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# Wage Profile and Monitoring under Adverse Selection

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# *Wage Profile and Monitoring under Adverse Selection*

**Abstract**

We consider wage profiles and monitoring under adverse selection and moral hazard, that is, extend the Lazear's shirking model to adverse selection. It is shown that workers with higher abilities are offered a steeper wage profile (high total payment) and frequent monitoring. As the self-selection theory shows, workers with high ability get information rent. If the rate of monitoring them were low, the low ability type would pretend to be the high ability type. Hence, to order the low ability type, high monitoring rate is offend to the high ability type.

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# 1. Introduction

As Lazear (1979) (1981) shows, upward-sloping age-earnings profiles are often incentives when firm managers face difficulty in monitoring the workers. When monitoring is difficult or the cost is high, the firm can decrease the cost by offering upward-sloping wage profiles. Even if workers are unlikely to be monitored, the possibility of high rewards lost due to monitoring encourages them to make efforts. Upward-sloping wage profiles prevent employees' moral hazard, and thus this is called the shirking hypothesis, which is supported by empirical studies: Medoff and Abraham (1980), Lazear and Moore (1984), Krueger (1991), and Okazaki (1993). We expand the idea to asymmetric information on workers' abilities, and consider wage profiles and monitoring under adverse selection and moral hazard.

We consider the relationship between delayed compensation schemes and monitoring on the shirking hypothesis by a two-period model where a firm can commit itself to the second period. In the first period, employees are required to provide the agreed upon level of effort for acquiring skills, but a firm cannot observe each of their efforts level without monitoring. To give incentives for skill accumulation, the firm has two options: delayed compensation schemes and frequent monitoring. Note that when workers are risk averse, the more upward-sloping wage profiles are offered, the more payment cost to compensate the reservation utility at least raises. On the other hand, monitoring cost is increasing rapidly as monitoring rate increases. When monitoring is difficult, the firm will offer the more delayed compensation schemes instead of frequent monitoring. The firm has, therefore, a motive to increase payment in the second period. If monitoring cost is low, a high monitoring rate leads to a flatter wage profile. Accordingly, the extent of the difficulty of monitoring has much influence on the slope of the wage profile and monitoring rate.

Under asymmetric information on workers' types, wage profile and monitoring should play a role as a self-selection mechanism as well as an incentive scheme. As Baron and Myerson (1982) and Laffont and Tirole (1986) (1987) show, a firm gives information rent to workers with high abilities in a self-selection mechanism. On the other hand, unless high ability workers are compensated sufficient levels of information rent, they are willing to pretend to be low ability ones. Those with low abilities have an incentive to choose the contract for the high ability worker in order to extract high information rent if the monitoring rate of the high type is not sufficiently high. Hence, a high monitoring rate is specified in the contract for the high ability workers. In this case, monitoring prevents workers from pretending to be another types of workers. In this model, monitoring plays two roles under adverse selection and moral hazard. One effect is to encourage workers to make efforts. The other is to discourage them from pretending to be other types. It is shown that, under adverse selection and moral hazard, the latter effect is more crucial. Therefore, wage plays a role of compensating information rent for high ability workers. Monitoring, on the other hand, deters the low ability workers to pretend the high ability workers.

Under typical adverse selection problems, there is a kind of incentive compatibility for truth-telling. Since this paper also examines the problem of workers' incentives for skill accumulation, we have another kind of incentive compatibility to consider. Thus, a type of worker has four ways to choose his action: (1) he chooses the contract for his own type and provides the agree upon level of effort; (2) he tells his own type truthfully but shirks; (3) he chooses the contract for another type and shirks; (4) he pretends to be the other type and then makes efforts at the required level. Therefore, the firm must encourage a type not only to choose his own contract but also to make the appropriate efforts required.

This paper is organized as follows. In section 2, we explain the model and consider the incentive compatibility to prevent adverse selection and moral hazard. Section 3 investigates characteristics of wage profile and monitoring rate after a benchmark analysis of a non-adverse selection case. Conclusions are examined in section 4.

## 2. The Model

We will consider a two-period model to analyze the optimal wage profile and monitoring under asymmetric information of employees' actions and types.

### Utility function and distribution of workers' ability

Workers work for two periods. In the first period, a worker is required to deliver efforts to acquire firm specific skills. If he makes some efforts $e$ in the first period, he will be a skilled worker in the second period and produce outputs $e$, otherwise he produces nothing. His effort level $e$ is a negative factor in his utility function. Workers' utility function is $U(w, e) = u(w) + kj(e)$, where $u'>0$, $u''<0$, $j'>0$, $j''>0$ and $j(0)=0$, $j'(0)=0$. The coefficient $k$ implies workers' ability, and is a negative constant, $k<0$. Let $w_t$ denote the wage in the period $t$ ($t$=1,2). No effort is required in the second period.[1]

The coefficient $k$ is uniformly distributed between $\underline{k}$ and $\bar{k}$. The firm and all workers have common knowledge about the distribution of workers' types. However, the firm cannot observe workers' abilities while a worker knows only his own ability and does not know the real abilities of the others. The number of a type $k$ is normalized to 1.

**Monitoring cost**

It costs the firm to monitor employees. The firm cannot distinguish earnest employees that have made efforts from shirking ones without monitoring. Define the cost to monitor a type $k$ as $h(m(k))$, where $m(k)$ ($0 \leq m \leq 1$) denotes the monitoring rate of a type $k$ and the following conditions are assumed to avoid a corner solution: $h'>0$, $h''>0$, $h(0)=0$, $\lim_{m \to 1} h(m) = +\infty$, $h'(0)=0$, and $\lim_{m \to 1} h'(m) = +\infty$. Workers can observe the monitoring rate, also. If a shirking worker is detected by monitoring, he will be dismissed at the end of the first period receiving no additional payment such as severance pay.[2]

**Timing of decisions and actions**

Order of actions and decisions by workers and the firm is as follows:

① In the first period, the firm specifies the range of workers to employ.

② The firm offers a bunch of contracts to each type worker: monitoring rate, wage profile, and required skills level of each contract are specified.

③ Each type chooses his own contract and provides the agreed upon level of effort (since the incentive compatibility and individual rationality are satisfied on the equilibrium).

④ At the end of the first period, the firm monitors each type at each of the offered monitoring rates after paying the first period wage. If shirking employees are caught, they are dismissed. If they have provided the agreed upon level of effort, they continue working in the firm. Unless workers are monitored, they are kept employed in the firm regardless of having shirked or not.

We assume that the firm can commit itself to the second period and to the monitoring. You might think that the firm has an incentive to monitor no type $k$ or to pay the second period wage equivalent to the reservation wage after they have made efforts. This implies a deviation from the contract which was implicitly made with the type $k$ at the beginning of the first period. However, the firm will follow the contract by reputation effect. If the firm were to deviate from the contract with the type $k$, all of the type $k$ of later generations would punish the firm, that is, continue shirking perpetually or choose other firms. Since the firm would lose its own credibility, the firm has no incentive to deviate from the implicit contract if its discount factor is sufficiently large.[3]

**Incentive compatibility**

$\underline{k}$ ($\bar{k}$) is the worst (best) type. As we show later, every type is not always employed by the firm, and thus the range of the employed workers' types is $[k^*, \bar{k}]$, where $k^*$ is the boundary of workers' types employed by the firm. Decision on the range of employed types will be taken up later. At the moment, it makes sense to consider that $k^*$ and employment range are exogenously given.

Wage, the monitoring rate and the required efforts level of the type $k$ are denoted as $w_1(k)$, $w_2(k)$, $m(k)$ and $e(k)$. The employee's utility in the spot external labor market or the reservation utility is zero regardless of workers' abilities. Let us define the reservation wage $\overline{w}$ $(>0)$; it satisfies $U(\overline{w}, 0) = 0$. Let $\delta$ denote a discount factor. When the firm employs workers from $\bar{k}$ to $k^*$, using the revelation principle by Myerson (1979), individual rationality and incentive compatibility are represented as follows:

$$IR_1: \forall k \in [k^*, \bar{k}] \quad u(w_1(k)) + kj(e(k)) + \delta u(w_2(k)) \geq 0 \qquad \text{...(1)}$$

$$IR_2: \forall k \in [k^*, \bar{k}] \quad u(w_2(k)) \geq 0 \qquad \text{...(2)}$$

$$IC_1: \forall k, \hat{k} \in [k^*, \bar{k}] \quad u(w_1(k)) + kj(e(k)) + \delta u(w_2(k)) \geq u(w_1(\hat{k})) + \delta(1 - m(\hat{k}))u(w_2(\hat{k})) \qquad \text{...(3)}$$

$$IC_2: \forall k, \hat{k} \in [k^*, \bar{k}] \quad u(w_1(k)) + kj(e(k)) + \delta u(w_2(k)) \geq u(w_1(\hat{k})) + kj(e(\hat{k})) + \delta u(w_2(\hat{k})) \qquad \text{...(4)}$$

If the firm were to monitor all of the type $k$ employees, no one would shirk. The monitoring cost would be huge, however, causing the firm's profit to be negative. On the other hand, if the firm monitors no type $k$, no one will make efforts. Hence, the firm should choose an optimal monitoring rate $m^*(k)$ $(\forall k \in [k^*, \bar{k}] \ \ 0 < m^*(k) < 1)$.

$IR_1$ represents the individual rationality for two periods. The left hand side expresses the utility which a type $k$ gets when he chooses a contract for that type $k$ and provides the agreed upon level of effort in the first period. $IR_1$ means that the expected utility must exceed the reservation wage for two periods.

$IR_2$ is the individual rationality in the second period. Since workers can quit the firm freely, in order to continue employing them regardless of what second period state is realized, the firm must offer them wages higher than the reservation wage.

$IC_1$ under $\hat{k}=k$ means that a type $k$ has an incentive to make efforts when he chooses the contract for that type $k$, that is, tells his own type truthfully. Unless a shirking worker is detected by monitoring, he keeps employment relationship to the firm, receives $w_2(k)$, and is regarded as a skilled worker with the probability (1-$m$). He is necessarily dismissed if he is caught by the firm manager, and when his utility is zero. The probability is $m$. $IC_1$ under $\hat{k}=k$ implies that the utility for a diligent employee must exceed that for a shirking one.

Consider $IC_1$ under $\hat{k}\neq k$. To induce a type $k$ not only to choose the contract for that type $k$ but also to provide the agreed upon level of effort required to the type $k$, it is necessary for the type $k$ to have no incentive to pretend to be another type $\hat{k}$. Note that the type $k$ can choose whether or not to provide the effort level required of the other type $\hat{k}$ when he pretends to be the type $\hat{k}$. If $IC_1$ holds, the type $k$ never pretends to be any other type $\hat{k}$ and shirks. The right hand of $IC_1$ refers to the utility of the type $k$ when he chooses the contract for the type $\hat{k}$ and then shirks.

In a similar manner, a type $k$ may pretend to be the other type $\hat{k}$ and then make efforts required of the type $\hat{k}$. The right hand of $IC_2$ expresses the utility when the type $k$ pretends to be the diligent type $\hat{k}$. $IC_2$ implies that the type $k$ has no incentive to do so. In the case of $\hat{k}=k$, it is trivial.

$IC_2$ is a typical incentive compatibility for truth telling under adverse selection. In this paper, since workers' incentives for skill accumulation are considered, the other incentive compatibility $IC_1$ appears.

To simplify, gross utility, other except the efforts cost of type $k$, is represented as follows:

$$GU(k)\equiv u(w_1(k))+\delta u(w_2(k)).$$

The expected utility of the type $k$ is expressed by $GU(k)+kj(e(k))$, if he makes efforts. By this notation, we can simplify $IC_2$ as follows:

$$IC_2:\forall k,\hat{k}\in[k^*,\bar{k}] \qquad GU(k)+kj(e(k))\geq GU(\hat{k})+kj(e(\hat{k})) \qquad ...(4)'$$

**Skills and production function**

Next consider the production function. Any type of worker supplies no labor input in the first period regardless of shirking or not. The first period is for training. If he makes efforts $e$ to acquire skills, his labor inputs will be $e$ in the second period. The skilled type $k$ supplies $e(k)$ labor inputs in the firm. Hence, total labor inputs $L$ of all skilled workers are equal to $\int_{k^*}^{\bar{k}} e(k)dk$. Production function is $y=L$. The firm cannot observe outputs of each worker only the total output. This means that, by knowing the total output, the firm cannot observe and conjecture whether a worker has provided the agreed upon level of effort. Hence, the firm must monitor some workers from every type to encourage them to make efforts.[4]

The firm faces the following problem:

***Problem***

$$\max_{\substack{w_1(k),w_2(k),\\ m(k),e(k)}} \int_{k^*}^{\bar{k}} [\delta e(k) - w_1(k) - \delta w_2(k) - h(m(k))]dk$$

$$s.t. \quad IC_1, IC_2, IR_1, IR_2.$$

To solve this problem, we consider whether each of the constraint conditions is binding or not.

***Lemma 1***

$IR_2$ for any type $k \in [k^*, \bar{k}]$ is not binding.

***Proof***

If it is binding, $IC_1$ never holds, that is, no one makes efforts.■

Lemma 1 implies that the second period wage of any employed type which is higher than the reservation wage gives that employee incentives for skill accumulation. Furthermore, we use the first order approach to simplify $IC_2$.[5]

***Lemma 2***

$IC_2$ is satisfied if and only if

$$\forall k \in [k^*, \bar{k}]$$

$$① GU(k) + kj(e(k)) = GU(k^*) + k^* j(e(k^*)) + \int_{k^*}^{k} j(e(\tilde{k}))d\tilde{k},$$

and $② \dfrac{de(k)}{dk} \geq 0$.

This proof is omitted because it is a well-known result of the mechanism design theory.[6] Since the high type worker makes efforts at a low cost, the firm induces him to make more efforts than the low type. The optimal contract yields a high-powered incentive to the high type.

If the firm offers different contracts to separate difficult types, their efforts levels will not be equivalent. To the contrary, if the efforts level of different types are identical, it holds by $IC_2$ that $GU(k)=GU(\hat{k})$ under $k \neq \hat{k}$. This is a bunching case. In this case, the firm offers the identical contract to the different types $k$ and $\hat{k}$ because, except for the efforts cost, it gives equal gross utility to the type $k$ and $\hat{k}$, i.e., wage profile and monitoring rate are equivalent among them.

Next, we show that some constraint conditions are not binding.

***Lemma 3***

$IR_1$ and $IC_1$ are binding for the lowest type employed $k^*$, and are not for any type $k \in (k^*, \bar{k}]$.

***Proof***

See Appendix.

Monitoring plays two roles. One is to give incentives for skill accumulation, and the other is to induce workers to choose the appropriate contract for themselves. Lemma 3 implies that the latter role of monitoring is more crucial than the former. The incentive compatibility for higher types is not binding. The monitoring rate of each type is determined at the level which the lowest ability type employed $k^*$ have no incentive for pretending to be another type and shirking.

***Lemma 4***

$GU(k) \geq GU(\hat{k})$ *under* $k > \hat{k}$, and the inequality holds strictly if $e(k) > e(\hat{k})$.

***Proof***

See Appendix.

Lemma 4 implies that, except for efforts cost, workers' gross utility increases with ability of workers. Although this result is intuitive, we represent it as a lemma because it is necessary to prove the later proposition.

Lemmas 1, 2, and 3 allow us to simplify the above constraints and problem. Binding constraints are summarized as follows:

$$\forall \hat{k} \in [k^*, \bar{k}] \quad 0 = u(w_1(\hat{k})) + \delta(1-m(\hat{k}))u(w_2(\hat{k})) \quad \text{...(5)}$$

$$\forall k \in [k^*, \bar{k}] \quad GU(k) + kj(e(k)) = \int_{k^*}^{k} j\left(e(\tilde{k})\right) d\tilde{k} \quad \text{...(6)}$$

$$\forall k \in [k^*, \bar{k}] \quad \dot{e}(k) \geq 0. \quad \text{...(7)}$$

(6) under $k=k^*$ is identical to $IR_1$ for the type $k^*$. The Lagrange function is represented as follows:

$\lambda(k)$, $\mu(k)$, and $\beta(k)$ are Lagrangian multipliers. The following first order conditions on $w_1$, $w_2$, *and* $m$ hold:

$w_1(k)$: $-1+(\lambda(k)-\mu(k))u'(w_1(k))=0$ ...(8)

$w_2(k)$: $-1+\{\lambda(k)-(1-m(k))\mu(k)\}u'(w_2(k))=0$ ...(9)

$m(k)$: $-h'(m(k))+\delta\mu(k)u(w_2(k))=0$ ...(10)

So far, the boundary of employed type $k^*$ and employment range are exogenously given. By lemma 3 and the envelope theorem, the boundary of employed types is determined as follows:

$$-\delta e(k^*)+w_1(k^*)+\delta w_2(k^*)+h(m(k^*))+j(e(k^*))\int_{k^*}^{\bar{k}}\lambda(k)dk=0 \qquad ...(11)$$

or $k^*=\underline{k}$.

Equation (11) implies that the marginal benefit of employing the type $k^*$ is equivalent to the marginal cost of employing him. The cost is constituted by the payment and monitoring for type $k^*$ and the increase of information rent of any employed higher type caused by the marginal employment. The firm offers $w_1=w_2=m=e=0$ to the unemployed type $k\in[\underline{k}, k^*)$. Since the type $k\in[\underline{k}, k^*)$ has no incentive to make contracts with the firm and to pretend to be other employed types, the firm can specify the range of types to employ.

Next, we show that employment range is $[k^*, \bar{k}]$.

***Lemma 5***

Employment range is $[k^*, \bar{k}]$, where $k^*$ satisfies equation (11) or $k^*=\underline{k}$.

***Proof***

To prove this proposition, it is sufficient to show that a higher type $k$ is never excluded from employment while a lower type $\hat{k}$ is employed under $k>\hat{k}$.

The type $k$ receives the reservation utility when he does not make a long-run contract with the firm. However, if he pretends to be the lower type $\hat{k}$, his utility will be more than just the reservation utility:

$$GU(\hat{k})+kj(e(\hat{k}))>GU(\hat{k})+\hat{k}j(e(\hat{k}))\geq 0.$$

Hence, the firm cannot exclude the type $k$ since he is willing to pretend to be the lower employed type $\hat{k}$. Employment range is $[k^*, \bar{k}]$, where $k^*$ is the critical point of employment and satisfies equation (11) or $k^*=\underline{k}$. ■

When the high type $k$ pretends to be the low type $\hat{k}$, information rent of an employed type higher than this type $\hat{k}$ is not affected by the actions of this type $k$. Hence, the firm's profit increases by pretense of type $k$ when the firm employs the lower type $\hat{k}$. The firm is willing to employ the higher type $k$ while the lower type $\hat{k}$ is employed.

As lemma 5 indicates, under adverse selection, workers with very low abilities are not employed by the firm and then work in the spot labor market. If no information asymmetry on workers' types exists, any type will be always employed due to constant marginal productivity of labor. Accordingly, the information asymmetry leads to the dual labor market.

## 3. Wage Profile and Monitoring

**A benchmark analysis: no adverse selection**

Here, for the convenience of later discussion, based on Eguchi (1998), a benchmark analysis under symmetric information on workers' types is useful. In this case, IR and IC are represented as follows:

$$IR_1^B: \; u(w_1) + kj(e^*) + \delta u(w_2) \geq 0$$

$$IR_2^B: \; u(w_2) \geq 0$$

$$IC^B: \; u(w_1) + kj(e^*) + \delta u(w_2) \geq u(w_1) + \delta(1-m)u(w_2)$$

Denote the optimal efforts level required as $e^*$. Since there is no problem of adverse selection, little needs to be said on the determination of the efforts level required. Hence, it is sufficient to consider the cost minimization problem given $e^*$ as a benchmark. The firm will minimize the per capita cost $w_1 + \delta w_2 + h(m)$ subject to $IR_1^B$, $IR_2^B$ and $IC^B$.

Obviously, $IR_2^B$ is not binding, but $IC^B$ and $IR_1^B$ are always binding. Furthermore, it holds that $w_1 < \overline{w}$. Unless this is true, no worker makes efforts. Hence, to induce an employee to deliver efforts, an upward-sloping wage profile is realized: $w_1 < \overline{w} < w_2$. The existence of "hostage" encourages workers to provide the required efforts level.

From the view of incentives for skill accumulation, the firm offers an upward-sloping wage profile and a positive monitoring rate. A more upward-sloping wage profile encourages employees to make efforts even under a low monitoring rate. In this case, a worker is unlikely to be detected by monitoring, even if he is shirking. However, the second period wage lost on dismissal is high, and thus he is unwilling to shirk. Note that the steeper the wage profile, the higher the wage cost is because of concavity of utility function. On the other hand, a high monitoring rate clearly forces workers to provide the needed efforts level under a flatter wage profile. In this case, for the firm, monitoring cost is high while wage cost is low.

Under adverse selection, the above relationship between wage profile and monitoring is modified. In the self-selection mechanism, the high type worker gets higher utility as information rent than the low type. However, information rent and information asymmetry on workers' efforts level cause the low type worker to choose the contract for the high type. Hence, to discourage the low type from pretending to be the high type, the firm should offer a high monitoring rate in the contract for the high type worker. Indeed, the high type is offered a more upward-sloping wage profile and more frequent monitoring, as we show later.

**Wage profile and monitoring under adverse selection**

As the following proposition shows, wage profiles are upward sloping for any employed type $k \in [k^*, \bar{k}]$.

***Proposition 1***

The first period wage of any type is lower than the reservation wage:
$w_1(k) < \bar{w}$ *for any* $k \in [k^*, \bar{k}]$.

***Proof***

See Appendix.

By lemma 1 and proposition 1, the wage profile for any type $k \in [k^*, \bar{k}]$ is upward sloping similar to the above benchmark. If $w_1(k) \geq \bar{w}$ for a type *k*, the lowest type employed *k** would pretend to be the type *k* and then shirk. Although this result seems to be intuitive from the above benchmark analysis, there is an important point to note. In the self-selection mechanism, the high type gets a higher utility as information rent, and thus a high total wage is paid to him. However, as proposition 1 indicates, the first period wage of the high type never exceeds the reservation wage even if the high type obtains a huge total wage as information rent, so that the firm is willing to attach considerable weight to the second period wage. Indeed, as proposition 2 shows, the high type worker gets the second period wage more often than the low type.

***Proposition 2***

It holds that $w_2(k) > w(\hat{k})$ if $e(k) > e(\hat{k})$ under $k > \hat{k}$.

***Proof***

Consider any two different types *k* and $\hat{k}$ $(k > \hat{k})$.

Suppose that the second period wage of the high type *k* is not higher than that of the low type $\hat{k}$: $w_2(k) \leq w_2(\hat{k})$. In this case, two sets of wage profiles can be offered: ① $w_1(k) \leq w_1(\hat{k})$ *and* $w_2(k) \leq w_2(\hat{k})$; ② $w_1(k) > w_1(\hat{k})$ *and* $w_2(k) \leq w_2(\hat{k})$.

By $e(k) > e(\hat{k})$, ① $w_1(k) \le w_1(\hat{k})$ *and* $w_2(k) \le w_2(\hat{k})$ does not satisfy $IC_1$. The high type $k$ will pretend to be the low type $\hat{k}$.

Next consider ② $w_1(k) > w_1(\hat{k})$ *and* $w_2(k) \le w_2(\hat{k})$. In this case, the inequalities are satisfied as follows:

$$\lambda(\hat{k}) - \mu(\hat{k}) < \lambda(k) - \mu(k)$$

$$\lambda(\hat{k}) - (1 - m(\hat{k}))\mu(\hat{k}) \ge \lambda(k) - (1 - m(k))\mu(k).$$

These inequalities are introduced by FOC (8) and (9). From these inequalities we have

$$m(\hat{k})\mu(\hat{k}) > m(k)\mu(k). \qquad \text{...(12)}$$

Using (10) and (12), the following inequality is obtained,

$$\frac{h'(m(k))m(k)}{u(w_2(k))} < \frac{h'(m(\hat{k}))m(\hat{k})}{u(w_2(\hat{k}))}. \qquad \text{...(13)}$$

Since it is assumed that $w_2(k) \le w_2(\hat{k})$, (13) means $m(k) < m(\hat{k})$. Hence, it holds that

$$m(k)u(w_2(k)) < m(\hat{k})u(w_2(\hat{k})). \qquad \text{...(14)}$$

On the other hand, since $IC_1$ of the lowest ability type employed $k^*$ is binding, we obtain the following equations:

$$GU(k) = \delta m(k)u(w_2(k)), \text{ and} \qquad \text{...(15)}$$

$$GU(\hat{k}) = \delta m(\hat{k})u(w_2(\hat{k})). \qquad \text{...(15)'}$$

Using the inequalities (15), (15)' and lemma 4, we find that

$$m(k)u(w_2(k)) > m(\hat{k})u(w_2(\hat{k})). \qquad \text{...(16)}$$

Note that inequality (14) contradicts (16). Thus, it holds that $w_2(k) > w_2(\hat{k})$.■

Proposition 2 implies that the high type worker gets a higher wage than the low type in the second period. Proposition 2 indicate that the high ability type $k$ receives total wage more than the low ability type $\hat{k}$. If $w_1(k) \ge w_1(\hat{k})$, it is intuitive. Even if $w_1(k) < w_1(\hat{k})$, concavity of utility function yields to large payment to the high ability type. By concavity of utility function, the steeper wage profile, the more total payment increases on compensating identical utility level. Hence, since the high ability type receives high gross utility by lemma 4, the total wage of the high type exceeds that of the low type.

***Proposition 3***

It holds that $\forall k,\ \hat{k} \in [k^*, \bar{k}]$ $\ m(k) > m(\hat{k})$ if $e(k) > e(\hat{k})$ under $k > \hat{k}$.

***Proof***

It is obtained by (8), (9) and (10) that

$$h'(m(k))\cdot m(k)=\delta\frac{u(w_2(k))}{u'(w_2(k))}\left(1-\frac{u'(w_2(k))}{u'(w_1(k))}\right). \qquad ....(17)$$

Differentiate (17) respect with to $k$,

$$\{h''(m)\cdot m+h'(m)\}\dot{m}=\delta\dot{w}_2\left\{1-\frac{u'(w_2)}{u'(w_1)}-\frac{u(w_2)u''(w_2)}{(u'(w_2))^2}\right\}+\delta\dot{w}_1\frac{u(w_2)u''(w_1)}{(u'(w_1))^2},$$

where $\dot{m}\equiv\frac{dm(k)}{dk}$, $\dot{w}_1\equiv\frac{dw_1(k)}{dk}$, *and* $\dot{w}_2\equiv\frac{dw_2(k)}{dk}$. The concavity of utility function, the convexity of monitoring cost function, and $w_1 < w_2$ leads to the following three inequalities:

$$\{h''(m)\cdot m+h'(m)\}>0,$$

$$1-\frac{u'(w_2)}{u'(w_1)}-\frac{u(w_2)u''(w_2)}{(u'(w_2))^2}>0,$$

$$\frac{u(w_2)u''(w_1)}{(u'(w_1))^2}<0.$$

Using proposition 2 and the above three inequalities, it holds that $w_1(k)>w_1(\hat{k})$ *if* $m(k)\le m(\hat{k})$.

Here, suppose that $m(k)\le m(\hat{k})$. We will show that it leads to a contradiction. Consider $\text{IC}_1$ (5):

$$0=u(w_1(\hat{k}))+\delta(1-m(\hat{k}))u(w_2(\hat{k}))$$
$$=u(w_1(k))+\delta(1-m(k))u(w_2(k)).$$

Since $w_1(\hat{k})<w_1(k)$, the following inequality should hold,

$$(1-m(\hat{k}))u(w_2(\hat{k}))>(1-m(k))u(w_2(k)). \qquad ....(18)$$

However, by proposition 2, (18) indicates $m(k)>m(\hat{k})$. This is a contradiction. ■

As the mechanism design theory shows, the high type gets a higher utility as information rent in the self-selection mechanism. Hence, the high type worker receives a higher wage in the second period. However, workers' efforts levels are not observed without monitoring. With a lower monitoring rate in the contract of the higher type, the lower type is willing to choose the contract for the higher type and then shirk. To prevent the pretense of the lower type, the firm must monitor the higher type at the high rate.

Monitoring plays two roles. One effect is encouraging workers to make efforts. The other is discouraging them from choosing other contracts and then shirking. As the above benchmark analysis shows, the substitutive relationship between an upward-sloping wage profile and monitoring is caused by the former. The latter is complementary and discourages the low type worker from choosing the contract for the high type and then shirking. Under adverse selection and moral hazard, the latter effect is more crucial.

Here, consider the monitoring cost function. Denote total monitoring cost of the firm as *TC(m)*. We have assumed throughout that the total monitoring cost function is additively separable:

$$TC = \int_{k^*}^{\bar{k}} h(m(k))dk . \quad ...(19)$$

The cost function implies that the monitoring rate of a type $k$ is independent of the determination of the monitoring rates of the other types. On the contrary, it is significant to consider the case whereby the monitoring rate is a perfect substitution for each monitoring rate of all workers' types:

$$TC = h\left(\int_{k^*}^{\bar{k}} m(k)dk\right). \quad ...(20)$$

This monitoring cost function is also reasonable. Obviously, under this monitoring function, all of the above results hold. The monitoring cost function (19) is not crucial.

So far, we have considered $e(k)$, $w_2(k)$, $m(k)$ and the critical point of employment $k^*$. However, the only information on $w_1(k)$ is found in proposition 1. In general, $w_1(k)$ of the high ability type $k$ may or may not be lower than that of the low ability type $\hat{k}$. The high type $k$ has more information rent so that total payment of the high type is high. From the view of decreasing payment cost, a flatter wage profile is desirable. This has an effect of raising $w_1(k)$ for the high type. However, a high wage in the first period leads to a high monitoring rate as shown by (5). Since the first period wage is certain for shirking workers, the attraction for them is canceled out by frequent monitoring, and hence the monitoring cost increases. This effect decreases the first period wage. The latter effect is directly opposite the former. Since these two effects exist, the difference between the first period wages among the employed types is ambiguous.

## 4. Conclusion

There is an inquiry on wage profiles under adverse selection. Salop and Salop (1976) consider wage profiles under asymmetric information on voluntary quit rates of workers. A firm cannot tell workers with a high voluntary quit rate from ones with a low voluntary quit rate. The firm is willing to hire the workers with a low quit rate because of a fixed training cost on the firm's side. Competition among firms for employment of the low quit rate type leads to two kinds of wage profiles, and separates the low quit rate type from the high quit rate type. They have shown that the low quit rate type necessarily chooses a more upward-sloping wage profile. However, they have not considered workers' incentives for skill accumulation and monitoring cost instantaneously.

We have obtained important results by considering an optimal contract under asymmetric information on workers' abilities and actions. The existence of the incentive problem leads to an upward-sloping wage profile for any type worker, and the second period wage of the high type is higher than that of the low type, i.e., wage of the high type worker increases more rapidly than that of the low type. Furthermore, it has been shown that the monitoring rate of the high type is higher than that of the low type. If the rate to monitor the high type worker were to be lowered, the low type worker would have an incentive to choose the contract for the high type and then shirk. Hence, the high type, who has high payment as information rent, are monitored at the high rate. Payment plays a role on workers' truth-telling incentives in the self-selection model, and monitoring workers to deter workers with lower ability from pretending these with higher ability.

## Appendix

***Proof of lemma 3***

Lemma 2 implies that the utility of workers with higher ability is higher than that of workers with lower ability. Ability of the type $k^*$ is the lowest among the employed workers. Hence, it is clear that $IR_1$ is not binding for the type $k \in (k^*, \bar{k}]$. For the lowest type employed $k^*$, it is always binding since the firm maximizes its profit.

Next, suppose that $IC_1$ for a type $k \in (k^*, \bar{k}]$ is binding:

$$\exists k \in (k^*, \bar{k}]\ \exists \hat{k} \in [k^*, \bar{k}]\quad GU(k) + kj(e(k)) = u(w_1(\hat{k})) + \delta(1 - m(\hat{k}))u(w_2(\hat{k})).$$

Using lemma 2, this assumption leads to the following inequality,

$$GU(k^*) + k^* j(e(k^*)) < u(w_1(\hat{k})) + \delta(1 - m(\hat{k}))u(w_2(\hat{k})).$$

$IC_1$ for the type $k^*$ is not satisfied. It is a contradiction. Hence, $IC_1$ for the type $k$ $(\neq k^*)$ is not binding.

Finally, consider the case wherein $IC_1$ for the type $k^*$ is not binding:

$$\forall k \in (k^*, \bar{k}]\quad \forall \hat{k} \in [k^*, \bar{k}]\quad GU(k) + kj(e(k)) > GU(k^*) + k^* j(e(k^*))$$
$$> u(w_1(\hat{k})) + \delta(1 - m(\hat{k}))u(w_2(\hat{k})).$$

The first inequality is obtained by lemma 2. The inequality means that the firm increases its profit by decreasing the monitoring rate of the type $\hat{k}$ in the right hand while other constraints are satisfied. Consequently, $IC_1$ for the type $k^*$ is always binding.■

***Proof of lemma 4***

By $IC_2$, $GU(k)+kj(e(k)) \geq GU(\hat{k})+kj(e(\hat{k}))$. Using lemma 2, we see that $GU(k) \geq GU(\hat{k})$ *if* $k > \hat{k}$. If $e(k) > e(\hat{k})$, it holds that $GU(k) > GU(\hat{k})$.■

***Proof of proposition 1***

Suppose that $\exists\ k \in [k^*, \bar{k}]$ $w_1(k) \geq \bar{w}$. Consider (5):

$$0 = u(w_1(k)) + \delta(1-m(k))u(w_2(k)).$$

It holds that $m(k)<1$ on the equilibrium. If $m(k)=1$, a huge monitoring cost leads to a negative profit for the firm. Since it holds by lemma 1 that $\delta(1-m(k))u(w_2(k)) > 0$, the right hand of this inequality is strictly larger than 0. This is a contradiction. ■

## Notes

1. We assume that workers face liquidity constraint for borrowing funds because of the imperfect financial market. Since the optimal wage profile is upward as we prove later, workers save nothing in the first period. We can ignore the financial market.

2. Shapiro and Stiglitz (1984), a well-known model of efficiency wage, consider a similar monitoring technology.

3. After a type $k$ has made efforts, the firm has no incentive for monitoring. If the firm does not monitor the type $k$, or if the firm monitors them at a rate less than the rate decided *ex ante*, all of the type $k$ of later generations would believe that the firm does not incur the monitoring cost decided *ex ante* in the contract. Since the earnest type $k$ receives less utility than the shirking one and the incentive compatibility does not hold, there would be perpetually shirking. Hence, the firm monitors them at the decided rate. Consider an indefinitely repeated game, where perfect information on the firm's actions is assumed. If all workers of any generation choose the trigger strategy, a firm will be averse to betrayal. Hence, in this paper, it is sufficient to consider only 'on the equilibrium path'. Bull (1987) shows that a long-run labor contract can be regarded as the subgame perfect Nash equilibrium of the infinitely repeated game. Macleod and Malcomson (1989) point out that a long-run labor contract involving implicitly agreed factors is enforceable. The implicit factor is something not explicitly written on the contract, for instance, the bonus payment. In this paper, the dismissal rates contingent on the state and/or the second period wage can be regarded as implicit factors.

4. As Holmstrom (1982) shows, when total outputs are observed, a group punishment scheme leads to the first best allocation without monitoring. However, we assume that total outputs are not verifiable since we are willing to pay close attention to the relationship between monitoring rate and wage profiles. When total outputs are not verifiable, the firm will not use the group punishment scheme without monitoring. If the firm chooses the group punishment scheme without monitoring in this environment, employees will have no incentive for skill accumulation because the firm has no legal recourse. Workers live for only two periods although the firm runs forever, and thus workers are always shirking. Actually, firms cannot usually make

workers take joint responsibility when a single worker's responsibility is not clear and verifiable. In particular, as the number of employees increases, responsibility of each worker is more obscure under imperfect monitoring.

5. In this case, it is clear that the SOC of the truth telling is strictly satisfied because the single crossing property condition holds.

6. See Salanie (1997), Fudenberg and Tirole (1991, ch7), Mas-Colell, Whinston and Green (1995, ch23), or Laffont and Tirole (1993, ch1).

## References


**Baron, David P. and Myerson, Roger B.** "Regulating a Monopolist with Unknown Cost", *Econometrica* 50 (1982): 911-930.

**Bull, Clive** "The Existence of Self-Enforcing Implicit Contracts" *Quarterly Journal of Economics,* 101 (1987): 147-159.

**Eguchi, Kyota** "Wage Profile, Employment Adjustment and Monitoring Cost", *Japanese Economic Review*, forthcoming, 1998.

**Fudenberg, Drew and Tirole, Jean** "Game Theory", Cambridge, Massachusetts: *MIT Press,* 1991.

**Holmstrom, Bengt** "Moral Hazard in Teams", *Bell Journal of Economics,* 13 (1982): 324-330.

**Krueger, Alan B.** "Ownership, Agency, and Wages: An Examination of Franchising in the Fast Food Industry", *Quarterly Journal of Economics,* 106 (1991): 75-101.

**Lazear, Edward P.** "Why is There Mandatory Retirement?", *Journal of Political Economy*, 87 (1979): 1261-1284.

---------- "Agency, Earnings Profiles, Productivity and Layoffs", *American Economic Review*, 71 (1981): 606-620.

**---------- and Moore, Robert L.** "Incentives, Productivity, and Labor Contracts", *Quarterly Journal of Economics,* 99 (1984): 275-296.

**Laffont, Jean-J. and Tirole, Jean** "Using Cost Observation to Regulate Firms", *Journal of Political Economy*, 94 (1986): 614-641.

**---------- and ---------** "Auctioning incentive Contracts", *Journal of Political Economy*, 95 (1987): 921-937.

**---------- and ---------** "A Theory of Incentives in procurement and Regulation", Cambridge, Massachusetts: *MIT Press*, 1993.

**Macleod, Bentley W. and Malcomson, James M.** "Implicit Contracts, Incentive Compatibility, and Involuntary Unemployment" *Econometrica*, 57 (1989): 447-480.

**Mas-Colell, Andrew; Whinston, Michael D.; and Green, Jerry R.** "Microeconomic Theory", Oxford: *Oxford University Press,* 1995.

**Medoff, James L. and Abraham, Katharing G.** "Experience, Performance, and Earnings", *Quarterly Journal of Economics,* 95 (1980): 703-736.

**Myerson, Roger B.** "Incentive Compatibility and the Bargaining Problem, *Econometrica,* 47 (1979): 61-73.

**Okazaki, Keiko** "Why is the Earnings Profile Upward-Sloping? The Sharing Model vs the Shirking Model", *Journal of the Japanese and International Economies,* 7 (1993): 297-314.

**Salanie, Bernard** "The Economics of Contracts", Cambridge, Massachusetts: *MIT Press*, 1997.

**Salop, Joanne and Salop, Steven** "Self-Selection and Turnover in the Labor Market", *Quarterly Journal of Economics*, 90 (1976): 619-627.

**Shapiro, Carl and Stiglitz, Joseph E.** "Equilibrium Unemployment as a Worker Discipline Device", *American Economic Review*, 74 (1984): 433-444.