CIRJE-F-53

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June 1999

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May, 1999
This Version: June 29, 1999

Abstract

We investigate multiple regions in which coordination games are exclusively played by their participants. For every region, there exist a number of immobile individuals locked into this region. There also exist mobile individuals who look out for chances to move into more beneficial regions, but the ranges of regions into which they can move may be limited. All individuals intend to maximize their payoffs in a self-fulfilling way but they sometimes choose non-optimal strategies with a small probability. It is shown that when there exist sufficiently many individuals who are mobile in limited ways, all regions except the least productive region are well coordinated in the long-run of adjustment dynamics. This possibility result holds irrespective of how pessimistic individuals are. On the other hand, when the ranges of regions into which mobile individuals can move are expanded too much, all regions except the most productive region fall into coordination failure and the distributive inequality between immobile and mobile individuals increases very badly. Moreover, we argue that the policy interventions in the least productive region give the powerful spillover effect on facilitating coordination in the other regions.

Key Words: Coordination Failure, Multiple Regions, Mobility, Random Perturbations, Efficiency, Inequality.
JEL Classification Numbers: C72, C73, D20, D60, J60, P00, R00.

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1. Introduction

Coordination is one of the central problem for a group of individuals (players) whose activities are interacted with complementarities. Within a large enterprise, for example, there would be needed to coordinate many parts of the organization, share ideas about how to improve products, reduce manufacturing costs, and coordinate R&D efforts.[1] The problems of coordination are modeled by so-called coordination games, in which multiple Nash equilibria exist and are Pareto-ranked each other. When a group falls into coordination failure, its participants play the Pareto-inferior equilibrium. Coordination failure is difficult to solve because of its self-enforcing equilibrium property. For example, when an enterprise is horizontally expanded, it might be hard for the stuffs in its central office to persuade many divisions to switch their activities into the Pareto-superior equilibrium.

A group, or a region (enterprise, organization, alliance, industry, state, etc.), consists of *immobile* individuals and *mobile* individuals: An immobile individual is locked into her home region and have the region-specific skill which gives the participants' activities in this region values that can not be replaced. On the other hand, a mobile individual has the more general skill and can move into another more attractive region instead of persuading others to coordinate their activities, i.e., choose "*exit*" instead of making "*voice*", by using the terminology of Hirschman (1970). However, the range of regions into which a mobile individual can move may be limited because of various reasons such as her limited ability to adapt herself to different regions, the limited opportunity of training the region-specific jobs, the government's regulation on mobility, and so on.

Most of the previous works on coordination games have assumed that there exist no

[1] For the works on the problem of coordination in internal organization, see Milgrom and Roberts (1992). The problem of coordination is ubiquitous in groups such as organizations, corporate alliances, industries, states, and so on: Among firms which produce different components of a complementary system, there would be needed for collaboration to makes their products compatible each other (see Shapiro and Varian (1999)). In a closed economy, the coordination of investments among multiple sectors is essential for industrialization because the investment of one sector gives the significant effect of spillover on enlarging the size of the market in other sectors (see Murphy, Schleifer and Vishny (1989)). Coordination failure induced by complementarities has been one of the important themes in the literature of Keynesian macroeconomics (see Cooper and John (1988)).

mobile individuals, and have specialized studies of a single region.[2] Contrary to these works, this paper examines multiple regions in which coordination games are exclusively played by their participants, and clarify the effect of mobility among regions on the individuals' long-run behaviors.

We model the individuals' dynamic adjustment by a Markov chain with random perturbations. According to this Markov chain, all individuals intend to maximize their payoffs in a self-fulfilling way, but they sometimes experiment and choose non-optimal strategies with a small probability. Hence, the individuals can get rid of the basin of attraction of a Nash equilibrium and shift to another Nash equilibrium when many of them experiment at a time by chance. We focus on the long-run distribution of strategy profile induced by this Markov chain. The individuals' long-run behaviors are described by one of the Nash equilibrium on the support of this long-run distribution, which are called *stochastically stable* profiles.

We argue that mobility plays the significant role in facilitating coordination: Suppose that for every region except the least productive region, there exists so many mobile individuals who can move into both this region and the less productive neighborhood region but can not move into the more productive regions. Then, according to the unique stochastically stable profile, the individuals succeed to coordinate each other in all regions except the least productive region. Of particular importance, this possibility result holds irrespective of how pessimistic individuals are. This is in contrast to the case that there exists no mobile individuals, where all regions fall into coordination failure in the long-run whenever individuals are pessimistic.

The logical core of this possibility result is as follows. When a region falls into coordination failure, all individuals mobile up to this region move into the less productive but well coordinated regions, and therefore, the organization in this region becomes slimmer. This makes the switch into the Pareto-superior equilibrium easier because only a small number of experiments is enough for this switch. However, making the organization slimmer is not sufficient for solving coordination failure permanently. It is the most crucial that such mobile individuals come back to this region as soon as coordination failure is solved, and therefore, the size of population in the case of coordination success is much greater than the size of population in the case of coordination failure.

We argue also that in order to make multiple regions coordinated in the long-run, it is necessary that the ranges of regions into which mobile individuals can move should be restricted to a certain extent: Suppose that all mobile individuals can move into every

[2] There are exceptions such as Ely (1999).

region, i.e., they are globally mobile. Then, according to the unique stochastically stable profile, the individuals fail to coordinate each other in all regions except the most productive region.

This impossibility result has the following negative implication with respect to equity: Suppose that the ranges of regions into which mobile individuals can move are expanded too much, and therefore, all mobile individuals become globally mobile. Then, in the long-run, all mobile individuals participate in the most productive and well coordinated region and their payoffs increase, whereas all other regions fall into coordination failure and the payoffs for the immobile individuals locked into these regions decrease. Hence, the distributive inequality between immobile and mobile individuals increases very badly.

Moreover, we argue that the policy interventions in the least productive region give the powerful spillover effect on facilitating coordination in the other regions: When either individuals are made more optimistic about the least productive region or some immobile individuals locked into the least productive region are made mobile, coordination failure in the other regions is much easier to solve in the long-run. This possibility result holds even though this policy intervention does the least productive region itself no good.

The technical aspect of this paper is related to the works on equilibrium selection in evolutionary game theory such as Kandori, Mailath and Rob (1993, 1995) and Young (1995), which have studied stochastic stability in their respective Markov chains with random perturbations.[3] One of the crucial differences is that most works in evolutionary game theory have assumed that players are not necessarily rational and play according to the law of inertia. Moreover, Kandori, Mailath and Rob (1993, 1995), Young (1995), and Ellison (1993) have emphasized that the unique stochastically stable profile is the risk-dominant Nash equilibrium, which is not necessarily payoff-dominant.[4]

Ely (1999), which has examined mobility among multiple regions in evolutionary

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[3] Kandori (1997) is the excellent survey on evolutionary game theory in economics. We have a lot of textbooks on evolutionary game theory in economics such as Weibull (1995), Vega-Redondo (1996), Samuelson (1997), Young (1998), and Fudenberg and Levine (1998).

[4] Some works have emphasized that the emergence of risk-dominance crucially depends on the specifications of the model such as players' adjustment rules. See Bergin and Lipman (1995). Matsui (1991) and Kim and Sobel (1995) have found that when players communicate by using unused "cheap talk" messages in a random matching play, efficient allocations are selected by evolutionary pressures. Ely (1999) has investigated mobility among multiple regions and has shown that there exists a region which is coordinated in the long-run and in which all individuals participate.

game theory, is closely related to this paper. However, on the contrary to this paper, Ely has assumed that there exists no immobile individuals, all individuals are globally mobile, and the games exclusively played in regions are the same each other.

The organization of this paper is as follows. Section 2 defines a non-cooperative game. Section 3 defines a Markov chain with random perturbations and stochastic stability. Section 4 gives a characterization of stochastically stable profiles, and shows coordination failure without mobility. Section 5 presents the main results with mobility.

Section 6 focuses on the situation in which all regions are of almost the same productivity but mobile individuals have heterogeneous non-pecuniary preferences over these regions. We show that when there exist sufficiently many mobile individuals, all regions including the least productive region are well coordinated in the long-run, even though all mobile individuals are globally mobile.

2. The Model

We consider a repeated situation in which there exist n individuals (players) each of which decides which region to participate in and chooses an action in every period.

Multiple Regions

There exist m regions in the world. Let $M \equiv \{1,...m\}$ denote the set of regions, and its element is denoted by $r \in M$. In every period, an individual $i \in N \equiv \{1,...,n\}$ decides which region to participate in among a non-empty subset $M_i \subset M$. Whichever region this individual participates in, she chooses an action between action c (*"cooperative"*) and action d (*"defective"*). For every $r \in M$, we denote by $n(r)$ the number of the individuals who participate in region r, denote by $n(r,c)$ the number of individuals who participate in region r and choose action c, and denote by $k(r) \equiv \frac{n(r,c)}{n(r)}$ the proportion of the participants in region r who choose action c.

Mobility

An individual i is said to be *immobile* if M_i is a singleton. For every $r \in M$, an immobile individual i is said to be *locked into region* r if $M_i = \{r\}$. We denote by $n^{(r)}$ the number of immobile individuals i who are locked into region r. We assume that $n^{(r)}$ is sufficiently large for every $r \in M$. An individual i is said to be *mobile* if M_i is not a singleton. For every $r \in M/\{1\}$, an individual i is said to be *mobile up to region* r if M_i is not a singleton, $r \in M_i$, and $r' \notin M_i$ for all $r' > r$. An individual mobile up to region r can move into region r and some region less productive than r, but can not move into every region more productive than r. We denote by $n^{[r]}$ the number of individuals i who are mobile up to region r.

Coordination Games

When an individual i participate in region $r \in M_i$ and chooses an action $a_i \in \{c,d\}$, she obtains payoff $v_i^r(a_i, k(r))$, in which the effect of strategic interaction is summarized by the proportion $k(r)$ in this region.[5] The strategic interaction in a single region is

[5] It is more precise to define $k(r)$ as the proportion among the participants other than individual

described by the following *coordination game*: All participants' choosing action c and all participant's choosing action d are *only* pure strategy Nash equilibria, and these are strict Nash equilibria, i.e., for every $r \in M$ and every $i \in N$ such that $r \in M_i$,

$$v_i^r(c,1) > v_i^r(d,1) \text{ and } v_i^r(d,0) > v_i^r(c,0).$$

The former *Pareto-dominates* the latter, i.e., for every $r \in M$ and every $i \in N$ such that $r \in M_i$,

$$v_i^r(c,1) > v_i^r(d,0).$$

The single-period game is described by the following n-person non-cooperative game $G = (N, (S_i, u_i)_{i \in N})$. The set of strategies for player $i \in N$ is given by

$$S_i \equiv M_i \times \{c, d\}.$$

We denote $s_i = (r_i, a_i) \in S_i$, $S \equiv \underset{i \in N}{\times} S_i$, and $s = (s_i)_{i \in N} \in S$. The payoff function for player i is given by

$$u_i(s) \equiv v_i^{r_i}(a_i, k(r_i, s)),$$

where $n(r, s)$ is the number of players $i \in N$ such that $r_i = r$, $n(r, c, s)$ is the number of players $i \in N$ such that $s_i = (r_i, a_i)$, and $k(r, s) \equiv \dfrac{n(r, c, s)}{n(r, s)}$. It is straightforward from the definition of Nash equilibrium and the definition of G that *a strategy profile* $s \in S$ *is a Nash equilibrium in* G *if and only if there exists* $\alpha^{(s)}: M \to \{c, d\}$ *such that for every* $i \in N$*,*

$$a_i = \alpha^{(s)}(r_i),$$

and for every $i \in N$ *and every* $r \in M_i / \{r_i\}$*,*

$$v_i^{r_i}(\alpha^{(s)}(r_i), k(r_i)) > v_i^r(\alpha^{(s)}(r), k(r)), \tag{1}$$

where

$$k(r) = \begin{cases} 1 & \text{if } \alpha^{(s)}(r) = c \\ 0 & \text{if } \alpha^{(s)}(r) = d \end{cases}.$$

Hence, a Nash equilibrium in G is simply represented by $\alpha: M \to \{c, d\}$: For every α, there exists the unique Nash equilibrium $s = (r_i, a_i)_{i \in N} \in S$ such that $\alpha^{(s)} = \alpha$, because $(r_1, \ldots, r_n)$ is uniquely determined by inequalities (1). We will denote a Nash equilibrium by $\alpha = \alpha^{(s)}$ instead of s. The set of all Nash equilibria in G, denoted by S^{NE}, is equivalent to the set of all possible α.

We define the *efficient Nash equilibrium* $\alpha^* \in S^{NE}$ by

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i. However, for convenience, $k(r)$ is defined as the proportion among all participants including herself in this paper. This gives no substantial change because the numbers of the lock-in participants are assumed sufficiently large.

$$\alpha^{*}(r) = c \text{ for all } r \in M,$$

which implies that all regions are coordinated. We define the *near-efficient Nash equilibrium* $\alpha^{**} \in S^{NE}$ by

$$\alpha^{**}(r) = c \text{ for all } r \in M / \{1\},$$

and

$$\alpha^{**}(1) = d,$$

which implies that region 1 falls into coordination failure but all other regions are coordinated. For every subset $D \subset M$, we define the *D-efficient Nash equilibrium* $\alpha^{*D} \in S^{NE}$ by

$$\alpha^{*D}(r) = c \text{ for all } r \in D,$$

and

$$\alpha^{*D}(r) = d \text{ for all } r \in M / D,$$

which implies that all regions in D are coordinated but all regions in M / D fall into coordination failure. We define the *inefficient Nash equilibrium* $\alpha^{+} \in S^{NE}$ by

$$\alpha^{+}(r) = d \text{ for all } r \in M,$$

which implies that all regions fall into coordination failure. We define the *near-inefficient Nash equilibrium* $\alpha^{++} \in S^{NE}$ by

$$\alpha^{++}(r) = d \text{ for all } r \in M / \{m\},$$

and

$$\alpha^{++}(m) = c,$$

which implies that only region m is coordinated.

3. Stochastic Stability

We analyze players' dynamic adjustment by a discrete Markov chain with *random perturbations*. When a strategy profile s has been chosen in the current period, player i anticipates that the proportion of the participants in a region r who choose action c in the next period is equal to $\rho_i(r,s)\in[0,1]$. Player i intends to maximize the payoff $v_i^{r_i'}(a_i',\rho_i(r_i',s))$ with respect to $s_i'=(r_i',a_i')$. We denoted by $\eta_i^{(s)}=s_i'=(r_i',a_i')\in S_i$ *the best response for player* i *associated with* $\rho_i(\cdot,s)$, where we assume

$$a_i'=c \text{ if } v_i^{r_i'}(c,\rho_i(r_i',s))\geq v_i^{r_i'}(d,\rho_i(r_i',s)),$$

and

$$a_i'=d \text{ otherwise.}$$

Let $\rho(\cdot,s)\equiv(\rho_i(\cdot,s))_{i\in N}$ and $\eta^{(s)}\equiv(\eta_i^{(s)})_{i\in N}\in S$.

Player i sometimes *experiments* and randomizes her strategies with a positive but small probability $\varepsilon>0$. Player $i's$ strategy undergoes a transition according to the time-homogeneous probability function $p^{(i,\varepsilon)}:S\times S_i\to[0,1]$ such that

$$p^{(i,\varepsilon)}(s,\eta_i^{(s)})=1-\varepsilon,$$

and

$$p^{(i,\varepsilon)}(s,s_i'')=\frac{\varepsilon}{2|M_i|-1} \text{ for all } s_i''\neq\eta_i^{(s)},$$

where $p^{(i,\varepsilon)}(s,s_i')$ is the probability that player i chooses strategy s_i', given that strategy profile s has been chosen in the last period. A strategy profile undergoes a transition according to $p^{(\varepsilon)}:S\times S\to[0,1]$ defined by

$$p^{(\varepsilon)}(s,s')=\prod_{i\in N}p^{(i,\varepsilon)}(s,s_i') \text{ for all } (s,s')\in S^2.$$

We assume that for every $r\in M$, there exists a *threshold* $\hat{k}(r)\in[0,1]$ such that for every $i\in N$,

$$\rho_i(r,s)=\begin{cases}1 & \textit{if } k(r,s)\geq\hat{k}(r)\\ 0 & \textit{if } k(r,s)<\hat{k}(r)\end{cases}.$$

When the proportion of the individuals who have participated in region r and chosen action c in the previous period is more than or equal to (less than) the threshold $\hat{k}(r)$, every individual anticipates that *all* participants in region r choose action c (action d, respectively). For every $r\in M$, the individuals are said to be *pessimistic about region* r if $\hat{k}(r)>\frac{1}{2}$, whereas they are *optimistic about region* r if $\hat{k}(r)<\frac{1}{2}$. We must note that *for every* $s\in S$, $\eta^{(s)}=(\eta_i^{(s)})_{i\in N}$ *is a Nash equilibrium such that for every* $r\in M$,

$$\alpha^{(\eta^{(s)})}(r) = \begin{cases} c & \text{if } k(r,s) \geq \hat{k}(r) \\ d & \text{if } k(r,s) < \hat{k}(r) \end{cases}.$$

The following proposition says that the existence of such thresholds is a plausible assumption when all individuals form rational expectations.

Proposition 1: *Suppose that for every* $r \in M$*, there exists* $\hat{\rho}(r) \in (0,1)$ *such that for every* $i \in N$ *satisfying* $r \in M_i$*,*

$$v_i^r(c,k) \geq v_i^r(d,k) \text{ if } k \geq \hat{\rho}(r),$$

and

$$v_i^r(c,k) < v_i^r(d,k) \text{ otherwise.}$$

Moreover, suppose that $\rho_i(r,s)$ *depends* s *only through* $k(r)$*, is non-decreasing with respect to* $k(r)$*, and is rational in the sense that for every* $r \in M$ *and every* $s \in S$*,*

$$\rho_i(r,s) = k(r,\eta^{(s)}).$$

Then, for every $r \in M$*, there exists such a threshold* $\hat{k}(r)$*.*

Proof: See the Appendix.

We define the *stationary distribution* of $p^{(\varepsilon)}$, $f^{(\varepsilon)}: S \to [0,1]$, by

$$f^{(\varepsilon)}(s) \equiv \sum_{s' \in S} f(s') p^{(\varepsilon)}(s',s) \text{ for all } s \in S \text{, and } \sum_{s \in S} f^{(\varepsilon)}(s) = 1.$$

We must note that for every $\varepsilon \in (0,1)$, the stationary distribution is *uniquely* determined. We focus on the stationary distribution of a Markov chain such that ε is close to zero, which is represented by the *limit distribution* $f: S \to [0,1]$ defined by

$$f(s) \equiv \lim_{\varepsilon \downarrow 0} f^{(\varepsilon)}(s) \text{ for all } s \in S.$$

A strategy profile $s \in S$ is said to be *stochastically stable* if $f(s) > 0$.

For properties of stochastic stability, we recommend the readers to see Freidlin and Wentzell (1984), Young (1998), and Fudenberg and Levine (1998, Chapter 5). We know that there exists the unique limit distribution and there exists a stochastically stable profile. We know also that a stochastically stable profile is either a point in a limit cycle or a Nash equilibrium. Since there exists no limit cycle in our model, one gets that *if a strategy profile* $s \in S$ *is stochastically stable, it is a strict Nash equilibrium in* G.[6]

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[6] Most works in evolutionary game theory assume that players behave according to the fictitious play dynamics and its variants which our dynamics on the basis of rational expectation does not belong to. However, we can prove these properties in the same way.

4. Basic Results

We present a characterization of stochastic stability from the graph-theoretical viewpoint. We define the *cost of transition* from a Nash equilibrium $\alpha \in S^{NE}$ to another Nash equilibrium $\alpha' \in S^{NE} / \{\alpha\}$ by

$$c(\alpha,\alpha') \equiv \sum_{r \in M} n(r,\alpha)\theta(r,\alpha,\alpha'),$$ [7]

where

$$\theta(r,\alpha,\alpha') \equiv \begin{cases} 0 & \text{if } \alpha(r) = \alpha'(r) \\ \hat{k}(r) & \text{if } \alpha(r) \neq \alpha'(r) \text{ and } \alpha(r) = d \\ 1 - \hat{k}(r) & \text{if } \alpha(r) \neq \alpha'(r) \text{ and } \alpha(r) = c \end{cases}.$$

We must note that $c(\alpha,\alpha')$ approximates the minimal number of experiments necessary for transition from α to α'. For every $\alpha \in S^{NE}$, we define the set of Nash equilibria which minimize the cost of transition from α by

$$\Xi(\alpha) \equiv \arg\min_{\alpha' \in S^{NE}/\{\alpha\}} c(\alpha,\alpha').$$

For every collection of directed edges $\lambda \subset S^{NE} \times S^{NE}$, *the cost of* λ is defined by

$$C(\lambda) \equiv \sum_{(\alpha,\alpha') \in \lambda} c(\alpha,\alpha').$$

For every $\alpha \in S^{NE}$, we denote by $\Lambda(\alpha)$ the set of collections of directed edges which are *trees* whose roots are α.

Proposition 2: *If a Nash equilibrium* α *is stochastically stable, then there exists* $\lambda \in \Lambda(\alpha)$ *such that*

$$C(\lambda) \leq C(\lambda') \text{ for all } \lambda' \in \bigcup_{\alpha' \in S^{NE}} \Lambda(\alpha').$$

Proof: See the Appendix.

Proposition 2 implies that a stochastically stable profile is equivalent to the Nash equilibrium which is reachable from the other Nash equilibria with the minimum number of experiments, i.e., which has the least-cost tree.[8] In general, determining the least-cost tree is a very complex problem of graph theory. However, the existence of a large number

[7] We sometimes write $c(s,s')$ instead of $c(\alpha,\alpha')$ when $\alpha^{(s)} = \alpha$ and $\alpha^{(s')} = \alpha'$.

[8] It is sometimes criticized in evolutionary game theory that the speed of convergence to stochastically stable profiles is very slow. This criticism is calmed in this paper by the assumption that individuals never behave according to the law of inertia.

of mobile individuals makes this problem much easier to solve. In this paper, we will prove all the results by constructing the least-cost trees.[9]

The following proposition says that when all individuals are immobile and pessimistic, all regions fall into coordination failure.

Proposition 3: *Suppose that all individuals are immobile. Then, there exists the unique stochastically stable strategy profile* $\alpha \in S^{NE}$*, and for every* $r \in M$*,*

$$\alpha(r) = c \text{ if } \hat{k}(r) < \frac{1}{2},$$

and

$$\alpha(r) = d \text{ if } \hat{k}(r) > \frac{1}{2}.$$

Proof: Suppose $m = 1$. Let α' and α'' denote the Nash equilibria such that $\alpha'(1) = c$ and $\alpha''(1) = d$. We must note that $S^{NE} = \{\alpha', \alpha''\}$, $c(\alpha', \alpha'') = n^{(1)}(1 - \hat{k}(1))$, $c(\alpha'', \alpha') = n^{(1)}\hat{k}(1)$, and that $\lambda' = \{(\alpha'', \alpha')\}$ ($\lambda'' = \{(\alpha', \alpha'')\}$) is the only tree with root α' (the only tree with root α'', respectively). Obviously,

$$C(\lambda'') = c(\alpha', \alpha'') = n^{(1)}(1 - \hat{k}(1))$$
$$> n^{(1)}\hat{k}(1) = c(\alpha'', \alpha') = C(\lambda') \text{ if } \hat{k}(1) < \frac{1}{2},$$

and

$$C(\lambda'') < C(\lambda') \text{ if } \hat{k}(1) > \frac{1}{2}.$$

Hence, a stochastically stable profile α is characterized by

$$\alpha(1) = c \text{ if } \hat{k}(1) < \frac{1}{2}, \text{ and } \alpha(1) = d \text{ if } \hat{k}(1) > \frac{1}{2}.$$

Next, suppose $m \geq 2$. Since there exists no mobile individual, we can divide the model into m independent models each of which has only a single region. By applying the above arguments to each of these models, one gets that if α is stochastically stable, then for every $r \in M$,

$$\alpha(r) = c \text{ if } \hat{k}(r) < \frac{1}{2}, \text{ and } \alpha(r) = d \text{ if } \hat{k}(r) > \frac{1}{2}.$$

Q.E.D.

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[9] Ellison (1995) has provided sufficient conditions for stochastic stability in evolutionary game theory. The models in this paper do not necessarily satisfy Ellison's conditions. All the theorems provide sufficient conditions which may not be necessary. We conjecture that these theorems hold even in wider classes. We present intuitions of the proofs on the basis of much weaker conditions.

5. Homogeneous Orderings

In this section, we focus on the case in which mobile individuals have *homogeneous* preference orderings over regions: That is, we assume that for every $i \in N$, *both* $v_i^r(d,0)$ *and* $v_i^r(c,1)$ *are increasing with respect to* $r \in M_i$. Hence, the more right-hand-sided a region is, the more productive it is, and therefore, the more attractive it is for all mobile individuals. We assume also that for every $r \in M / \{1\}$ and every individual i mobile up to region r,

$$r - 1 \in M_i,$$

and

$$v_i^r(d,0) < v_i^{r-1}(c,1).$$

That is, every individual mobile up to a region r can move into the less productive neighborhood region $r-1$ and prefers the coordination success in region $r-1$ to the coordination failure in region r.

5.1. Mobility Facilitates Coordination

The following theorem says that when for every $r \in M / \{1\}$, there exist sufficiently many individuals mobile up to region r, all regions except region 1 are coordinated in the long-run.

Theorem 4: *Suppose that for every* $r \in M$ *and every* $r' \in M / \{1\}$,

$$(n^{[r']} + n^{(r')})(1 - \hat{k}(r')) > n^{(r)}\hat{k}(r). \quad (2)$$

Then, the efficient Nash equilibrium α^* *is the unique stochastically stable profile if* $\hat{k}(1) < \frac{1}{2}$, *whereas the near-efficient Nash equilibrium* α^{**} *is the unique stochastically stable profile if* $\hat{k}(1) > \frac{1}{2}$.

Proof: For every $\alpha \in S^{NE}$, fix $\beta^{(\alpha)} \in \Xi(\alpha)$ arbitrarily. We must note that for every $\alpha \in S^{NE}$, there exists $r^{(\alpha)} \in M$ such that

$$\beta^{(\alpha)}(r^{(\alpha)}) \neq \alpha(r^{(\alpha)}),$$

and

$$\beta^{(\alpha)}(r) = \alpha(r) \text{ for all } r \in M / \{r^{(\alpha)}\}.$$

For every $\alpha \in S^{NE}$, we define a collection of directed edges by

$$\lambda(\alpha) \equiv \{(\alpha', \beta^{(\alpha')}) : \alpha' \neq \alpha\}.$$

We must note that for every $\alpha \in S^{NE}$ and every $\lambda \in \Lambda(\alpha)$,

$$C(\lambda(\alpha)) \leq C(\lambda).$$

Inequalities (2) implies that for every $\alpha \in S^{NE} / \{\alpha^*\}$,

$$\alpha(r^{(\alpha)}) = d \text{ and } c(\alpha, \beta^{(\alpha)}) = n^{(r^{(\alpha)})}\hat{k}(r^{(\alpha)}), \tag{3}$$

and

$$\beta^{(\beta^{(\alpha^*)})} = \alpha^*, \text{ i.e., } r^{(\alpha^*)} = r^{(\beta^{(\alpha^*)})}. \tag{4}$$

Hence, $\lambda(\alpha^*)$ is a tree with root α^* and $\lambda(\beta^{(\alpha^*)})$ is a tree with root $\beta^{(\alpha^*)}$. However, equality (4) implies that for every $\alpha \in S^{NE} / \{\alpha^*, \beta^{(\alpha^*)}\}$, $\lambda(\alpha)$ is not a tree, and for every $\lambda \in \Lambda(\alpha)$, there exists $(\alpha', \alpha'') \in \lambda$ such that either $\alpha' = \alpha^*$ or $\alpha' = \beta^{(\alpha^*)}$, and

$$c(\alpha', \alpha'') = (n^{[r]} + n^{(r)})(1 - \hat{k}(r)) \text{ for some } r \in M / \{1\}.$$

This, together with inequalities (2) and equalities (3), implies that for every $\alpha \in S^{NE} / \{\alpha^*, \beta^{(\alpha^*)}\}$,

$$\min[C(\lambda(\alpha^*)), C(\lambda(\beta^{(\alpha^*)}))] < C(\lambda(\alpha)).$$

Hence, every stochastically stable profile is either α^* or $\beta^{(\alpha^*)}$.

Suppose $\hat{k}(1) < \frac{1}{2}$. Then,

$$n^{(1)}(1 - \hat{k}(1)) > n^{(1)}\hat{k}(1),$$

which, together with inequalities (2), equalities (3), equalities (4), and equality

$$c(\alpha^*, \beta^{(\alpha^*)}) = \min[n^{(1)}(1 - \hat{k}(1)), \min_{r \in M/\{1\}} (n^{[r]} + n^{(r)})(1 - \hat{k}(r))], \tag{5}$$

implies

$$c(\alpha^*, \beta^{(\alpha^*)}) > c(\beta^{(\alpha^*)}, \alpha^*).$$

Hence, one gets

$$C(\lambda(\alpha^*)) < C(\lambda(\beta^{(\alpha^*)})),$$

and therefore, α^* is the unique stochastically stable profile.

Suppose $\hat{k}(1) > \frac{1}{2}$. Then,

$$n^{(1)}(1 - \hat{k}(1)) < n^{(1)}\hat{k}(1),$$

which, together with inequalities (2), equalities (3), and equality (5), implies

$$r^{(\alpha^*)} = 1, \text{ i.e., } \beta^{(\alpha^*)} = \alpha^{**},$$

and

$$c(\alpha^*, \alpha^{**}) > c(\alpha^{**}, \alpha^*).$$

Hence,

$$C(\lambda(\alpha^*)) > C(\lambda(\alpha^{**})),$$

and therefore, α^{**} is the unique stochastically stable profile.

Q.E.D.

The intuition of this proof is as follows: Suppose that in a period, a region $r \in M / \{1\}$ falls into coordination failure but region $r-1$ is well coordinated. Then, all individuals mobile up to region r move into region $r-1$, and therefore, the number of the participants in region r is equal to $n^{(r)}$, i.e., the number of individuals locked into region r. Hence, the minimum number of experiments necessary for solving coordination failure is as large as $n^{(r)}\hat{k}(r)$. Next, suppose that region r solves coordination failure. Then, all individuals mobile up to region r come back to region r, and therefore, the number of the participants in region r is equal to $n^{[r]} + n^{(r)}$, i.e., the total number of individuals who are either mobile up to or locked into region r. Hence, the minimum number of experiments necessary for falling into coordination failure is as large as $(n^{[r]} + n^{(r)})(1-\hat{k}(r))$. Inequalities (2) imply that it is larger than $n^{(r)}\hat{k}(r)$, i.e., the minimum number of experiments necessary for solving coordination failure. This implies that it is more likely to solve coordination failure than to fall into coordination failure.

Theorem 4 implies that *the role of mobility, or marketization, among multiple regions* is crucial in coordination. This is in contrast to previous works which have neglected mobility among regions: Cooper and John (1988) have argued that the marketization *within* a closed region plays a very limited role. Murphy, Schleifer and Vishny (1989) have argued that the role of the government's big push is crucial. Krugman (1991) and Matsuyama (1991) have emphasized that the roles of history and expectation are crucial. On the contrary to these works, Theorem 4 indicates that many regions can be well coordinated in the long-run *irrespective of how pessimistic all individuals are and irrespective of which history has been given at the beginning.*

5.2. Global Mobility

In this subsection, we argue that in order to make multiple regions well coordinated in the long-run, it is necessary that the ranges of regions into which mobile individuals can move should be restricted to a certain extent.

An individual i is said to be *globally mobile* if $M_i = M$. A globally mobile individual is mobile up to region m and can move into every region. The following theorem says that when all mobile individuals are globally mobile and all individuals are pessimistic, all regions except region m fall into coordination failure in the long-run.

Theorem 5: *Suppose that every mobile individual is globally mobile, $n^{(r)}\hat{k}(r)$ is decreasing with respect to r,*

$$n^{(m)}\hat{k}(m) > n^{(r)}(1-\hat{k}(r)) \text{ for all } r \in M, \tag{6}$$

$$(n^{[m]} + n^{(r')})(1-\hat{k}(r')) > n^{(r)}\hat{k}(r) \textit{ for all } r \in M \textit{ and } \textit{all } r' \in M / \{1\}, \quad (7)$$

and there exists $\tilde{r} \in M$ *such that for every globally mobile individual* i,

$$v_i^r(c,1) > v_i^m(d,0) \textit{ for all } r \geq \tilde{r},$$

and

$$v_i^r(c,1) < v_i^m(d,0) \textit{ for all } r < \tilde{r}.$$

Then, the near-inefficient Nash equilibrium α^{++} *is the unique stochastically stable profile.*

Proof: The definition of $\tilde{r}$ implies that every globally mobile individual always participates in some region r such that $r \geq \tilde{r}$. For every $\alpha \in S^{NE}$, fix $\beta^{(\alpha)} \in \Xi(\alpha)$ arbitrarily, and define $r^{(\alpha)} \in M$ and $\lambda(\alpha) \subset S^{NE} \times S^{NE}$ in the same way as the proof of Theorem 4. Since $n^{(r)}\hat{k}(r)$ is decreasing with respect to r and inequalities (6) hold, one gets

$$n^{(r')}\hat{k}(r') > n^{(r)}(1-\hat{k}(r)) \text{ for all } r \in M \text{ and all } r' \in M. \quad (8)$$

Inequalities (7) and (8) imply that for every $\alpha \in S^{NE}$,

$$r^{(\alpha)} = m-1, \ \alpha(r^{(\alpha)}) = c \text{ and } c(\alpha, \beta^{(\alpha)}) = n^{(m-1)}\hat{k}(m-1)$$
$$\text{if either } \alpha = \alpha^{+} \text{ or } \alpha = \alpha^{++},$$
$$r^{(\alpha)} = m, \ \alpha(r^{(\alpha)}) = c \text{ and } c(\alpha, \beta^{(\alpha)}) = n^{(m)}\hat{k}(m) \leq n^{(m-1)}\hat{k}(m-1)$$
$$\text{if } \alpha = \alpha^{*\{r\}} \text{ for some } r \in \{\tilde{r}, \ldots, m-1\},$$
$$\alpha(r^{(\alpha)}) = c \text{ and } c(\alpha, \beta^{(\alpha)}) = n^{(r^{(\alpha)})}(1-\hat{k}(r^{(\alpha)})) \leq n^{(m-1)}\hat{k}(m-1)$$
$$\text{otherwise,}$$

and

$$\beta^{(\beta^{(\alpha^{++})})} = \alpha^{++}.$$

Hence, $\lambda(\alpha)$ is not a tree for every $\alpha \in S^{NE} / \{\alpha^{++}, \beta^{(\alpha^{++})}\}$, whereas $\lambda(\alpha^{++})$ is a tree with root α^{++}. Since

$$c(\alpha^{++}, \beta^{(\alpha^{++})}) = n^{(m-1)}\hat{k}(m-1) \geq c(\alpha, \beta^{(\alpha)}) \text{ for all } \alpha \in S^{NE},$$

one gets in the same way as the proof of Theorem 4 that

$$C(\lambda(\alpha^{++})) \leq C(\lambda(\alpha)) < C(\lambda) \text{ for all } \alpha \in S^{NE} / \{\alpha^{++}, \beta^{(\alpha^{++})}\}$$
$$\text{and all } \lambda \in \Lambda(\alpha).$$

Moreover, inequalities (8) say that

$$c(\alpha^{++}, \beta^{(\alpha^{++})}) = n^{(m-1)}\hat{k}(m-1)$$
$$> n^{(m-1)}(1-\hat{k}(m-1)) = c(\beta^{(\alpha^{++})}, \alpha^{++}).$$

Hence, one gets in the same way as the proof of Theorem 4 that

$$C(\lambda(\alpha^{++})) < C(\lambda(\beta^{(\alpha^{++})})) \leq C(\lambda) \text{ for all } \lambda \in \Lambda(\beta^{(\alpha^{++})}),$$

and therefore, α^{++} is the unique stochastically stable profile.

Q.E.D.

The intuition of this proof is as follows. Suppose that in a period, a region $r \in M / \{m\}$ falls into coordination failure but region m is coordinated. Then, all globally mobile individuals move into region m. Next, suppose that region r solves coordination failure and region m is still coordinated. Then, no mobile individual come back to region r. Hence, irrespective of whether region r is coordinated or not, the number of participants in region r is as large as $n^{(r)}$, i.e., the number of lock-in individuals, as long as region m is coordinated. Since $\hat{k}(r) > \frac{1}{2}$, i.e., all individuals are pessimistic, one gets that $n^{(r)}(1-\hat{k}(r))$, i.e., the minimum number of experiments necessary for falling into coordination failure, is less than $n^{(r)}\hat{k}(r)$, i.e., the minimum number of experiments necessary for solving coordination failure. Hence, it is less likely to solve coordination failure than to fall into coordination failure in all regions except region m.[10]

The comparison between Theorems 4 and 5 indicates that the government's excessive policy to expand the ranges within which mobile individuals can move gives the following negative effect on the distributive equity: Consider the situation in which all regions except region 1 are well coordinated at the outset. Suppose that mobile individuals are provided with the opportunity of learning the general skill, and becomes globally mobile. Then, in the long-run, all mobile individuals participate in the most productive and well coordinated region and their payoffs increase, whereas all other regions fall into coordination failure and the payoffs for the immobile individuals in these regions decrease. Hence, we can conclude that *when mobile individuals become globally mobile, the distributive inequality between mobile and immobile individuals increases very badly.*

5.3. Least Productive Region

In this subsection, we argue that the policy interventions in the least productive region give the powerful spillover effect on facilitating coordination in the other, more productive regions.

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[10] Theorem 5 does not imply that globally mobile individuals never participate in regions other than region m. Whenever a region $r \geq \tilde{r}$ is coordinated and the individuals fail to coordinate from region $r+1$ through region m then all globally mobile individuals participate in region r.

For every $r \in M / \{1\}$, an individual is said to be *locally mobile between region* r *and region* $r-1$ if $M_i = \{r, r+1\}$. A locally mobile individuals between region r and region $r-1$ is mobile up to region r and can never move into every region less productive than region $r-1$. The following theorem says that when all mobile individuals are locally mobile and inequalities (2) in Theorem 4 do not hold for $r = 1$, all regions may fall into coordination failure in the long-run.

Theorem 6: *Suppose*

$$\begin{aligned} & n^{(1)}\hat{k}(1) > (n^{[2]} + n^{(2)})\hat{k}(2) \\ & > \cdots > (n^{[m]} + n^{(m)})\hat{k}(m) > (n^{[m]} + n^{(m)})(1 - \hat{k}(m)) \\ & > \cdots > (n^{[2]} + n^{(2)})(1 - \hat{k}(2)) > n^{(2)}\hat{k}(2) \\ & > \cdots > n^{(m)}\hat{k}(m) > (n^{[2]} + n^{(1)})(1 - \hat{k}(1)). \end{aligned} \tag{9}$$

Then, the inefficient Nash equilibrium α^+ *is the unique stochastically stable profile.*

Proof: Inequalities (9) imply that

$$r^{(\alpha^+)} = m \text{ and } c(\alpha^+, \beta^{(\alpha^+)}) = (n^{[m]} + n^{(m)})\hat{k}(m),$$

for every $\alpha \in S^{NE}$,

$$r^{(\alpha)} = 1 \text{ and } c(\alpha, \beta^{(\alpha)}) \le (n^{[2]} + n^{(1)})(1 - \hat{k}(1)) < (n^{[m]} + n^{(m)})\hat{k}(m)$$
$$\text{if } \alpha(1) = c,$$

and for every $r \in M / \{1\}$,

$$r^{(\alpha)} = r \text{ and}$$
$$c(\alpha, \beta^{(\alpha)}) = (n^{[r]} + n^{(r)})(1 - \hat{k}(r)) < (n^{[m]} + n^{(m)})\hat{k}(m)$$
$$\text{if } \alpha(r') = c \text{ for all } r' \ge r \text{ and } \alpha(r') = d \text{ for all } r' < r,$$

and

$$r^{(\alpha)} = r \text{ and } c(\alpha, \beta^{(\alpha)}) = n^{(r)}\hat{k}(r) < (n^{[m]} + n^{(m)})\hat{k}(m)$$
$$\text{if } \alpha(1) = d, \ \alpha(r-1) = c, \text{ and } \alpha(r') = d \text{ for all } r' \ge r.$$

We must note that $\lambda(\alpha^+)$ is a tree with root α^+, and

$$c(\alpha^+, \beta^{(\alpha^+)}) > c(\alpha, \beta^{(\alpha)}) \text{ for all } \alpha \in S^{NE} / \{\alpha^+\}.$$

Hence,

$$C(\lambda(\alpha^+)) < C(\lambda) \text{ for all } \alpha \in S^{NE} / \{\alpha^+\} \text{ and all } \lambda \in \Lambda(\alpha),$$

and therefore, α^+ is the unique stochastically stable profile.

Q.E.D.

Inequalities (9) imply that $n^{(r)}\hat{k}(r)$ and $(n^{[r]}+n^{(r)})\hat{k}(r)$ are decreasing and $(n^{[r]}+n^{(r)})(1-\hat{k}(r))$ is increasing with respect to r, and, of particular importance, imply that inequalities (2) for all $r' \in M/\{1\}$ hold for every $r \in M/\{1\}$, but do not hold for $r=1$. Hence, Theorem 6 implies that all regions may fall into coordination failure in the long-run even if inequalities (2) for all $r' \in M/\{1\}$ hold for every $r \in M/\{1\}$.

The intuition of this proof is as follows. Even if a region 2 falls into coordination failure in a period, it is unlikely that the individuals mobile up to region 2 move into region 1, because it is unlikely that region 1 is coordinated enough to attract these individuals. Hence, it is unlikely that the size of population in region 2 in the case of coordination failure is smaller than in the case of coordination success, and therefore, region 2 falls into coordination failure in the long-run. In the some way, we can check that every region falls into coordination failure in the long-run.

The comparison between Theorems 4 and 6 indicates that either when individuals are made more optimistic about the least productive region, or when some immobile individuals locked into the least productive region are made mobile, coordination failure in the other regions is much easier to solve in the long-run: Consider the situation in which the government intervenes in region 1 and either makes individuals more optimistic about region 1, or makes some individuals locked into region 1 mobile. Then, $n^{(1)}\hat{k}(1)$ decreases and inequality (2) holds not only for every $r \in M/\{1\}$ and every $r' \in M/\{1\}$ but also for $r=1$ and every $r' \in M/\{1\}$. Hence, as Theorem 4 has shown, all regions except region 1 is coordinated in the long-run.

We must note that this possibility result holds even though individuals may still be pessimistic about region 1 after this policy intervention, i.e., even though this policy intervention does the least productive region itself no good in the long-run.

6. Heterogeneous Orderings

In the previous section, we have assumed that the individuals have homogeneous preference orderings over regions. However, this assumption is not plausible when all regions are of almost the same productivity: When individuals have heterogeneous preferences on non-pecuniary, region-specific traits such as culture and living environment, it might be more natural to assume that individuals have heterogeneous preference orderings over regions.

In this section, we argue that the heterogeneity plays the significant role in facilitating coordination. That is, it is shown that when there exist sufficiently many mobile individuals with heterogeneity, all regions including the least productive region are well coordinated in the long-run, even though all mobile individuals are globally mobile.

We assume that:

(i) All mobile individuals are globally mobile.

(ii) For every one-to-one mapping $\mu:\{1,\ldots,m\}\to M$, there exists $\frac{n^{[m]}}{m!}$ globally mobile individuals such that

$$v_i^{\mu(m)}(c,1)>\cdots>v_i^{\mu(1)}(c,1)>v_i^{\mu(m)}(d,0)>\cdots>v_i^{\mu(1)}(d,0).$$

(iii) There exist $\hat{k}$ and $\hat{n}$ such that for every $r\in M$,

$$\hat{k}(r)=\hat{k} \text{ and } n^{(r)}=\hat{n}.$$

Assumptions (ii) and (iii) imply that the model considered in the following theorem is symmetric. Moreover, Assumption (ii) implies that for every region, there exist the same number of globally mobile individuals who regard this region as being the most attractive in potential. Assumption (ii) implies also that whenever there exists a coordinated region then no globally mobile individual participates in any region which falls into coordination failure.

Theorem 7: *If*

$$\frac{n^{[m]}}{\hat{n}}>(\frac{2\hat{k}-1}{1-\hat{k}})m,$$

then the efficient Nash equilibrium α^* *is the unique stochastically stable profile. For every* $h\in\{1,\ldots,m\}$, *if*

$$(\frac{2\hat{k}-1}{1-\hat{k}})(h+1)>\frac{n^{[m]}}{\hat{n}}>(\frac{2\hat{k}-1}{1-\hat{k}})h,$$

then, for every $D \subset M$ such that $|D| = h$, the D-efficient Nash equilibrium α^{*D} is stochastically stable. Moreover, if

$$\frac{n^{[m]}}{\hat{n}} < \frac{2\hat{k} - 1}{1 - \hat{k}},$$

then the inefficient Nash equilibrium α^{+} is the unique stochastically stable profile.

Proof: For every $r \in M$, for every $h \in \{0, \ldots, m\}$, for every $D \subset M$ such that $|D| = h$, for $\alpha = \alpha^{*D}$, and for every $\alpha' \in S^{NE} / \{\alpha\}$ such that $\alpha(r) \neq \alpha'(r)$ and $\alpha(r') = \alpha'(r')$ for all $r' \in M / \{r\}$, one gets that

$$c(\alpha, \alpha') = \hat{n}\hat{k} \text{ if } \alpha(r) = d, \tag{10}$$

and

$$c(\alpha, \alpha') = (\hat{n} + \frac{n^{[m]}}{h})(1 - \hat{k}) \text{ if } \alpha(r) = c. \tag{11}$$

We define $h^{*} \in \{0, \ldots, m\}$ by

$$h^{*} = 0 \text{ if } \frac{n^{[m]}}{\hat{n}} < \frac{2\hat{k} - 1}{1 - \hat{k}},$$

$$h^{*} = m \text{ if } \frac{n^{[m]}}{\hat{n}} > (\frac{2\hat{k} - 1}{1 - \hat{k}})m,$$

and

$$(\frac{2\hat{k} - 1}{1 - \hat{k}})(h^{*} + 1) > \frac{n^{[m]}}{\hat{n}} > (\frac{2\hat{k} - 1}{1 - \hat{k}})h^{*} \text{ otherwise.}$$

For every $\alpha \in S^{NE}$, fix $\beta^{(\alpha)} \in \Xi(\alpha)$ arbitrarily, and define $r^{(\alpha)} \in M$ and $\lambda(\alpha) \subset S^{NE} \times S^{NE}$ in the same way as the proof of Theorem 4. Equalities (10) and (11) and the definition of h^{*} imply that for every $\alpha \in S^{NE}$,

$$\alpha(r^{(\alpha)}) = c \text{ if } h > h^{*},$$

$$\alpha(r^{(\alpha)}) = d \text{ if } h \leq h^{*},$$

and that for every $D \subset M$ such that $|D| = h^{*}$,

$$c(\alpha^{*D}, \beta^{(\alpha^{*D})}) \geq c(\alpha, \beta^{(\alpha)}) \text{ for all } \alpha \in S^{NE}, \tag{12}$$

and

$$\beta^{(\alpha^{*D})} = \alpha^{*D'} \text{ for some } D' \subset M \text{ such that } |D'| = h^{*}.$$

The last equalities imply that for every $D \subset M$ such that $|D| \neq h^{*}$, $\lambda(\alpha^{*D})$ is not a tree. On the other hand, for every $D \subset M$ such that $|D| = h^{*}$, by specifying $\beta^{(\alpha)}$ such that

$$r^{(\alpha)} \notin D \text{ if } h > h^{*},$$

and

$$r^{(\alpha)} \in D \text{ if } h \leq h^{*},$$

one gets that $\lambda(\alpha^{*D})$ is a tree with root α^{*D}. These, together with inequalities (12), imply that for every $D \subset M$ such that $|D| = h^*$, and for every $D' \subset M$ such that $|D'| \neq h^*$,

$$C(\lambda(\alpha^{*D})) \leq C(\lambda(\alpha^{*D'}) < C(\lambda) \text{ for all } \lambda \in \Lambda(\alpha^{*D'}).$$

Hence, one gets from Proposition 1 that α^{*D} is stochastically stable if and only if $|D| = h^*$, and therefore, we have completed the proof of Proposition 6.

Q.E.D.

The intuition of this proof is as follows: For every region $r \in M$, there exists $\frac{n^{[m]}}{m}$ globally mobile individuals who regard region r the most attractive. This implies that when coordination failure is solved in a region r, at least $\frac{n^{[m]}}{m}$ globally mobile individuals always come back to this region irrespective of what happens in the other regions. Hence, in every region including region 1, the size of population in the case of coordination success is much greater than the size of population in the case of coordination failure, and therefore, it is more likely to solve coordination failure than to fall into coordination failure in the long-run.

References

Bergin, J. and B. Lipman (1995): "Evolution with State-Dependent Mutations," *Econometrica* 64, 943-956.

Cooper, R. and A John (1988): "Coordinating Coordination Failures in Keynesian Models," *Quarterly Journal of Economics* 103, 441-464.

Ellison, G. (1993): "Learning, Local Interaction, and Coordination," *Econometrica* 61, 1047-1071.

Ellison, G. (1995): "Basin of Attraction and Long-run Equilibria," mimeo, MIT.

Ely, J. (1999): "Local Conventions", Mimeo.

Freidlin, M. and A. Wentzell (1984): *Random Perturbations of Dynamical Systems*, Berlin: Springer-Verlag.

Fudenberg, D. and D. Levine (1998): *The Theory of Learning in Games*, Cambridge: MIT Press.

Hirschman, A. (1970): *Exit, Voice and Loyalty: Responses to Decline in Firms, Organizations, and States*, Cambridge: Harvard University Press.

Kandori, M. (1997): Evolutionary Game Theory in Economics," in Kreps, D. and K. Wallis (eds.), *Advances in Economics and Econometrics: Theory and Applications: Seventh World Congress, Vol. 1*, Cambridge University Press.

Kandori, M., G. Mailath, and R. Rob (1993): "Learning, Mutation, and Long-run Equilibria in Games," *Econometrica* 61, 29-56.

Kandori, M. and R. Rob (1995): "Evolution of Equilibria in the Long-Run: A General Theory and Applications," *Journal of Economic Theory* 65, 383-414.

Kim Y. and J. Sobel (1995): "An Evolutionary Approach to Pre-Play Communication," *Econometrica* 63, 1181-1194.

Krugman, P. (1991): "History versus Expectation," *Quarterly Journal of Economics* 56, 651-667.

Matui, A. (1991): "Cheap-Talk and Coordination in a Society," *Journal of Economic Theory* 54, 245-258.

Matsuyama, K. (1991): "Increasing Returns, Industrialization, and Indeterminacy of Equilibrium," *Quarterly Journal of Economics* 56, 617-650.

Milgrom, P. and J. Roberts (1992): *Economics, Organization and Management*, Prentice-Hall.

Murphy, K., A. Schleifer, and R. Vishny (1989): "Industrialization and the Big Push," *Journal of Political Economy* 97, 1003-1025.

Samuelson, L. (1997): *Evolutionary Games and Equilibrium Selection*, Cambridge: MIT

Press.

Shapiro, C. and H. Varian (1999): *Information Rules*, Harvard Business School Press.

Vega-Redondo, F. (1996): *Evolution, Games, and Economic Behavior*, New York: Oxford University Press.

Weibull, J. (1995): *Evolutionary Game Theory*, Cambridge: MIT Press.

Young, P. (1993): "The Evolution of Conventions," *Econometrica* 61, 57-84.

Young, P. (1998): *Individual Strategy and Social Structure*, Princeton University Press.

Appendix

Proof of Proposition 1: Since $\rho_i(r,s) = k(r,\eta^{(s)})$ for all $i \in N$, one gets that for every $i \in N$ and for $\eta_i^{(s)} = (r_i', a_i')$,

$$a_i' = c \text{ if } r_i' = r \text{ and } k(r,\eta^{(s)}) \geq \hat{\rho}(r),$$

and

$$a_i' = d \text{ if } r_i' = r \text{ and } k(r,\eta^{(s)}) < \hat{\rho}(r).$$

This, together with the definition of $k(r,\eta^{(s)})$, implies that

$$\rho_i(r,s) = 1 \text{ for all } i \in N \text{ if } k(r,\eta^{(s)}) \geq \hat{\rho}(r),$$

and

$$\rho_i(r,s) = 0 \text{ for all } i \in N \text{ otherwise.}$$

Since $\rho_i(r,s)$ depends s only through $k(r)$ and is non-decreasing with respect to $k(r)$, one gets that there exists $\hat{k}(r)$ such that

$$\rho_i(r,s) = 1 \text{ for all } i \in N \text{ if } k(r,s) \geq \hat{k}(r),$$

and

$$\rho_i(r,s) = 0 \text{ for all } i \in N \text{ otherwise.}$$

Q.E.D.

Proof of Proposition 2: For every $s \in S$ and every $s' \in S / \{s\}$, we denote by $e(s,s')$ the minimal number of experiments necessary for transition from s to s' within one period, which is equal to the number of individuals i such that $s_i' \neq \eta_i^{(s)}$. Since $\eta^{(s)}$ and $\eta^{(s')}$ are Nash equilibria, $c(\eta^{(s)},\eta^{(s')})$ approximates $e(s,s')$.

For every $\alpha \in S^{NE}$ and every $\alpha' \in S^{NE} / \{\alpha\}$, we denote by $\vec{e}(\alpha,\alpha')$ the minimum of $\sum_{t=0}^{T-1} e(s^t,s^{t+1})$ with respect to all finite sequences on S, $(s^1,\ldots,s^T)$, such that $\alpha^{(s^1)} = \alpha$ and $\alpha^{(s^T)} = \alpha'$. For every finite sequence $(s^1,\ldots,s^T)$ on S, we specify a finite sequence on S^{NE}, $(\alpha^1,\ldots,\alpha^T)$, by

$$\alpha^t = \alpha^{(\eta^{(s^t)})} \text{ for all } t = 1,\ldots,T.$$

Since $\sum_{t=0}^{T-1} c(\alpha^t,\alpha^{t+1})$ approximates $\sum_{t=0}^{T-1} e(s^t,s^{t+1})$, one gets that there exists a finite sequence on S^{NE}, $(\alpha^1,\ldots,\alpha^T)$, such that $\alpha^1 = \alpha$, $\alpha^T = \alpha'$, and $\sum_{t=0}^{T-1} c(\alpha^t,\alpha^{t+1})$ approximates $\vec{e}(\alpha,\alpha')$. This implies that for every $\alpha \in S^{NE}$ and every $\lambda \in \Lambda(\alpha)$, there exists a collection of directed edges $\Delta \subset S^{NE} \times S^{NE}$ such that $C(\Delta)$ approximates $\sum_{(\alpha'',\alpha''') \in \lambda} \vec{e}(\alpha'',\alpha''')$ and a subset of Δ is a tree with root α. This implies that for every

$\alpha \in S^{NE}$, if

$$\min_{\lambda \in \Lambda(\alpha)} \sum_{(\alpha'',\alpha''') \in \lambda} \vec{e}(\alpha'',\alpha''') \leq \min_{\lambda' \in \Lambda(\alpha')} \sum_{(\alpha'',\alpha''') \in \lambda'} \vec{e}(\alpha'',\alpha''') \text{ for all } \alpha' \in S^{NE},$$

then there exists $\lambda \in \Lambda(\alpha)$ such that $C(\lambda)$ approximates $\sum_{(\alpha'',\alpha''') \in \lambda} \vec{e}(\alpha'',\alpha''')$. Based on Freidlin and Wentzell (1984), and in the same way as Kandori, Mailath and Rob (1993) and Young (1993), one gets that if α is stochastically stable, then there exists $\lambda \in \Lambda(\alpha)$ such that

$$\sum_{(\alpha'',\alpha''') \in \lambda} \vec{e}(\alpha'',\alpha''') \leq \min_{\lambda' \in \Lambda(\alpha')} \sum_{(\alpha'',\alpha''') \in \lambda'} \vec{e}(\alpha'',\alpha''') \text{ for all } \alpha' \in S^{NE}.$$

From these observations, we have proved that if α is stochastically stable, there exists $\lambda \in \Lambda(\alpha)$ such that

$$C(\lambda) \leq C(\lambda') \text{ for all } \lambda' \in \bigcup_{\alpha' \in S^{NE}} \Lambda(\alpha').$$

Q.E.D.