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An Economic Analysis of Public Transfers

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Abstract

This paper develops simple models of public transfers. The sources of income inequality are differences in ability and in luck. The government employs a redistribution policy due to altruistic motives in the case of ability differences. We consider the case where the government reoptimizes income transfers after it observes the outcome of private activities. When the source of income inequality is differences in luck, the economy creates a mutual insurance or provides public goods due to risk sharing motives. We derive a paradoxical result that a more able individual would not enjoy higher welfare than a less able individual. We also investigate how public transfers react to increases in income level and income inequality.

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1. Introduction

When the size of a national economy becomes large, it is likely to have a large amount of public transfers. As shown in Table 1, an increase in transfer payments of the general government is the most important factor in making the government bigger in the developed countries during the last decade. Economic growth itself may have a positive effect on income transfers. At the same time an increase in income inequality due to differences in ability and/or luck may induce more public transfers. The recent experience from some countries suggests that the sources of income inequality are mainly differences in ability and in luck, and would be important to explain the degree of income transfers. By incorporating ex-ante ability and/or ex-post income differences into simple models with altruistic and risk sharing motives, this paper examines possible reasons why the large economy has a big government in terms of public transfers.

In the field of public economics there have been intensive attempts to investigate the positive and normative aspects of public transfers. There are at least two useful formulations to examine theoretically the mechanism of income redistribution. First of all, the literature on optimal income taxation mainly treats differences in income as being due to differences in ex-ante ability. Among others, Mirrlees (1971) has presented a very important theoretical framework of optimal income redistribution due to altruism. Section 2 first briefly summarizes both his analytical framework and some policy implications. Section 2 then investigates the case where the government reoptimizes income transfers after it observes the ex-post income differences from private activities. This would raise a problem of time inconsistency, leading to some paradoxical results.

Second, the source of ex-post income inequality may be due to differences in luck. Some income transfers may hence be caused by risk sharing motives. Public assistance and social insurance programs are such examples. Although transfers could be provided by private interfamily transfers, the idea of public transfers as risk sharing motivated safety nets has gained wide acceptance. A pervasive network of public safety nets could also be provided by public goods. Section 3 investigates policy implications of income transfers and public spending when the source of inequality is due to differences in luck. Finally, section 4 concludes this paper.

2. Optimal Income Transfers

2.1 Mirrlees' Model

2.1.1 Theoretical Framework

It is widely believed from the equity viewpoint that an important role of the government is to redistribute income among individuals. The social welfare based on an altruistic motive increases when income is redistributed from the rich to the poor. A progressive income tax is a direct means of effective redistribution in order to meet objectives of equity.

The Mirrlees model of optimal income taxes and transfers assumes that the economy is competitive and that households in the economy differ only in the levels of ex-ante skill in employment. A household's level of skill determines its wage and hence income. The skill level is exogenously given private information, which is not known to the government. The value of wage per hour, w, gives the relative effectiveness of the labor supplied per unit of time, so a high w household is more effective in production. The distribution of wage rates is exogenously given and public information, which is known to the government.

A household's supply of labor is denoted by ℓ and private consumption of the good by c. The supply of labor is limited by $0 \leq \ell \leq 1$ and $c \geq 0$. Denote the supply of effective labor of a household with ability w by $y(w) \equiv w\ell(w)$.

Normalizing the price of the consumption good at 1, $y(w)$ is also the household's pre-tax income in units of consumption.

All households have the same strictly concave utility function. This is an assumption that permits interpersonal comparability. This common utility function is denoted as

(1) $U = U(c, \ell)$

The analysis of optimal income taxation can only provide a qualitative characterization of the properties that the optimal tax function will have. It is not possible to calculate quantitatively the tax function without precisely stating the functional forms of utility, production, social welfare, and skill distribution. The complexity of the general model of optimal income taxation has led to considerable interest in the restricted case of linear income taxation[1].

2.1.2 Linear Income Tax System

The linear tax system may be described by just two parameters; the marginal tax rate and the lump-sum subsidy. In the linear tax function, the marginal tax rate is constant and there is an identical lump-sum tax or subsidy for all households. The linear tax structure corresponds to a negative income tax scheme, in which all households below a given income level receive a subsidy from the tax system and a flat tax system.

Let us briefly summarize the standard results in the case of linear taxation. Under the linear tax system a household with ability w supplying ℓ units of labor will pay tax of amount

(2) $T = -\alpha + (1 - \beta)w\ell$

where α is a lump-sum subsidy if positive (and a lump-sum tax if negative) and $1 - \beta$ is the marginal tax rate. The government's optimization problem is to

1 . For the recent development of optimal nonlinear taxation, see Diamond (1998).

choose two parameters of the linear tax system to maximize the social welfare subject to raising the required revenue.

There are several useful results on the optimal tax rate when various parameters change (see Ihori (1981) (1987)). The standard conjectures may be summarized as follows:

(1) The optimal marginal tax rate increases with the government's inequality aversion.

(2) The optimal marginal tax rate decreases with the elasticity of labor supply.

(3) The optimal marginal tax rate increases with the spread in abilities.

(4) The optimal marginal tax rate increases with the government's needs.

All of the conjectures (1)-(4) are not always analytically valid.

First of all, if the welfare weights were increased on the high-ability households so that equity was given less weight, the optimal tax rate would decrease. The optimal tax rate is bounded above by the optimal tax under a maxi-min social welfare function and this rate is bounded by the revenue-maximizing tax rate. It is now recognized that we can confirm conjecture (1).

Second, it is also shown that we may not obtain clear-cut analytical results in general with respect to (2) and (3). An increase in the disincentive effect of taxation would not always reduce the optimal tax rate. The effect of a mean-preserving increase in the spread of the skill distribution cannot be unambiguously signed. An increase in differences in wages would not necessarily lead to more transfers from the rich to the poor.

Finally, it is also interesting to note that we cannot always confirm analytically conjecture (4). For example, in the case of an educational investment model due to Shishinski (1971), when the social welfare function is given by the maximin criterion, an increase in the tax revenue requirement should be financed by a decrease in the income guarantee, while the marginal rate should be kept constant. In such a case a less progressive tax structure is desirable.

2.2 Alternative Redistribution Scheme

2.2.1 Theoretical Framework

The standard Mirrlees model assumes that after the government has chosen the optimal structure of taxation, individuals determine their labor supply and earn labor income. The government would commit the pre-determined tax-transfer policy. It is, however, possible and sometimes plausible in the real world to assume that the government can reoptimize the tax and transfer structure after it observes the outcome of private activities, that is, differences in ex-post income. If the government can determine additional public transfers, it would impose a very progressive transfer policy. In this section, we would like to investigate the optimal redistribution scheme in such a situation. We examine the case where the size of population, n, is not so large.

The structure of a game is as follows: At the first stage the government determines a tax-transfer policy and individuals determine their labor supply and earn labor income. At the second stage the government can reoptimize the tax and transfer structure to maximize social welfare after observing labor income of all individuals. For simplicity, we assume that the utility function (1) can be additively separated into two parts.

(3) $$U(c, \ell) = W(c) + V(\ell)$$

This two stage game can be solved backwards. At the second stage, the labor supply of all individuals is given. Hence, the social welfare can be affected only through changes in disposable income, namely, private consumption. Considering equation (2), private consumption is given as

(4) $$c_i = \alpha + \beta y_i$$

From the government budget constraint we also have

(5) $\alpha = (1-\beta)G$

where $G \equiv \sum_{i=1}^{n} y_i / n$ is the average income in the economy. Substituting (5) into (4), we obtain

(6) $c_i = (1-\beta)G + \beta y_i$

As before the social welfare is assumed to be represented by the utilitarian criterion. Since $V(\ell)$ is fixed, the optimization problem is to maximize the sum of utilities from private consumption

$$\sum_{i=1}^{n} W[(1-\beta)\bar{y} + \beta y_i]$$

Since the marginal utility from private consumption is decreasing, the optimal level of β is given as zero.

(7) $\beta^* = 0$

Equation (7) means perfect equalization of after-tax income (or private consumption). This is a very aggressive redistribution policy. This equation also implies that an increase in mean-preserving differences in wages would lead to more transfers from the rich to the poor. If it is true that differences in ability enlarge with economic growth, this policy may explain the positive relation between income transfers and the size of the national economy.

2.2.2 The First Stage Game

At the first stage the individual anticipates the optimal tax rule (7) and determines labor supply endogenously. We consider the case where the size of population, n, is not so large and hence an individual can affect the average income in the economy G by changing his labor supply. Substituting (7) into (5), we have under the perfect income equalization policy

(8) $c_i = G = \sum_{i=1}^{n} g_i$

where g_i denotes the individual's own income divided by population or normalized labor supply

(9) $$g_i \equiv y_i / n = w_i \ell_i / n$$

g_i may be regarded as his contribution to the average income. The individual's optimization problem is to maximize (3) subject to (8) and (9).

Denote by H the initial amount of time available for each individual. Then, the time constraint of individual i is given as

(10) $$H = x_i + \frac{n}{w_i} g_i$$

where x_i is the amount of leisure for individual i. Therefore, the optimization problem may reduce to the standard model of voluntary provision of public good, g_i.

Maximize $W(G) + z(x_i)$ subject to (10)

where $z(x_i) \equiv V(H - \ell_i)$ represents benefits from enjoying leisure. $n / w_i \equiv p_i$ may now be regarded as the personalized cost of providing the public good, which is the average income here. An individual can raise the average income by providing a larger amount of his own labor supply (and hence his own income), which may be regarded as voluntary provision of the public good. When w_i is large, he can contribute the public good at a low cost in terms of time.

Following the conventional literature on the public good provision, we assume that each individual determines his labor supply at Nash conjectures. In other words, we assume that each individual determines his supply of public good, treating the others' normalized labor supply g_i, prices p_i and H as given. We will exclude binding contracts or cooperative behavior among the agents (individuals) and will explore the outcome of non-cooperative Nash behavior. Substituting (8) into (10), we have

(10)' $$x_i + p_i G = H + p_i \sum_{j \neq i} g_j$$

In this Cournot-Nash model, define the expenditure function:

Minimize $E^i = x_i + p_i G$ subject to $U^i = U$.

Then, the following equation will determine U^i as a function of real income, $H + p_i \sum_{j \neq i} g_j$, which contains actual income and the externalities from the other individuals' provision of the public good.

(11) $$E(U^i, p_i) = H + p_i \sum_{j \neq i} g_j$$

By a variant of Shephard's Lemma we know

(12) $$G = G(U^i, p_i)$$

where G^i ($\equiv \partial E^i / \partial p_i$) is the compensated demand function for the public good of individual i.

By definition we have

(13) $$E^j = H + p_j(G - g_j)$$

From (11), (12) and (13) the voluntary provision model at the first stage may be summarized by the following n+1 equations.

(14) $$\sum_{i=1}^{n} \prod^i (p_i) E(U^i, p_i) = H \sum_{i=1}^{n} \prod^i (p_i) + \prod^i (p_i) G$$

(15) $$G = G(U^1, p_1) = G(U^2, p_2) = \ldots = G(U^n, p_n)$$

where $\prod^i (p_i) = p_1 p_2 \cdots p_{i-1} p_{i+1} \cdots p_n$. Equation (14) comes from (13). Namely, multiply (13) for E^1 by Π^1, multiply (13) for E^2 by Π^2, and so on, then sum the equations to get (14). Equation (15) comes from (12). These n+1 equations determine $U^1, U^2, \ldots, U^n$ (and G) as a function of $H, p_1, p_2, \ldots, p_n$. We assume the existence of a Nash equilibrium with $g_i > 0$.[2]

2. In order to present the results in the simplest way and in their strongest form, we assume that non-negativity constraints on providing public goods are non-binding in equilibrium. As remarked by Bergstrom et al. (1986) and Boadway et al. (1989), this assumption is relatively weak in some of the situations we analyze.

It is well recognized that public goods are under-supplied in a Nash equilibrium. When the government employs an aggressive redistribution policy, individuals would reduce labor supply, resulting in lower average income and welfare.

2.2.3 Paradoxical Result

It should be stressed that the perfect redistribution policy given as (7) does not imply that welfare is equalized among individuals. For simplicity we assume n=2 from now on. The economy may be separated into two groups; the rich and poor, the urban and rural, or the old and young. As shown in Ihori (1996), we have a seemingly paradoxical result that a less able individual can enjoy higher welfare than a more able individual.

The intuition behind this result is as follows. Suppose initially $w_1 = w_2$ and $U^1 = U^2$ but now individual 1's ability w_1 declines for some exogenous reasons. A decrease in individual 1's ability, w_1, reduces his labor supply and g_1, which will hurt individual 2. Individual 2 will then enjoy less leisure due to the negative income effect and hence will react to raise g_2. This reaction will raise the average income and hence benefit individual 1. We thus have $U^1 > U^2$. An individual with higher productivity does not necessarily enjoy higher welfare. An individual with lower productivity (lower wage) can enjoy higher welfare, which is a seemingly paradoxical result.[3] We would likely have the paradoxical result when the marginal propensity to consume leisure is high and the price elasticity of private consumption is high.

3 . If both p_1 and p_2 decrease at the same time, we would expect that the welfare of individual 1 will normally be raised. At least one individual gains by the decrease in p_1 and p_2. When preferences are identical, both individuals gain. See Ihori (1996).

Without the perfect equalization of income policy (7) an increase in wages (an increase in the productivity of providing the public good) is normally beneficial. We have shown, however, that this result is not necessarily valid in the perfect equalization scheme. Our analysis suggests that an individual may not have a strong incentive to reduce the marginal cost of providing the public good (or labor supply) as an increase in wages may reduce (not raise) the welfare of the individual[4].

It should be also noted that the public good (or labor supply) at the Nash equilibrium is under-provided due to the free-riding effect. The perfect income equalization policy is not efficient.

2.2.4 Transfer among Jurisdictions in Japan

The above analytical framework may also be applied to the income transfer among jurisdictions in Japan. The central government actually imposes redistribution transfers after it observes the outcome of regional economic activities. The local allocation tax plays an important role in standardizing the level of public services among local jurisdictions in Japan. This program equalizes the fiscal capacity among local governments by supplementing the shortage of tax revenue. The allocation tax enables local governments to provide public services at the level prescribed by the national government. When a local government does not maintain the level prescribed for public services, or has paid an excessive amount for the services, the

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4 . The positive spillover effect of the other individuals on individual 1 becomes large if the number of individuals is large. In other words, when the number of individuals is greater, an increase in p_1 is more likely to raise welfare of individual 1.

national government may reduce the local allocation tax for that local government.

Basic financial revenue is defined as general revenue which can be appropriated to meet the basic financial need. Basic financial revenue is calculated as the sum of the local transfer tax and a prescribed percentage of the standardized local tax revenue, 80% for prefectures and 75% for municipalities. Since some of government deficits are financed by the local allocation tax, almost all of the tax revenue is included in the calculation of basic financial revenue, which approximately corresponds to (7). Thus, such redistribution transfers may be regarded as the solution of the second stage game in section 2.2.1. Since any increase in local tax revenue would reduce the local allocation tax by the same amount, this would discourage local governments from developing their areas and enlarging the tax base within their territories. Local governments may not have a strong incentive to raise the technology of collecting local taxes since such an effort will reduce their own welfare. Thus, the central government has controlled the local tax system, resulting in little freedom of collecting local taxes for local governments in Japan. So long as the first stage game of local governments is assumed away, the extreme progressive transfer policy would work.

3. Risk and Redistribution

3.1 Mutual Insurance

3.1.1 Theoretical Framework

This section investigates positive aspects of income transfers when the source of inequality is differences in luck. Individuals need not passively accept risks that income and hence consumption would reduce in uncertain situations. There are several measures to protect against uncertainty. Typically, as Varian (1980) among others stressed, an insurance transfer to form safety nets implies connections between individuals to share effective income against income uncertainty. The mutual insurance will minimize the expected welfare loss of emergency costs.

For simplicity we consider a two-person economy and two states A and B. Assuming the additively separable utility function, individual i's expected utility W_i is given as

(16) $$W_i = (1-\alpha)V(c_i^A) + \alpha V(c_i^B)$$

where c_i is private consumption of individual i (i=1,2). c_i is subject to uncertainty. In state A, which occurs at the probability of 1-α, individual i enjoys c_i^A. In state B, which occurs at the probability of α, individual i cannot enjoy c_i^A but can enjoy c_i^B. α indicates the probability of an economically disruptive emergency or a bad state like being unemployed. Subscript i refers to the individual.

Individual i's budget constraint in each state is given as

(17-1) $$c_i^A = Y_i - ps_i$$

(17-2) $$c_i^B = (1-\pi_i)Y_i - ps_i + s_i = c_i^A - \pi_i Y_i + s_i$$

where Y_i is exogenously given income of individual i. Inequality of Y_i is due to differences in ability. π_i (>0) indicates the net quantity of resources lost to each dollar of private income during the period of contingency when unemployment or natural disaster actually occurs -- resources lost by reason of being diverted to a job search effort or being cut off because of disruptions in production activities. Thus, π is called the penalty ratio. Inequality of π_i, which is due to differences in luck, results in ex-post income differences. s is insurance return in the event of an emergency with p now the price of insurance that is, the premium per dollar of insurance coverage. ps indicates an individual's premium paid to the government (an insurance company in the case of private insurance) during the state of A.

It is assumed that uncertainty is restricted to income; the insurance premium paid from an individual to a supply agent (or the government) in state B is risk free and is not subject to the penalty. We assume that each consumer determines their insurance demand, treating exogenous parameters α, π and insurance price p as given.

The government budget constraint is given as

(18) $$p(s_1 + s_2) = \alpha(s_1 + s_2)$$

Or

(18)' $$p = \alpha$$

The left-hand side of (18) is insurance revenue and the right-hand side is insurance payment in event of state B. p is determined so as to satisfy the balanced budget. This corresponds to the zero profit condition for the private insurance company[5].

From (17-1,2) the budget constraint may be rewritten as

(19) $$pc_i^B + \rho c_i^A = (1 - p\pi_i)Y_i$$

where $\rho \equiv 1 - p$. (17-2) means that the effective rate of return on insurance premium ps is $(1-p)/p$. The price of insurance, p, also means the price of consumption in state B, while $1-p$ means the price of consumption in state A. Effective income in the left hand side of (19) evaluates emergency costs $\pi_i Y_i$ using p, the price of consumption in state B.

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5 . We could consider the case where two individuals demand and supply the insurance without incorporating the government; $s_1 + s_2 = 0$ instead of (18). See Ihori (1997).

3.1.2 Perfect Equalization

From the utility maximizing behavior and (18)' it is easy to derive

(20) $c_i^A = c_i^B = (1 - \alpha\pi_i)Y_i$

Or

(20)' $s_i = \pi_i Y_i$

Public insurance can realize the perfect equalization of consumption between states for all individuals. We obtain the similar equalization result as in the case of differences in ability. Since differences in luck occur as exogenous shocks, it is not necessary to investigate the first stage game as in section 2.

Without using public insurance the government could realize the perfect equalization by imposing income taxes and transfers. In such a case, the relevant tax structure will be given as

(22) $T_i(Y) = \alpha\pi_i Y_i + (Y - Y_i) = Y - (1 - \alpha\pi_i)Y_i$

In state A ($Y = Y_i$) the government taxes $\alpha\pi_i Y$ and in state B ($Y = (1 - \pi_i)Y_i$) the government transfers $\pi_i(1 - \alpha)Y_i$. It should be noted that from (22) this tax system is person specific and the effective marginal tax rate on income is one.

An increase in Y_i will raise welfare of individual i, while an increase in π_i will reduce welfare. From (20)', both economic growth and an increase in the penalty ratio would raise the insurance demand for safety nets and contribute more transfers in terms of risk sharing. Since an increase in the penalty ratio induces (ex post) income inequality, it would have a positive effect on public transfers. An increase in α, the probability of a bad state, will raise insurance contributions, αs_i, but reduce insurance payments, $(1 - \alpha)s_i$, thus keeping the size of public transfer, s_i, constant.

3.2 Voluntary Contributions of Public Goods

3.2.1 Theoretical Framework

Individuals may also agree to allocate some fraction of income to public goods (for example, safety nets provided by unemployment benefits, military preparedness for natural disaster, foreign aid) to reduce regional, domestic and international tension and to avoid random emergency costs[6].

There are two types of public goods when we allow for risk sharing motives. First of all, publicly provided safety nets may be regarded as a public good, which is a standard idea since Olson and Zeckhauser (1966). Safety nets may well be nonexcludable among individuals. For example, benefits as provided by unemployment insurances are nonexcludable among workers if being unemployed is random. The interdependence of economic activities among individuals collectivizes the security of the individuals and hence increases nonexcludability of safety nets. Second, some of the government's consumption, which is mainly used for the (ex post) poor, may be regarded as a public good since it would also benefit the (ex post) rich with risk averse motives. When all individuals are willing to accept the tax burden for public goods in uncertain situations, we say that they have risk sharing motives.

We now develop a model of voluntary acceptance of tax burden for a public good, G. We employ a contribution game among various individuals who would accept, voluntarily, their tax burden in order to gain the redistributional benefits of government spending on public transfers. We believe that the critical point of formulating a public transfer process is to clarify how individuals are willing to accept their tax burden, which is used for income redistribution.

6 . As explained in Sandler (1993), the notion of public good is a useful theoretical idea that is a reasonable approximation under some circumstances such as environmental pacts, the United Nations, and common markets. Ihori (1994) presented a model of jurisdictions with local public goods. Alesina and Perotti (1995) investigated economic risk and political risk in fiscal unions. Domestic transfers among individuals may be also regarded as a type of public goods.

Then, when incorporating public goods, the utility function (16) may be rewritten as

(16)' $W_i = (1-\alpha)V(c_i^A) + \alpha V(c_i^B) + \alpha U(G)$

It might be plausible to assume that the public good G is more beneficial when the emergency occurs than when it does not. In this paper we consider the case where the benefit of the public good is dependent on the state of nature, relevant only for the emergency state. The qualitative results are almost the same even if G is assumed to be beneficial for both states.

Since G is a pure public good, G is given by

(23) $G = g_i + \sum_{j \neq i} g_j$

where g_i is the tax burden voluntarily accepted by individual i (=1,2).

Individual i's budget constraint in each state is now given by

(17-1)' $c_i^A = Y_i - g_i$

(17-2)' $c_i^B = (1-\pi_i)Y_i - g_i = c_i^A - \pi_i Y_i$

where the relative price of public goods in terms of private consumption is assumed to be unity. Substituting (17-2)' into (16), we have

(16)'' $W_i = (1-\alpha)V(c_i^A) + \alpha V(c_i^A - \pi_i Y_i) + \alpha U(G)$

We assume as in section 2 that each individual in a noncooperative setting determines his tax burden provision, treating the other individual's tax burden as given[7]. Considering (23), (17-1)' may be rewritten as

(24) $c_i^A + G = Y_i + \sum_{i \neq j} g_j$

3.2.2 Equilibrium and Welfare

Let us now define the following expenditure function:

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7 . This public good model is a standard private provision model. See Cornes and Sandler (1996) among others.

$$Min \ E_i \equiv c_i^A + G \quad subject \ to \ W_i \geq \overline{W_i}$$

Then we have

(25) $$E(W_i, \pi_i Y_i) = Y_i + \sum_{i \neq j} g_j$$

which summarizes the private optimization behavior. By conducting a similar manipulation as in section 2.2, the two individual model with voluntary provision of the public good will be summarized by

(26) $$E(W_1, \pi_1 Y_1) + E(W_2, \pi_2 Y_2) = Y_1 + Y_2 + G(W_1, \pi_1 Y_1)$$

(27) $$G(W_1, \pi_1 Y_1) = G(W_2, \pi_2 Y_2)$$

where G( ) is the compensated demand function for the public good. (27) means that each individual demands the same amount of the public good in equilibrium.

From (27) it is easy to see that so long as emergency costs are not equal between individuals, that is,

$$\pi_1 Y_1 \neq \pi_2 Y_2$$

then, expected welfare is not equalized between individuals either.

$$W_1 \neq W_2$$

Generally, the public good provision with risk sharing motives cannot attain equalization of welfare between individuals. Namely, if emergency costs for individual 1 are greater than for individual 2 ($\pi_1 Y_1 > \pi_2 Y_2$), then welfare for individual 1 is less than welfare for individual 2 ($W_1 < W_2$). This result comes from the property that the compensated demand function for G is increasing with the emergency costs; $G_\pi \equiv \partial G / \partial \pi Y > 0$. See Appendix.

Let us examine the welfare effect of an increase in π_1. An increase in π_1 will reduce the total effective income, hurting both individuals. This is called the total income effect. An increase in π_1 raises the demand for the public good by individual 1, inducing more provision of g_1 and less provision of

g_2. This hurts individual 1 but benefits individual 2, which is called the relative burden effect. Thus, individual 1 loses but the welfare effect on individual 2 is ambiguous. Individual 2 loses when the total income effect dominates the relative burden effect.

3.2.3 Paradoxical Result

We may derive a seemingly paradoxical result as in section 2.2.3. Suppose that individual 1 is more able than individual 2 and hence individual 1 earns higher income than individual 2 for each state.

$$Y_1 > Y_2,\ (1-\pi_1)Y_1 > (1-\pi_2)Y_2$$

And, suppose also that the emergency cost is higher for individual 1 than for individual 2.

$$\pi_1 Y_1 > \pi_2 Y_2$$

Then, from (27) we have a paradoxical result that expected welfare is higher for individual 2, who is less able, than for individual 1, who is more able. This is because individual 1 would agree to accept more tax burden than individual 2, which benefits individual 2.

3.2.4 Identical Individuals

Suppose two individuals are ex-ante identical. Then the system will reduce to

(28) $$2E(W,\pi Y) = 2Y + G(W,\pi Y)$$

We have

(29) $$\frac{dW}{dY} = \frac{2(1-E_\pi)+G_\pi}{2E_W - G_W}$$

(30) $$\frac{dW}{d\pi} = \frac{-2E_\pi + G_\pi}{2E_W - G_W}$$

where subscripts associated with $E()$ and $U()$ are partial derivatives. In this setting the size of G may be regarded as the degree of redistributional spending due to risk sharing motives. An increase in Y will normally increase W, stimulating G. Thus, economic growth would likely contribute more government spending on public transfers. On the contrary, an increase in $\pi_1 = \pi_2$ would normally reduce W. Thus, it may reduce G, contributing less spending on safety nets.

If an increase in the penalty ratio is positively associated with economic growth in the real economy, the insurance demand model considered in 3.1 may well explain the increase in income transfers. On the other hand, if an increase in national income itself is important in the real economy, the public good demand model considered in this section can also explain the increase in government spending on income transfers.

3.3 Mutual Insurance and Public Goods

3.3.1 Theoretical Framework

This subsection incorporates both the mutual insurance demand and the public good demand at the same time. Individual i's budget constraint is now rewritten as

(31-1) $c_i^A = Y_i - ps_i - g_i$

(31-2) $c_i^B = (1 - \pi_i)Y_i - ps_i + s_i - g_i$

Hence, we have

(32) $pc_i^B + \rho c_i^A + g_i = (1 - p\pi_i)Y_i$

The game is a single-shot one stage game. Before the game the equilibrium price of insurance $p = \alpha$ is announced and binding commitments regarding insurance are determined in the insurance market. Then two individuals simultaneously choose the tax burden g_i and have Nash conjectures about the choice of the other's tax burden. Finally nature decides which state (A or B) occurs.

Considering (23), (32) may be rewritten as

(32)' $$pc_i^B + \rho c_i^A + G = (1 - p\pi_i)Y_i + \sum_{i \neq j} g_j$$

Let us now define the following expenditure function:

$$Min\, E_i \equiv pc_i^B + \rho c_i^A + G \quad subject\ to\ W_i \geq \overline{W_i}$$

Then in place of (25) we have

(33) $$E(W_i, p) = (1 - p\pi_i)Y_i + \sum_{i \neq j} g_j$$

We also have *G(W, p)*; the compensated demand function for the public good.

The model with both mutual insurance and public spending will then be summarized by

(34) $$E(W_1, p) + E(W_2, p) = Y_1 + Y_2 - p(Y_1\pi_1 + Y_2\pi_2) + G(W_1, p)$$

(35) $$G(W_1, p) = G(W_2, p)$$

(34) and (35) correspond to (26) and (27), respectively.

3.3.2 Welfare Implications

Since preferences are identical between individuals, from (35) we always have the perfect welfare equalization result between individuals.

(36) $$W_1 = W_2$$

Since we incorporate the insurance demand, we also have (20) as in section 3.1. Private consumption is perfectly equalized between states as well. Thus, we have derived two types of perfect equalization results in this model.

Once we incorporate both the mutual insurance demand and the public good demand at the Nash solution, expected welfare is equalized, irrespective of differences in income or the penalty ratio. In such a case the divergence between the effective income ($(1 - p\pi_i)Y_i$) does not matter, but both the total effective income $Y_1 + Y_2 - p(\pi_1 Y_1 + \pi_2 Y_2)$ and the total emergency costs $\pi_1 Y_1 + \pi_2 Y_2$ do matter. This is because both individuals face the same price of insurance and different emergency costs have the income effect only[8].

Considering (35) the model will reduce to

(36) $$2E(W,p) = Y_1 + Y_2 - p(Y_1\pi_1 + Y_2\pi_2) + G(W,p)$$

Thus, we have

(37) $$\frac{dW}{dY_1} = \frac{1 - p\pi_1}{2E_W - G_W} > 0$$

(38) $$\frac{dW}{d\pi_1} = \frac{-pY_1}{2E_W - G_W} < 0$$

Economic growth (an increase in income level) benefits welfare, raising both the public good and insurance demand. On the contrary, an increase in the penalty ratio (an increase in income inequality) hurts welfare, raising the insurance demand, while reducing the public good demand. See Table 2. If we observe a negative relation between the mutual insurance and the public good in the real economy, then it might be due to changes in the penalty ratio.

4. Conclusion

This paper has reviewed analytical results of optimal income transfers and also developed simple models of income transfers when the sources of income inequality are differences in ability and in luck. The existence of the altruistic and mutual insurance motives are crucial to obtaining welfare equalization. We have first investigated the optimal redistribution policy in the case of ability differences. It is shown that when the government reoptimizes income transfers, after it observes the outcome of private activities, a

8 . This is an extension of the neutrality result. See Shibata (1971), Warr (1983), and Ihori (1992).

paradoxical result that a more able individual would not enjoy higher welfare than a less able individual would occur.

We have then considered positive aspects of public transfers in the case where the economy creates a mutual insurance as a safety net. We have explored the interdependence through the voluntary acceptance of tax burden for public transfers based on the risk sharing motive. We have derived a similar paradoxical result as in the optimal income tax scheme.

If an increase in the penalty ratio (or an increase in ex-post income inequality) is relevant in the real economy, the insurance demand model may well explain the increase in public transfers in the developed countries. On the other hand, if an increase in national income is important, the public good demand model may also explain the increase in government spending on income transfers.

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Table 1 Government Spending (per GDP)

		government consumption	government investment	transfer payments	others	total
Japan	1975	10.0	5.3	7.8	3.6	26.7
	1993	9.5	6.5	12.0	5.9	33.9
US	1975	18.6	2.1	11.1	1.3	33.1
	1993	17.1	1.6	13.2	4.2	36.2
UK	1975	22.0	4.7	9.9	8.6	45.3
	1993	22.0	1.8	14.6	6.1	44.6
Germany	1975	20.5	3.6	17.6	6.6	48.3
	1993	17.8	2.2	15.8	12.6	48.4
France	1975	16.6	3.7	17.4	5.7	43.4
	1993	19.3	3.6	23.6	6.3	52.8
Sweden	1975	23.8	4.3	14.2	5.5	47.8
	1993	28.0	3.1	25.2	16.2	72.4

Table 2: Comparative Statics: Effects on Public Transfers

	an increase in income level	an increase in income inequality
altruistic model: section 2.1	+	?
altruistic model: section 2.2	+	+
risk sharing model: section 3.1	+	+
risk sharing model: section 3.2	+	?
risk sharing model: section 3.3	+ (insurance demand) + (public good demand)	+ (insurance demand) - (public good demand)

Appendix

With the public good the first order condition is written as

$$(1-\alpha)V_c^A + \alpha V_c^B = U_G \qquad \textbf{(A1)}$$

where $U_G \equiv dU/dG$. Let us investigate properties of expenditure and compensated demand functions. Totally differentiating (16)" and (A1), we have

$$\begin{bmatrix} (1-\alpha)V_{cc}^A + \alpha V_{cc}^B & -U_{GG} \\ (1-\alpha)V_c^A + \alpha V_c^B & U_G \end{bmatrix} \begin{bmatrix} dc^A \\ dG \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} dW + \begin{bmatrix} \alpha V_{cc}^B \\ \alpha V_c^B \end{bmatrix} \pi_i Y_i \qquad \text{(A2)}$$

Hence we have

$$c_W^A = \frac{1}{\bar{\gamma}} U_{GG} > 0 \qquad \text{(A3)}$$

$$G_W \equiv \frac{\partial G}{\partial W} = \frac{1}{\bar{\gamma}}[(1-\alpha)V_{cc}^A + \alpha V_{cc}^B] > 0 \qquad \text{(A4)}$$

$$c_\pi^A \equiv \frac{\partial c^A}{\partial \pi Y} = \frac{1}{\bar{\gamma}}[\alpha V_{cc}^B U_G + \alpha V_c^B U_{GG}] > 0 \qquad \text{(A5)}$$

$$G_\pi \equiv \frac{\partial G}{\partial \pi Y} = \frac{\alpha}{\bar{\gamma}}\{[(1-\alpha)V_{cc}^A + \alpha V_{cc}^B]V_c^B - [(1-\alpha)V_c^A + \alpha V_c^B]V_{cc}^B\} \qquad \text{(A6)}$$

$$E_\pi \equiv \frac{\partial E}{\partial \pi Y} = \frac{\alpha}{\bar{\gamma}}\{V_c^B U_{GG} + [(1-\alpha)V_{cc}^A + \alpha V_{cc}^B]V_c^B\} > 0 \qquad \text{(A7)}$$

$$E_W = c_W^A + G_W > 0 \qquad \text{(A8)}$$

where $\bar{\gamma} \equiv [(1-\alpha)V_{cc}^A + \alpha V_{cc}^B]U_G + [(1-\alpha)V_c^A + \alpha V_c^B]U_{GG} < 0$. Subscripts denote partial derivatives. Suppose the relative risk aversion is constant ($\frac{cV_{cc}}{V_c} = -\lambda$). Then (A6) may be rewritten as

$$\frac{\partial G}{\partial \pi Y} = \frac{\alpha\lambda V_c^A V_c^B \alpha(1-\alpha)}{c^A c^B}(c^A - c^B) > 0 \qquad \text{(A9)}$$