an open economy, these assumptions would be plausible.[5]

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4 . The natural environment does not necessarily deteriorate. The national environment, if left alone, may tend naturally to re-generate. In other words, δ may be negative in a case of renewable resources such as forests and fishes which grow by self-reproductive. The analytical results would be valid so long as $\delta + \rho$ is positive.

5 . In order to justify the assumption that each country's income is exogenously given, we have to further assume that the world interest rate is equal to the discount rate, ρ. Although the gap between the initial capital stock held by each small country and the steady state capital stock determined by the world interest rate immediately disappears due to the instantaneous inflow (or outflow) of capital from (or to) the rest of world, the accumulation (or deccumulation) of foreign assets owned by each country takes place so long as the world interest rate and the discount rate are not equal.

3. Cooperative behavior: Pareto efficient outcome

Pareto efficient outcomes for the differential game are found from solving the following problem:

$$Max \ \sum_{i=1}^{n}\int_0^{\infty}\{U_1^i(c_i)+U_2^i(G)\}e^{-\rho t}dt \tag{6}$$

subject to (4), (5) and the initial stock of environment G_0. Before solving this problem, substitute (5) into (2) and (4). (2) may be rewritten as follows:

$$U_1^i(c)+U_2^i(G)=\tilde{\alpha}^i+\tilde{\beta}_1^i g_i-\frac{\gamma_1^i}{2}g_i^2+\beta_2^i G-\frac{\gamma_2^i}{2}G^2 \tag{7}$$

and (4) may be rewritten as follows:

$$\dot{G}=\sum_{i=1}^{n}\tilde{\varepsilon}_i g_i-A-\delta G \tag{8}$$

where

$$\tilde{\alpha}^i\equiv\alpha+\beta_1^i Y_i-\frac{\gamma_1^i}{2}Y_i^2$$

$$\tilde{\beta}_1^i\equiv-\beta_1^i+\gamma_1^i Y_i \ (<0)$$

$$\tilde{\varepsilon}_i\equiv\varepsilon_i+\sigma_i \ (>0)$$

$$A\equiv\sum_{i=1}^{n}\varepsilon_i Y_i \ (>0)$$

We assume that $\tilde{\beta}_1^i<0$, which comes from the customary assumption that the marginal utility of consumption is positive.

After substituting (5) into (6), the current value Hamiltonian is given by

$$H\equiv\sum_{i=1}^{n}\{U_1^i(Y-g_i)+U_2^i(G)\}+\mu[\sum_{i=1}^{n}\tilde{\varepsilon}_i g_i-A-\delta G] \tag{9}$$

where μ is the shadow price associated with the accumulation of environmental stock. Hence, the first order conditions are given as

$$\frac{\partial H}{\partial g_i} = \frac{\partial U_1^i}{\partial c_i}(-1) + \mu\tilde{\varepsilon}_i = 0 \tag{10}$$

$$\dot{\mu} - \rho\mu = -\frac{\partial H}{\partial G} = -\sum_{i=1}^{n} \frac{\partial U_2^i}{\partial G} + \mu\delta \tag{11}$$

We assume that countries are identical with respect to preferences as well as income. Since

$$U_c \equiv \frac{\partial U_1}{\partial c} = \beta_1 - \gamma_1 c$$

equation (10) reduces to

$$g_i = Y - \frac{1}{\gamma_1}(\beta_1 - \mu\tilde{\varepsilon}) \tag{12}$$

Thus, (11) reduces to

$$\dot{\mu} = (\rho + \delta)\mu - nU_G \tag{13}$$

where $U_G \equiv \partial U_2 / \partial G = \beta_2 - \gamma_2 G$.

At the steady state from the condition (13) = 0 we have

$$\mu = \frac{n(\beta_2 - \gamma_2 G)}{\rho + \delta} \tag{14}$$

Substituting (14) into (12), we also have

$$g_i^P = \kappa_1^P + \kappa_2^P G \tag{15}$$

where

$$\kappa_1^P \equiv Y - \frac{\beta_1}{\gamma_1} + \frac{\beta_2}{\gamma_1}\frac{n\tilde{\varepsilon}}{\rho + \delta} > 0$$

$$\kappa_2^P \equiv -\frac{\gamma_2}{\gamma_1}\frac{n\tilde{\varepsilon}}{\rho + \delta} < 0$$

From the condition (8) = 0 and using the assumption of symmetry in the long run,

$$\sigma n Y = n\tilde{\varepsilon}c + \delta G \tag{16}$$

Substituting (5) and (15) into (16) and rearranging yields

$$\overline{G}^{P} = \frac{n\tilde{\varepsilon}\kappa_1^{P} - A}{\delta - n\tilde{\varepsilon}\kappa_2^{P}} \tag{17}$$

On the other hand, from (10) and the condition (13)=0 we have

$$n\frac{U_G}{U_c} = \frac{\rho + \delta}{\tilde{\varepsilon}} \tag{18}$$

Equations (16) and (18) together determine the steady state values of G [given explicitly by (17)] and c. Figure 1 shows the feasibility condition (16) at the steady state as line AB. The Pareto efficient solution at the steady state is given by point P, where (18) is satisfied on line AB. Equation (18) can be viewed as a dynamic version of familiar Samuelson's rule for the provision of public goods. Since an increase in one unit of g will directly improve G by the amount of ε, and indirectly improve G by the amount of σ thereby reducing private consumption by the same amount, the total utility of public goods (relative to that of private consumption) increases by the amount of $\tilde{\varepsilon}(\equiv \varepsilon + \sigma)$ multiplied by U_G / U_c. Thus, $n\tilde{\varepsilon}U_G / U_c$ means the total marginal benefit of environmental expenditure (g) over all countries, while $\rho + \delta$ means the marginal cost of environmental expenditure.

4. The open-loop strategies

Let us investigate the open-loop strategies in a decentralized world. This type of Nash equilibrium concept presumes that the contribution made by each country in the quality of environment at each point in time is only conditioned on the initial stock of environment, $G(0)$, and that each country precommits itself to a path of contribution. It follows that the expected contributions of the others do not depend on past or current stocks or on past or current contributions of each country.

The problem is formulated as follows: Country i maximizes (3) subject to (5), (8) and the exogenously given $G(0)$ and $g_j(t)$ $j \neq i$ at time 0. Under the symmetric assumption the first order conditions are as follows

$$U_c(-1) + \mu\tilde{\varepsilon} = 0 \tag{19}$$

$$\dot{\mu} - \rho\mu = -U_G + \delta\mu \tag{20}$$

Recalling that

$$U_c \equiv \beta_1 - \gamma_1 c_i = \beta_1 - \gamma_1(Y - g_i)$$

$$U_G \equiv \beta_2 - \gamma_2 G$$

equation (19) reduces to

$$g_i = Y - \frac{1}{\gamma_1}(\beta_1 - \mu\tilde{\varepsilon}) \tag{21}$$

and equation (20) reduces to

$$\dot{\mu} = (\rho + \delta)\mu - (\beta_2 - \gamma_2 G) \tag{22}$$

At the steady state the condition (22)=0 implies

$$\mu = \frac{\beta_2 - \gamma_2 G}{\rho + \delta} = \frac{U_G}{\rho + \delta} \tag{23}$$

Substituting (23) into (21), we have

$$g_i^O = \kappa_1^O + \kappa_2^O G \tag{24}$$

where

$$\kappa_1^O \equiv Y - \frac{\beta_1}{\gamma_1} + \frac{\beta_2}{\gamma_1}\frac{\tilde{\varepsilon}}{\rho + \delta} > 0$$

$$\kappa_2^O \equiv -\frac{\gamma_2}{\gamma_1}\frac{\tilde{\varepsilon}}{\rho + \delta} < 0$$

Compare between the steady state stock of environment under the Pareto efficient solution ($\overline{G}^P$) and that under the open-loop Nash equilibrium ($\overline{G}^O$). The steady state stock of $\overline{G}^O$ is given as

$$\overline{G}^O = \frac{n\tilde{\varepsilon}\kappa_1^O - A}{\delta - n\tilde{\varepsilon}\kappa_2^O} \tag{25}$$

From (19) and (23) in the steady state we have

$$\frac{U_G}{U_c} = \frac{\rho + \delta}{\tilde{\varepsilon}} \tag{26}$$

Thus, as shown in Figure 1, the open-loop steady-state equilibrium is given by point O, which satisfies (26) on line AB so long as $n > 1$; U_G / U_c given by (26) is greater than U_G / U_c given by (18). Equation (26) means that the *per-capita* marginal benefit of g is equal to the marginal cost of g, while (18) means that the *total* marginal benefit of g is equal to the marginal cost of g. As the marginal rate of substitution of G with respect to c is greater at point O than at point P, G is too little and c is too much at point O compared with the Pareto efficient allocation. In the open-loop Nash equilibrium each country sticks to the optimal strategies that were chosen at the starting point and completely ignores the information on the actual stock of environment that unravels during the transition towards the steady state. It follows that this equilibrium can be regarded as a dynamic counterpart of the static Nash equilibrium. Thus, the resulting under-provision of the environmental stock in the steady state is a dynamic version of the well-known under-provision of the voluntarily supplied public good at the static Nash equilibrium [Bergstrom et. al. (1986)].

5. Perfect Nash equilibrium

The closed-loop (or subgame perfect) Nash equilibrium allows each country to condition its contribution to public goods on the current and past stocks of environment. The subgame perfect Nash equilibrium requires that for each subgame the relevant part of the set of strategies be in Nash equilibrium. The subgame-perfect or feedback Nash equilibrium can be found by using dynamic programming.

Let $V^i(G)$ be the value of country i of the game that starts at $G(0) = G$. Using the value function approach the feedback equilibrium strategies must satisfy the following Jacobi-Bellman condition:

$$\rho V^i(G) = \underset{g_i}{Max}[\tilde{\alpha} + \tilde{\beta}_1 g_i - \frac{\gamma_1}{2} g_i^2 + \beta_2 G - \frac{\gamma_2}{2} G^2 + V_G^i\{\tilde{\varepsilon} \sum_{i=1}^{n} g_i - A - \delta G\}] \tag{27}$$

Since the right hand side of (27) is concave with respect to g_i, the g_i that maximizes it is given by

$$g_i = \frac{1}{\gamma_1}(\tilde{\beta}_1 + V_G^i \tilde{\varepsilon}) \tag{28}$$

Consider the quadratic value function

$$V^i(G) = \theta_0 + \theta_1 \gamma_1 G + \frac{\theta_2}{2} \gamma_1 G^2 \tag{29}$$

Differentiating this value function yields

$$V_G^i = \theta_1 \gamma_1 + \theta_2 \gamma_1 G$$

Substituting this into (28), we have

$$g_i^S = \kappa_1^S + \kappa_2^S G \tag{30}$$

where

$$\kappa_1^S \equiv \frac{\tilde{\beta}_1}{\gamma_1} + \theta_1 \tilde{\varepsilon}$$

$$\kappa_2^S \equiv \tilde{\varepsilon} \theta_2$$

Substitute (29), (30), and V_G^i into the functional equation (27) to obtain

$$\rho[\theta_0 + \theta_1\gamma_1 G + \frac{1}{2}\gamma_1\theta_2 G^2] = \tilde{\alpha} + \tilde{\beta}_1[\frac{\tilde{\beta}_1}{\gamma_1} + (\theta_1 + \theta_2 G)\tilde{\varepsilon}] + \beta_2 G$$

$$-\frac{\gamma_1}{2}\{\frac{\tilde{\beta}_1}{\gamma_1} + (\theta_1 + \theta_2 G)\tilde{\varepsilon}\}^2 - \frac{\gamma_2}{2}G^2 + (\gamma_1\theta_1 + \gamma_1\theta_2 G)\{n\tilde{\varepsilon}g_i - \delta G - A\}$$

Or equivalently

$$0 = -\rho[\theta_0 + \gamma_1\theta_1 G + \frac{1}{2}\gamma_1\theta_2 G^2] + \tilde{\alpha} + \frac{\tilde{\beta}_1^2}{\gamma_1} + \tilde{\beta}_1(\theta_1 + \theta_2 G)\tilde{\varepsilon} + \beta_2 G -$$

$$\frac{\gamma_1}{2}\{(\frac{\tilde{\beta}_1}{\gamma_1})^2 + \frac{2\tilde{\beta}_1}{\gamma_1}(\theta_1 + \theta_2 G)\tilde{\varepsilon} + (\theta_1^2 + 2\theta_1\theta_2 G + \theta_2^2 G^2)\tilde{\varepsilon}^2\} - \frac{\gamma_2}{2}G^2 + \quad (31)$$

$$(\gamma_1\theta_1 + \gamma_1\theta_2 G)[n\tilde{\varepsilon}\frac{\tilde{\beta}_1}{\gamma_1} + n(\theta_1 + \theta_2 G)\tilde{\varepsilon}^2 - \delta G - A]$$

Since this equation must be satisfied for every possible G, the constant term and each of the coefficients of the G-terms on the right hand side of this equation must be zero. This requirement generates the following system of equations in the value function parameters. Because of the assumed symmetry of value functions, an identical system of equations is generated for each country. The equation corresponding to the coefficient of the G^2-term is given by

$$-\frac{\rho}{2}\gamma_1\theta_2 - \frac{\gamma_1}{2}\theta_2^2\tilde{\varepsilon}^2 - \frac{\gamma_2}{2} + \gamma_1\theta_2 n\theta_2\tilde{\varepsilon}^2 - \gamma_1\theta_2\delta = 0$$

Rearranging yields

$$\gamma_1\tilde{\varepsilon}^2(\frac{2n-1}{2})\theta_2^2 - \gamma_1(\delta + \frac{\rho}{2})\theta_2 - \frac{\gamma_2}{2} = 0 \quad (32)$$

Applying the quadratic formula, we have

$$\theta_2 = \frac{(\delta + \frac{\rho}{2}) \pm \sqrt{(\delta + \frac{\rho}{2})^2 + \frac{\gamma_2}{\gamma_1}\tilde{\varepsilon}^2(2n-1)}}{\tilde{\varepsilon}^2(2n-1)} \quad (33)$$

Let denote these two roots λ_1 and λ_2, respectively. They are real, because

$$(\delta+\frac{\rho}{2})^2+\frac{\gamma_2}{\gamma_1}\tilde{\varepsilon}^2(2n-1)>0$$

Moreover, it is clear that one root is positive and the other root is negative, i.e.,

$$\lambda_1<0<\lambda_2$$

As shown later (from the stability analysis in Appendix 1), λ_1 is the only root for θ_2.

The equation corresponding to the coefficient of the G-term is written as

$$-\rho\gamma_1\theta_1+\tilde{\beta}_1\theta_2\tilde{\varepsilon}+\beta_2-\frac{\gamma_1}{2}2\frac{\tilde{\beta}_1}{\gamma_1}\theta_2\tilde{\varepsilon}-\frac{\gamma_1}{2}2\theta_1\theta_2\tilde{\varepsilon}^2+\gamma_1\theta_1 n\theta_2\tilde{\varepsilon}^2$$

$$-\gamma_1\theta_1\delta+\gamma_1\theta_2 n\tilde{\varepsilon}\frac{\tilde{\beta}_1}{\gamma_1}+\gamma_1\theta_2 n\theta_1\tilde{\varepsilon}^2-\gamma_1\theta_2 A=0$$

Or equivalently

$$-\rho\gamma_1\theta_1+\beta_2+(2n-1)\gamma_1\theta_1\theta_2\tilde{\varepsilon}^2-\gamma_1\theta_1\delta+\theta_2 n\tilde{\varepsilon}\tilde{\beta}_1-\gamma_1\theta_2 A=0 \tag{34}$$

Substituting λ_1 into θ_2 in (34) yields

$$\theta_1=\frac{\beta_2+\lambda_1 n\tilde{\varepsilon}\tilde{\beta}_1-\gamma_1\lambda_1 A}{(\rho+\delta)\gamma_1-(2n-1)\gamma_1\lambda_1\tilde{\varepsilon}^2} \tag{35}$$

Since $\tilde{\beta}_1\equiv-\beta_1+\gamma_1 Y<0$, we can show that $\lambda_1 n\tilde{\varepsilon}\tilde{\beta}_1-\gamma_1\lambda_1 A>0$ and hence $\theta_1>0$.

Considering (30), we have

$$\kappa_1^S=\frac{\tilde{\beta}_1}{\gamma_1}+\theta_1\tilde{\varepsilon}=Y-\frac{\beta_1}{\gamma_1}+\tilde{\varepsilon}\frac{\beta_2+\lambda_1 n\tilde{\varepsilon}\tilde{\beta}_1-\gamma_1\lambda_1 A}{(\delta+\rho)\gamma_1-(2n-1)\gamma_1\lambda_1\tilde{\varepsilon}^2}>0$$

$$\kappa_2^S=\tilde{\varepsilon}\lambda_1=\frac{(\delta+\frac{\rho}{2})-\sqrt{(\delta+\frac{\rho}{2})^2+\frac{\gamma_2}{\gamma_1}\tilde{\varepsilon}^2(2n-1)}}{\tilde{\varepsilon}(2n-1)}<0$$

At the steady state we have as in the previous sections

$$\overline{G}^S=\frac{n\tilde{\varepsilon}\kappa_1^S-A}{\delta-n\tilde{\varepsilon}\kappa_2^S} \tag{36}$$

Equation (28) can be rewritten as

$$U_c = V_G \tilde{\varepsilon} = (\theta_1 \gamma_1 + \theta_2 \gamma_1 G)\tilde{\varepsilon} \tag{37}$$

As shown in Appendix 2, we have

$$\frac{(\rho + \delta)V_G}{U_G} < 1 \text{ and hence } \frac{U_G}{U_c} > \frac{\rho + \delta}{\tilde{\varepsilon}} \tag{38}$$

at the closed-loop steady-state solution.

As shown in Figure 1, the closed-loop steady-state solution is given by point S, which satisfies the second inequality in (38). Since the marginal rate of substitution of G at point S is greater than that at point O, G is too little and c is too much at point S, compared with point O. The free riding problem is further aggravated when players' contributions are conditional on the observable current collective contributions compared with that at the open-loop solution. Without commitment the resulting quality of environment is lower than that with the enforceable commitment case. The economic insight behind the result is almost the same as that of Fershtman and Nitzan (1991), except that the equilibrium values of these endogenous variables are also sensitive to changes in the parameters of utility function. For example, in any strategies the steady-state value of g falls with γ_2. In other words, the greater γ_2, or the greater the concavity of the utility function with respect to G [or the faster the marginal utility of G is to decline], the smaller the steady state level of G. Moreover, setting $\gamma_2 = 0$ yields

$$\overline{G}^P > \overline{G}^O = \overline{G}^S$$

This result corresponds to Theorem 4 in Fershtman and Nitzan in which the technology of production is linear.

6. Optimal Pigovian consumption taxes

We now introduce consumption taxes and investigate the normative role of tax policy to internalize the free riding problem[6]. With a consumption tax rate, τ, the budget constraint (5) is rewritten as

$$(1+\tau)c_i + g_i = Y_i + T_i \tag{39}$$

where T_i is a lump-sum transfer given to country i. The overall government budget constraint in the world means

$$\sum_{i=1}^{n} \tau c_i = \sum_{i=1}^{n} T_i \tag{40}$$

Revenue from consumption taxes is uniformly returned to the private sector in a lump-sum way. Since redistribution of income between countries is well known to be neutral, we can focus on the substitution effect of consumption taxes .[7]

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[6] . The world government actually does not exist. Nevertheless, we think that it is important to investigate the world-wide Pigovian taxation in the following sense. First, in order to combat global environmental issues (or global commons pollution) such as acid rain, global warming, or ozone depletion, many governments have been negotiating, bargaining, and making considerable efforts to reach international agreements on how to undertake worldwide environmental policies for a long time. European Union has already adopted several common environmental agreements which apply equally to its member countries. The global ban of the use of chlorofluorocabons is one of the most successful agreement. Such international agreements and cooperation can be thought of as substitutes for the world government. Second, in a real world different countries independently impose different taxes and voluntarily spend different environmental improving expenditures in controlling transboundary or global commons pollution. However, to effectively control those types of pollution international cooperation is indispensable. Our hypothetical world government is a convenient vehicle which would provide a reference point on how to coordinate those countries in achieving this end. Third, some of environmental problems are country specific and many regional governments may voluntarily spend environmental improving expenditures. In such a situation the central government can impose the widespread Pigovian taxes.

[7] . As for the neutrality theorem, see Shibata (1971) and Warr (1983). Buchholz and Konrad (1995) and Ihori (1996) discussed several interesting cases where the neutrality

Under the symmetric assumption we now have

$$\tilde{\varepsilon}(\tau) \equiv \frac{\varepsilon}{1+\tau} + \sigma$$

$$A(\tau) \equiv \frac{n\varepsilon}{1+\tau}(Y+T)$$

First, we consider the open-loop strategies. In the open-loop case we now have in place of (19)

$$-U_c \frac{1}{1+\tau} + \mu\tilde{\varepsilon}(\tau) = 0 \qquad (41)$$

Thus, at the open-loop solution in place of (26)

$$\frac{U_G}{U_c} = \frac{\rho+\delta}{(1+\tau)\tilde{\varepsilon}(\tau)} \qquad (42)$$

In order to realize the Pareto efficient allocation at the open-loop steady-state solution, considering (18), we need the following equality:

$$\frac{\rho+\delta}{(1+\tau)\tilde{\varepsilon}(\tau)} = \frac{\rho+\delta}{\tilde{\varepsilon}n}$$

Thus, the optimal Pigovian consumption tax rate at the open-loop steady-state solution, τ^O, is determined from

$$\tau^O = \frac{(\varepsilon+\sigma)(n-1)}{\sigma} \qquad (43)$$

which is increasing with the number of countries (noting that if n=1, $\tau^O = 0$) and the degree of environmental degradation, while decreasing with the degree of productivity from environmental expenditures. These properties are intuitively plausible.

Next, let us consider the perfect Nash equilibrium. At the closed-loop solution with the Pigovian consumption tax we now have

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result does not hold.

$$\tilde{\beta}_1(\tau) \equiv -\frac{\beta_1}{1+\tau} + \frac{\gamma_1(Y+T)}{(1+\tau)^2}$$

$$\tilde{\gamma}_1(\tau) \equiv \frac{\gamma_1}{(1+\tau)^2}$$

and in place of (37)

$$U_c = V_G[\varepsilon + (1+\tau)\sigma] \tag{44}$$

Thus, in order to realize the Pareto efficient allocation at the closed-loop steady-state solution, considering (18) and (44), we have the following condition:

$$\frac{(\varepsilon + \sigma)n}{\varepsilon + \sigma(1+\tau)} = \pi \tag{45}$$

where

$$\pi \equiv \frac{(\rho + \delta)V_G}{U_G} < 1 \quad \text{from (38)}$$

The optimal Pigovian consumption tax rate at the closed-loop steady-state solution, τ^S, is then determined from

$$\tau^S = \frac{(\varepsilon + \sigma)(n - \pi)}{\pi\sigma} \tag{46}$$

Note that (46) is not an explicit solution for τ^S as π is dependent on the steady state levels of c and G and hence on τ^S. Nevertheless, since $\pi < 1$, we can say that the optimal Pigovian consumption tax rate is smaller at the open-loop solution than at the closed-loop solution; that is, $\tau^O < \tau^S$. Intuitively, since the free-riding is more severe at the closed-loop solution than that at the open-loop solution, a higher rate of the consumption tax is required to rectify it at the former solution.[8]

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[8]. For analytical simplicity the model includes only one consumption good which is also the polluting good, so the consumption tax is a Pigovian tax in the sense that the tax internalizes the negative externality generated by consumption of that good.. This may

7. Adjustment speed

As shown in Appendix 1, the adjustment speeds of G under the Pareto efficient path, the open-loop path, and the closed-loop path are respectively given as

$$D^P = \frac{\rho - \sqrt{\rho^2 + 4\{(\rho+\delta)\delta + \frac{\gamma_2}{\gamma_1}\tilde{\varepsilon}^2 n^2\}}}{2} \qquad (47)$$

$$D^O = \frac{\rho - \sqrt{\rho^2 + 4\{(\rho+\delta)\delta + \frac{\gamma_2}{\gamma_1}\tilde{\varepsilon}^2 n\}}}{2} \qquad (48)$$

$$D^S = n\frac{(\delta + \frac{\rho}{2}) - \sqrt{(\delta + \frac{\rho}{2})^2 + \frac{\gamma_2}{\gamma_1}\tilde{\varepsilon}^2(2n-1)}}{2n-1} - \delta \qquad (49)$$

From these equations we can show the followings. At any strategies the speed of adjustment in absolute value increases with n, $\gamma_2, \varepsilon, \sigma$, and δ but decreases with γ_1 and ρ. The opposite effect of γ_1 reflects the fact that the larger γ_1, the less willing each country is to accept larger variations in g instantaneously. The larger ρ, the more is the future utilities discounted and the weaker is the incentive of each country to accumulate the stock of environment thus resulting in the lower speed of adjustment.

Comparing (47) and (48), it is easy to see in the absolute value

$$|D^P| > |D^O|$$

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not be the usual way to think about a consumption tax. Indeed, if the model had a vector of differentially-polluting consumption goods, then we would not be thinking in terms of a general consumption tax but need different Pigovian taxes on differentially-polluting consumption goods. In the latter case the Pigovian tax may be thought of as an excise tax rather than a consumption tax.

The adjustment speed of a Pareto efficient path is greater in the absolute value than that of an open-loop equilibrium path. Since the marginal utility of g is summed over all countries under the Pareto efficient path, there are stronger incentives to accumulate (or deaccumulate) G thereby accelerating the adjustment speed of G compared with the open-loop solution.

Let us then compare (48) and (49). From (48) we have

$$D^O - \frac{\rho}{2} = -\sqrt{(\frac{\rho}{2}+\delta)^2 + \frac{\gamma_2}{\gamma_1}\tilde{\varepsilon}^2 n}$$

On the other hand

$$D^S - \frac{\rho}{2} = \frac{1-n}{2n-1}(\delta + \frac{\rho}{2}) - \sqrt{\frac{n^2}{(2n-1)^2}(\frac{\rho}{2}+\delta)^2 + \frac{\gamma_2}{\gamma_1}\tilde{\varepsilon}^2 \frac{n^2}{2n-1}}$$

It is tedious but straightforward to show (see Appendix 3) that

$$(D^O - \frac{\rho}{2})^2 > (D^S - \frac{\rho}{2})^2$$

which means

$$\left|D^O\right| > \left|D^S\right|$$

Unlike the open-loop equilibrium, since each country knows that when G is increasing, her larger contribution leads to the smaller future contributions of the others, this anticipation forces each country to depress her contribution and hence the adjustment speed of G. When G is decreasing, the result is reversed. Thus, in either case the adjustment speed of the closed-loop equilibrium path is smaller in the absolute value than that of the open-loop equilibrium path.

Let us then examine the impact of Pigovian consumption taxes on the adjustment speed under open-loop and closed-loop paths. In the open-loop equilibrium with the consumption tax rate the adjustment speed (48) is rewritten as

$$D^O = \frac{\rho - \sqrt{\rho^2 + 4\{(\rho+\delta)\delta + \frac{\gamma_2}{\gamma_1}\tilde{\varepsilon}(\tau)^2(1+\tau)^2 n\}}}{2} \tag{50}$$

Since $\tilde{\varepsilon}(\tau)(1+\tau)$ is increasing with τ, the adjustment speed (or the absolute value) of D^O is increasing with the consumption tax rate.

In the closed-loop equilibrium the adjustment speed is rewritten as

$$D^S = n\frac{(\delta + \frac{\rho}{2}) - \sqrt{(\delta + \frac{\rho}{2})^2 + \frac{\gamma_2}{\gamma_1(\tau)}\tilde{\varepsilon}(\tau)^2(2n-1)}}{2n-1} - \delta \tag{51}$$

Since $\tilde{\varepsilon}(\tau)^2 / \gamma_1(\tau)$ is increasing with τ, the adjustment speed is again increasing with the consumption tax rate. This is due to the substitution effect of Pigovian consumption taxes; that is, a higher rate of the consumption tax makes private consumption more expensive relative to environmental expenditures (g) thereby raising g and thus the speed of accumulation of G.

We have shown that the adjustment speed of the Pareto efficient path is greater than that of the open or closed-loop equilibrium path. An increase in the Pigovian consumption tax can raise the adjustment speed in the non-cooperative equilibrium close to the level associated with a Pareto efficient path and hence it is desirable in terms of the adjustment speed. For example, suppose $G(0) < \overline{G}^P$. Then, an introduction of (or an increase in) τ will raise the speed of accumulation of G as well as the steady state level of G. The dynamic movement of G becomes closer to the Pareto efficient path. On the contrary, if $G(0) > \overline{G}^P$, an introduction of (or an increase in) τ will raise the speed of deterioration of environmental quality although the steady state level of G is raised by consumption taxes. In such a case, world-wide consumption taxation has two different impacts of the dynamic movement of G; short-run deterioration and long-run improvement of environmental quality.

8. Conclusion

We have shown that the free riding problem of environmental externalities is aggravated when countries' contributions are conditional on the observable collective contributions. Without commitment lower contributions, environmental quality, and welfare are made relative to the enforceable commitment case.

The world government may internalize the free riding problem by introducing Pigovian consumption taxes at either open- or closed-loop solution. The optimal level of the world-wide consumption tax at the open-loop solution is increasing with the number of countries and the degree of environmental degradation, while decreasing with the degree of environmental improvement from environmental expenditures. The optimal Pigovian consumption tax rate is smaller at the open-loop solution than at the closed-loop solution.

We have explicitly derived the adjustment speed of environmental quality under alternative solutions and also shown that the adjustment speed of the Pareto efficient path is greater than either that of the open- or closed-loop equilibrium path under the plausible utility function. An increase in the consumption tax will raise the adjustment speed in the non-cooperative equilibrium. When the initial level of environmental quality is very high, Pigovian consumption taxation has two different impacts; short-run deterioration and long-run improvement of environmental quality. According to recent scientific investigations [e.g. Intergovernmental Panel on Climate Change (1990)], the stock of environment, such as ozone layer, forests, or air quality is quickly diminishing on a global scale over the last two decades. In the light of this observation, in using the world-wide Pigovian consumption tax to curb deterioration of the global environment, such as ozone depletion, acid rain, or global warming, there is a dilemma between accelerating the speed of deterioration during the transition and achieving the Pareto optimal level of environmental stocks in the steady state.

A model as simple as the one presented in this paper is bound to have many limitations. Among the more serious ones, calling for further research, are the absence of capital accumulation and heterogeneous agents or countries. Although our conjecture is that the main results would be the same, so long as each country is a small country in a growing world economy and the world interest rate is equal to the discount rate, a more rigorous and full fledged dynamic analysis is needed in order to treat more general cases. In so doing, we have to explicitly incorporate two state variables of environmental quality and capital stock into the present model. Another extension would incorporate heterogeneous countries or agents with respect to environmental parameters, income levels, or Pigovian taxes. Admittedly, the comparative static and comparative dynamic results obtained here rest on crucially on our specification of the utility function. In order to see the robustness of our results, therefore, it is desirable to carry out the present analysis under more general utility functions, although it may not be possible to get closed-form, analytical solutions for a closed-loop Nash equilibrium.

Appendix 1: Stability

Let us investigate the stability property of the Pareto efficient path. The system is given by

$$\begin{bmatrix} \dot{\mu} \\ \dot{G} \end{bmatrix} = \begin{bmatrix} \rho+\delta & n\gamma_2 \\ \dfrac{\tilde{\varepsilon}^2 n}{\gamma_1} & -\delta \end{bmatrix} \begin{bmatrix} \mu \\ G \end{bmatrix} + \begin{bmatrix} -\beta_2 n \\ \tilde{\varepsilon} n (Y - \dfrac{\beta_1}{\gamma_1}) - A \end{bmatrix} \tag{A1}$$

The characteristic equation is given by

$$\{(\rho+\delta)-q\}(-\delta-q)-\frac{\gamma_2}{\gamma_1}\tilde{\varepsilon}^2 n^2 = 0 \tag{A2}$$

We have

$$q = \frac{\rho \pm \sqrt{\rho^2 + 4\{(\rho+\delta)\delta + \dfrac{\gamma_2}{\gamma_1}\tilde{\varepsilon}^2 n^2\}}}{2} \tag{A3}$$

That is, the two roots are real, one positive and the other negative. Thus, if we take the stable solution and use the initial condition, we get the equilibrium path

$$G(t) = \overline{G}^P + (G(0) - \overline{G}^P)e^{D^P t}$$

where

$$D^P \equiv \frac{\rho - \sqrt{\rho^2 + 4\{(\rho+\delta)\delta + \dfrac{\gamma_2}{\gamma_1}\tilde{\varepsilon}^2 n^2\}}}{2} < 0 \tag{A4}$$

The system is globally asymptotic stable.

The system of the open-loop equilibrium path is given by

$$\begin{bmatrix} \dot{\mu} \\ \dot{G} \end{bmatrix} = \begin{bmatrix} \rho+\delta & \gamma_2 \\ \dfrac{\tilde{\varepsilon}^2 n}{\gamma_1} & -\delta \end{bmatrix} \begin{bmatrix} \mu \\ G \end{bmatrix} + \begin{bmatrix} -\beta_2 \\ \tilde{\varepsilon} n (Y - \dfrac{\beta_1}{\gamma_1}) - A \end{bmatrix} \tag{A5}$$

The characteristic equation is given as

$$\{(\rho+\delta)-q\}(-\delta-q)-\frac{\gamma_2}{\gamma_1}\tilde{\varepsilon}^2 n=0 \tag{A6}$$

Applying the quadratic formula, we have

$$q=\frac{\rho \pm \sqrt{\rho^2+4\{(\rho+\delta)\delta+\frac{\gamma_2}{\gamma_1}\tilde{\varepsilon}^2 n\}}}{2} \tag{A7}$$

Thus, we get the equilibrium path

$$G(t)=\overline{G}^O+(G(0)-\overline{G}^O)e^{D^O t}$$

where

$$D^O \equiv \frac{\rho - \sqrt{\rho^2+4\{(\rho+\delta)\delta+\frac{\gamma_2}{\gamma_1}\tilde{\varepsilon}^2 n\}}}{2}<0 \tag{A8}$$

This path is globally asymptotic stable.

Finally, let us investigate the closed-loop equilibrium path. After substituting (30), the solution of homogeneous part of

$$\dot{G}+(\delta-n\tilde{\varepsilon}\kappa_2^S)G=0$$

is

$$G(t)=Me^{D^S t} \tag{A9}$$

where M is the constant of integration and since $\kappa_2^S<0$, we know that

$$D^S \equiv n\tilde{\varepsilon}\kappa_2^S-\delta<0 \tag{A10}$$

We get the feedback Nash equilibrium contribution path, which is given as

$$G(t)=\overline{G}^S+(G(0)-\overline{G}^S)e^{D^S t} \tag{A11}$$

Note that the negativity of D^S guarantees that the equilibrium path is globally asymptotic stable.

Appendix 2: Derivation of (38)

Let us compare $U_G/(\rho+\delta)$ and V_G. We investigate the sign of

$$Q \equiv V_G - \frac{U_G}{\rho+\delta} = \theta_1\gamma_1 + \theta_2\gamma_1 G - [\frac{\beta_2}{\rho+\delta} - \frac{\gamma_2}{\rho+\delta}G]$$

From (34)

$$\begin{aligned}(\rho+\delta)\gamma_1\theta_1 - \beta_2 &= \theta_2\theta_1\gamma_1\tilde{\varepsilon}^2(2n-1) + \theta_2 n\tilde{\varepsilon}\tilde{\beta}_1 - \gamma_1\theta_2 A \\ &= \theta_2[\theta_1\gamma_1\tilde{\varepsilon}^2(n-1) + \theta_1\gamma_1\tilde{\varepsilon}^2 n + n\tilde{\varepsilon}\tilde{\beta}_1 - \gamma_1 A]\end{aligned} \tag{A12}$$

Hence

$$\gamma_1\theta_1 - \frac{\beta_2}{\rho+\delta} = \frac{\theta_2}{\rho+\delta}[\theta_1\gamma_1\tilde{\varepsilon}^2(n-1) + \theta_1\gamma_1\tilde{\varepsilon}^2 n + n\tilde{\varepsilon}\tilde{\beta}_1 - \gamma_1 A] \tag{A12)'$$

From κ_1^S, κ_2^S and (36) we also know

$$\theta_1\gamma_1\tilde{\varepsilon}^2 n + n\tilde{\varepsilon}\tilde{\beta}_1 - \gamma_1 A = \gamma_1(\delta - n\tilde{\varepsilon}^2\theta_2)G \tag{A13}$$

Substituting (A13) into (A12)', and then the resulting expression into Q, we obtain

$$Q = \frac{\theta_2}{\rho+\delta}[\theta_1\gamma_1\tilde{\varepsilon}^2(n-1) + \gamma_1(\delta - n\tilde{\varepsilon}^2\theta_2)G] + [\theta_2\gamma_1 + \frac{\gamma_2}{\rho+\delta}]G \tag{A14}$$

Notice that from (32) we have

$$\tilde{\varepsilon}^2(2n-1)\theta_2^2 = (2\delta+\rho)\theta_2 + \frac{\gamma_2}{\gamma_1} \tag{A15}$$

Substituting (A15) into (A14), we have

$$\begin{aligned}Q &= \frac{\theta_2\gamma_1\tilde{\varepsilon}^2(n-1)}{\rho+\delta}[\theta_1 + \frac{1}{\tilde{\varepsilon}^2(n-1)}(2\delta - n\tilde{\varepsilon}^2\theta_2 + \rho + \frac{\gamma_2}{\gamma_1\theta_2})G] \\ &= \frac{\theta_2\gamma_1\tilde{\varepsilon}^2(n-1)}{\rho+\delta}[\theta_1 + \theta_2 G]\end{aligned} \tag{A16}$$

which is negative since $V_G > 0$ and $\theta_2 < 0$. $Q<0$ means (38).

Appendix 3: Adjustment speed

Let us compare $D^O - \frac{\rho}{2}$ and $D^S - \frac{\rho}{2}$.

$$(D^O - \frac{\rho}{2})^2 - S_1 = \frac{2n(n-1)}{(2n-1)^2}[(\delta + \frac{\rho}{2})^2 + \frac{\gamma_2}{\gamma_1}\tilde{\varepsilon}^2 \frac{2n-1}{2}] \quad \text{(A17)}$$

where

$$S_1 \equiv \frac{(n-1)^2}{(2n-1)^2}(\delta + \frac{\rho}{2})^2 + \frac{n^2}{(2n-1)^2}(\delta + \frac{\rho}{2})^2 + \frac{n^2}{2n-1}\frac{\gamma_2}{\gamma_1}\tilde{\varepsilon}^2$$

On the other hand we have

$$(D^S - \frac{\rho}{2})^2 = S_1 + S_2 \quad \text{(A18)}$$

where

$$S_2 \equiv \frac{2n(n-1)}{(2n-1)^2}(\delta + \frac{\rho}{2})\sqrt{(\delta + \frac{\rho}{2})^2 + \frac{\gamma_2}{\gamma_1}\tilde{\varepsilon}^2(2n-1)}$$

Since $(D^O - \frac{\rho}{2})^2 - S_1 > S_2$, we obtain $(D^O - \frac{\rho}{2})^2 > (D^S - \frac{\rho}{2})^2$.

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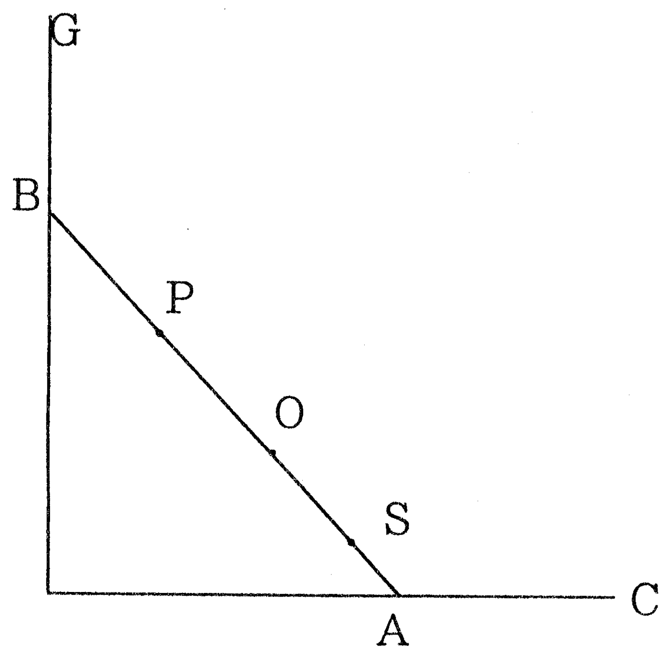

Figure 1